SSC-186

The Effect of Ship Stiffness Upon the Structural Response of a Cargo Ship to an Impulsive Load

SHIP STRUCTURE COMMITTEE

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ADDRESS CORRESPONDENCE TO:

SECRETARY
SHIP STRUCTURE COMMITTEE
U.S. COAST GUARD HEADQUARTERS
WASHINGTON, D.C. 20591

September 1968

Dear Sir:

The Ship Structure Committee sponsored a project to study the dynamic effects resulting from an impulsive load on a ship and to determine how these effects tend to vary with the stiffness of the ship's girder. The results of this work are described in the enclosed report "On the Effect of Ship Stiffness upon the Structural Response of a Cargo Ship to an Impulsive Load" by Manley St. Denis.

The report is being distributed to individuals and groups associated with or interested in the work of the Ship Structure Committee. Comments concerning this report are solicited.

Sincerely,

D. B. Henderson

D. B. Henderson
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure Committee

SSC-186

Final Report

on

Project SR-173

"Ship Stiffness Studies"

to the

Ship Structure Committee

THE EFFECT OF SHIP STIFFNESS UPON THE STRUCTURAL RESPONSE OF A CARGO SHIP TO AN IMPULSIVE LOAD

by

Manley St. Denis
&
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National Engineering Science Company

under

Department of the Navy
Naval Ship Engineering Center
Contract Nobs 94321

U. S. Coast Guard Headquarters
Washington, D. C.

September 1968

ABSTRACT

The purpose of the study is to set up a computer program to investigate the dynamic effects resulting from an impulsive loading on a ship and to determine how these effects tend to vary with the stiffness of the hull girder. The hull is treated as a Timoshenko beam and the solution is obtained by finite difference technique. Two codes are written: an explicit one, which is more efficient for short durations, and an implicit one, which is superior for long durations of impulse. Application is made to a dry cargo ship. Limited analysis of her response to a unit impulse indicates that, in general, reduced hull rigidity tends to be beneficial.

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1. INTRODUCTION

1.1 Purpose

The purpose of this study is to investigate the dynamic effects resulting from an impulsive loading on a ship and to determine how these effects tend to vary with the stiffness of the ship's girder. To a first approximation, such a stiffness is interpreted to be proportional to the second central moment of area of the cross section through the ship's longitudinally continuous structure. This proportionality is somewhat modified if shear rigidity is taken into account.

1.2 Scope

The inputs to the study are:

a) The geometry of the hull and the grade and basic disposition of the structural material
b) The time and space distribution of the hydrodynamic impulse

The desired results are:

a) Response of the ship's structure in the elastic modes
b) Maximum dynamic bending moment amidships

A comparison is made between a standard cargo ship for which pertinent data are available (the SS WOLVERINE STATE) and equivalent ships of reduced stiffness.

1.3 Background

The structural design of merchantmen has long been an empirical process. Such a process has the virtuous claim of reliability of insurance against structural distress from all environmental conditions save the extraordinarily extreme. However, such claim is valid and tenable only so long as one does not exceed the range of experience upon which the empirical rules have been established. Indeed, as new experience is accumulated, it should be interpreted to provide a wider statistical basis and the rules should be examined for possible modification to insure good design practice. But, more urgently, the introduction of new structural materials or the consideration of greater principal dimensions or of different proportions of hull geometry and shape of hull all demand the prudent reassessment of the empirical rules of structural design. In the absence of experience sufficient to provide new empirical rules covering the contemplated changes, such reassessment can best be made upon the judicious interpretation of available knowledge in metallurgy and structural dynamics. The proposed study is aimed at providing the insight and basis for such reassessment. The specific problem to be examined is as follows:

Given a basic ship designed according to established rules, how should her scantlings be modified to insure that, upon the introduction of higher strength steels or of corrosion-preventing coatings, her structural performance under the impulsive loading of the sea will remain unaltered from that of the basic ship in spite of the decreased stiffness of the ship's girder?

1.4 Overview of the Problem

In its simplest form, the problem consists in solving for the impulse-induced stresses in a beam of variable geometry and dynamic properties along its length. The beam is hollow, internally cross-stiffened, and not of shallow depth with respect to its length; its boundary conditions are free-free.

The assumption is made that the hollow, cross-stiffened beam which is the hull behaves as if it were homogeneous. The consequence of this assumption is that the transverse distribution of stress intensity across the decks at a generic cross section is uniform (unless influenced by longitudinal discontinuities such as hatch openings). This appears to be a reasonable assumption so long as the loading on the hull is sufficiently slow and the longitudinal gradient of the bending moment is sufficiently low. These conditions do not obtain under impulsive loading of short duration.

But determination of the transverse gradient of stress intensity or shear lag requires the prior knowledge of the bending moment or deflection curve and since these are obtained as solutions to the problem, an iterative method is called for at each consecutive step of the process. While it appears to be feasible to prepare a code that will take shear lag into account, the process will necessarily be considerably lengthier.

The effect of shear lag is to reduce the section moduli, hence the elastic restoration, over the hull length. Such a reduction does not appear to be quite sensitive to changes in hull rigidity so that, so long as one is interested in comparative analysis, a first order approximation to the influence of rigidity of hull on shear and bending moment can perhaps be obtained upon disregard of shear lag. Eventually, such hypothesis must be tested.

Since the beam is hollow, question also arises as to the importance of the local response of structure in way of the loading. The possible effect of a local response is to modify the intensity and distribution of the loading. Such question will not be considered. It is assumed that local response can be taken into account by a proper definition of impulse loading.

An additional number of complications exists. The impulsive stresses are additive to an underlying base-line stress field which results from residual, thermal and bending stresses (both static and dynamic). Moreover, these stresses are not distributed throughout the ship in a gradually changing pattern but are subject to high local magnifications from structural discontinuities (such as from hatch openings, super-structure endings, etc.).

These complications emphasize the point that the effects of impulse-induced stresses are meaningful only when considered in the context of all coexisting stress pattern.

The ship system is in dynamic equilibrium under the action of an impulsive excitation and of inertial, damping and restoring reactions. The excitation is specified in terms of arbitrary parameters. The reactions and boundary conditions are defined in terms of given physical factors (mass, material distributions, etc.) and of the motions of the system. The statements of equilibrium of the forces and moments is a set of partial differential equations in the axial coordinate (x) and time (t). The two dependent variables are the vertical displacement $y(x,t)$ and the cross-sectional rotation $\gamma(x,t)$. In considering what terms should be included in the dynamical equations governing the motion of the hull, the following observations are pertinent:

a) Inertial Reaction

The essential term is that of transverse (or translational) inertia. In addition, consideration is given to the rotary (or rotatory, or rotational) component of inertia. This component is introduced because it is anticipated that it may have a significant effect on the results; this argument is made because the depth of the ship is not negligible in comparison with her length. To be sure, rotary inertia may have but little effect on the elastic behavior of a ship when she is subjected to gradual wave action, for, in this case, the dominant mode of deformation is that of the ship as a whole, since the ship's structure has ample time to adjust itself to the transient loading. However, when the hydrodynamic loading is impulsive in nature, local deformations tend to predominate at least for an initial period following the impulse, and it is in the analysis of these that the rotary inertia of the hull sections need be taken into account.

b) Restoration

The primary restoration is flexural; however, the relation between flexural moment and deflection is somewhat modified by the effect of shear flexibility. The argument for including shear is the same as the one made in the previous paragraph for considering rotary inertia. The coefficients of the inertial and restoring reactions are measurable physical quantities with exception of the shape factor, which is derivable by structural analysis and which is of importance principally for the higher modes and local loading.

c) Damping

Ordinarily, when analyzing the transient response of systems under impulsive action, the damping term is omitted from the basic equation describing the dynamic equilibrium. The reason for this is that the amount of damping (either structural or hydrodynamic) which may be present is quite small with the consequence that the behavior of the system during the important time interval immediately following impulse is validly described by conservative differential equations. To be exact, this argument is tenable only when gradually changing loading is considered.

However, when the loading is impulsive, high velocities of deformation can be generated locally with the result that damping could become of significance at least in the region of the impulse.

Inclusion of the damping term leads to a set of nonlinear equations of motion. Admittedly, in a numerical solution, a nonlinearity introduces only a complication but not conceptual difficulty.

But the rigorous determination of damping is in itself a difficult task which does not promise to yield rewards commensurate with the effort. For the proposed analysis, it appears that the solution should first be sought upon neglect of damping and, then, if feasible, allow for damping in a simple empirical manner. Comments on the damping coefficients are contained in Appendix A.

d) Excitation

The excitation is impulsive, and the values of the parameters defining the impulse will be given as the outcome of the parallel study on ship response. Thus, initially at least, the loading is assumed to be independent of the response. Although the validity of such an assumption should eventually be examined, it does appear that the uncoupling of excitation from response will not lead to errors of any consequence so long as only overall structural performance is considered. For the analysis of local structural performance, the coupling must be considered.

During slamming, or other dynamic loading, the ship experiences a transient hydrodynamic loading of short duration in a localized area. Such a loading can be built up from a continuous sequence of impulse distributions over the localized surface of the ship. Such distributions are resolvable into vertical and horizontal components which can be separately considered.

Of the two, the vertical component of the transient loading is the most important. This is fortunate because the vibrations excited by this component are wholly in the vertical plane; there is no coupling with horizontal or torsional modes. On the other hand, the horizontal component of the loading does give rise to torsional vibrations. Thus, the first step in the solution is to determine the response of the ship to a unit vertically directed impulse arbitrarily located along her longitudinal axis.

Once such a response is known, the work can be extended to yield the response to any vertical transient excitation. It is believed that further extension of the analysis to the case of horizontally directed transients should await the successful solution to the vertical case.

1.5 Philosophy of Solution

The analysis of shock, in contrast to that of vibration, is a problem in propagation as against one of stationarity, of transient fluctuations as against steady state response. It follows that the techniques for solution to be applied in the analysis of shock are likely to, and indeed do, differ from those employed in the analysis of steady state vibration. Since this point is not always apparent, some brief remarks are made relative to the distinction.

Following Crandall (1956), physical problems are divisible into problems of equilibrium, of characteristic values (normal modes) and of propagation. The first, are problems of steady state, an example of which, relevant to the present study, is that of determining the stress intensities and deflections when the hull structure is in static balance under the action of weight and buoyancy. These problems are time invariant, i. e., time does not enter as a variable in the problem and the problem is stated by a set of one or more ordinary differential equations which are to be solved subject to certain boundary conditions. The second are problems of steady regime, an example of which would be the forced response of the hull to a steady alternating excitation by the propeller. The statement of the problem is, for a discrete or lumped system, by a set of one or more ordinary differential equations in which time is the independent variable or, for distributed systems, by a set of one or more partial differential equations in space and time. But time does not enter into the problem as a parameter, i. e., initial conditions are not specified, these being irrelevant and a solution is sought satisfying only a set of boundary conditions. The third are initial value problems, an example of which is the transient or unsteady response of a hull to hydrodynamic impact. Here time does enter as an essential parameter in describing the time-characteristics of the transient excitation. The statement of the problem is as for the previous case, but now a solution is to be sought which satisfies the initial conditions in addition to the boundary conditions.

1.6 Procedure

The first step toward the solution is to set down the differential equations governing the motion of the system. This step is carried out in Appendix B. The equations are set up on the assumption that the material obeys Hooke's law and that the hollow built-up hull of the ship, behaves very much like a homogeneous beam and on the further consideration that shear deflection and rotary inertia may be important. Additional assumptions are made relative to the geometry of the ship and to the distribution of her dynamic properties. As to the first, the hull is slender with respect to length and symmetric with respect to the longitudinal centerplane. As to the second, the dynamic properties vary gradually over the length.

The equations of motion are a set of two simultaneous partial differential equations, the independent coordinates of which are the longitudinal distance along the ship and time, while the dependent coordinates are the vertical displacement and the rotation of the cross-section. The last pair are the generalized coordinates of the system.

Since the coefficients entering into the equations are of arbitrary distribution along the ship, resort must be had to numerical methods of integration. From a practical standpoint, three general methods are available to solve the system of equations in hand:

a) Method of trial solutions with undetermined parameters
b) Method of finite differences
c) Mixed technique (based on the Runge-Kutta method).

Since each method has advantages and disadvantages, it becomes first of all necessary to examine them within the context of the problem to be solved.

The method of trial solutions with undetermined parameters (see, e. g., Faedo, 1947/49) consists in replacing the continuous propagation process by one described approximately by a limited family of suitably chosen functions. In essence, by so doing, the problem of continuous propagation (infinite number of degrees of freedom) is reduced to one of propagation in a system having a finite number of degrees of freedom. The success of the method rests critically on the selection of the family of functions to be employed. A good choice can be made if the solution is known for a simpler, comparable system (e. g., uniform beam). Unfortunately, this appears to be wanting, and without guidance in this regard, the method fails to give good promise.

The method of finite differences consists in writing the differential equations and the initial and boundary conditions as finite difference equations and finite difference ratios and then, starting with the initial conditions, marching the solution thru in such manner that the boundary conditions are always satisfied. In this process, time is held fixed at a generic instant while the profile of the solution is developed along the axis of distance. When the whole profile has been obtained, the time is incremented. This method is simple to apply.

When setting up the finite difference analog for obtaining the solution to motion of the system, two choices are possible: the analog may be written either in explicit or in implicit form.

In the explicit analog, the recurrence formula is an explicit expression for the value of the dependent variables corresponding to an incremented instant of time based on the values of these variables for previous times. Each step in the solution yields the dependent variables at one point in space and time.

In the implicit analog, the recurrence formula is a system of 2N algebraic equations in 2N unknowns, where N is the number of segments into which the ship is subdivided. Their solution yields the dependent variables at the incremented instant of time for all the stations.

The method of finite difference (both explicit and implicit forms) is developed in Appendix C.

The foregoing comparison does not reveal any evident computational superiority of one analog over the other. However, the explicit analog is stable only for a time increment which must be determined and which may be zero. To express it differently, the explicit analog may not have a numerically stable solution, or,at best, it may not be possible to determine the time increment for a stable solution. As against this, the implicit analog is always stable.

The numerical stability of the solutions for both the explicit and implicit forms of the finite difference equations is examined in Appendix D.

Whether an explicit or implicit analog is to be preferred depends, in part, also on external factors such as the duration of the impulse, the extent over which applied and the number of segments in which the ship is to be subdivided to provide a reasonable step representation of the ship's dynamic properties.

The mixed technique consists in replacing the partial differential equations describing the motion of the system by a set of ordinary differential equations in time by introducing finite spatial differences. The ordinary differential equations (two to each spatial point) are then solved by a fourth order Runge-Kutta Method. Such a technique is more stable than the explicit finite difference method and has a smaller truncation error than either the explicit or implicit finite difference analogs. The mixed technique analog and numerical stability of the method are analyzed in Appendix E.

Of the coefficients entering into the equations of motion, three need elaboration. These are: the coefficient of hydrodynamic inertia, the coefficient damping and the shear factor. The remaining coefficient, that of restoration is immediately derivable from the geometry of the hull.

When calculating the response of the hull to impact, allowance must be made for the inertial effect of the surrounding water. A direct solution of the problem of the hydrodynamic inertia of a buoyant hull of arbitrary form is not in hand and in practice an estimate is obtained by integrating the effect of partial solutions to the problem. The calculation is in three steps:

a) Determination of the hydrodynamic inertia distribution over the length of the hull on the basis that the flow over a vertical cross section is two-dimensional (cross-flow hypothesis)
b) Correction for aspect ratio
c) Correction for free surface.

The hydrodynamic mass obtained on the assumption of two-dimensional flow must be corrected for aspect ratio, i. e., for deviations from the cross-flow hypothesis because of the presence of three-dimensional flow. This correction can be strong. Methods for making this correction have been developed for a hull in continuous, sinusoidal, steady-state deformation over its whole length. However, for the problem in hand, the deformation pattern is not this and cannot be determined in advance. Pending the derivation of a method to cope with this aspect, no correction has been introduced.

The hydrodynamic inertia must also be modified because of the presence of the free surface. A method for analyzing the influence of the free surface on the hydrodynamic inertia of a heaving cylinder is due to Ursell (1949). It is readily apparent that for the durations of impulse considered herein, the free surface has no effect.

The problem of determining the hydrodynamic inertia is discussed in Appendix F. The shear factor is discussed in Appendix G.

The computer program with explanatory notes is contained in Appendix I. The program has been written in Fortran IV language for a Control Data Corporation 3600 computer.

Both the explicit and implicit techniques have been programmed. The mixed technique, although investigated and found to be promising, has not been programmed.

The reason for developing two programs is to provide freedom in selecting the time increment. The explicit and implicit programs are complementary. As brought out above, and developed in Appendix D, the explicit analog is subject to certain limitations: if the numerical solution is to be valid, the time increment to be used must not exceed a certain critical value. On the other hand, the implicit analog is always numerically stable. Of course, this stability is bought at the price of greater complexity. Thus, the explicit solution is to be preferred if a sufficiently small time increment can be used or is called for by external reasons, otherwise, the implicit analog must be used.

The program for calculations reported herein has been set up so that the time increment is selected a priori. A test is first made whether the criterion of stability is met. If it is, the explicit program is selected, if the criterion of stability is not met, the implicit program is automatically selected.

2. RESULTS

Several numerical runs have been made on a Control Data Corporation 3600 computer. For these runs, the following inputs have been used:

Axial increment,	Δx	4.96 ft.
Time increment,	Δt	0.0001 to 0.02 sec.
Impulse duration	τ	0.001, 0.01, 0.1, 1 sec.
Impulse location		25 percent of the ship's length from the bow

Also, three sets of values of flexural and shear rigidities have been introduced, namely, normal, 75 percent of normal, and 50 percent of normal. The normal set of values corresponds to the ship as built.

The runs have been carried out for a sufficiently long duration to insure that the bending moment wave has reached the stern and has been reflected to amidships. Typical results are shown in Appendix J. From these, the summary Figs 1 and 2 have been derived.

Observations based on the results obtained from the limited number of computer runs made are as follows:

a) The bending moment at a generic station does not, in general, attain its maximum value during the first cycle of response but rather during the second or a later cycle.

b) The highest bending moments occur in the region of the bow forward of the quarter point where the impulse is applied.

c) High bending moments occur at the station where the impulse is applied.

d) High bending moments also occur in the region of the after quarter point and closer to the stern.

e) The bending moment amidships is not the critical one.

f) An increase in hull flexibility tends to reduce the bending moment at a generic station.

3. CONCLUSION

The specifications for the task called for the development of a computer program which would make it possible to determine the influence of ship flexibility on her response to an external loading impulsively applied.

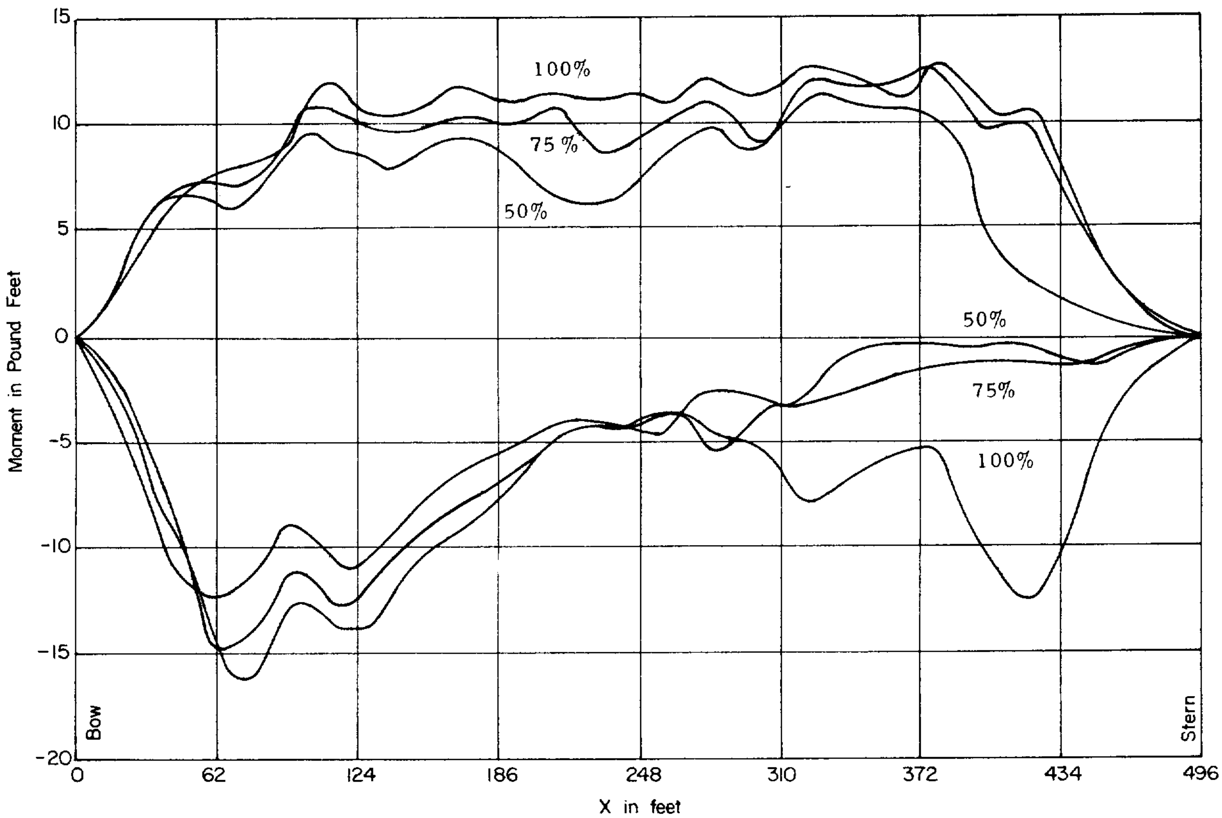


Fig. 1 Envelope of Maximum Bending Moments Along Length Of Ship Corresponding To A Unit Impulse of 0.1 Seconds Duration Applied At The Quarter Point Of The Hull Of The Wolverine State As Built (Normal Rigidity) And of Derived Hulls Of 75 And 50 Percent Normal Rigidities.

Two complementary programs have been developed: an explicit one, advantageous for short durations of impulse, and an implicit one, preferable for longer durations: the choice between the two is made automatically on the basis of a criterion of stability. In addition, a third possible program, based on a mixed technique, has been explored. Although this technique appears to be superior to the explicit technique, the corresponding program has not been written.

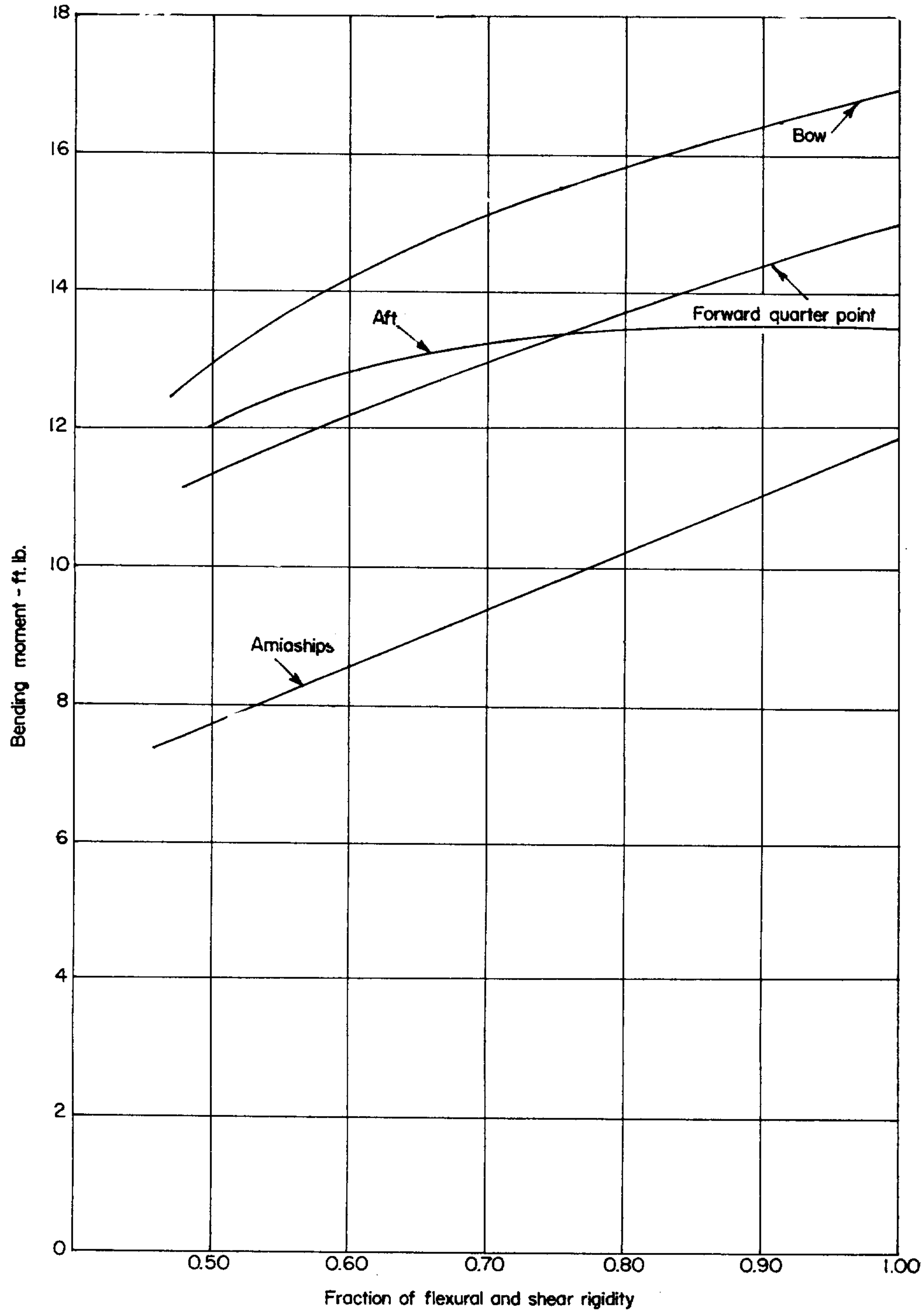


Fig. 2 Maximum Bending Moment At Various Locations Corresponding To A Unit Impulse Of 0.1 Seconds Duration Applied At The Forward Quarter Point Of The Hull Of The Wolverine State As Built (Normal Rigidity) And Of Derived Hulls Of 75 And 50 Percent Normal Rigidity.

Preliminary results indicate that the influence of structural flexibility is to reduce the maximum bending moment in the hull structure. It follows that if this flexibility is attained thru application of higher strength steels, the fraction of yield strength taken up by impulsive loading will be comparatively less than for medium steel and the factor of safety will experience a beneficial increase.

4. RECOMMENDATIONS

The following recommendations are made with the view of improving the program and obtaining an adequate amount of realistic data:

a) Extend the program to take into account the aspect of dynamic shear lag in the decks.

b) Extend the program to take into account the local behavior of the bottom structure in way of the loading.

c) Make production runs for hydrodynamic loadings having realistic characteristics. A sufficient quantity of experiment data appear to be in hand so that runs can be designed to yield meaningful results.

d) Extend the program to include damping, at least in a simplified form, to verify its influence on hull response.

ACKNOWLEDGMENT

The authors are indebted to both Mr. Lee Butler, who prepared the computer program, and Cdr. Richard Nielsen, U. S. Coast Guard, for his valuable suggestions on computer techniques.

APPENDIX A

REMARKS ON ENERGY DISSIPATION OR DAMPING

The dissipation of energy is of two types: structural and hydrodynamic. Structural damping is made up of material damping and slip damping. Hydrodynamic damping consists of radial wave dissipation and of viscous damping. Some remarks are presented on the inclusion of damping in the equations of motion.

The components of energy dissipation are variously expressed, but for inclusion in the equations of motion, it becomes necessary to relate them to a variable of the motion itself (displacement, velocity, acceleration).

a) <u>Material Dissipation</u>
The specific material dissipation (per unit volume per cycle) is expressed in terms of the amplitude of the stress intensity. A convenient empirical expression is

$$d_m \equiv c_m(\sigma_m) \cdot \sigma_m^{n(\sigma_m)}$$

where:

$d_m \equiv$ specific dissipation

$c_m(\sigma_m),\ n(\sigma_m) \equiv$ experimentally derived coefficients related to the material and to the amplitude of stress intensity. Below a critical stress these coefficients are constants and can be written simply as c_m and n.

One is faced with the problem of converting this expression to one written in terms of a dependent variable. The dependent variables being all functions of time, it is convenient to replace the given empirical expression for specific damping by one that is time dependent. Accordingly, write

$$d_m(t) = c_m(\sigma, t) \cdot \sigma^{n(\sigma, t)}$$

where $c_m(\sigma, t)$ is so chosen that over a cycle

$$\frac{1}{T} \int_0^T d_m(t)\, dt = d_m$$

If n does not change appreciably over the stress intensity range from zero to σ_m,

$$d_m(t) \cong \frac{1}{n(\sigma_m) + 1} \cdot d_m$$

As a consequence, the damping at any instant at the generic section is given by

$$\int \int d_m(t) \cdot dy\, d\zeta \quad .$$

where ζ is the vertical ordinate measured above the keel and where the integration extends over the cross sectional area of the longitudinally continuous structure. At the generic section x, the stress intensity at any point and instant is

$$\sigma(x, \zeta, t) = \frac{M(x, t)}{Z(x)} \cdot \frac{\zeta - \zeta_o(x)}{\zeta_o(x)} \equiv A(x, \zeta) \cdot M(x, t)$$

where:

M	≡	flexural moment
Z	≡	section modulus
ζ_o	≡	height of neutral axis above keel
$A(x, \zeta)$	≡	time-independent constant.

If this expression for stress intensity is introduced in the expression for instantaneous specific damping and the latter is integrated over the cross sectional area, the expression is derived for the instantaneous sectional damping in terms of bending moment. Since this bending moment is related to the curvature

$$M(x, t) = EI(x) \cdot \frac{\partial \gamma(x, t)}{\partial x}$$

the relation between instantaneous sectional damping and instantaneous curvature is established. A parallel expression can be derived relating instantaneous sectional damping to the instantaneous shear value.

b) <u>Slip Damping</u>
Slip damping varies with the type of construction (whether riveted or welded) and approximately with the third power of the load, hence bending moment, hence deflection. The same logic applies as in the preceding case, but the bending moment is now raised to a power which is empirically derived.

c) <u>Radial Wave Dissipation</u>
This component of hydrodynamic damping varies with (the first power of) the transverse velocity. It's value at any station is

$$b_w(x) \cdot \left[\frac{\partial y(x, t)}{\partial t} \right]$$

an experssion which is readily incorporated into the calculations. The coefficient $b_w(x)$ is calculable by the method given by St. Denis (1951) among others.

d) <u>Viscous Damping</u>
This component of hydrodynamic damping varies with the square of the transverse velocity and is given by

$$b_v(x) \cdot \left[\frac{\partial y(x, t)}{\partial t} \right]^2$$

where the damping coefficient

$$b_v(x) \equiv c_v \cdot \frac{1}{2} \rho \ell(x)$$

and $\ell(x)$ is the girth of the underwater hull at station x and c_V is an empirical coefficient of viscosity. As for the previous case, this component of damping is easily introduced into the equations of motion.

The overall conclusion regarding damping is that hydrodynamic and structural damping can be incorporated in the analysis without particular difficulty.

APPENDIX B

DIFFERENTIAL EQUATIONS OF MOTION

Introduction

The free vibrations of a free-free slender non-uniform beam with shear deformation and rotary inertia considered has been extensively discussed in the literature on dynamics, see e. g. Timoshenko, (1928). The behavior of such a beam when supported by buoyancy and loaded impulsively was first developed by Ormondroyd et al (1948). Their work was followed by that of Polachek (1961), McGoldrick (1961), Leibowitz (1962), Leibowitz and Kennard (1963), Andrews (1963), Leibowitz (1963), Leibowitz and Kennard (1964), Leibowitz and Greenspon (1964), Cuthill and Henderson (1965). The foregoing references, which deal with the overall problem of elastic response, are accompanied by complementary studies on specific aspects of the problem which will be mentioned when considering these latter aspects.

The equations of motion are set up by equating the excitation to the sum of the inertial and restoring reactions, their calculation being made on the basis that the ship is at rest prior to application of the exciting impulse. The boundary conditions at the ends are those of a free-free beam. Certain assumptions are made relative to the ship's geometry, namely, that she is slender with respect to length, that her dynamic properties (form, distribution of mass and of structural elements) do not vary rapidly along the length, and that she is symmetric with respect to the longitudinal centerplane. An additional assumption is made relative to the ship's material, namely, that it obeys Hooke's law of proportionality of stress and strain.

To derive the equations of motion, introduce a right-handed cartesian coordinate system, the origin of which is at the center of gravity of the ship, the x-axis being horizontal and longitudinally directed, its positive direction being sternward; the y-axis being vertically directed and positive upwards, see Fig. B-1.

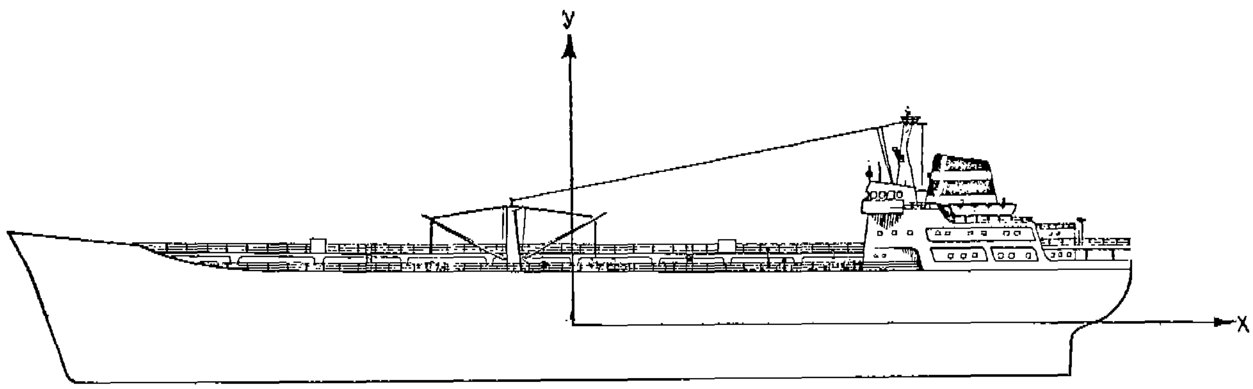


Fig. B-1 Coordinate System.

Dynamical Equations in Terms of Generalized Displacements

The dynamical equations are written in a convenient form in terms of a system of generalized coordinates $y(x,t)$ and $\gamma(x,t)$ as follows:

a) Force Equation

$$m_v(x)\,\frac{\partial^2 y(x,t)}{\partial t^2} - \frac{\partial}{\partial x}\left\{k(x)\cdot A(x)\cdot G\left[\frac{\partial y(x,t)}{\partial x} - \gamma(x,t)\right]\right\}$$

$$+ \;\rho g\, B(x)\cdot y(x,t) \;=\; p(x,t)$$

b) Moment Equation

$$r_v^2(x) \cdot m_v(x) \cdot \frac{\partial^2 \gamma(x,t)}{\partial t^2} - \frac{\partial}{\partial x}\left[EI(x) \cdot \frac{\partial \gamma(x,t)}{\partial x}\right]$$

$$- k(x) \cdot A(x) \cdot G\left[\frac{\partial y(x,t)}{\partial x} - \gamma(x,t)\right] = 0$$

In these equations:

r_v ≡ radius of gyration of the virtual mass about the lateral axis thru the dynamic center (ft)

m_v ≡ virtual mass per unit length (lb sec^2 ft^{-2})

γ ≡ angle of rotation of cross section measured positive when counter-clockwise (rad)

t ≡ time (sec)

EI ≡ flexural rigidity (lb ft^2)

E ≡ Young's modulus of the material (lb ft^{-2})

I ≡ second central moment of the cross sectional area of longitudinal structure (ft^4)

kAG ≡ shearing rigidity (lb)

- k ≡ shear factor, see Appendix G
- A ≡ cross sectional area of longitudinal structure (ft^2)
- G ≡ modulus of rigidity of the material (lb ft^{-2})
- y ≡ total vertical deflection (rigid plus elastic) of hull (ft)

$\left[\frac{\partial y(x,t)}{\partial x} - \gamma(x,t)\right]$ ≡ effective angle of shear deformation (rad)

- ρ ≡ mass density of water (lb $sec^2 ft^{-4}$)
- g ≡ acceleration of gravity (ft sec^{-2})
- B(x) ≡ local beam (ft)
- p(x, t) ≡ excitation (lb ft^{-1})

The following relations hold:

$$r_v^2(x) \equiv \frac{r^2(x) \cdot m(x) + r_h^2(x) \cdot m_h(x)}{m(x) + m_h(x)}$$

where:

- r ≡ radius of gyration of basic mass about the lateral axis thru the center of gravity of the section (ft)
- r_h ≡ radius of gyration of the hydrodynamic mass about the lateral axis thru the centroid of the section (ft)
- m ≡ basic mass per unit length (lb $sec^2 ft^{-2}$)
- m_h ≡ hydrodynamic mass per unit length (lb $sec^2 ft^{-2}$)

The excitation acting on the system is a vertically-directed rectangular pulse of duration $\tau \equiv t_1 - t_0$, which is considered as a parameter. The basic assumption of linearity permits taking the total impulse as unity without loss of generality. The impulse is applied in the longitudinal centerplane in the interval ξ_o to ξ_1, which is also considered as a parameter. It is defined as

$$I(\xi_o, \xi_1, t_o, t_1) = 1 \quad \text{(lb sec)}$$

The amplitude of the pulse is, consequently,

$$\frac{1}{[\xi_1 - \xi_0] \cdot [t_1 - t_0]} \quad (\text{lb ft}^{-1})$$

for

$$\xi_0 < x < \xi_1$$

$$t_0 < t < t_1$$

zero otherwise

The differential equations of motion can be solved in closed form and in terms of ordinary functions only if the coefficients entering into the equations are constants. If the coefficients are simply-defined, well-behaved variables of x alone, closed form solutions can be hoped for, but not in terms of ordinary functions, or, alternately, series solutions are feasible. If the coefficients are well-behaved variables of x, but of arbitrary distribution, resort must be had to numerical integration. If the coefficients are ill-behaved variables of x, no reliable solution can be expected.

The solution to these equations is sought on the assumption that the coefficients are arbitrary and well-behaved. To this end, the differential equations must be transformed into difference equations. This transformation is made in the next Appendix.

The assumption of well-behavior does permit obtaining a solution, but the validity of such assumption bears examination. Of the elastic and dynamic properties of the hull ($k(x)$, $A(x)$, $m(x)$, $r^2(x) \cdot m(x)$ $m_h(x)$, $r_h^2(x) \cdot m_h(x)$) all tend to vary gradually except $m(x)$ and, consequently, $r^2(x) \cdot m(x)$. In the lightship condition, sudden variations in mass occur at the ends of the machinery compartment and at the ends of No. 3 hold where the fixed ballast is located. Depending upon how the ship is loaded, additional steps in the distribution of mass occur at the ends of holds, see Fig. H-3 .

That the only discontinuities occur in the distribution of mass is fortunate, for in such case the effect is on the inertial reaction, hence second time derivative of the displacement. Because of the double integration, the displacement and its line derivatives (slope, curvature) result as reasonably well-behaved.

APPENDIX C
DIFFERENCE EQUATIONS OF MOTION

Transformation of Differential to Difference Equations

The conversion of a differential equation into a difference equation is made by introducing finite increments in the position (Δx) and time (Δt) of the dependent variables, $y(x, t)$, $\gamma(x, t)$. The derivative terms of $y(x, t)$, $\gamma(x, t)$ and of the dynamical properties obtaining at discrete points of the independent variables.

In a marching problem, the method of solution consists in finding the values of the independent variables for the next increment in time given a knowledge of the variables for positions and times up to the generic ones.

As a first step in the logic of conversion, it would appear necessary to establish the values of Δx and Δt. It is preferable, however, to delay discussion on this problem until after the difference equations have been set up and solved.

Boundary and Initial Conditions

The boundary conditions of the free-free beam are (for all times)

$$M_{1,h} = M_{N,h} = 0$$

$$Q_{1,h} = Q_{N,h} = 0$$

Consider the conditions at the end $x = -L/2$ i.e., $n = 1$. The condition of zero moment is written

$$\frac{EI_1}{(\Delta x)}\left[\gamma_{2,h} - \gamma_{0,h}\right] = 0$$

where the point 0, h is fictitious, and that of zero shear is

$$k_{1,h} \cdot A_{1,h} \cdot G \left[y_{2,h} - y_{0,h} - 2(\Delta x)\gamma_{1,h}\right] = 0$$

At the end $x = L/2$, $n = N$ the condition of zero moment is

$$\frac{EI_N}{(\Delta x)}\left[y_{N+1,h} - y_{N-1,h}\right] = 0$$

Where the point $N+1$ is fictitious, while the condition of zero shear gives

$$k_N \cdot A_N \cdot G'\left[y_{N+1,h} - y_{N-1,h} - 2(\Delta x)\,y_{N,h}\right] = 0$$

The Impulsive Load

Upon taking Δx sufficiently small and writing

$$\xi_o = i \cdot \Delta x \quad , \quad \xi_1 = [i + j] \cdot \Delta x$$

where i and j are integers, the unit impulse load amounts to

$$\frac{1}{j \cdot \Delta x \cdot \tau}$$

An arbitrary loading is represented by the array

$$p_{n,h}$$

where n denotes the station and h the instant

Computational Form of Equations in Terms of Generalized Coordinates

In this section finite difference expressions will be developed for the dynamical equations expressed in terms of generalized coordinates. However, in lieu of single order approximations to the derivatives, second order ones will be used.

If the deflection and rotation of a section at the point $n \cdot \Delta x$ and instant $h \cdot \Delta t$ be expanded in a Taylor series, the result is

$$y_{n+1,h} = y_{n,h} + (\Delta x)\frac{\partial y_{n,h}}{\partial x} + \frac{(\Delta x)^2}{2}\frac{\partial^2 y_{n,h}}{\partial x^2} + \frac{(\Delta x)^3}{6}\frac{\partial^3 y_{n,h}}{\partial x^3} + \frac{(\Delta x)^4}{24}\frac{\partial^4 y_{n,h}}{\partial x^4} +$$

$$+ \theta[(\Delta x)^5]$$

$$y_{n-1,h} = y_{n,h} - (\Delta x)\frac{\partial y_{n,h}}{\partial x} + \frac{(\Delta x)^2}{2}\frac{\partial^2 y_{n,h}}{\partial x^2} - \frac{(\Delta x)^3}{6}\frac{\partial^3 y_{n,h}}{\partial x^3} + \frac{(\Delta x)^4}{24}\frac{\partial^4 y_{n,h}}{\partial x^4} -$$

$$- \theta[(\Delta x)^5]$$

Upon subtracting the two equations and dividing the result by $2(\Delta x)$ one obtains

$$\frac{\partial y_{n,h}}{\partial x} = \frac{y_{n+1,h} - y_{n-1,h}}{2(\Delta x)} + \theta[(\Delta x)^2]$$

$$\cong \frac{y_{n+1,h} - y_{n-1,h}}{2(\Delta x)}$$

Upon summing the two equations and dividing the result by $(\Delta x)^2$ one has

$$\frac{\partial^2 y_{n,h}}{\partial x^2} = \frac{y_{n+1,h} - 2y_{n,h} + y_{n-1,h}}{(\Delta x)^2} + \theta[(\Delta x)^2]$$

$$\cong \frac{y_{n+1,h} - 2y_{n,h} + y_{n-1,h}}{(\Delta x)^2}$$

From the two foregoing equations,

$$\frac{\partial^3 y_{n,h}}{\partial x^3} \cong \frac{y_{n+1,h} - 3y_{n,h} + 3y_{n-1,h} - y_{n-2,h}}{(\Delta x)^3}$$

Upon introducing this expression in the first pair, subtracting and solving for $\partial y_{n,h}/\partial x$, the following second order approximation for the first derivative is obtained

$$\frac{\partial y_{n,h}}{\partial x} \cong \frac{1}{6(\Delta x)}\left[2y_{n+1,h} + 3y_{n,h} - 6y_{n-1,h} + y_{n-2,h}\right]$$

which has a residue of $\theta[(\Delta x)^4]$. This second order expression should allow for a more refined treatment of the boundary conditions. Of course, the time derivatives are similarly treated and result in parallel expressions.

It is possible to set up two finite difference analogs to the dynamical equations: an explicit one and an implicit one. In the first, the recurrence formula is an explicit expression for $y_{n,h+1}$ and $Y_{n,h+1}$ in terms of $y_{n,h-1}$, $y_{n,h}$, $Y_{n,h-1}$, $Y_{n,h}$. The implicit analog is constructed by applying the Taylor series approximation to $y_{n,h}$. The expansion yields

$$y_{n,h} = \frac{1}{4}\left[y_{n,h+1} + 2y_{n,h} + y_{n,h-1}\right] + \theta\left[(\Delta t)^2\right]$$

The first term on the right is the time averaged value

$$\bar{y}_{n,h} \cong \frac{1}{4}\left[y_{n,h+1} + 2y_{n,h} + y_{n,h-1}\right]$$

which is approximately $y_{n,h}$ and will be substituted for it. This expansion is applied to every term in the system with the exception of such terms that involve time derivatives. The result is a system of linear algebraic equations for $y_{n,h+1}$ and $\gamma_{n,h+1}$ which rest upon the knowledge of the terms for the discrete instants h-1 and h. There will be 2N equations for the same number of unknowns. This system of equations can be set up in a form for which the matrix of the coefficients will be of the block diagonal type. The two analogs are now specifically considered.

Explicit Analog

Application of the expressions for the derivatives given above yield the following finite difference analog to the dynamical equations in terms of generalized coordinates

a) Force equation

$$\frac{(m_v)_n}{(\Delta t)^2}\left[y_{n,h+1} - 2y_{n,h} + y_{n,h-1}\right]$$

$$- \frac{k_n A_n G}{(\Delta x)}\left[\frac{y_{n+1,h} - 2y_{n,h} + y_{n-1,h}}{\Delta x} - \frac{\gamma_{n+1,h} - \gamma_{n-1,h}}{2}\right]$$

$$- \frac{G}{2(\Delta x)}\left[k_{n+1} \cdot A_{n+1} - k_{n-1} \cdot A_{n-1}\right] \cdot \left[\frac{y_{n+1,h} - y_{n-1,h}}{2(\Delta x)} - \gamma_{n,h}\right]$$

$$+ \rho g B_n \cdot y_{n,h} = p_{n,h}$$

b) Moment equation

$$(r_v^2)_n \cdot (m_v)_n \cdot \frac{1}{(\Delta t)^2}\left[\gamma_{n,h+1} - 2\gamma_{n,h} + \gamma_{n,h-1}\right]$$

$$- \frac{EI_n}{(\Delta x)^2}\left[\gamma_{n+1,h} - 2\gamma_{n,h} + \gamma_{n-1,h}\right] - \frac{E}{4(\Delta x)^2}\left[I_{n+1} - I_{n-1} \cdot \gamma_{n+1,h} - \gamma_{n-1,h}\right]$$

$$- k_n A_n G \left[\frac{y_{n+1,h} - y_{n-1,h}}{2(\Delta x)} - \gamma_{n,h} \right] = 0$$

The terms in the unknowns $y_{n,h+1}$ and $\gamma_{n,h+1}$ are separated from these equations with the following result:

a) Force equation

$$y_{n,h+1} = 2 y_{n,h} - y_{n,h-1}$$

$$+ \frac{k_n A_n G}{(m_v)_n} \cdot \frac{(\Delta t)^2}{\Delta x} \left[\frac{y_{n+1,h} - 2y_{n,h} + y_{n-1,h}}{(\Delta x)} - \frac{\gamma_{n+1,h} - \gamma_{n-1,h}}{2} \right]$$

$$+ \frac{G}{2(m_v)_n} \cdot \frac{(\Delta t)^2}{\Delta x} \left[k_{n+1} A_{n+1} - k_{n-1} A_{n-1} \right] \cdot$$

$$\cdot \left[\frac{y_{n-1,h+1} - 2y_{n,h} + y_{n-1,h}}{\Delta x} - \frac{\gamma_{n+1,h} - \gamma_{n-1,h}}{2} \right]$$

$$- \frac{\rho g B_n}{(m_v)_n} (\Delta t)^2 \cdot y_{n,h} + p_{n,h}$$

b) Moment equation

$$\gamma_{n,h+1} = 2\gamma_{n,h} - \gamma_{n,h-1}$$

$$+ \frac{(\Delta t)^2}{(\Delta x)^2} \cdot \frac{E}{(r_v^2 \cdot m_v)_n} \Big\{ \gamma_{n+1,h} - 2\gamma_{n,h} + \gamma_{n-1,h}$$

$$+ \frac{1}{4}\left[I_{n+1} - I_{n-1}\right] \cdot \left[\gamma_{n+1,h} - \gamma_{n-1,h}\right]\Bigg\}$$

$$+ \frac{k_n A_n G}{(r_v^2 \cdot m_v)_n} \cdot (\Delta t)^2 \left[\frac{y_{n+1,h} - y_{n-1,h}}{2(\Delta x)} - \gamma_{n,h}\right],$$

Boundary Conditions for Explicit Analog

It is now necessary to express the boundary conditions in terms of discrete values of y and γ. As discussed above, the boundary condition at the point $x = -L/2$ is given by the second order approximations.

a) Condition of zero moment

$$\gamma_{2,h} - \gamma_{0,h} = 0$$

b) Condition of zero shear

$$y_{2,h} - y_{0,h} - 2(\Delta x)\,\gamma_{1,h} = 0$$

However, a better approximation to the boundary condition is given by using the fourth order approximation. This results in

$$\gamma_{1,h} = \frac{1}{2}\left[-3\gamma_{2,h} + 6\gamma_{3,h} - \gamma_{4,h}\right]$$

$$\gamma_{1,h} = \frac{1}{2}\left[-3y_{2,h} + 6y_{3,h} - y_{4,h}\right] + 3(\Delta x) \cdot \gamma_{2,h}$$

The parallel approximation holds at the end $x = L/2$.

Implicit Analog

If the Taylor series expansion for $y_{n,h}$ is introduced in the dynamical equations and made to apply to all the terms except those involving time derivatives and the excitation, the following equations result:

a) Force equation

$$(m_v)_n \cdot \frac{1}{(\Delta t)^2}\left[y_{n,h+1} - 2y_{n,h} + y_{n,h-1}\right]$$

$$- \frac{k_n A_n G}{4 \cdot (\Delta x)} \left\{ \frac{1}{(\Delta x)} \left\{ y_{n+1,h+1} - 2y_{n,h+1} + y_{n-1,h+1} \right.\right.$$

$$+ 2\left[y_{n+1,h} - 2y_{n,h} + y_{n-1,h}\right]$$

$$\left. + y_{n+1,h-1} - 2y_{n,h-1} + y_{n-1,h-1}\right\}$$

$$- \frac{1}{2} \left\{ \gamma_{n+1,h+1} - \gamma_{n-1,h+1} + 2\left[\gamma_{n+1,h} - \gamma_{n-1,h}\right]\right.$$

$$\left.\left. + \gamma_{n+1,h-1} - \gamma_{n-1,h-1}\right\}\right\}$$

$$- \frac{G}{8 \cdot (\Delta x)}\left[k_{n+1} A_{n+1} - k_{n-1} A_{n-1}\right]$$

$$\cdot \left\{ \frac{1}{2 \cdot (\Delta x)} \left\{ y_{n+1,h+1} - y_{n-1,h+1} + 2\left[y_{n+1,h} - y_{n-1,h}\right]\right.\right.$$

$$\left.\left. + y_{n+1,h-1} - y_{n-1,h-1} \right\} - \left[\gamma_{n,h+1} + 2\gamma_{n,h} + \gamma_{n,h-1}\right]\right\}$$

$$- \frac{\rho g}{4} B_n\left[y_{n,h+1} + 2y_{n,h} + y_{n,h-1}\right] = p_{n,h}$$

b) Moment equation

$$(r_v^2 \cdot m_v)_n \cdot \frac{1}{(\Delta t)^2}\left[\gamma_{n,h+1} - 2\gamma_{n,h} + \gamma_{n,h-1}\right]$$

$$- \frac{EI_n}{4(\Delta x)^2}\left\{ \gamma_{n+1,h+1} - 2\gamma_{n,h+1} + \gamma_{n-1,h+1}\right.$$

$$+ 2\left[\gamma_{n+1,h} - 2\gamma_{n,h} + \gamma_{n-1,h}\right]$$

$$\left. + \gamma_{n+1,h-1} - 2\gamma_{n,h-1} + \gamma_{n-1,h-1}\right\}$$

$$- \frac{E}{16\cdot(\Delta x)^2}\left[I_{n+1} - I_{n-1}\right]$$

$$\cdot \left\{ \gamma_{n+1,h+1} - \gamma_{n-1,h+1} + 2\left[\gamma_{n+1,h} - \gamma_{n-1,h}\right]\right.$$

$$\left. + \gamma_{n+1,h-1} - \gamma_{n-1,h-1} \right\}$$

$$- k_n A_n G \left\{ \frac{1}{8\cdot(\Delta x)} \left\{ y_{n+1,h+1} - y_{n+1,h+1} + 2\left[y_{n+1,h} - 2y_{n+1,h}\right]\right.\right.$$

$$\left. + y_{n+1,h-1} - y_{n-1,h-1}\right\}$$

$$\left. - \frac{1}{4}\left[\gamma_{n,h+1} + 2\gamma_{n,h} + \gamma_{n,h-1}\right]\right\} = 0$$

These equations form a system of 2N linear algebraic equations for the same number of unknowns. Note that these equations are of the form

$$a_n \gamma_{n-1,h+1} + b_n \gamma_{n,h+1} + c_n \gamma_{n+1,h+1}$$

$$+ \; d_n y_{n-1,h+1} + e_n y_{n,h+1} + f_n y_{n+1,h+1} = g_n$$

where:

$$n = 1, 2, \ldots N$$

To arrange this system of equations in a form such that the matrix of the coefficients will have its non-zero elements on a diagonal, alternate the equations and the unknowns in the fashion that follows and, to this end, let:

$$\gamma_{n,h+1} \equiv u_{2n-1,h+1}$$

$$y_{n,h+1} \equiv u_{2n,h+1}$$

Denote the coefficients of the first dynamical equation of the system by

$$a_n \equiv A_{2n-1}, \quad b_n \equiv B_{2n-1}, \ldots g_n \equiv G_{2n-1}$$

and those of the second dynamical equation by

$$a_n \equiv A_{2n}, \quad b_n \equiv B_{2n}, \ldots g_n \equiv G_{2n}$$

Then

$$A_r u_{r-2} + D_r u_{r-1} + B_r u_r + E_r u_{r+1} + C_r u_{r+2} + F_r u_{r+3} = G_r \; .$$

for r <u>odd</u>

$$A_r u_{r-3} + D_r u_{r-2} + B_r u_{r-1} + E_r u_r + C_r u_{r+1} + F_r u_{r+2} = G_r$$

for r <u>even</u>

where $r = 1, 2, \ldots 2N$

The matrix of coefficients for this system of equations will have the form

$$\begin{bmatrix}
B_1 & E_1 & C_1 & F_1 & & & & & 0 & & & \\
B_2 & E_2 & C_2 & F_2 & & & & & 0 & & & \\
A_3 & D_3 & B_3 & E_3 & C_3 & F_3 & & & 0 & & & \\
A_4 & D_4 & B_4 & E_4 & C_4 & F_4 & & & 0 & & & \\
0 & 0 & A_5 & D_5 & B_5 & E_5 & C_5 & F_5 & 0 & & & \\
0 & 0 & A_6 & D_6 & B_6 & E_6 & C_6 & F_6 & 0 & & & \\
 & & & \cdot & & & & & & & & \\
0 & 0 & & & & & & & A_{2N-1} & D_{2N-1} & B_{2N-1} & E_{2N-1} \\
0 & 0 & & & & & & & A_{2n} & D_{2N} & B_{2n} & E_{2N}
\end{bmatrix}$$

The matrix of the coefficients is a seven band diagonal matrix. The system of dynamical equations represented by the preceding expression is solvable by standard techniques for each time step h.

<u>Boundary Conditions for Implicit Analog</u>

The boundary conditions in the implicit analog are the same as for the explicit one with the exception that they must be averaged with respect to time in the same fashion as done for the explicit analog. This leads to the following:

a) Condition of zero moment at $x = -L/2$

$$y_{2,h+1} - y_{0,h+1} + 2\{y_{2,h} - y_{0,h}\}$$

$$+ y_{2,h-1} - y_{0,h-1} = 0$$

b) Condition of zero shear at $x = -L/2$

$$y_{2,h+1} - y_{0,h+1} + 2\{y_{2,h} - y_{0,h}\} + y_{2,h-1} - y_{0,h-1}$$

$$- 2(\Delta x)\{y_{1,h+1} + 2y_{1,h} + y_{1,h-1}\} = 0$$

and similarly for the station at $x = L/2$.

APPENDIX D
NUMERICAL STABILITY

General Case

The condition that the equations are computationally stable is satisfied if any of the dependent variables entering into the equations are subjected to a small disturbance, at any instant of time t, and this disturbance does not increase with each successive increment of time. The condition of stability is given by the relation

$$\left| \frac{\delta F(x, t + \Delta t)}{\delta F(x, t)} \right| < 1$$

where

$\delta F(x, t)$	$\equiv$	perturbation (or variation) in a generic dependent variable of x and t, at instant t
$\delta F(x, t + \Delta t)$	$\equiv$	perturbation of same at instant $t + \Delta t$

The equations of motion are weakly nonlinear in the dynamic buoyancy and hydrodynamic mass terms. However, the oscillations of the ship resulting from the application of the impulse are sufficiently small as to justify the assumption that both the restoration coefficient and the hydrodynamic mass distribution are independent of the motion. Furthermore, it is assumed that the dynamic properties of the ship (flexural and shear rigidities, virtual mass and moment of inertia, and restoration) vary slowly along the ship. Both of these assumptions lead us to consider the system of dynamical equations to be a linear one with constant coefficients, such consideration holding only for the analysis of numerical stability.

It may be observed that the validity of such an assumption can be tested by carrying out numerical computations employing either the explicit or the implicity method of solution and equating the time increment to the upper limit given from the solution for numerical stability if such a limit exists.

Explicit Analog

The finite difference explicit analog to the dynamical equations in terms of generalized coordinates and for constant coefficients is

$$(r_v^2)_n \cdot (m_v)_n \cdot \frac{1}{(\Delta t)^2}\left[\gamma_{n,h+1} - 2\gamma_{n,h} + \gamma_{n,h-1}\right]$$

$$- \frac{EI_n}{(\Delta x)^2}\left[\gamma_{n+1,h} - 2\gamma_{n,h} + \gamma_{n-1,h}\right]$$

$$-k_n A_n G\left[\frac{y_{n+1,h} - y_{n-1,h}}{2(\Delta x)} - \gamma_{n,h}\right] = 0$$

$$\frac{(m_v)_n}{(\Delta t)^2} \cdot \left[y_{n,h+1} - 2y_{n,h} + y_{n,h-1}\right]$$

$$- \frac{k_n \cdot A_n G}{(\Delta x)}\left[\frac{y_{n+1,h} - 2y_{n,h} + y_{n-1,h}}{\Delta x} - \frac{\gamma_{n+1,h} - \gamma_{n-1,h}}{2}\right]$$

$$+ \rho g B_n \cdot y_{n,h} = p_{n,h}$$

Consider, as previously, a solution for the homogeneous part of these dynamical equations in the form

$$\gamma_{n,h} \equiv C\lambda^h \cdot \exp(i n \varphi)$$

$$y_{n,h} \equiv D\lambda^h \cdot \exp(i n \varphi)$$

where $\exp(in\varphi)$ is a fundamental mode for certain general boundary conditions. Upon introducing these expressions into the homogeneous part of the dynamical equations, it results that

$$\left\{ \frac{2EI}{(\Delta x)^2} \left[1 - \cos\varphi\right] + kAG + \frac{r_v^2 \cdot m_v}{(\Delta t)^2}\left[\lambda - 2 + \frac{1}{\lambda}\right] \right\} C$$

$$- i \left\{ \frac{kAG}{\Delta x} \sin\varphi \right\} D = 0$$

$$\left\{ \frac{2kAG}{(\Delta x)^2} \left[1 - \cos\varphi\right] + \rho g B + \frac{m_v}{(\Delta t)^2}\left[\lambda - 2 + \frac{1}{\lambda}\right] \right\} D$$

$$+ i \left\{ \frac{kAG}{\Delta x} \sin\varphi \right\} C = 0$$

To obtain a non-trivial solution to this system of homogeneous equations in C and D, the determinant of the coefficients must vanish. Therefore,

$$\left\{ \frac{2EI}{(\Delta x)^2} \left[1 - \cos\varphi\right] + kAG + \frac{r_v^2 \cdot m_v}{(\Delta t)^2} \cdot \mu \right\}$$

$$\left\{ \frac{2kAG}{(\Delta x)^2} \left[1 - \cos\varphi\right] + \rho g B + \frac{m_v}{(\Delta t)^2} \cdot \mu \right\}$$

$$- \left[\frac{kAG}{\Delta x}\right]^2 \sin^2\varphi = 0$$

where

$$\mu \equiv \lambda - 2 + \frac{1}{\lambda}$$

This equation is a simple quadratic one and can be written

$$\mu^2 + 2\alpha\rho \left\{ \frac{I[1 - \cos\varphi]}{r_v^2 m_v} + \frac{kAG}{Em_v} + \frac{kAG \cdot (\Delta x)^2}{2E r_v^2 m_v} + \frac{\rho g B (\Delta x)^2}{2Em_v} \right\}$$

$$+ \alpha^2 \rho^2 \left[\frac{1}{E r_v^2 m_v^2} \right]$$

$$\cdot \left\{ 4IkAG[1 - \cos\varphi]^2 + \frac{2[kAG]^2 [1 - \cos\varphi](\Delta x)^2}{E} + \right.$$

$$\left. + 2\rho g BI [1 - \cos\varphi](\Delta x)^2 + \frac{kAG\rho g B (\Delta x)^4}{E} \right\}$$

where

$$\alpha \equiv \frac{E}{\rho} \frac{(\Delta t)^2}{(\Delta x)^2}$$

is a nondimensional form for the square of the velocity of propagation of uniaxial elastic stress waves.

The equation defining μ can be rewritten in the form

$$\lambda^2 - 2\left[1 - \frac{\mu}{2}\right]\lambda + 1 = 0$$

To inquire into the stability of the computations, consider the definitions of $Y_{n,h}$ and $y_{n,h}$. It is evident that an error introduced at one line of the computation will not grow as the computation proceeds provided that

$$|\lambda| < 1$$

If this condition is fulfilled, the equations are stable. From the condition that the determinant of the system of equations in C and D should vanish, it can be verified that μ is real and it follows from the equation in λ that

$$\left|1 + \frac{\mu}{2}\right| \leq 1$$

i.e.,

$$-4 \leq \mu \leq 0$$

But since the coefficients of the quadratic equation in λ are all positive, it follows that the condition

$$\mu < 0$$

is fulfilled and it only remains to fulfil the condition

$$\mu \geq -4$$

Examination of the quadratic equation for μ indicates that the smallest values of μ are obtained when $\varphi = \pi$. In this case

$$\mu_1 = -\frac{(\Delta t)^2}{m_v}\left[\frac{4kAG}{(\Delta x)^2} + \rho gB\right].$$

$$\mu_2 = -\frac{(\Delta t)^2}{r_v^2 m_v}\left[\frac{4EI}{(\Delta x)^2} + kGA\right]$$

The final statements of the stability conditions are

$$\Delta t \leq \frac{\Delta x}{\left[\frac{kAG}{m_v} + \frac{\rho g B (\Delta x)^2}{4 m_v}\right]^{1/2}}$$

$$\Delta t \leq \frac{\Delta x}{\left[\frac{EI}{r_v^2 m_v} + \frac{kAG(\Delta x)^2}{4 r_v^2 m_v}\right]^{1/2}}$$

The smaller of the two Δt's is the time increment for which the explicit analog is stable.

Implicit Analog

The finite difference implicit analog to the dynamical equations in terms of generalized coordinates and for constant coefficients is

$$(r_v \cdot m_v)_n \cdot \frac{1}{(\Delta t)^2}\left[\gamma_{n,h+1} - 2\gamma_{n,h} + \gamma_{n,h-1}\right]$$

$$- \frac{EI_n}{4(\Delta x)^2} \left\{\gamma_{n+1,h+1} - 2\gamma_{n,h+1} + \gamma_{n-1,h+1}\right.$$

$$+ 2\left[\gamma_{n+1,h} - 2\gamma_{n,h} + \gamma_{n-1,h}\right]$$

$$+ y_{n+1,\,h-1} - 2y_{n,\,h-1} + y_{n-1,\,h-1}\Bigg\}$$

$$-k_n A_n G \Bigg\{ \frac{1}{8 \cdot (\Delta x)} \Big\{ y_{n+1,\,h+1} - y_{n+1,\,h+1} - 2\Big[y_{n+1,\,h} - 2y_{n+1,\,h}\Big]$$

$$+ y_{n+1,\,h-1} - y_{n-1,\,h-1}\Big\}$$

$$- \frac{1}{4}\Big[y_{n,\,h+1} + 2y_{n,\,h} + y_{n,\,h-1}\Big]\Bigg\} = 0$$

$$(m_v)_n \cdot \frac{1}{(\Delta t)^2}\Big[y_{n,\,h+1} - 2y_{n,\,h} + y_{n,\,h-1}\Big]$$

$$- \frac{k_n A_n G}{4 \cdot (\Delta x)} \Bigg\{ \frac{1}{(\Delta x)} \Big\{ y_{n+1,\,h+1} - 2y_{n,\,h+1} + y_{n-1,\,h+1}$$

$$2\Big[y_{n+1,\,h} - 2y_{n,\,h} + y_{n-1,\,h}\Big]$$

$$+ y_{n+1,\,h-1} - 2y_{n,\,h-1} + y_{n-1,\,h-1}\Big\}$$

$$- \frac{1}{2}\Big\{ y_{n+1,\,h+1} - y_{n-1,\,h+1} + 2\Big[y_{n+1,\,h} - y_{n-1,\,h}\Big]$$

$$+ y_{n+1,\,h-1} - y_{n-1,\,h-1}\Big\}\Bigg\}$$

$$+ \frac{\rho g}{4} B_n\Big[y_{n,\,h+1} + 2y_{n,\,h} + y_{n,\,h-1}\Big] = P_{n,\,h}$$

If the expressions for $\gamma_{n,h}$ and $y_{n,h}$ are introduced into the homogenized form of these equations, the result is

$$\left\{ \frac{2EI}{(\Delta x)^2}\left\{1 - \cos\varphi\right\} + kAG + \frac{r_v^2 m_v}{(\Delta t)^2} \cdot \mu \right\} C$$

$$- i \left\{ \frac{kAG}{\Delta x} \sin\varphi \right\} D = 0$$

$$\left\{ \frac{2kAG}{(\Delta x)^2}\left\{1 - \cos\varphi\right\} + \rho g B + \frac{m_v}{(\Delta t)^2} \cdot \mu \right\} D$$

$$+ i \left\{ \frac{kAG}{\Delta x} \sin\varphi \right\} C = 0$$

where μ is now

$$\mu \equiv 4 \frac{\lambda - 2 + \frac{1}{\lambda}}{\lambda + 2 + \frac{1}{\lambda}}$$

Again, to obtain a non-trivial solution to this system of homogeneous equations, the determinant of the coefficients must vanish. Upon pursuing the same logic as for the explicit analog, the equation defining μ becomes

$$\lambda^2 - 2 \frac{4 + \mu}{4 - \mu} \cdot \lambda + 1 = 0$$

The condition

$$|\lambda| < 1$$

will be satisfied if, and only if,

$$\left| \frac{4 + \mu}{4 - \mu} \right| < 1$$

Since μ is real and negative, it follows that this condition holds for all Δt's. and Δx's. In other words, the implicit analog is unconditionally stable.

The limitations on the choice of both time and spacial increments imposed by truncation and round-off errors and inherent in both the explicit and implicit analogs remain to be considered.

APPENDIX E
MIXED TECHNIQUE

Analog

To apply to mixed technique, only the derivatives with respect to the variable x are expressed in terms of finite differences, while those with respect to the time variable are retained in their original form. Thus:

a) Force Equation

$$(m'_v)_n \cdot \frac{d^2 y_n(t)}{dt^2} - \frac{k_n A_n G}{(\Delta x)} \left\{ \frac{y_{n+1}(t) - 2y_n(t)}{(\Delta x)} - \frac{\gamma_{n+1}(t) - \gamma_{n-1}(t)}{2} \right\}$$

$$- \frac{G}{2(\Delta x)} \left\{ k_{n+1} A_{n+1} - k_{n-1} A_{n-1} \right\} \left\{ \frac{y_{n+1}(t) - y_{n-1}(t)}{2(\Delta x)} - \gamma_n(t) \right\}$$

$$+ \rho g B_n \cdot y_n(t) = p_n(t)$$

b) Moment Equation

$$(r_v^2)_n \cdot (m_v)_n \cdot \frac{d^2 \gamma_n(t)}{dt^2}$$

$$- \frac{EI_n}{(\Delta x)^2} \left\{ \gamma_{n+1}(t) - 2\gamma_n(t) + \gamma_{n-1}(t) \right\} - \frac{E}{4(\Delta x)^2} \left\{ I_{n+1} - I_{n-1} \right\} \cdot$$

$$\cdot\left\{\gamma_{n+1}(t) - \gamma_{n-1}(t)\right\} - k_n A_n G \left\{\frac{y_{n+1}(t) - y_{n-1}(t)}{2(\Delta x)} - \gamma_n(t)\right\} = 0$$

Introduce

$$\zeta_n(t) \equiv \frac{d\gamma_n(t)}{dt}$$

$$z_n(t) \equiv \frac{dy_n(t)}{dt}$$

This leads to the equation being expressed as follows:

a) <u>Force Equation</u>

$$(m_v)_n \cdot \frac{dz_n(t)}{dt} - \frac{k_n A_n G}{(\Delta x)}\left\{\frac{y_{n+1}(t) - 2y_n(t) + y_{n-1}(t)}{(\Delta x)} - \frac{\gamma_{n+1}(t) - \gamma_{n-1}(t)}{2}\right\}$$

$$- \frac{G}{2(\Delta x)}\left\{k_{n+1}A_{n+1} - k_{n-1}A_{n-1}\right\} \cdot \left\{\frac{y_{n+1}(t) - y_{n-1}(t)}{2(\Delta x)} - \gamma_n(t)\right\}$$

$$+ \rho g B_n \cdot y_n(t) = p_n(t)$$

b) <u>Moment Equation</u>

$$(r_v^2)_n \cdot (m_v)_n \cdot \frac{d\zeta_n(t)}{dt}$$

$$- \frac{EI_n}{(\Delta x)^2}\left\{\gamma_{n+1}(t) - 2\gamma_n(t) + \gamma_{n-1}(t)\right\} - \frac{E}{4(\Delta x)^2}\left\{I_{n+1} - I_{n-1}\right\} \cdot \left\{\gamma_{n+1}(t) - \gamma_{n-1}(t)\right\}$$

$$- k_n A_n G \left\{ \frac{y_{n+1}(t) - y_{n-1}(t)}{2(\Delta x)} - \gamma_n(t) \right\} = 0$$

Thus, the system now consists of four ordinary, first order differential equations for each discrete point (n). For the total of N discrete points there are 4N equations. This system of equations is integrated by the standard fourth order Runge-Kutta technique. This technique is outlined in Abramowitz and Stegun (1965) p. 897.

Stability Analysis for the Mixed Technique

If the assumption is made that the coefficients in the equations of motion are slowly varying and if finite differences are introduced for the derivatives with respect to x, one obtains

$$m_v \cdot \frac{d^2 y_n}{dt^2} = \frac{kGA}{(\Delta x)} \left\{ \frac{1}{(\Delta x)} \left\{ y_{n+1} - 2y_n + y_{n-1} \right\} - \frac{1}{2} \left\{ \gamma_{n+1} - \gamma_{n-1} \right\} \right\} - \rho g B y_n$$

$$r_v^2 m_v \frac{d^2 \gamma_n}{dt^2} = \frac{EI}{(\Delta x)^2} \left\{ \gamma_{n+1} - 2\gamma_n + \gamma_{n-1} \right\} + kGA \left\{ \frac{y_{n+1} - y_{n-1}}{2(\Delta x)} - \gamma_n \right\}$$

Introducing a general mode shape for the variation with respect to n in the form of $\exp(i n \varphi)$, one can write

$$\gamma_n \equiv f(t) \cdot \exp(i n \varphi)$$

$$y_n \equiv g(t) \cdot \exp(i n \varphi)$$

Upon substitution of γ_n and y_n into the equations of motion, the following set is obtained

$$m_v \cdot \frac{d^2 g(t)}{dt^2} = -\left\{ \frac{2kGA}{(\Delta x)^2} \left\{ 1 - \cos\varphi \right\} - \rho g B \right\} g(t) - i \frac{kGA}{(\Delta x)} \sin\varphi \cdot f(t)$$

$$r_v^2 m_v \cdot \frac{d^2 f(t)}{dt^2} = -\left\{ \frac{2EI}{(\Delta x)^2} \left\{ 1 - \cos\varphi \right\} - kGA \right\} f(t) + i \frac{kGA}{(\Delta x)} \sin\varphi \cdot g(t)$$

Reduce the order of the derivatives in these equations by introducing

$$h(t) \equiv \frac{df(t)}{dt}$$

$$j(t) \equiv \frac{dg(t)}{dt}$$

with the result that

$$\frac{dj(t)}{dt} = -\frac{1}{m_v}\left\{\frac{2kGA}{(\Delta x)^2}\left\{1 - \cos\varphi\right\} - \rho g B\right\} g(t) - i\,\frac{kGA}{m_v \cdot (\Delta x)} \sin\varphi \cdot f(t)$$

$$\frac{dh(t)}{dt} = -\frac{1}{r_v^2 m_v}\left\{\frac{2EI}{(\Delta x)^2}\left\{1 - \cos\varphi\right\} - kGA\right\} f(t) + \frac{kGA}{r_v^2 m_v \cdot (\Delta x)} \sin\varphi \cdot f(t)$$

Thus, one has a system of four first order ordinary differential equations in four unknown functions. The stability analysis for a fourth-order Runge-Kutta numerical scheme related to these equations will follow that of Abdel Karim (1966).

To this end, consider a system of ordinary differential equations of the form

$$\frac{dy_n(t)}{dt} = f_n(t, y_m) \qquad \text{where} \quad \begin{array}{l} n = 1,\ 2,\ \ldots\ N \\ m = 1,\ 2,\ \ldots\ N \end{array}$$

Solve for the characteristic value problem. The characteristic equation of the system is

$$\left| \frac{\partial f_n}{\partial y_m} - \lambda\, \delta_{mn} \right| = 0$$

where

$$\delta_{mn} \equiv \begin{cases} 0 & \text{for} \quad m = n \\ 1 & \text{for} \quad m = n \end{cases}$$

Let λ_n be the solutions of the characteristic equation. For a fourth order, Runge-Kutta process the numerical technique will be stable only if the criterion is satisfied that

$$\left| \sum_{r=0}^{4} \frac{(\Delta t \cdot \lambda_n)^r}{r!} \right| < 1 \qquad \text{for} \quad n = 1, 2, \ldots N$$

For the λ_n's which are pure imaginaries, this equation yields the condition

$$0 < \left| \Delta t \right| \lambda_n < 2\sqrt{2}$$

The characteristic equation of the system of the four ordinary differential equations is

$$\begin{bmatrix} \lambda & 0 & 1 & 0 \\ 0 & \lambda & 0 & 1 \\ -\frac{1}{r_v^2 m_v}\left\{\frac{2EI}{(\Delta x)^2}\{1 - \cos\varphi + kGA\}\right\} & i\,\frac{kGA}{r_v^2 m_v \cdot (\Delta x)} & \lambda & 0 \\ -i\,\frac{kGA}{m_v \cdot (\Delta x)} & -\frac{1}{m_v}\left\{\frac{2kGA}{(\Delta x)^2}\{1 - \cos\varphi\} + \rho g B\right\} & 0 & \lambda \end{bmatrix} = 0$$

Expansion of the determinant yields

$$\lambda^4 + \left\{ \frac{1}{r_v^2\, m_v} \left[\frac{2EI}{(\Delta x)^2} \left\{ 1 - \cos\varphi \right\} + kGA \right] + \frac{1}{m_v} \left[\frac{2kGA}{(\Delta x)^2} \left\{ 1 - \cos\varphi \right\} + \rho g B \right] \right\} \lambda^2$$

$$+ \frac{1}{r_v^2 \cdot m_v^2} \left[\frac{2EI}{(\Delta x)^2} \left\{ 1 - \cos\varphi \right\} + kGA \right] \cdot \left[\frac{2kGA}{(\Delta x)^2} \left\{ 1 - \cos\varphi \right\} + \rho g B \right] - \left[\frac{kGA}{(\Delta x)} \right]^2 \sin^2\varphi = 0$$

This equation is of the form

$$\lambda^4 + \left\{ a + b \right\} \lambda^2 + \left\{ ab - c^2 \right\} = 0$$

Where

$$a > 0, \quad b > 0, \quad ab - c^2 > 0$$

Solving the foregoing equation for λ^2 results in

$$\lambda^2 = -\frac{1}{2}\left\{ a + b \right\} \pm \frac{1}{2} \sqrt{ \left\{ a - b \right\}^2 + c^2 }$$

From the statement of inequalities, it follows that

$$a + b > \sqrt{ \left\{ a - b \right\}^2 + c^2 }$$

Hence

$$\lambda^2 < 0$$

The smallest value for λ^2 is, therefore

$$\lambda^2 = -\frac{1}{2}\{a + b\} - \frac{1}{2}\sqrt{\{a - b\}^2 + c^2}$$

One can also see that for $\varphi = \pi$, $|\lambda^2|$ will have a maximum value.

Hence,

$$\lambda_1 = \pm i\sqrt{\frac{1}{r_v^2\, m_v}\left\{\frac{4EI}{(\Delta x)^2} + kGA\right\}}$$

$$\lambda_2 = \pm i\sqrt{\frac{1}{m_v}\left\{\frac{4kGA}{(\Delta x)^2} + \rho gB\right\}}$$

The characteristic values are pure imaginaries, therefore, one can apply the criterion which gives

$$\Delta t < \sqrt{2}\,\frac{(\Delta x)}{\left[\frac{kAG}{m_v} + \frac{\rho gB \cdot (\Delta x)^2}{4m_v}\right]^{1/2}}$$

$$\Delta t < \sqrt{2}\,\frac{(\Delta x)}{\left[\frac{EI}{r_v^2 m_v} + \frac{kAG \cdot (\Delta x)^2}{4r_v^2\, m_v}\right]^{1/2}}$$

The smallest of the two time increments (Δt) will be the one for which the system is stable. Note that the time increment for the mixed technique is greater than that for the explicit finite difference method by a factor of $\sqrt{2}$

APPENDIX F

HYDRODYNAMIC INERTIA

Introduction

The direct determination of the hydrodynamic inertia of a floating hull of arbitrary form is mathematically untractable with the consequence that the hydrodynamic inertia is commonly determined on the basis of several assumptions.

The first assumption is that the cross flow hypothesis is valid according to which the flow over the hull can be considered to be two-dimensional. Thus, the hydrodynamic inertia of the hull is obtained by integrating along the length the hydrodynamic inertias of cylinders of varying cross section. One has, consequently,

$$m_h \equiv \int_L m_h'(x)\,dx$$

where the integration is over the length of the hull.

The second assumption is that the three dimensional flow that takes place, particularly at the ends, can be taken into account by an aspect ratio correction based on that obtaining for an ellipsoid of same mass and equal ratios of principal dimensions.

The third assumption is that the hydrodynamic mass of a body at the surface is, as an initial approximation, equal to one half that of the fully submerged body consisting of the original body and its reflected image above the waterline.

The fourth assumption is that the presence of the free surface can be taken into account by a correction based on the results obtained for a circular cylinder of diameter equal to the local breadth of the section.

The fifth assumption is that of sectional equivalence, i. e., a section of arbitrary shape has the same hydrodynamic mass as that corresponding to a mathematically tractable section having the same cross sectional area, breadth and draft.

Aspect Ratio Correction

The aspect ratio correction is made by applying Taylor's (1929) longitudinal inertia coefficient. But such a correction applies to the hydrodynamic inertia obtained for the whole body and what is required in the present application is a correction at each section. Proportioning of the correction to each section is made by the heuristic method that follows:

The aspect ratio correction in heave is approximated by the following formula of Pabst (see Blagoveshchensky, 1962).

$$J_z \equiv \frac{r}{\sqrt{1 + r^2}} \left[1 - 0.425 \frac{r}{1 + r^2} \right]$$

where r is the ratio of ship length to beam, L/B.

Consider an ellipsoid of semi-axes a, b and c where these axes are related to the ship's hull by

$$a \equiv L/2$$

$$b \equiv B/2$$

$$c \equiv H$$

The distribution of cross-sectional area is along the major (x) axis is

$$A(x) = \frac{\pi bc}{2} \left[1 - \frac{x^2}{a^2} \right]$$

Introduce a distribution of aspect ratio correction of the form

$$j_z(x) \equiv 1 - \left| \frac{x}{a} \right|^n$$

Upon weighting the aspect ratio correction at a point by the cross-sectional area at the same point, integrating over the longitudinal axis and normalizing by dividing by the volume, the overall aspect ratio correction is obtained, namely,

$$\frac{\int_0^a j_z(x) \cdot A(x) \cdot dx}{\int_0^a A(x) \cdot dx} = J_z$$

The cross-sectional area of the ellipsoid being given by

$$A(x) = \frac{\pi bc}{2} \left\{ 1 - \left[\frac{x}{a}\right]^2 \right\}$$

one has

$$\frac{\frac{\pi bc}{2} \int_0^a \left\{ 1 - \frac{x}{a}^n \right\} \cdot \left\{ 1 - \left[\frac{x}{a}\right]^2 \right\} dx}{\frac{\pi}{3} abc} \equiv J_z(n)$$

The integration yields

$$J_z(n) = 1 - 3 \frac{n + 2}{[n + 1][n + 3]}$$

A plot of $J_z(n)$ against n is given in Fig. F-1. From this plot, the value of n is derived corresponding to a given J_z and from it the distribution $j_z(x)$.

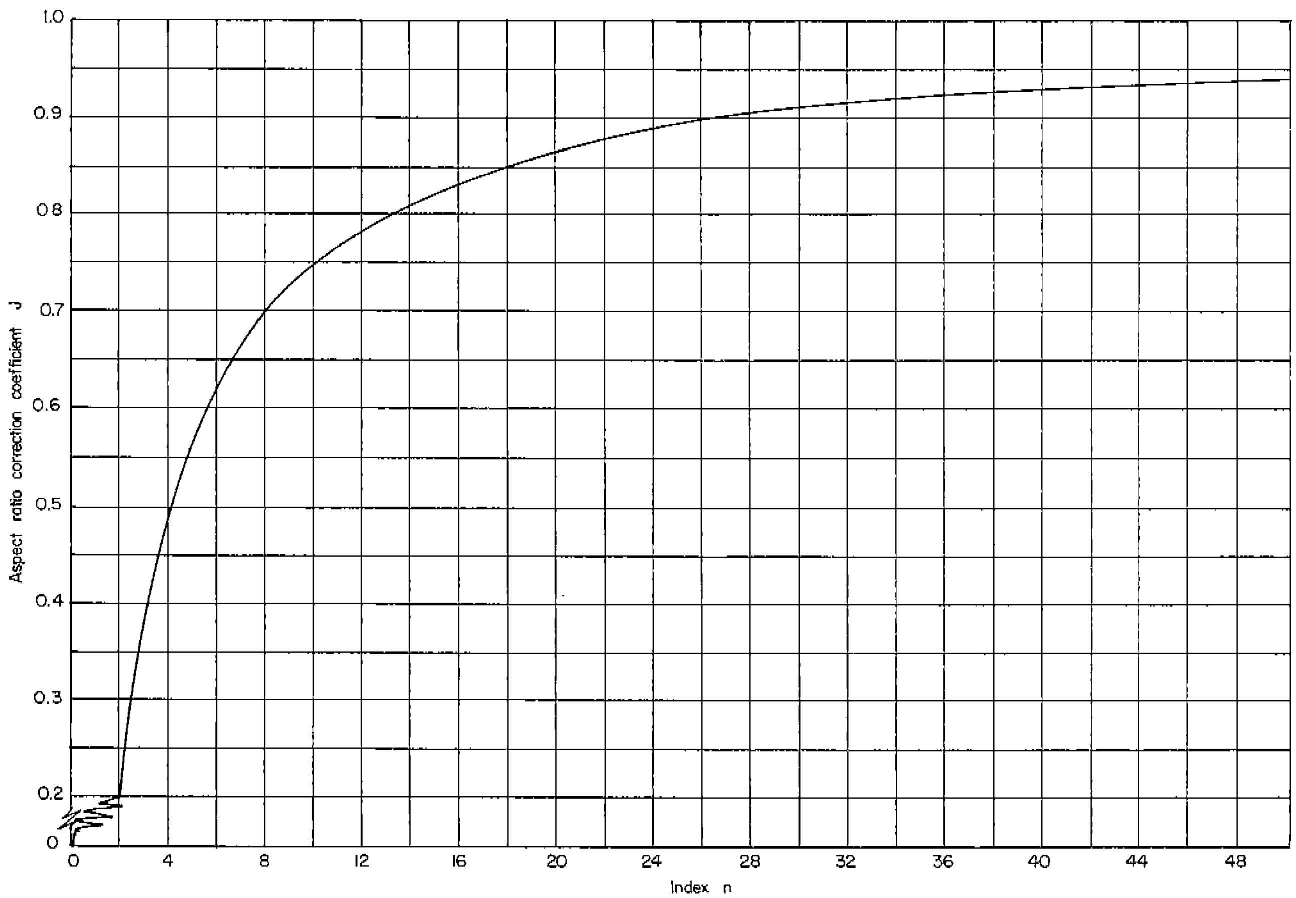


Fig. F-1 Aspect Ratio Correction Coefficient.

The hydrodynamic mass distribution is now taken to be

$$j_z(x) \cdot m'_h(x)$$

where $m'_h(x)$ is the hydrodynamic mass distribution based on two-dimensional flow. The hydrodynamic mass of the hull is by integration

$$m_h \equiv \int_L j_z(x) \cdot m'_h(x) \cdot dx$$

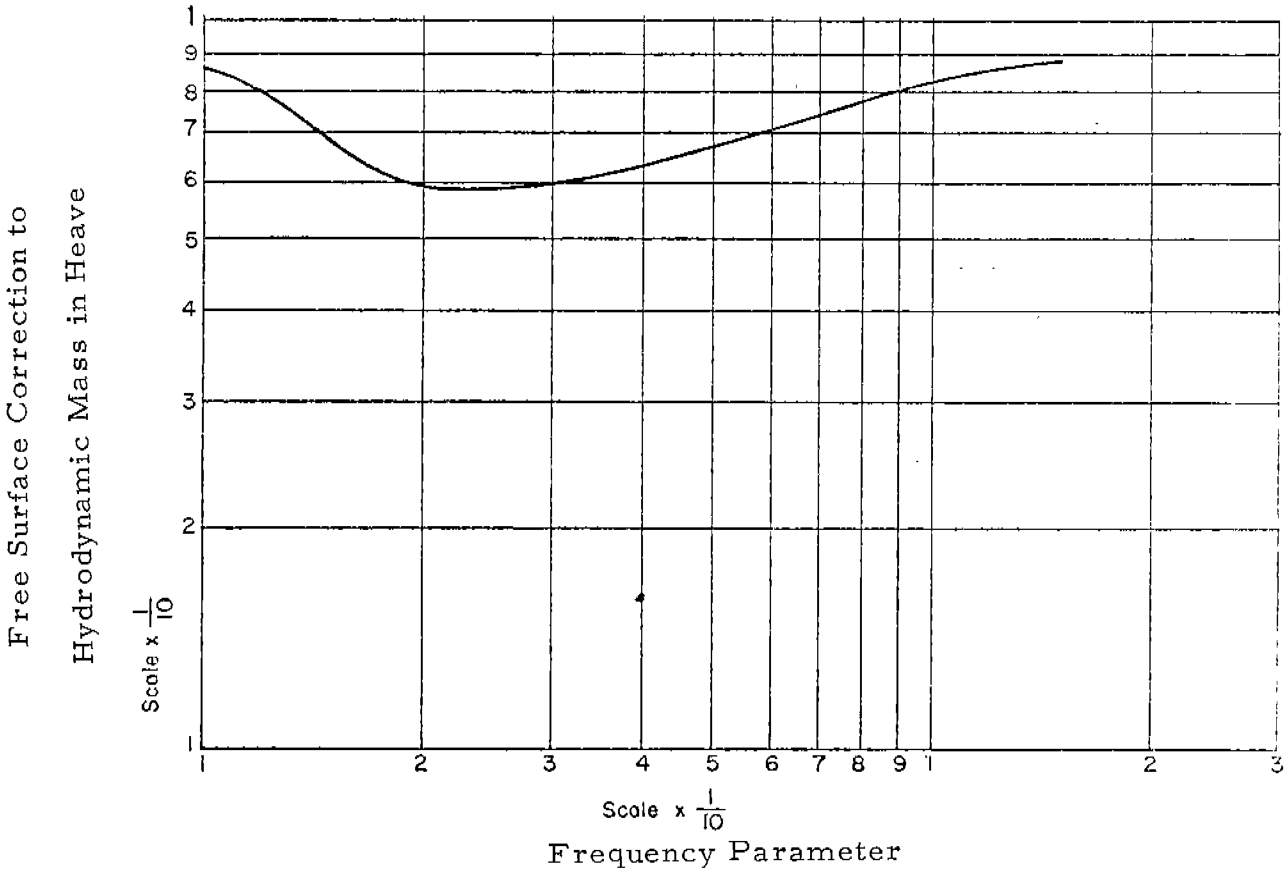


Fig. F-2 Free Surface Correction To Hydrodynamic Mass In Heave (After Ursell).

Free Surface Correction

An adjustment for the influence of the free surface on the hydrodynamic mass has been determined by Ursell (1949). The parameter on which the correction depends contains as a factor the frequency of oscillation (ω) and is

$$\frac{\omega^2 B}{2\pi g}$$

see Fig. F-2.

Such a correction can be readily applied if the motion of the hull can be expressed in terms of normal modes, but since the solution presented herein is not based on normal modes, Ursell's correction can be used only as a basis for a qualitative argument.

It is noted in this connection that the natural frequency of the two-noded vertical vibration of the hull calculated by Schlick's formula (Seward, 1944) lies between 7.7 and 8.9 rad sec^{-1} depending upon the loading (the former corresponding to full load, the latter to light load). These frequencies give values of the frequency parameter of 21 and 28 respectively, and Ursell's correction factor for these values is close to unity. Therefore, since in the lowest possible vibratory mode no correction for the influence of the free surface need be made, it follows, a fortiori, that no such correction need by made for the transient response when slamming occurs.

Sectional Equivalence

The sectional hydrodynamic inertia is obtained by application of the results of Landweber and de Macagno (1957) for Lewis (1929) type sections. The hydrodynamic mass per unit length of a section oscillating vertically is

$$m_h'(x) \equiv \frac{\pi\rho}{g} C_z(x) \cdot B^2(x)$$

where:

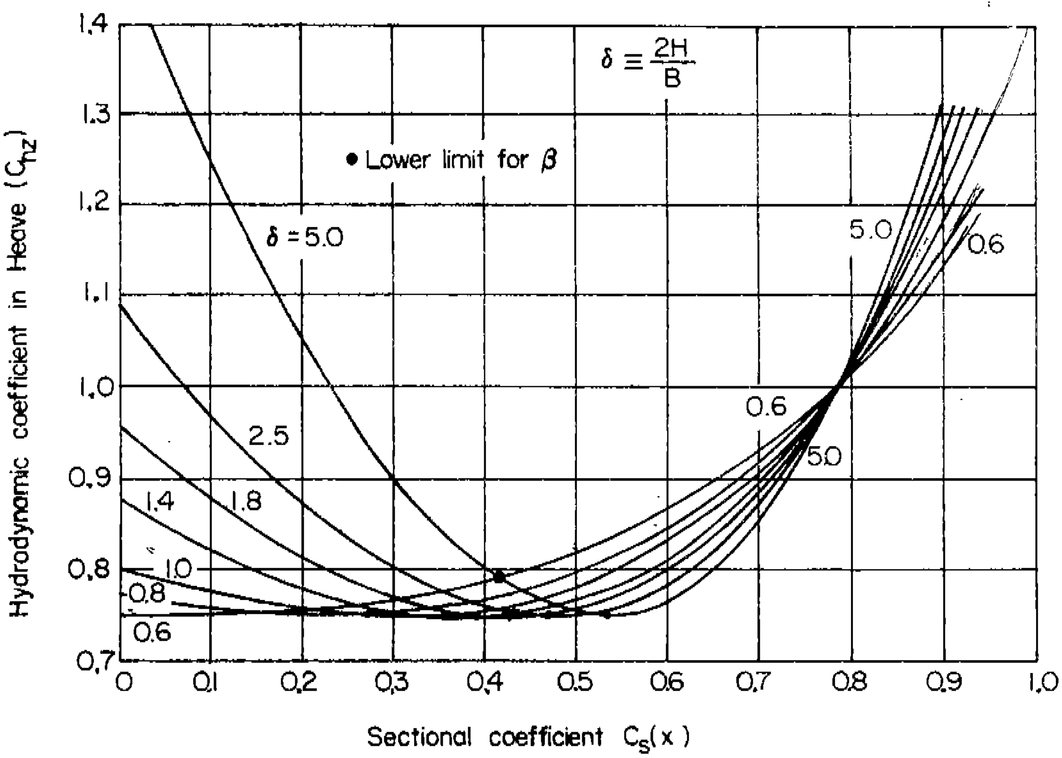
ρ	$\equiv$	mass density of water (lb sec^2 ft^{-4})
$C_z(x)$	$\equiv$	sectional inertia coefficient for vertical motion
$B(x)$	$\equiv$	local beam (ft)

The sectional inertia coefficient is obtained from Figs. F-3. This coefficient is a function of the sectional coefficient

$$C_s(x) \equiv \frac{A(x)}{B(x) \cdot H(x)}$$

and of the beam draft ratio, where

A	$\equiv$	sectional area (ft^2)
H	$\equiv$	draft (ft)



$$C_s(x) \equiv \frac{A(x)}{BH}$$

Fig. F-3 Hydrodynamic Coefficients Of Ship Forms (From Landweber And De Macagno).

APPENDIX G

SHEAR FACTOR

Consider a beam, not necessarily uniform, which is symmetric with reference to the plane xy, where x is measured longitudinally and y vertically, and which is symmetrically loaded. The mean vertical displacement of the beam is

$$\bar{y} = \frac{1}{A} \cdot \iint_A y \, dA$$

where A is the cross-sectional area and the mean rotation about a transverse is obtained from

$$\iint \bar{\gamma} \, y^2 \, dA = \iint_A u y \, dA$$

where u is the longitudinal displacement of the section. In both cases, the integration is over the cross-sectional area.

The second relation can also be written

$$\bar{\gamma} = \frac{1}{I} \cdot \iint_A u y \, dA$$

where I is the second central moment of area. This is the form given by Cowper (1966).

Upon introducing the strain-displacement relations

$$\epsilon_x \equiv \frac{\partial u}{\partial x} \qquad \text{and} \qquad \sigma_x \equiv E \epsilon_x$$

and differentiating with respect to x the second expression for mean angle of rotation, the slope gradient is obtained, namely,

$$\frac{\partial \bar{\gamma}}{\partial x} = \frac{1}{I} \cdot \int \int_A \frac{\partial u}{\partial x} \cdot y \cdot dA = \frac{1}{EI} \int \int_A \tau_x \cdot y \cdot dA = \frac{M}{EI}$$

or

$$M = EI \frac{\partial \bar{\gamma}}{\partial x}$$

which is the well-known expression for bending moment.

Consider the shear stress equation

$$\tau_{xy} \equiv \frac{QS}{Ih}$$

where Q is the shear and S is the first central moment of area. The mean angle of shear deformation of the cross section is

$$\epsilon_{xy} \equiv \frac{\partial \bar{y}}{\partial x} - \bar{\gamma}$$

The assumption that the shear strain energy is conserved results in the shear strain energy relation

$$\int \int_A \tau_{xy} \cdot \bar{\epsilon}_{xy} \cdot dA = \frac{1}{G} \int \int_A \tau_{xy}^2 \cdot dA$$

Therefore,

$$\left[\frac{\partial \bar{y}}{\partial x} - \bar{\gamma}\right] \int\int_A \tau_{xy} \cdot dA = \frac{Q^2}{I^2 G} \int\int_A \frac{S^2}{h^2} \cdot dA$$

or

$$\frac{Q}{kAG} = \frac{\partial \bar{y}}{\partial x} - \bar{\gamma}$$

where the shear factor k is given by

$$\frac{1}{k} \equiv \frac{A}{I^2} \int\int_A \frac{S^2}{h^2} \cdot dA$$

APPENDIX H
BASIC INPUTS

The basic inputs for evaluating the physical parameters of the ship are:

a) The body plan

b) The transverse structural sections

c) The weight and load distributions

The ship chosen for the study is the WOLVERINE STATE of the States Marine Lines.

The particulars of the ship are listed in Table H-1. The items of weight are given in Table H-2.

The body plan of this ship is shown in Fig. H-1. The midship section of the ship, as designed, is shown in Fig. H-2. The weight and load distributions are given in Fig. H-3. The hydrodynamic mass distribution is plotted in Fig. H-4.

The shear factor has been determined to be 0.91 in the midship region. Because of its small variation with change of shape forward and aft and of the relatively small influence of the shear component to the overall flexibility, the value of 0.91 was kept constant for the full length of the ship.

TABLE H-1

WOLVERINE STATE - PARTICULARS

Type	C4-S-B5 (machinery aft dry cargo vessel)
Length, overall (ft)	520
Length, between perpendiculars (ft)	496
Beam, molded (ft)	71.5
Depth, molded (ft)	43.5

Condition	Design	Light Operating
Draft, molded (ft)	30.0	18.0
Displacement (tons)	20,000	11,130
Block Coefficient	0.654	0.610
Longitudinal Coefficient	0.664	0.628
Waterplane Coefficient	0.752	0.685

Machinery - Two State Turbine	
Design power (hp)	9,000
Normal propeller speed (rpm)	80 to 85
Normal operating speed (knots)	16 to 17

Builder: Sun Shipbuilding & Dry Dock Co., Chester, Pennsylvania
Owner: States Marine Lines

TABLE H-2

WOLVERINE STATE - WEIGHT CURVE

Hull	Weight Per 5 Ft. Interval (lb)
$1 < n < 29$	$85,400 + 1930\ n$
$29 < n < 69$	141,100
$69 < n < 100$	$141,100 - 605\ n - 69$
Machinery	
$71 < n < 84$	123,200
Fuel Oil + Reserve Feed	
$71 < n < 84$	448,000
Deep Tanks	
$26 < n < 36$	134,400
Cargo	
#1 Hold	159,000
2 Hold	389,800
3 Hold	304,600
4 Hold	492,800
5 Hold	457,000
6 Hold	441,300
7 Hold	147,900

Note: The index n indicates the station. These are spaced 5 ft apart. n = 1 is at the foreperpendicular

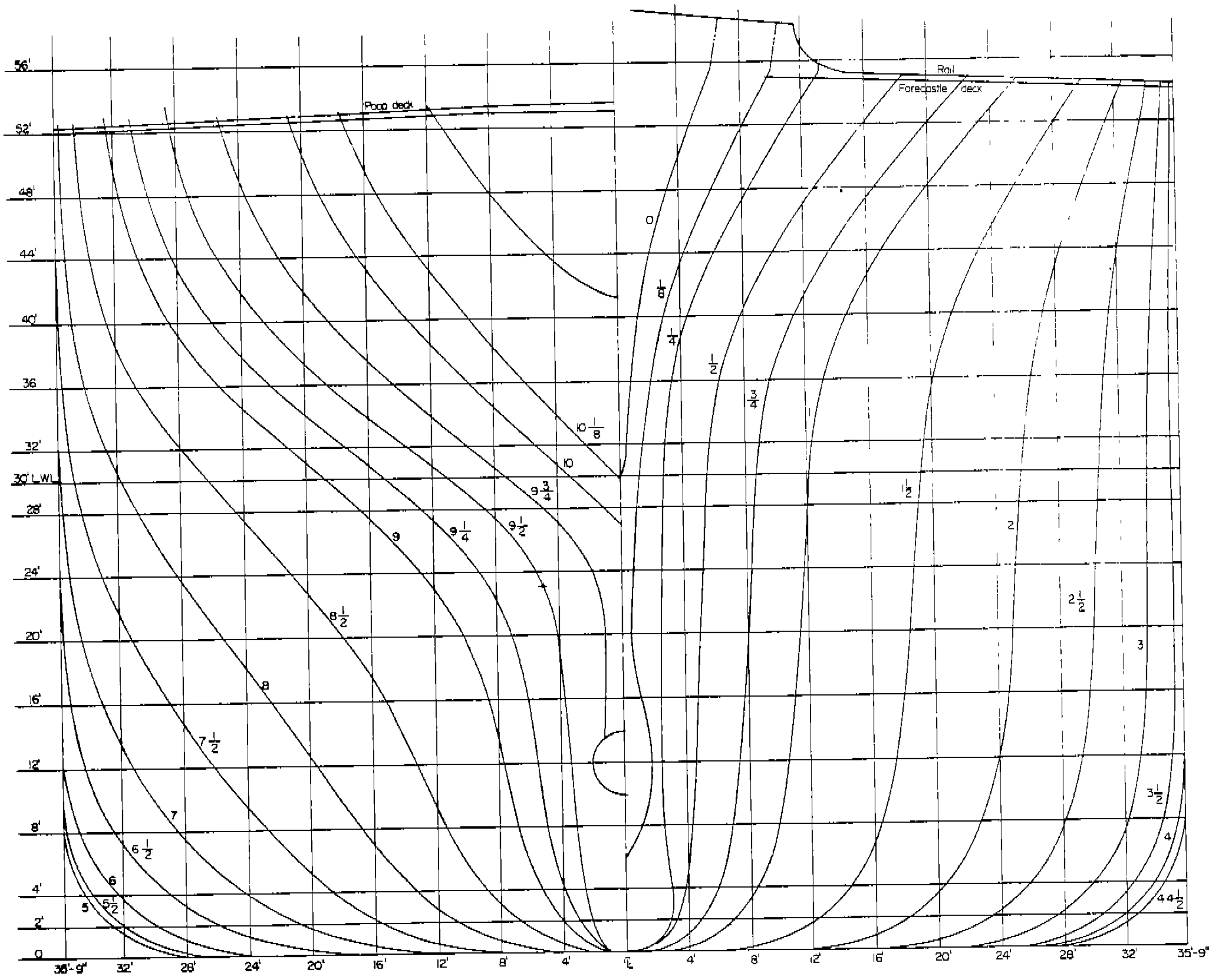
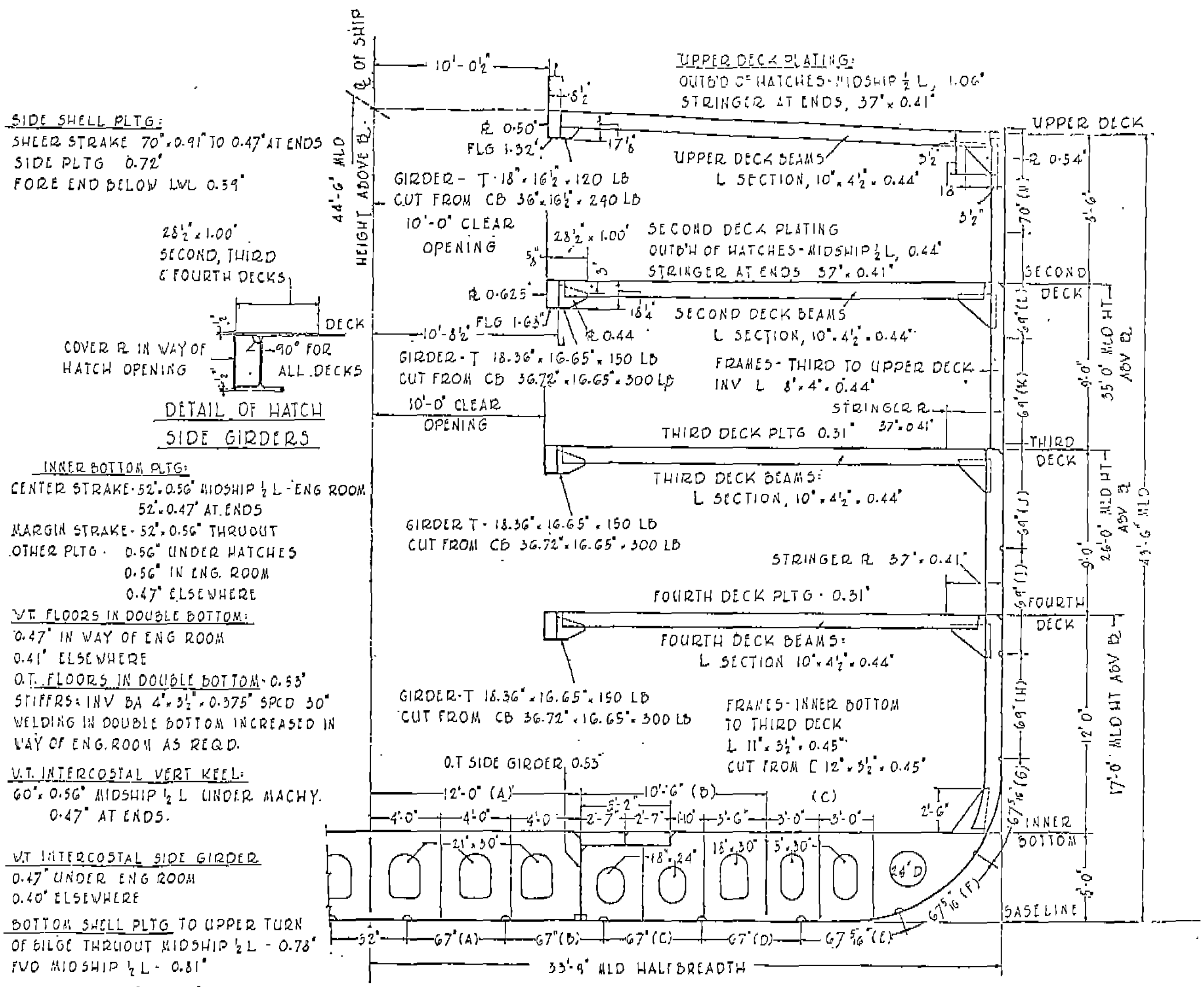


Fig. H-1 Wolverine State Body Plan.



SECTION IN WAY OF HATCH

Fig. H-2 Wolverine State Midship Section.

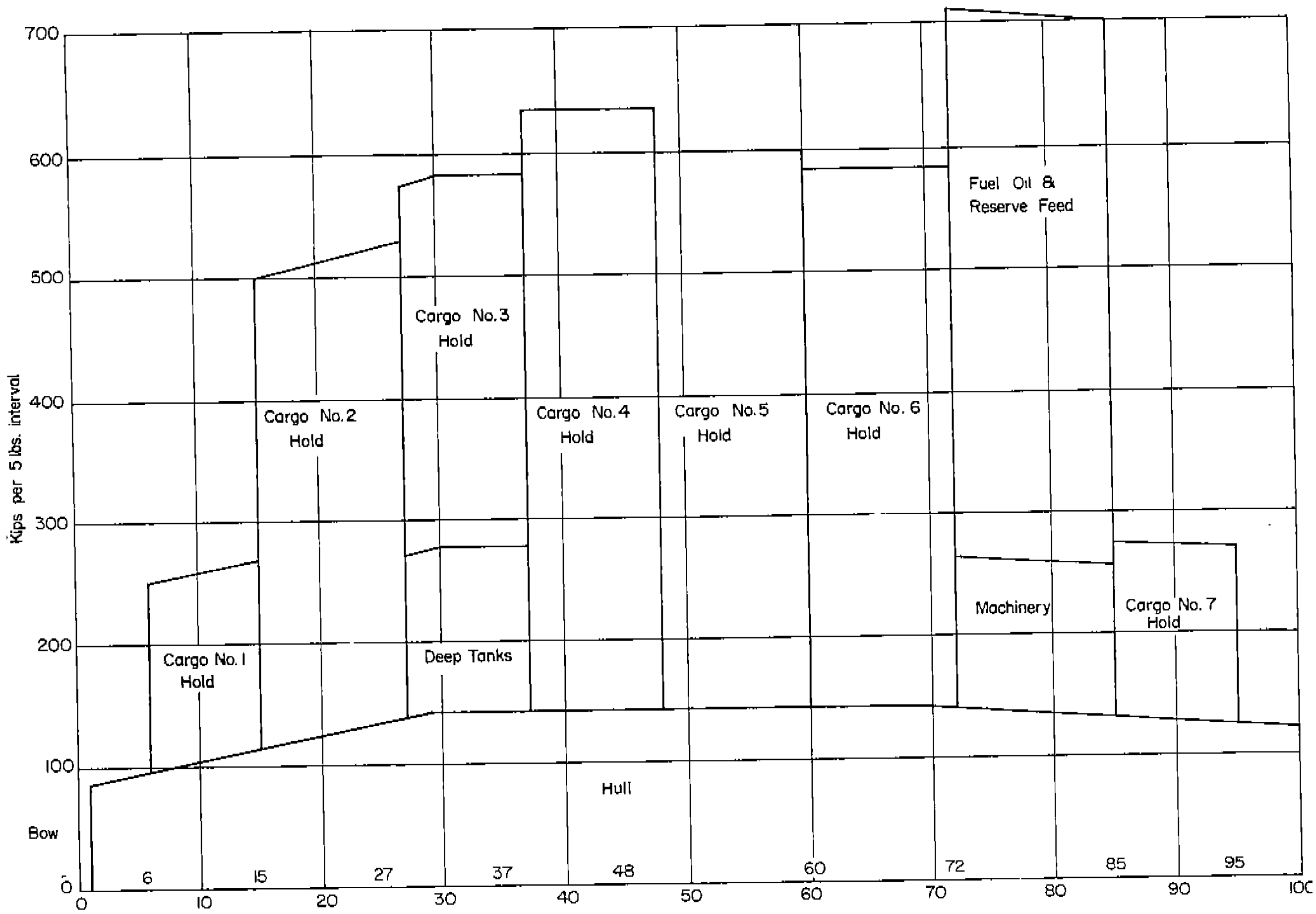


Fig. H-3 Wolverine State Weight Curve.

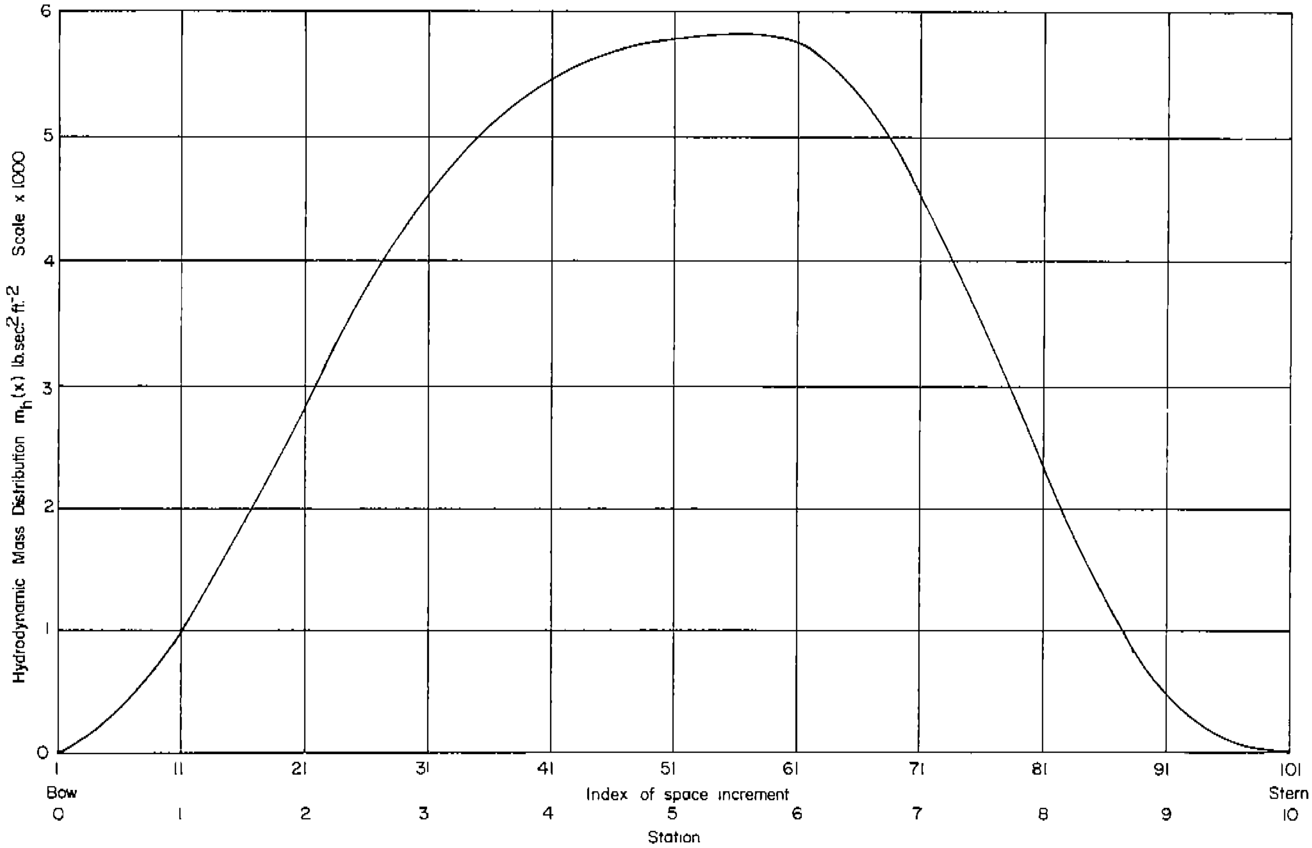


Fig. H-4 Wolverine State - Hydrodynamic Mass Distribution.

APPENDIX I
PROGRAM

Program Components

DRIVER	Main routine to handle general flow, namely, calls all pertinent subroutines, determines, which finite difference scheme to use (implicit or explicit), and determines at which times output is desired.
INPUT	Subroutine to handle ship input and initial input Actual FORTRAN input statement may change in accordance with the format of the user's data.
PARAM	Subroutine to compute coefficients in the pertinent differential equations. The statements in this program may change in accordance with the changes made in subroutine INPUT.
STABLE	Subroutine to check the stability criterion and compute the proper time stepsize if necessary.
CONST	Subroutine to initialize all constants used in program.
INITL	Subroutine to set up initial conditions (zero deflection and zero angle).
FORCE	Subroutine to compute the impulsive load, $p(x, t)$

GENER	Subroutine to generate the matrix coefficients in the implicit technique.
BNDRY	Subroutine to set up boundary conditions (has two entry points, one for each different technique).
COMPUT	Subroutine to compute γ and y for the explicit technique.
SOLN	Subroutine to perform a Gaussian elimination to arrive at a solution for γ and y for the implicit technique.
FICTP	Subroutine compute γ and y at the fictitious points in the implicit technique.
OUTE	Subroutine to handle all output for both difference schemes (two entry points).
XIMPULSE	Subroutine to handle all load inputs.

SUBROUTINE INPUT

Card No.	Quantity	Format
1	Information card	9 A 8
2	M Control card	10 I 5
	M(1) = 0	
	M(2) = 0 - Normal boundary conditions 1 - Refined Boundary conditions	
	M(3) = n - Print every $n \cdot \Delta t$	
	M(4) = Unspecified	
	M(5) = Logic control for computing K	Zero presently
	M(6) = Logic control for computing EI	Zero presently
	M(7) = Logic control for computing m_v	Zero presently
	M(8) = Logic control for computing $r_v^2 m_v$	Zero presently
	M(9) = n - use for last run as number of cross sections in input	
	M(10) = 0 - there are XNN stations 1 - there are M(9) stations	
3	DT = Δt	5 E 14. 6
	DX = Δx	
	SC - If $y + \gamma > SC$ stop program	
	FINIS - Time to stop	
	XNN - Total number of stations including fictitious ones; maximum number = 204	
4	RSTL - Mass density of steel $\rho = 15.2$ (lb sec^2 ft^{-4})	5 E 14. 6
	RSEA - Mass density of sea water $\rho = 2.0$ (lb sec^2 ft^{-4})	
	GC - Shear modulus GC = 1.66×10^9 (lb ft^{-2})	
	GE - Elastic modulus GE = 4.32×10^9 (lb ft^{-2})	
	TAU -Time duration of impulsive load profile	

5	CK - Shear coefficient	5 E 14. 6
6	RI - Radius of gyration	5 E 14. 6

The following data input cards depend on the number of stations M(9) which are declared on Card 2

7	CB - Half beam b(x)	7 E 10. 5
8	CHZ - Hydrodynamic inertia coefficient in heave	
9	CI - Second central moment of cross sectional area, I.	
10	CA - Shear area, A	
11	CK - Shear factor K	
12	SM - Weight curve, (tons ft $^{-1}$)	

The number of ship weight stations depends on M(10) declared in Card 2.
If M(10) = 0 there are XNN stations
If M(10) = 1 there are M(9) stations

SUBROUTINE XIMPULSE

All loading inputs are read in this subroutine. This routine is called at TIME = 0 and thereafter for each time duration TAU.

Card No.	Quantity	Format
13	NP - Number of points in NPS(I)	1 I 5
14	NPS - Actual number of points where load is applied maximum of 200	10 I 5
15	P(X) - Force	5 E 14. 6
16	TAUC - Start of force application	5 E 14. 6
17	XTAU - Duration of load application	5 E 14. 6
18	CF - Circular frequency of load - set to zero if load is impulsive	5 E 14. 6

```
      PROGRAM SHIP
C     DRIVER - PROGRAM SHIP - MAIN ROUTINE TO HANDLE GENERAL FLOW
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CE(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,F,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2
       COMMON /A/ TAUC,PX,XTAU,XTDX,NP
       DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
 1111 CALL INPUT
       TAUD=TAU
      CALL XIMPULSE
      CALL PARAM
      CALL STABLE
      CALL CONST
       IF (M(1)) 10,10,11
C     EXPLICIT SCHEME
   10 CALL INITL
   50 CALL BNDRY
      TIME = TIME+DT
       IF(TIME-TAUD)360,370,370
  370 TAUD=TAUD+TAU
C DESIGNATE A NEW LOAD PROFILE
      CALL XIMPULSE
  360 CONTINUE
       IC = IC+1
       DO 51 I=1,N
      U(I,1)=U(I,2)
   51 U(I,2)=U(I,3)
   62 CALL FORCE
   61 CALL COMPUT
      DO 52 I=3,N2,2
      A1= ABS(U(I,3))+ABS(U(I+1,3))
      IF (A1-SC) 52,53,53
   53 CALL EROR
   52 CONTINUE
      IF(IC-MMM3-1) 55,56,56
   56 MMM3= MMM3+M3
      CALL BNDRY
      CALL OUTE
   55 IF (TIME-FINIS) 50,1111,1111
C     IMPLICIT SCHEME
   11 CALL INITL
  500 CONTINUE
      DO 501 I=1,N
      U(I,1)=U(I,2)
```

```
  501 U(I,2)=U(I,3)
  601 CALL GENER
      CALL BNDRYI
      TIME=TIME+DT
      IF(TIME-TAUD)3600,3700,3700
 3700 TAUD=TAUD+TAU
C DESIGNATE A NEW LOAD PROFILE
      CALL XIMPULSE
 3600 CONTINUE
  602 CALL FORCE
      IC=IC+1
      CALL SOLN
      CALL FICTP
      IF (IC-MMM3-1) 504,505,505
  505 MMM3=MMM3+M3
      CALL OUTI
  504 IF (TIME-FINIS) 500,1111,1111
      END


      SUBROUTINE INPUT
C THIS ROUTINE READS ALL SHIP DATA
      DIMENSION NXP(100),PUTIN(110,7)
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2,MM
       COMMON /A/ TAUC,PX,XTAU,XTDX,NP
         COMMON /b/ XNPS
       DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
       DIMENSION XNPS(204)
      READ(5,1) (HEADER(I),I=1,9)
    1 FORMAT (9A8)
      IF (HEADER(1))2,3,2
    3 CALL EXIT
    2 READ(5,4)(M(I),I=1,10)
    4 FORMAT (14I5)
    5 FORMAT (5E14,6)
      READ(5,5)DT,DX,SC,FINIS,XNN,RSTL,RSEA,GC,CE,TAU
      NN=XNN
      MM=NN-1
      M9=M(9)
```

```
      READ (5,4) (NXP(I),I=1,M9)
      READ (5,5) CK  (2)
      READ (5,5) RI  (2)
      READ (5,40)  (PUTIN(I,1),I=1,M9)
      READ (5,40)  (PUTIN(I,2),I=1,M9)
      READ (5,40)  (PUTIN(I,3),I=1,M9)
      READ (5,40)  (PUTIN(I,4),I=1,M9)
   40 FORMAT (7E10,5)
       IF(M(10))39,39,41
   39 READ (5,40) ( SM(I)   ,I=2,MM)
      GO TO 42
   41 READ (5,40) (PUTIN(I,5),I=1,M9)
   42 CONTINUE
      DO 30 I=1,M9
      LM=NXP(I)
      CB (LM)=PUTIN(I,1)
      CHZ(LM)=PUTIN(I,2)
      CI (LM)=PUTIN(I,3)
      CA (LM)=PUTIN(I,4)
      IF(M(10))30,30,44
   44  SM(LM)=PUTIN(I,5)
   30 CONTINUE
C THE SHIP DATA IS INTERPOLATED HERE
      M91=M9-1
      DO 31 I=1,M91
      LM1=NXP(I+1)-1
      LM=NXP(I)+1
      FAT=LM1-LM+2
      XJ1=LM-1
      I1=LM1+1
      I2=LM-1
      DO 32 J=LM,LM1
      XJ=J
      WOW=(XJ-XJ1)/FAT
      CB (J)=(CB (I1)-CB (I2))*WOW+CB (I2)
      IF(M(10))59,59,51
   51  SM(J)=( SM(I1)- SM(I2))*WOW* SM(I2)
   59 CHZ(J)=(CHZ(I1)-CHZ(I2))*WOW+CHZ(I2)
      CI (J)=(CI (I1)-CI (I2))*WOW+CI (I2)
   32 CA (J)=(CA (I1)-CA (I2))*WOW+CA (I2)
   31 CONTINUE
C   THE FOLLOWING LOOP SETS THE ARRAY ELEMENTS EQUAL
      DO  517  I = 3,MM
      CK  (I) = CK  (I-1)
  517 RI  (I) = RI  (I-1)
C     WRITE INITIAL OUTPUT
      WRITE (6,21)
      WRITE (6,1) (HEADER(I),I=1,9)
   21 FORMAT (1H1)
      WRITE (6,22) DT,DX,TAU,XNN,SC,FINIS
```

```
   22 FORMAT(//10H DELTA T =,E15.7/10H DELTA X =,E15.7/6H TAU =,E15.7/
     119H NO. OF INTERVALS =,E15.7/18H STABILITY BOUND =,E15.7/14H TIME
     1TO END =,E15.7)
      WRITE (6,23) (M(I),I=1,10)
   23 FORMAT (//17H M CONTROL CARD =,10I5)
      PRINT 4551,CE,GC
 4551 FORMAT(/,* ELASTIC MODULAS   SHEAR MODULAS LB/SQ. FT. *,/2E16.2,/)
      PRINT 230
  230 FORMAT(/,* HALF BREADTH     AREA       HEAVE COEF.   SHIP WT./FT.
     1 SECOND MOMENT   SHEAR COEF.   GYRATION RADIUS *,/)
      WRITE (6,9)(CB(I),CA(I),CHZ(I), SM(I), CI(I),CK(I),RI (I),I=2,MM)
    9 FORMAT (7E13.4)
      RETURN
      END


      SUBROUTINE PARAM
C     ROUTINE TO COMPUTE COEFFICIENTS IN DIFFL. EQNS.
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2,MM
       COMMON /A/ TAUC,PX,XTAU,XTDX,NP
       DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
      MM=NN-1
      DO 1 I=2,MM
      CAYB(I)=2.*CB(I)*RSEA*32.2
      IF (M(7)) 3,2,3
    2 CONTINUE
C     CAN INSERT
C     MULTIPLY  SM BY 2240/ G    FOR S.MASS/F.S.
    3 VM(I)= SM(I)/32.2*2240. +1.570796327*RSEA*CHZ(I)*CB(I)**2
      IF (M(6)) 4,5,4
    5 EI(I)=CE*CI(I)
C     CAN INSERT
    4 IF (M(5)) 6,7,6
C     CAN INSERT
    6 CONTINUE
    7 IF(M(8)) 8,9,8
C     CAN INSERT
    9 CONTINUE
    8 CAYGA(I)=CK(I)*CA(I)*GC
      RI(I)*RI(I)* SM(I)*2240./32.2
```

```
1 CONTINUE
  EI(1)=EI(2)
  CAYGA(1)=CAYGA(2)
  EI(NN)=EI(MM)
  CAYGA(NN)=CAYGA(MM)
  RETURN
  END
```

```
      SUBROUTINE EROR (I)
      WRITE (6,1) I
1      FORMAT(1I5)
      CALL EXIT
      RETURN
      END
```

```
      SUBROUTINE CONST
C     THIS ROUTINE SETS UP CONSTANTS AND INITIALIZES LOGIC CONTROLS
C     USED THROUGHOUT PROGRAM
      DIMENSION X(205)
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2,MM
      COMMON X,OUTPLOT,DSCALE,VSCALE,BMSCALE,XSCALE
       COMMON /A/ TAUC,PX,XTAU,XTDX,NP
         COMMON /B/ XNPS
       DIMENSION XNPS(204)
      DIMENSION OUTPLOT(400)
       DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
      CALL PLOTS (OUTPLOT,400,20,10)
      N=2*NN
      N1=N-1
      N2=N1-1
      N3=N2-1
      N4=N3-1
      N5=N4-1
      N6=N5-1
      N7=N6-1
      RFD=M(2)
       D2X=2.*DX
```

```
      D3X=3,*DX
      DX24=4,*DX2
      D2T=2,*DT
      DT2=DT*DT
      IC=0
      TIME=0,0
      M3=M(3)*JC
      MMM3=M3
      STP=0,0
      X(1)=0,0
      DO 100 I=2,MM
      I1=I+1
      IM1=I-1
      X(I)=X(IM1)+DX
      P(I)=0,0
      R1=EI(I1)-EI(IM1)
      R2=CAYGA(I1)-CAYGA(IM1)
      S1(I)=-(R1-4,*EI(I))/(16,*DX2)
      S2(I)=-(,5*EI(I)/DX2+CAYGA(I)/4,+RI(I)/DT2)
      S3(I)=-S1(I)+,5*EI(I)/DX2
      S4(I)=-CAYGA(I)/(8,*DX)
       S5(I)=-R2/(8,*DX)
      S6(I)=-(R2-4,*CAYGA(I))/(16,*DX2)
      S7(I)=,5*CAYGA(I)/DX2+VM(I)/DT2
      S8(I)=-S6(I)+,5*CAYGA(I)/DX2
      S9(I)=R1/(16,*DX2)
  100 S10(I)=R2/(8,*DX)
      XSCALE = 8,/ X(MM)
      DSCALE = 10,**7
       DSCALE=5*DSCALE
      VSCALE = 3,/1,
      BMSCALE =12,/X(MM)*6,
      RETURN
      END
```

```
      SUBROUTINE STABLE
C     THIS ROUTINE COMPUTES THE PROPER TIME MESHSIZE
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2,MM
       COMMON /A/ TAUC,PX,XTAU,XTDX,NP
       DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
       DX2=DX*DX
   20 TEM=1000.
      DO 1 I=2,MM
       Q1=DX/SQRT((CAYGA(I)*CAYB(I)*DX2/4.)/VM(I))
      Q2=DX/SQRT((EI(I)*CAYGA(I)*DX2/4.)/RI(I))
      IF (Q1-Q2) 2,3,3
    2 SAV=Q1
      GO TO 4
    3 SAV=Q2
    4 IF (SAV-TEM) 5,1,1
    5 TEM=SAV
    1 CONTINUE
      JC=1
    8 IF (DT-TEM) 6,6,7
    7  JKC=JC-2
      IF(JKC)9,10,10
    9 DT = DT/2.
      JC= 2*JC
      GO TO 8
   10 M(1)=1
      GO TO 21
    6 M(1)=0
   21 WRITE (6,25) M(1),DT,TEM
   25 FORMAT (///18H CODE FOR SCHEME =,1I3,10X,4HDT =,1E16.8,10X,11HCRIT
     1ERION =,1E16.8)
      RETURN
      END


      SUBROUTINE INITL
C     THIS ROUTINE SETS UP THE INITIAL CONDITIONS
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
```

```
      1,CI(204),CA(204),HEADER(9)
       COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
      1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
      1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
      1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2
        COMMON /A/ TAUC,PX,XTAU,XTDX,NP
        DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
       DO 1 I=1,N
       U(I,3)=0,0
       U(I,2)=0,0
     1 CONTINUE
       RETURN
       END


       SUBROUTINE FORCE
C THE FORCE IS DETERMINED WITH RESPECT TO TIME HERE
       DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
      1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
      1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
      18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
      1,CI(204),CA(204),HEADER(9)
       COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
      1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
      1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
      1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2,MM
        COMMON /A/ TAUC,PX,XTAU,XTDX,NP
        COMMON /B/ CF
        DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204),CF(204)
C      ROUTINE TO COMPUTE SMALL P
       PIE=3,1416
       DO 1 I=2,MM
       IF(CF(I))901,901,5
   901 CONTINUE
        IF(TIME-TAUC(I))9,4,4
     4 IF (TIME-XTAU(I)-TAUC(I))5,9,9
     9 P(I)=0,0
       GO TO 1
     5 P(I)=XTDX(I)
        IF(CF(I))1,1,6
C A PERIODIC FORCE IS CALCULATED HERE
     6 ARG= 2,*PIE*CF(I)*TIME
       P(I)=P(I)*SIN(ARG)
     1 CONTINUE
    20 FORMAT(4E13,3)
       RETURN
       END
```

```
      SUBROUTINE GENER
C     GENERATE MATRIX COEFFICIENTS
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2,MM
      DO 1  I=3,N2,2
      J=I+1
      K=J/2
      I1=I-1
      I2=I-2
      I4=I+2
      I5=J+2
      I3=J-2
      A(I2)=S1(K)
      A(I1)=-S4(K)
      B(I2)=S2(K)
      B(I1)=S5(K)
      C(I2)=S3(K)
      C(I1)=-A(I1)
      D(I2)=S4(K)
      D(I1)=S6(K)
      E(I2)=0.0
      E(I1)=-(S7(K)+.25*CAYB(K))
      F(I2)=A(I1)
      F(I1)=S8(K)
      G(I2)=RI(K)*(U(I,1)-2.*U(I,2))/DT2-.25*EI(K)*(2.*(U(I4,2)-2.*U(I,
     12)+U(I2,2))+U(I4,1)-2.*U(I,1)+U(I2,1))/DX2-S9(K)*(2.*(U(I4,2)-U(I2
     1,2))+U(I4,1)-U(I2,1))+S4(K)*(2.*(U(I5,2)-U(I3,2))+U(I5,1)-U(I3,1))
     1+.25*CAYGA(K)*(2.*U(I,2)+U(I,1))
      T1=(U(I5,2)-U(I3,2))/D2X
      T2=(U(I4,2)-U(I2,2))/D2X
      G(I1)=-P(K)+.25*CAYB(K)*(2.*U(J,2)+U(J,1))+VM(K)*(U(J,1)-
     12.*U(J,2))/DT2+S4(K)*((2.*(U(I5,2)-2.*U(J,2)+U(I3,2))+U(I5,1)-2.*
     1U(J,1)+U(I3,1))*2./DX-(2.*(U(I4,2)-U(I2,2))+U(I4,1)-U(I2,1)))
     1-S10(K)*(2.*T1+(U(I5,1)-U(I3,1))/D2X-(2.*U(I,2)+U(I,1)))
    1 CONTINUE
      RETURN
      END


      SUBROUTINE BNDRY
C     THIS ROUTINE SETS UP THE BOUNDARY CONDITIONS
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
```

```
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2
       COMMON /A/ TAUC,PX,XTAU,XTDX,NP
       DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
C     EXPLICIT SCHEME
   11 IF (RFD) 1,2,1
C     NORMAL ANALOGUE
    2 U(N1,3)=U(N5,3)
      U(N,3)=U(N4,3)+D2X*U(N3,3)
      U(1,3)=U(5,3)
      U(2,3)=U(6,3)-D2X*U(3,3)
      GO TO 50
C     REFINED CONDITIONS
    1 U(N1,3)=.5*(6.*U(N5,3)-U(N7,3)-3.*U(N3,3))
      U(N,3)=.5*(6.*U(N4,3)-U(N6,3)-3.*U(N2,3))+D3X*U(N3,3)
      U(1,3)=.5*(6.*U(5,3)-U(7,3)-3.*U(3,3))
      U(2,3)=.5*(6.*U(6,3)-U(8,3)-3.*U(4,3))-D3X*U(3,3)
      GO TO 50
C     IMPLICIT SCHEME
      ENTRY BNDRYI
      C(1)=C(1)+A(1)
      C(2)=C(2)+A(2)
      F(1)=F(1)+D(1)
      F(2)=F(2)+D(2)
      B(1)=B(1)-D2X*D(1)
      B(2)=B(2)-D2X*D(2)
      G(1)=G(1)-A(1)*(2.*(U(5,2)-U(1,2))+U(5,1)-U(1,1))-D(1)*(2.*(U(6,2)
     1-U(2,2))+U(6,1)-U(2,1)-D2X*(2.*U(3,2)+U(3,1)))
      G(2)=G(2)-A(2)*(2.*(U(5,2)-U(1,2))+U(5,1)-U(1,1))-D(2)*(2.*(U(6,2)
     1-U(2,2))+U(6,1)-U(2,1)-D2X*(2.*U(3,2)+U(3,1)))
      A(N4)=A(N4)+C(N4)
      A(N5)=A(N5)+C(N5)
      D(N4)=D(N4)+F(N4)
      D(N5)=D(N5)+F(N5)
      B(N4)=B(N4)+D2X*F(N4)
      B(N5)=B(N5)+D2X*F(N5)
      G(N4)=G(N4)-C(N4)*(2.*(U(N1,2)-U(N5,2))+U(N1,1)-U(N5,1))-F(N4)*(2.
     1*(U(N,2)-U(N4,2))+U(N,1)-U(N4,1)+D2X*(2.*U(N3,2)+U(N3,1)))
      G(N5) =G(N5)-C(N5)*(2.*(U(N1,2)-U(N5,2))+U(N1,1)-U(N5,1))-F(N5)*(2
     1.*(U(N,2)-U(N4,2))+U(N,1)-U(N4,1)+D2X*(2.*U(N3,2)+U(N3,1)))
   50 RETURN
      END
```

```
      SUBROUTINE COMPUT
C     THIS ROUTINE COMPUTES THETA AND OMEGA FROM THE EXPLICIT DIFFERENCE
C     ANALOGUE
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
       COMMON /A/ TAUC,PX,XTAU,XTDX,NP
       DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2
      DO 1 I=3,N2,2
      J=I+1
      K=J/2
      T1=(U(J+2,2)-U(J-2,2))/D2X
      T3=U(I+2,2)-U(I-2,2)
      T2=T3/D2X
      T4=T1-U(I,2)
      U(I,3)=2,*U(I,2)-U(I,1)+DT2*((EI(K)*(U(I+2,2)-2,*U(I,2)+U(I-2,2))+
     1,25*(EI(K+1)-EI(K-1))*T3)/DX2+CAYGA(K)*T4)/RI(K)
      U(J,3)=2,*U(J,2)-U(J,1)+DT2*(P(K)-CAYB(K)*U(J,2)+CAYGA(K)*
     1((U(J+2,2)-2,*U(J,2)+U(J-2,2))/DX-,5*T3)/DX+(CAYGA(K+1)-CAYGA(K-1)
     1)*T4/D2X)/VM(K)
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SOLN
C     THIS ROUTINE EVALUATES GENERATING SEQUENCES AND COMPUTES
C     SOLUTION VECTOR
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2
C     FIRST FORM 5-BAND DIAGONAL MATRIX FROM 7-BAND MATRIX
      DO 1 I=5,N2,2
      I1=I-1
      I2=I-2
      I3=I-3
      I4=I-4
```

```
      FM=A(I1)/A(I2)
      B(I1)=B(I1)-FM*B(I2)
      C(I1)=C(I1)-FM*C(I2)
      D(I1)=D(I1)-FM*D(I2)
      E(I1)=E(I1)-FM*E(I2)
      F(I1)=F(I1)-FM*F(I2)
      G(I1)=G(I1)-FM*G(I2)
      FM=F(I4)/F(I3)
      B(I4)=B(I4)-FM*B(I3)
      C(I4)=C(I4)-FM*C(I3)
      D(I4)=D(I4)-FM*D(I3)
      E(I4)=E(I4)-FM*E(I3)
      G(I4)=G(I4)-FM*G(I3)
    1 CONTINUE
C     EVALUATE SEQUENCES
      E(1)=E(1)/B(1)
      C(1)=C(1)/B(1)
      G(1)=G(1)/B(1)
      E(2)=E(2)-E(1)*B(2)
      C(2)=C(2)-C(1)*B(2)
      G(2)=G(2)-G(1)*B(2)
      B(2)=0.0
      DO 2 K=1,N7,2
      K1=K+1
      K2=K+2
      K3=K+3
      E(K1)=E(K1)-E(K)*B(K1)
      C(K1)=(C(K1)-C(K)*B(K1))/E(K1)
      F(K1)=F(K1)/E(K1)
      G(K1)=(G(K1)-G(K)*B(K1))/E(K1)
      D(K2)=D(K2)-E(K)*A(K2)
      B(K2)=B(K2)-C(K)*A(K2)-C(K1)*D(K2)
      G(K2)=(G(K2)-G(K)*A(K2)-G(K1)*D(K2))/B(K2)
      E(K2)=(E(K2)-F(K1)*D(K2))/B(K2)
      C(K2)=C(K2)/B(K2)
      B(K3)=B(K3)-C(K1)*D(K3)
      E(K3)=E(K3)-F(K1)*D(K3)
    2 G(K3)=G(K3)-G(K1)*D(K3)
      E(N4)=E(N4)-E(N5)*B(N4)
      G(N4)=(G(N4)-G(N5)*B(N4))/E(N4)
C     COMPUTE SOLUTION
      V(N2)=G(N4)
      V(N3)=G(N5)-E(N5)*V(N2)
      DO 3 I=4,N4,2
      J=N4-I+4
      V(J)=G(J-2)-F(J-2)*V(J+2)-C(J-2)*V(J+1)
      V(J-1)=G(J-3)-C(J-3)*V(J+1)-E(J-3)*V(J)
C     CHECK FOR AN UNSTABLE SOLUTION
      IF (ABS(V(J))+ABS(V(J-1))-SC) 3,4,4
    4 CALL EROR (1)
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FICTP
C     THIS ROUTINE COMPUTES VALUES FOR THE DISPLACEMENTS AT THE FOUR
C     FICTITIOUS POINTS
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2
       COMMON /A/ TAUC,PX,XTAU,XTDX,NP
       DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204)
      DO 1 I=3,N2
    1 U(I,3)=V(I)
      U(1,3)=U(5,3)+2.*(U(5,2)-U(1,2))+U(5,1)-U(1,1)
      U(2,3)=U(6,3)+2.*(U(6,2)-U(2,2))+U(6,1)-U(2,1)-D2X*(U(3,3)+2.*
     1U(3,2)+U(3,1))
      U(N1,3)=U(N5,3)-(2.*(U(N1,2)-U(N5,2))+U(N1,1)-U(N5,1))
      U(N,3)=U(N4,3)-(2.*(U(N,2)-U(N4,2))+U(N,1)-U(N4,1))+D2X*(U(N3,3)
     1+2.*U(N3,2)+U(N3,1))
      RETURN
      END


      SUBROUTINE OUTE
C     THIS ROUTINE HANDLES OUTPUT FOR EXPLICIT SCHEME
      DIMENSION X(205)
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      DIMENSION WO(204),THO(204),XMO(204),SHO(204),WV(204),THV(204)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2,MM
      COMMON X,OUTPLOT,DSCALE,VSCALE,BMSCALE,XSCALE
      DIMENSION OUTPLOT(400) ,XX(205)
      TIO= TIME - DT
      J = 0
      DO 1 I=3,N2,2
      L=I+1
      J=J+1
      K=L/2
      WO(J)=U(L,2)
      THO(J)=U(I,2)
      XMO(J)=EI(K)*(U(I+2,2)-U(I-2,2))/D2X
```

```
      SHO(J)=CAYGA(K)*((U(L+2,2)-U(L-2,2))/D2X-U(I,2))
      WV(J)=(U(L,3)-U(L,1))/D2T
      THV(J)=(U(I,3)-U(I,1))/D2T
    1 CONTINUE
    6 CONTINUE
      WRITE (6,2) TIO
    2 FORMAT (7H1TIME =,1F9.4//9X,1HX,13X,1HY,8X,5HGAMMA,8X,6HMOMENT,7X,
     15HSHEAR,8X,5HY-DOT,6X,9HGAMMA-DOT/)
    3 FORMAT (7E13.4)
      WRITE (6,3) (X(I),WO(I),THO(I),XMO(I),SHO(I),WV(I),THV(I),I=1,J)
      DMAX=DMIN=VMAX=VMIN=BMAX=BMIN=0.
      DO 25 I=1,J
      XX(I)=X(I)*XSCALE
      DMAX = MAX1F(DMAX,WO(I))
      DMIN=MIN1F(DMIN,WO(I))
      VMAX=MAX1F(VMAX,SHO(I))
      VMIN=MIN1F(VMIN,SHO(I))
      BMAX=MAX1F(BMAX,XMO(I))
   25 BMIN=MIN1F(BMIN,XMO(I))
      IF((DMAX+DMIN).LT.0.)     DMAX = DMIN
      IF((VMAX+VMIN).LT.0.)     VMAX = VMIN
      IF((BMAX+BMIN).LT.0.)     BMAX = BMIN
      CALL SYMBOL(3.5,-.25,.14, 9HX IN FEET   ,0.,9)
      CALL NUMBER(7.5,-.25,.14,X(J),0.,4HF5.0)
      CALL PLOT(8.,0.,3)
      CALL SYMBOL(6.,0.,.07,3,0.,-2)
      CALL SYMBOL(4.,0.,.07,3,0.,-2)
      CALL SYMBOL(2.,0.,.07,3,0.,-2)
      CALL PLOT(0.,0.,2)
      CALL SYMBOL(-.17,1.,.14,5,0.,-1)
      CALL SYMBOL(-.1,1.25,.14,17H- SHEAR IN POUNDS,90.,17)
      CALL NUMBER(-.1,4.5,.14,VMAX,90.,5HE10.2)
      CALL SYMBOL(0.,6.1,.14,49HSHIP STRUCTURE RESPONSE TO IMPULSE LOAD
     +- TIME = ,0.,49)
      CALL NUMBER(5.9,6.1,.14,TIO,0.,4HF5.3)
      CALL SYMBOL(6.5,6.1,.14,6H, DT= ,0.,6)
      CALL NUMBER(7.2,6.1,.14,DT,0.,4HF6.3)
      CALL PLOT(8.,0.,3)
      CALL PLOT(8.,6.,2)
      CALL PLOT(0.,6.,2)
      CALL SYMBOL(0.,4.5,.07,3,0.,-2)
      CALL SYMBOL(0.,3.0,.07,3,0.,-2)
      CALL SYMBOL(0.,1.5,.07,3,0.,-2)
      CALL PLOT(0.,0.,2)
      CALL SYMBOL(-.67,1.,.14,0,0.,-1)
      CALL SYMBOL(-.6,1.25,.14,22H- MOMENT IN POUND-FEET,90.,22)
      CALL NUMBER(-.6,4.5,.14,BMAX,90.,5HE10.2)
      CALL PLOT(-.5,6.,3)
      CALL SYMBOL(-.5,4.5,.07,3,0.,-2)
```

```
      CALL SYMBOL(-.5,3.0,.07,3,0.,-2)
      CALL SYMBOL(-.5,1.5,.07,3,0.,-2)
      CALL PLOT(-.5,0.,2)
      CALL SYMBOL(-1.17,1.,.14,4,0.,-1)
      CALL SYMBOL(-1.1,1.25,.14,15H- DEFL, IN FEET,90.,15)
      CALL NUMBER(-1.1,4.5,.14,DMAX,90.,5HE10.2)
      CALL PLOT(-1.,6.,3)
      CALL SYMBOL(-1.,4.5,.07,3,0.,-2)
      CALL SYMBOL(-1.,3.0,.07,3,0.,-2)
      CALL SYMBOL(-1.,1.5,.07,3,0.,-2)
      CALL PLOT(-1.,0.,2)
      CALL PLOT(0.,0.,2)
      CALL SYMBOL(-1.,-.25,.14,4HTAU=,0.,4)
      CALL NUMBER(-.4,-.25,.14,TAU,0.,4HF5.2)
      YY = WO * DSCALE + 3.
      CALL SYMBOL(XX,YY,.07,4,0.,-1)
      DO  100 I = 2,J
      YY = WO(I) * DSCALE + 3.
      IF((I/10)*10-I)          GO TO  50
      CALL SYMBOL(XX(I),YY,.07,4,0.,-2)
      GO TO 100
 50   CALL PLOT(XX(I),YY,2)
100   CONTINUE
      YY = XMO(J)* BMSCALE + 3.
      CALL SYMBOL(XX(J),YY,.07,0,0.,-1)
      DO  200 I = 2,J
      K =  J-I+1
      YY = XMO(K) * BMSCALE + 3.
      IF((I/10)*10-I)          GO TO 150
      CALL SYMBOL(XX(K),YY,.07,0,0.,-2)
      GO TO 200
150   CALL PLOT(XX(K),YY,2)
200   CONTINUE
      YY = SHO * VSCALE + 3.
      CALL SYMBOL(XX,YY,.07,5,0.,-1)
      DO  300 I = 2,J
      YY = SHO(I) * VSCALE + 3.
      IF((I/10)*10-I)          GO TO 250
      CALL SYMBOL(XX(I),YY,.07,5,0.,-2)
      GO TO 300
250   CALL PLOT(XX(I),YY,2)
300   CONTINUE
      CALL PLOT(12.,0.,-3)
      RETURN
      ENTRY OUTI
      TIO=TIME-DT
      J=0
      DO 5 I=3,N2,2
      L=I+1
      J=J+1
      K=L/2
```

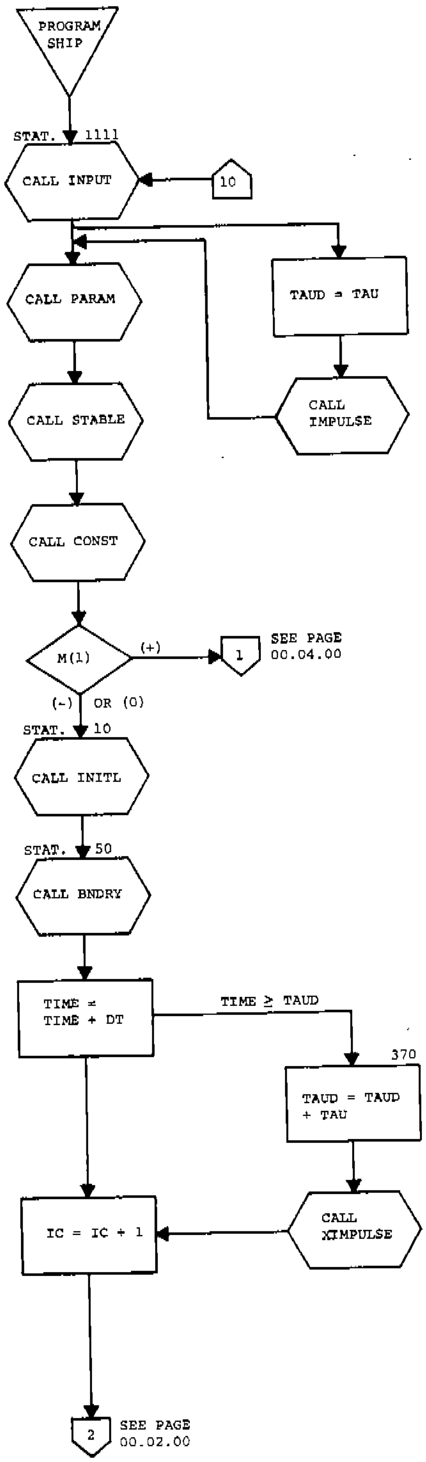
```
      I1=I+2
      I2=I-2
      I3=L+2
      I4=L-2
      WO(J)=(U(L,3)+2,*U(L,2)+U(L,1))/4,
      THO(J)=(U(I,3)+2,*U(I,2)+U(I,1))/4,
      XMO(J)=EI(K)*(U(I1,3)-U(I2,3)+2,*(U(I1,2)-U(I2,2))+U(I1,1)-U(I2,1)
     1)/(4,*D2X)
      SHO(J)=CAYGA(K)*((U(I3,3)-U(I4,3)+2,*(U(I3,2)-U(I4,2))+U(I3,1)-
     1U(I4,1))/D2X-(U(I,3)+2,*U(I,2)+U(I,1)))/4,
      WV(J)=(U(L,3)-U(L,1))/D2T
      THV(J)=(U(I,3)-U(I,1))/D2T
    5 CONTINUE
      GO TO 6
      END




      SUBROUTINE XIMPULSE
C THIS ROUTINE READS ALL FORCE DATA.
      DIMENSION M(10),CAYGA(204),CAYB(204),VM(204),EI(204),RI(204),
     1S1(204),S2(204),S3(204),S4(204),S5(204),S6(204),S7(204),S8(204),
     1S9(204),S10(204),U(408,3) ,A(408),B(408),C(408),D(408),E(408),F(40
     18),G(408),P(204),V(204),NPS(200),CB(204),CHZ(204), SM(204),CK(204)
     1,CI(204),CA(204),HEADER(9)
      COMMON M,CB,CHZ, SM,CK,CI,CA,TIME,DT,IC,N,N1,N2,N3,N4,N5,N6,N7,
     1NN,SC,M3,MMM3,FINIS,DX,DX2,JC,CAYGA,CAYB,VM,EI,RI,P,RFD,D2X,D3X,
     1DX24,D2T,TAU,TDX,STP,S1,S2,S3,S4,S5,S6,S7,S8,S9,S10,U,A,B,C,D,E,
     1F,G,T1,T2,T3,T4,V,RSTL,RSEA,GC,CE,NPS,DT2,MM
      COMMON /A/ TAUC,PX,XTAU,XTDX,NP
      COMMON /B/ CF
      DIMENSION XTAU(204),TAUC(204),PX(204),XTDX(204),CF(204)
      DIMENSION XNPS(204)
    5 FORMAT (5E14,6)
    6 FORMAT(1I5/(10I5))
C     LOAD LOCATION
      READ (5,6)NP,(NPS(I),I=1,NP)
C     FORCE
      READ(5,5)  ( PX(I),I=1,NP)
C     START OF LOAD APPLICATION
      READ(5,5)  (TAUC(I),I=1,NP)
C     DURATION OF LOAD
      READ(5,5) (XTAU(I),I=1,NP)
C     CIRCULAR FREQUENCY
      READ(5,5)  ( CF(I),I=1,NP)
      DO 440  I=1,NP
  440 XNPS(I)=DX*NPS(I)
      PRINT 920
      WRITE(6,20) (XNPS(I),PX(I),TAUC(I),XTAU(I),CF(I),I=2,MM)
```

```
20 FORMAT(5E13,4)
      J=1
      JT=1
      DO 442 I=2,MM
      K=NPS(JT)
      IF (I-K) 9,8,9
    8 PX(I)=PX(J)
      XNPS(I)=DX*NPS(J)
      TAUC(I)=TAUC(J)
      XTAU(I)=XTAU(J)
      CF(I)=CF(J)
      JT=JT+1
      J=J+1
      GO TO 442
    9 PX(I)=0.0
      TAUC(I)=0.0
      XTAU(I)=0.0
      CF(I)=0.
      XNPS(I)=DX*I
  442 CONTINUE
      PRINT 920
  920 FORMAT(/,* LOCATION      FORCE          START            DURATION
     1FREQUENCY *,/)
      JT=1
      DO 542 I=2,MM
      K=NPS(JT)
      IF (I-K) 19,18,19
   18 XTDX(I)=PX(I)/DX/XTAU(I)
      JT=JT+1
      GO TO 542
   19 XTDX(I)=0.
  542 CONTINUE
      WRITE(6,21) (XNPS(I),XTDX(I),I=2,MM)
   21 FORMAT(2E13,4)
      RETURN
      END
```

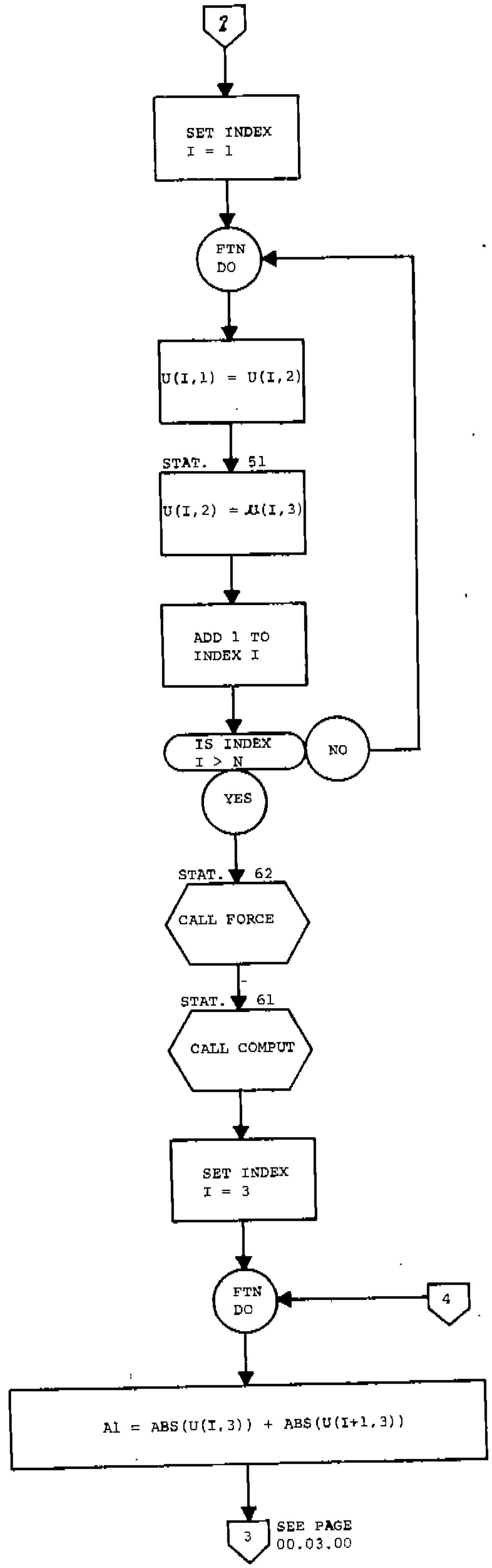
The following pages are the logic flow charts. Please note the following:

a) Page 00. 07. 00 is identical to Page 00. 08. 00.

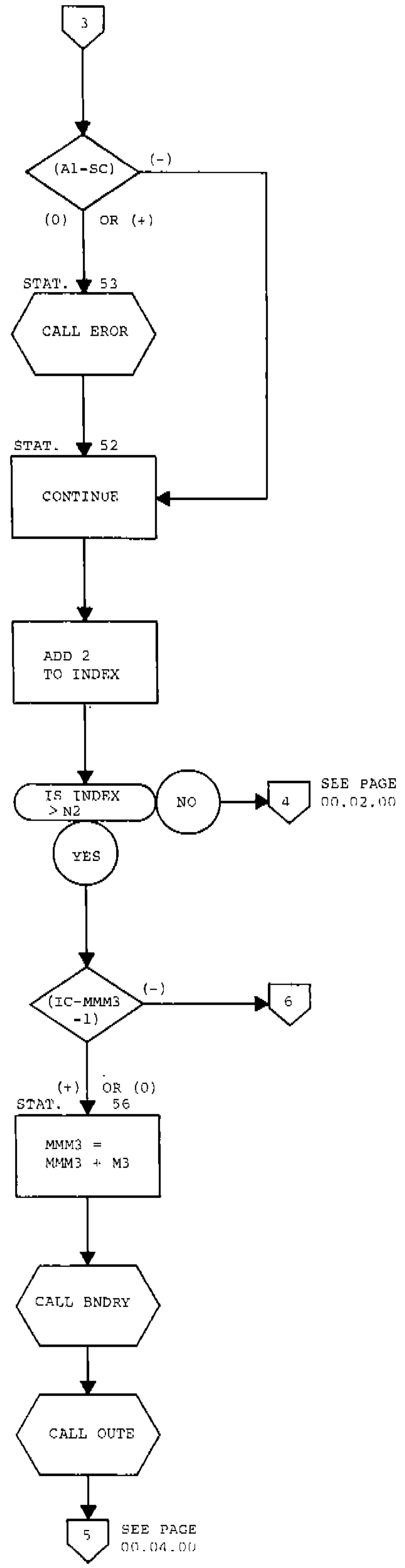
b) Page 00. 16. 00 is intentionally missing.

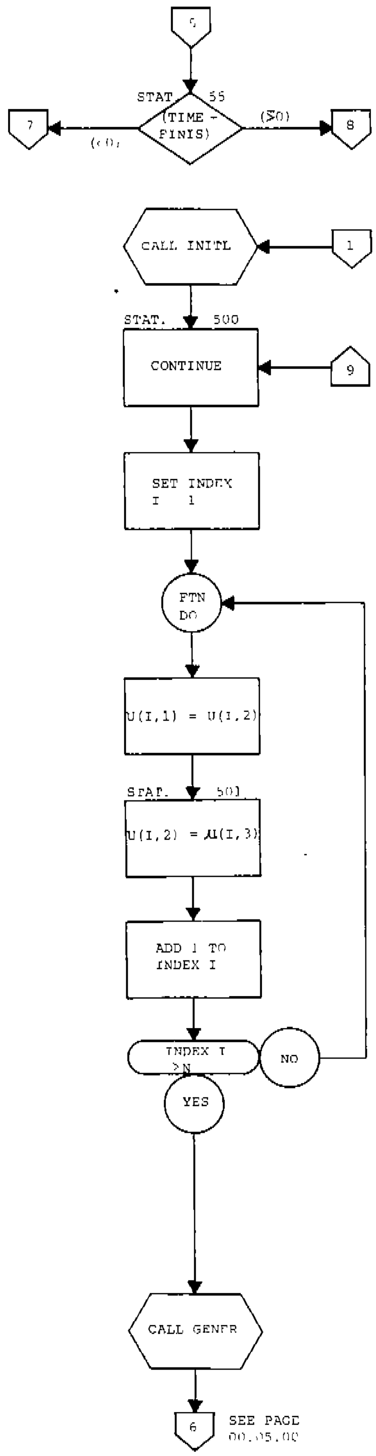


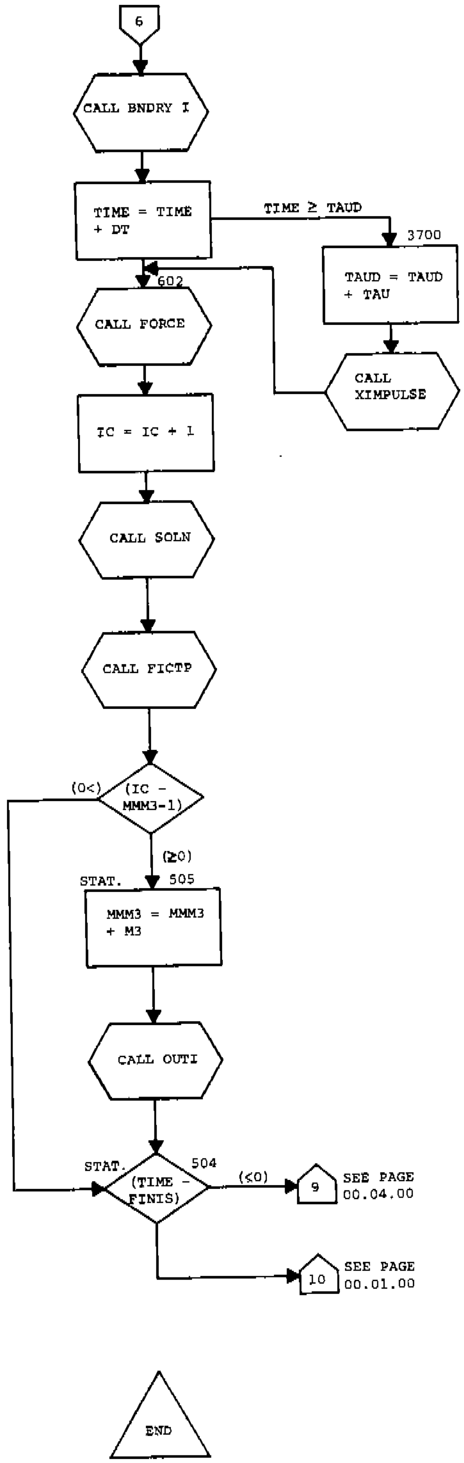
PAGE 00.01.00

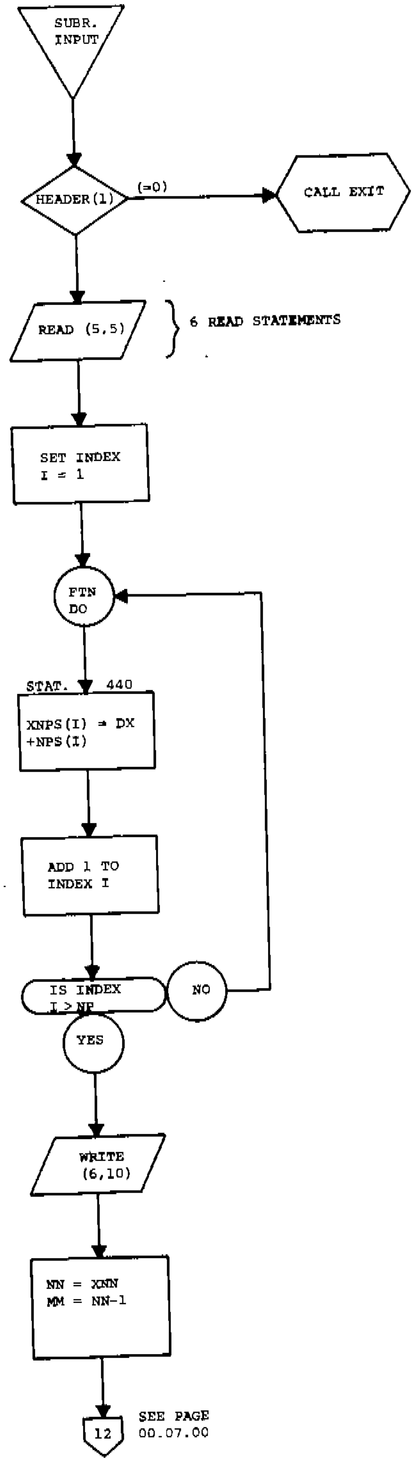


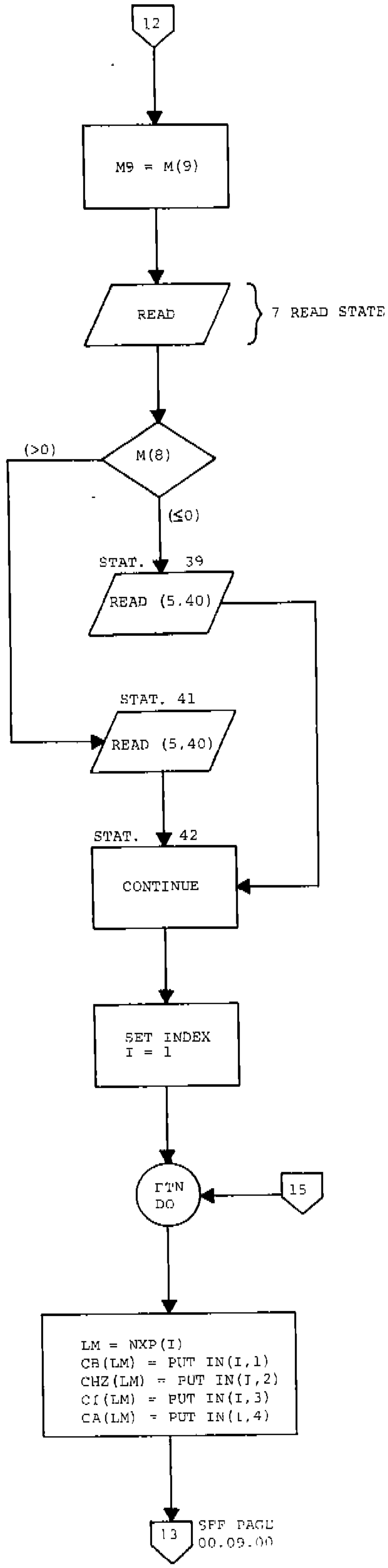
PAGE 00.02.00

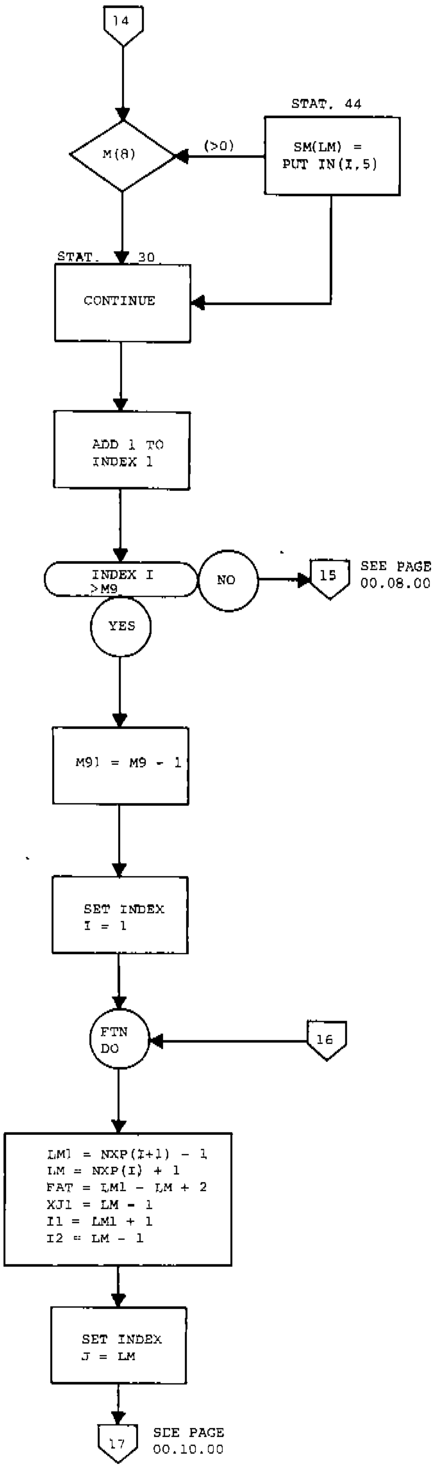

3
(A1-SC)
(-)
(0)
OR (+)
STAT. 53
CALL EROR
STAT. 52
CONTINUE
ADD 2
TO INDEX
IS INDEX
> N2
NO
4
SEE PAGE
00.02.00
YES
(IC-MMM3
-1)
(-)
6
(+)
OR (0)
STAT. 56
MMM3 =
MMM3 + M3
CALL BNDRY
CALL OUTE
5
SEE PAGE
00.04.00

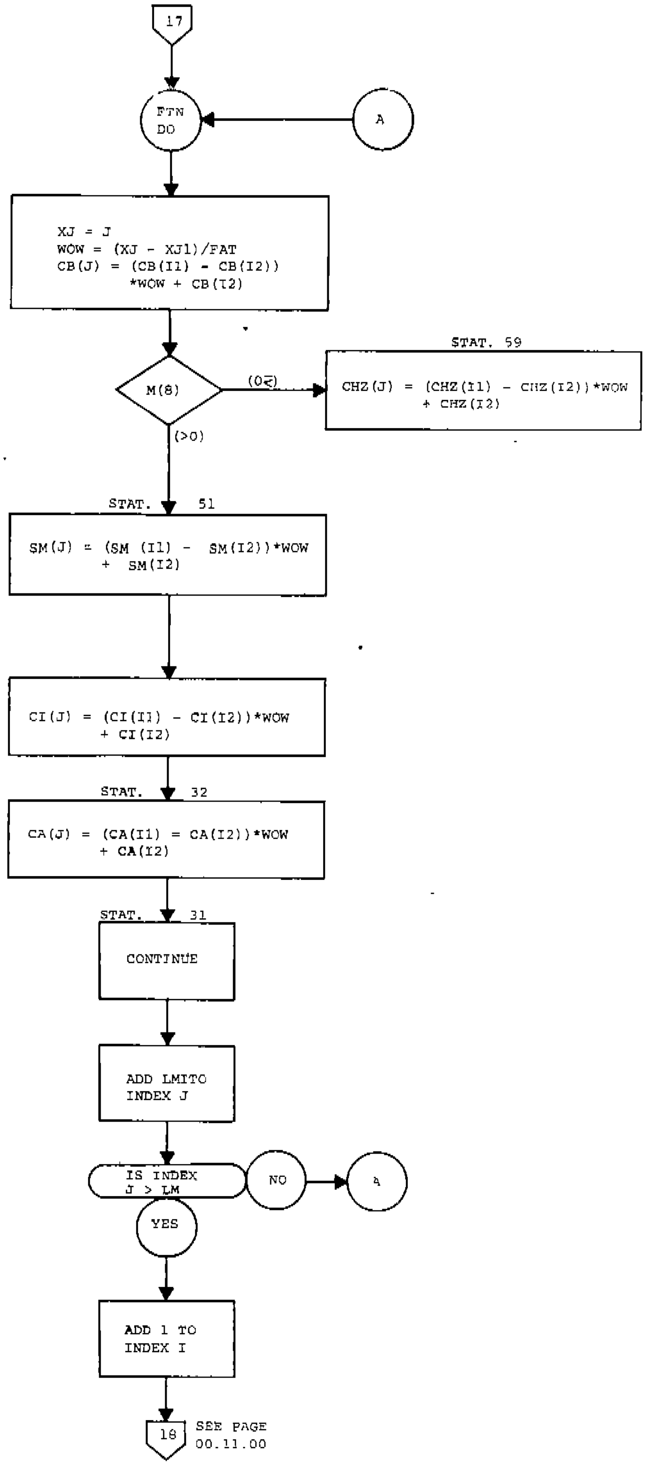

STAT
55
(TIME -
FINIS)
7
8
CALL INITL
1
STAT.
500
CONTINUE
9
SET INDEX
I 1
FTN
DO
U(I,1) = U(I,2)
U(I,2) = U(I,3)
ADD 1 TO
INDEX I
INDEX I
>N
NO
YES
CALL GENER
6
SEE PAGE
00.05.00

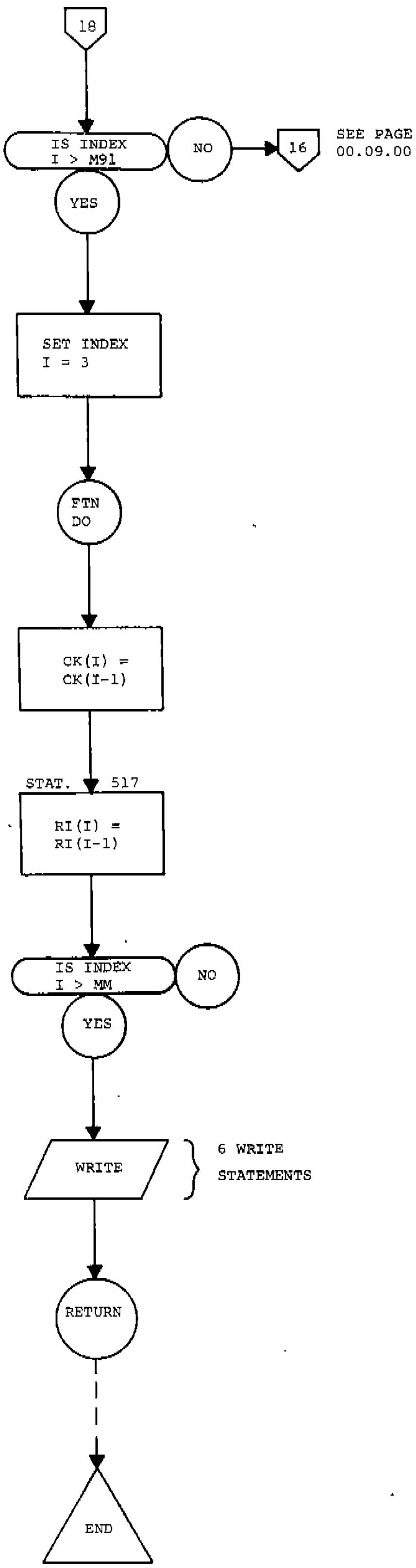

6
CALL BNDRY I
TIME = TIME + DT
TIME ≥ TAUD
3700
TAUD = TAUD + TAU
CALL XIMPULSE
602
CALL FORCE
IC = IC + 1
CALL SOLN
CALL FICTP
(0<)
(IC - MMM3-1)
(≥0)
STAT. 505
MMM3 = MMM3 + M3
CALL OUTI
STAT. 504
(TIME - FINIS)
(<0)
9
SEE PAGE 00.04.00
10
SEE PAGE 00.01.00
END

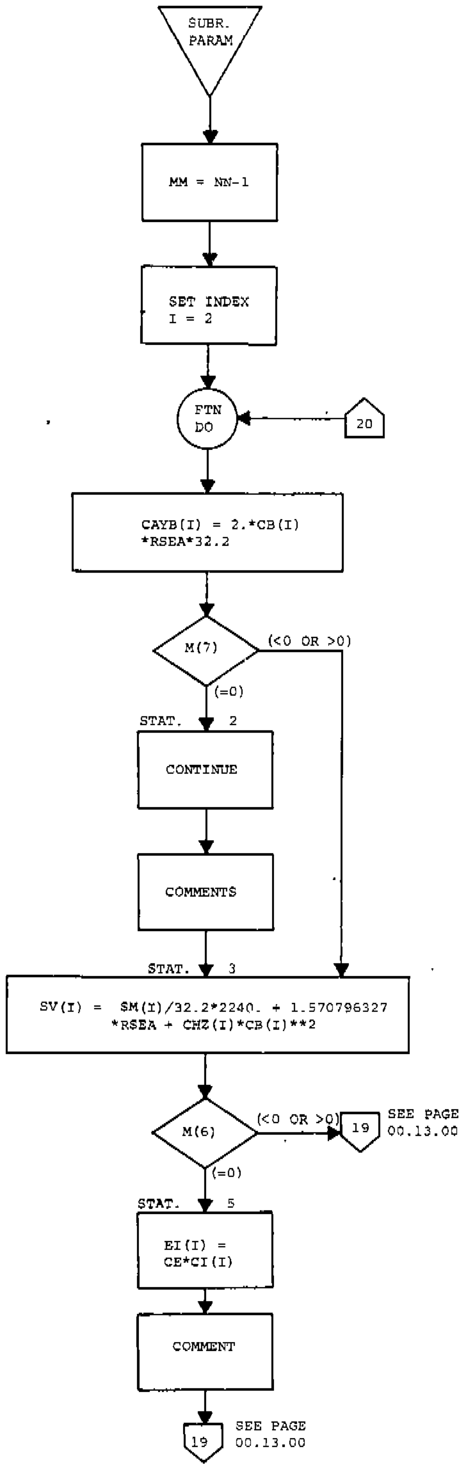

SUBR. INPUT
HEADER(1)
(=0)
CALL EXIT
READ (5,5)
6 READ STATEMENTS
SET INDEX I = 1
FTN DO
STAT. 440
XNPS(I) = DX +NPS(I)
ADD 1 TO INDEX I
IS INDEX I > NP
NO
YES
WRITE (6,10)
NN = XNN
MM = NN-1
12
SEE PAGE 00.07.00

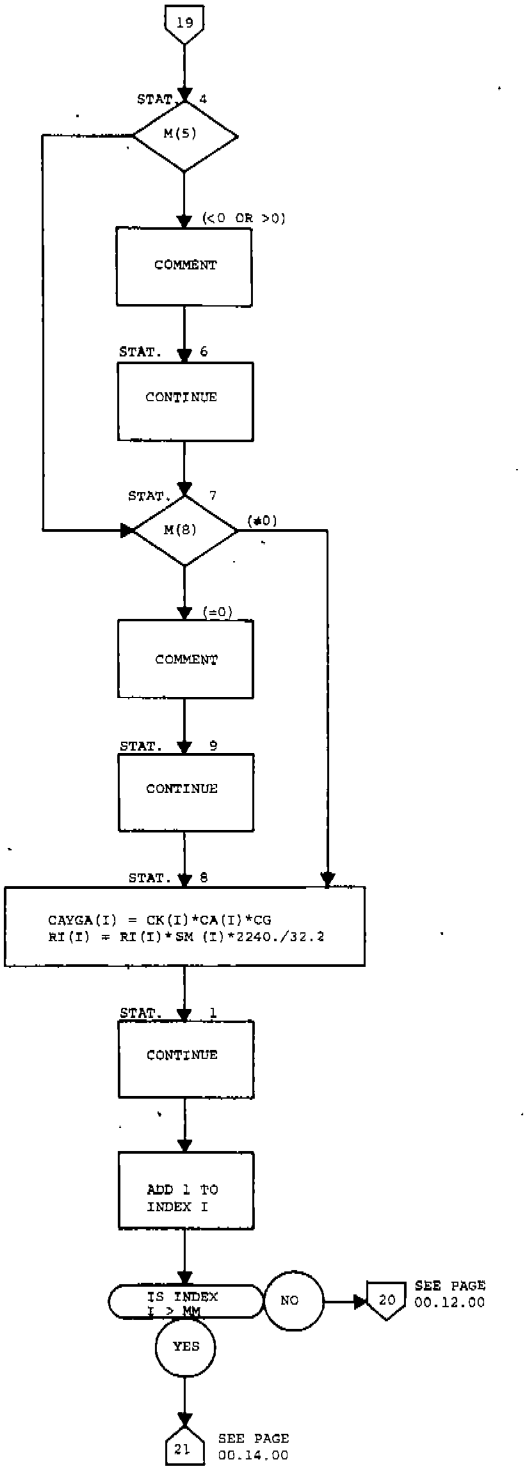

12
M9 = M(9)
READ
7 READ STATE
M(8)
(>0)
(≤0)
STAT. 39
READ (5.40)
STAT. 41
READ (5,40)
STAT. 42
CONTINUE
SET INDEX
I = 1
FTN
DO
15
LM = NXP(I)
CB(LM) = PUT IN(I,1)
CHZ(LM) = PUT IN(I,2)
CI(LM) = PUT IN(I,3)
CA(LM) = PUT IN(I,4)
13
SEE PAGE
00.09.00


14
M(8)
(>0)
STAT. 44
SM(LM) =
PUT IN(I,5)
STAT. 30
CONTINUE
ADD 1 TO
INDEX 1
INDEX I
>M9
NO
15
SEE PAGE
00.08.00
YES
M91 = M9 - 1
SET INDEX
I = 1
FTN
DO
16
LM1 = NXP(I+1) - 1
LM = NXP(I) + 1
FAT = LM1 - LM + 2
XJ1 = LM - 1
I1 = LM1 + 1
I2 = LM - 1
SET INDEX
J = LM
17
SEE PAGE
00.10.00

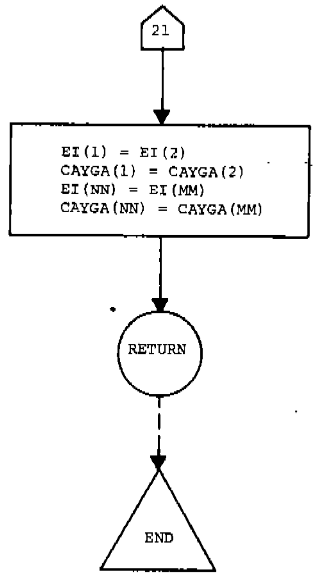

17
FTN
DO
A
XJ = J
WOW = (XJ - XJ1)/FAT
CB(J) = (CB(I1) - CB(I2))
*WOW + CB(I2)
M(8)
(0≷)
STAT. 59
CHZ(J) = (CHZ(I1) - CHZ(I2))*WOW
+ CHZ(I2)
(>0)
STAT. 51
SM(J) = (SM (I1) - SM(I2))*WOW
+ SM(I2)
CI(J) = (CI(I1) - CI(I2))*WOW
+ CI(I2)
STAT. 32
CA(J) = (CA(I1) = CA(I2))*WOW
+ CA(I2)
STAT. 31
CONTINUE
ADD LMITO
INDEX J
IS INDEX
J > LM
NO
A
YES
ADD 1 TO
INDEX I
18
SEE PAGE
00.11.00

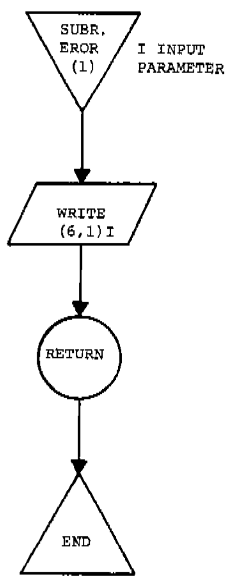

18
IS INDEX
I > M91
NO
16
SEE PAGE
00.09.00
YES
SET INDEX
I = 3
FTN
DO
CK(I) =
CK(I-1)
STAT. 517
RI(I) =
RI(I-1)
IS INDEX
I > MM
NO
YES
WRITE
6 WRITE
STATEMENTS
RETURN
END


SUBR.
PARAM
MM = NN-1
SET INDEX
I = 2
FTN
DO
20
CAYB(I) = 2.*CB(I)
*RSEA*32.2
M(7)
(<0 OR >0)
(=0)
STAT. 2
CONTINUE
COMMENTS
STAT. 3
SV(I) = SM(I)/32.2*2240. + 1.570796327
*RSEA + CHZ(I)*CB(I)**2
M(6)
(<0 OR >0)
19
SEE PAGE
00.13.00
(=0)
STAT. 5
EI(I) =
CE*CI(I)
COMMENT
19
SEE PAGE
00.13.00

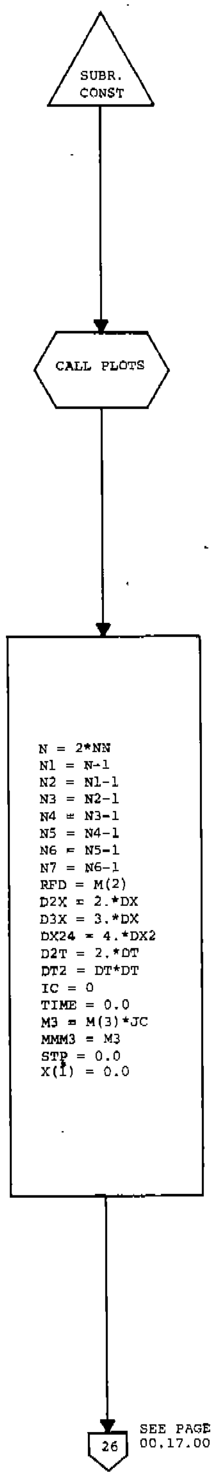


PAGE 00.13.00

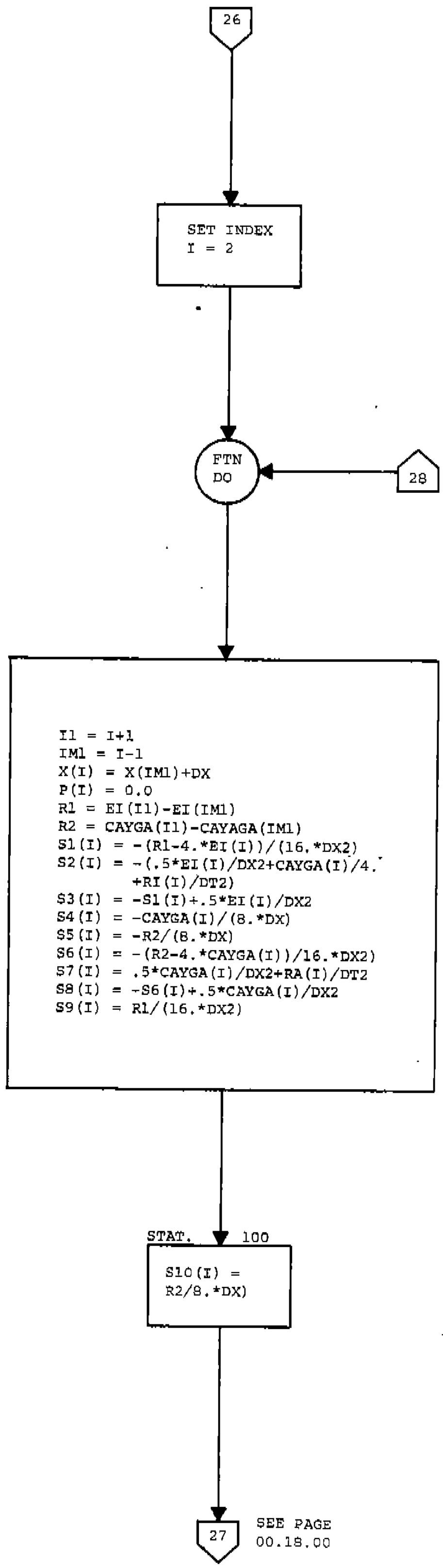




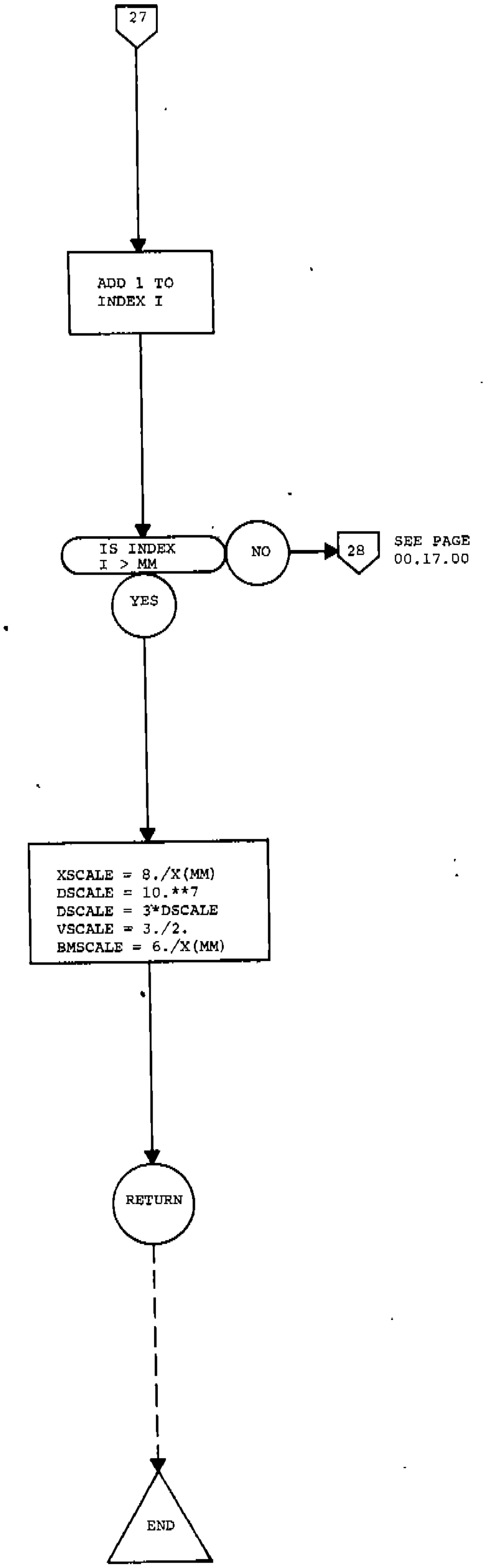
PAGE 00.14.00

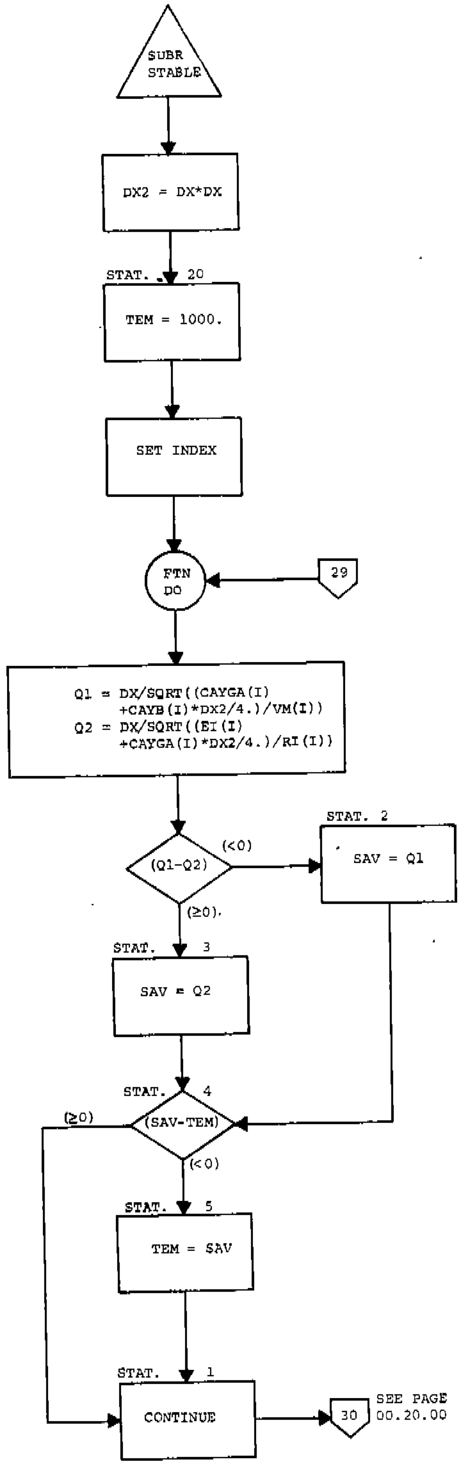


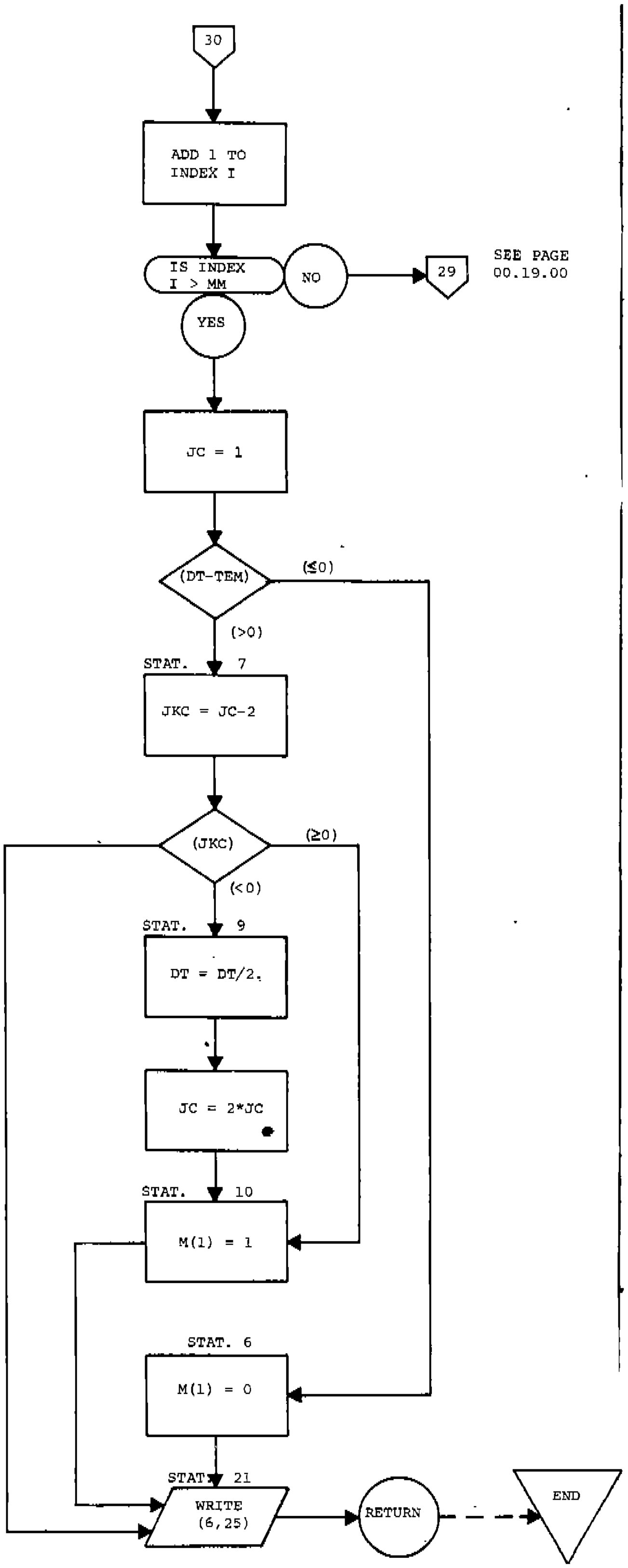
PAGE 00.15.00



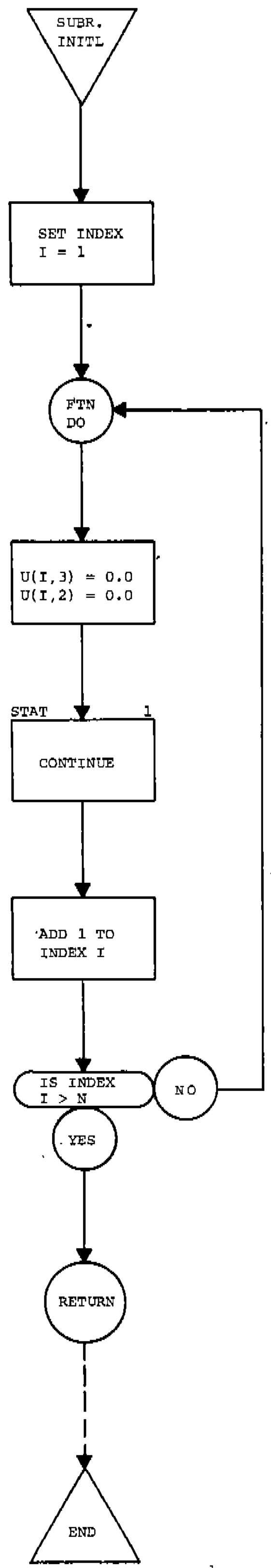
PAGE 00.17.00


27
ADD 1 TO INDEX I
IS INDEX I > MM
NO
28
SEE PAGE 00.17.00
YES
XSCALE = 8./X(MM)
DSCALE = 10.**7
DSCALE = 3*DSCALE
VSCALE = 3./2.
BMSCALE = 6./X(MM)
RETURN
END

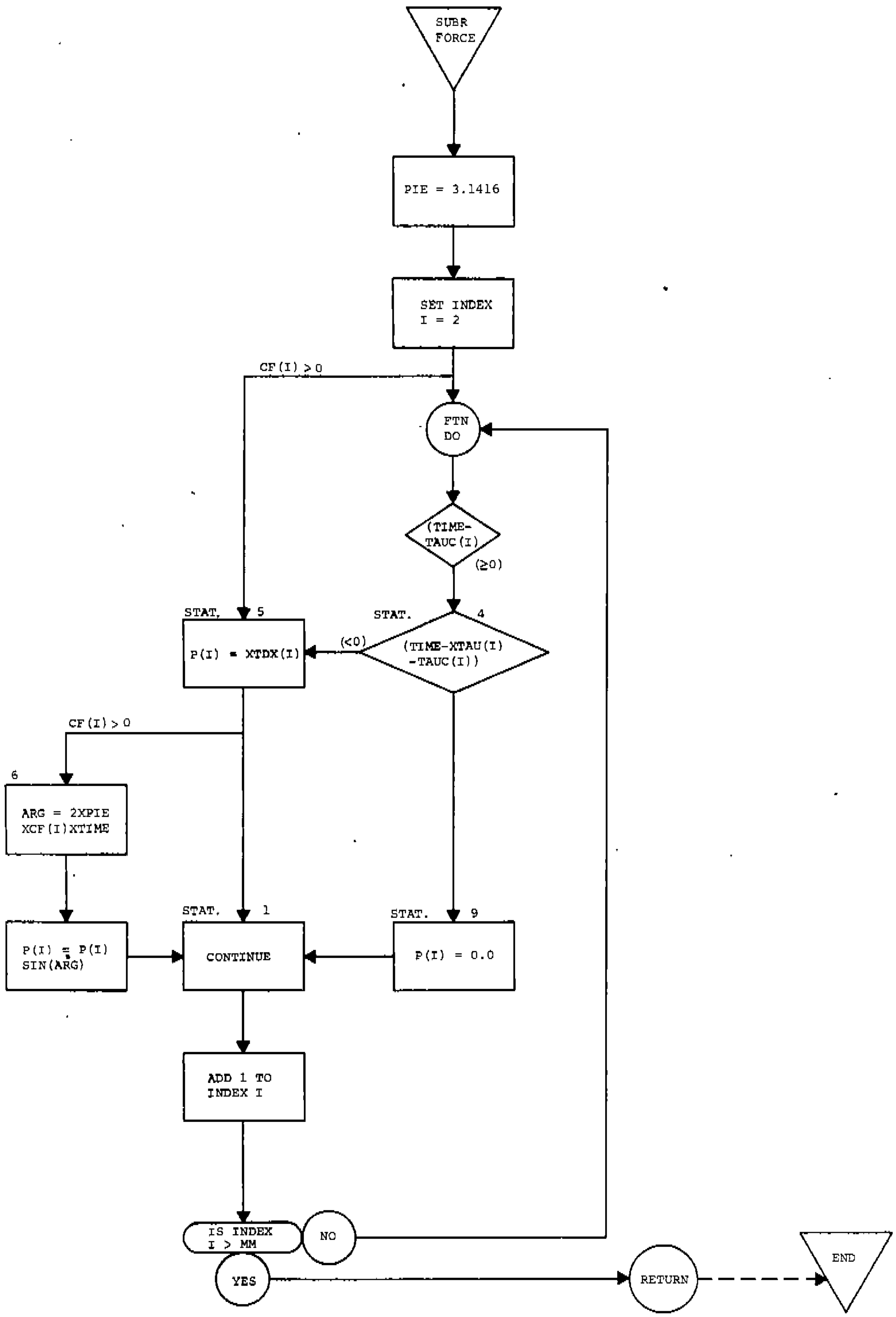

SUBR STABLE
DX2 = DX*DX
STAT. 20
TEM = 1000.
SET INDEX
FTN DO
29
Q1 = DX/SQRT((CAYGA(I)
+CAYB(I)*DX2/4.)/VM(I))
Q2 = DX/SQRT((EI(I)
+CAYGA(I)*DX2/4.)/RI(I))
(Q1-Q2)
(<0)
(≥0)
STAT. 2
SAV = Q1
STAT. 3
SAV = Q2
STAT. 4
(SAV-TEM)
(≥0)
(<0)
STAT. 5
TEM = SAV
STAT. 1
CONTINUE
30
SEE PAGE 00.20.00

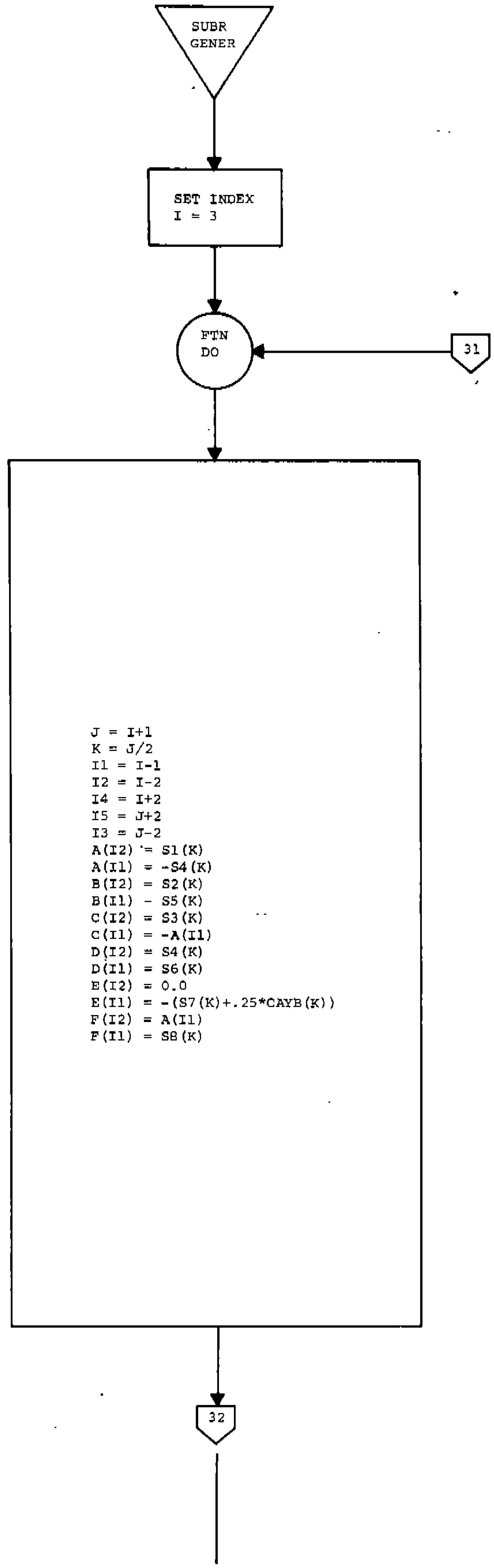


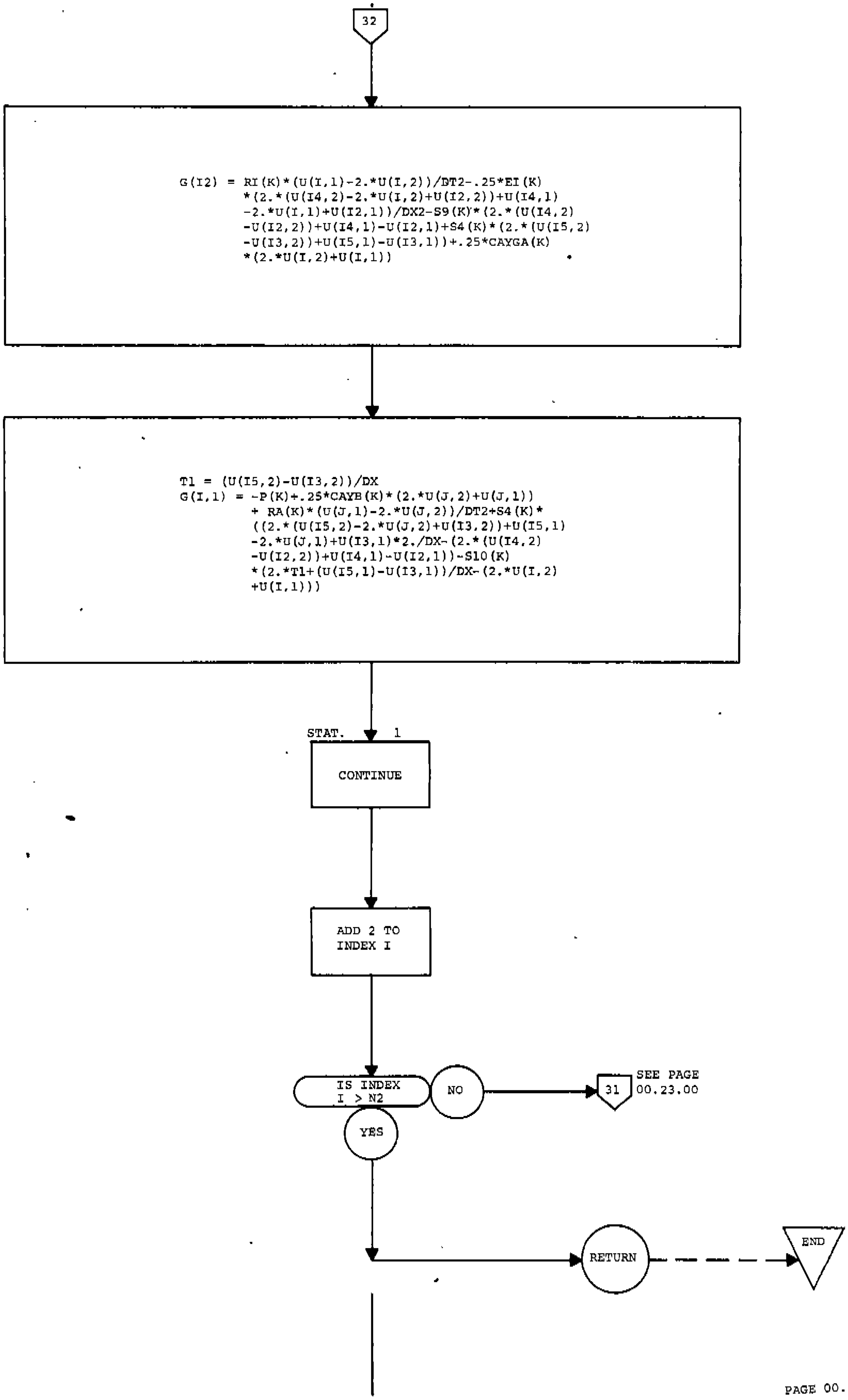
PAGE 00.20.00

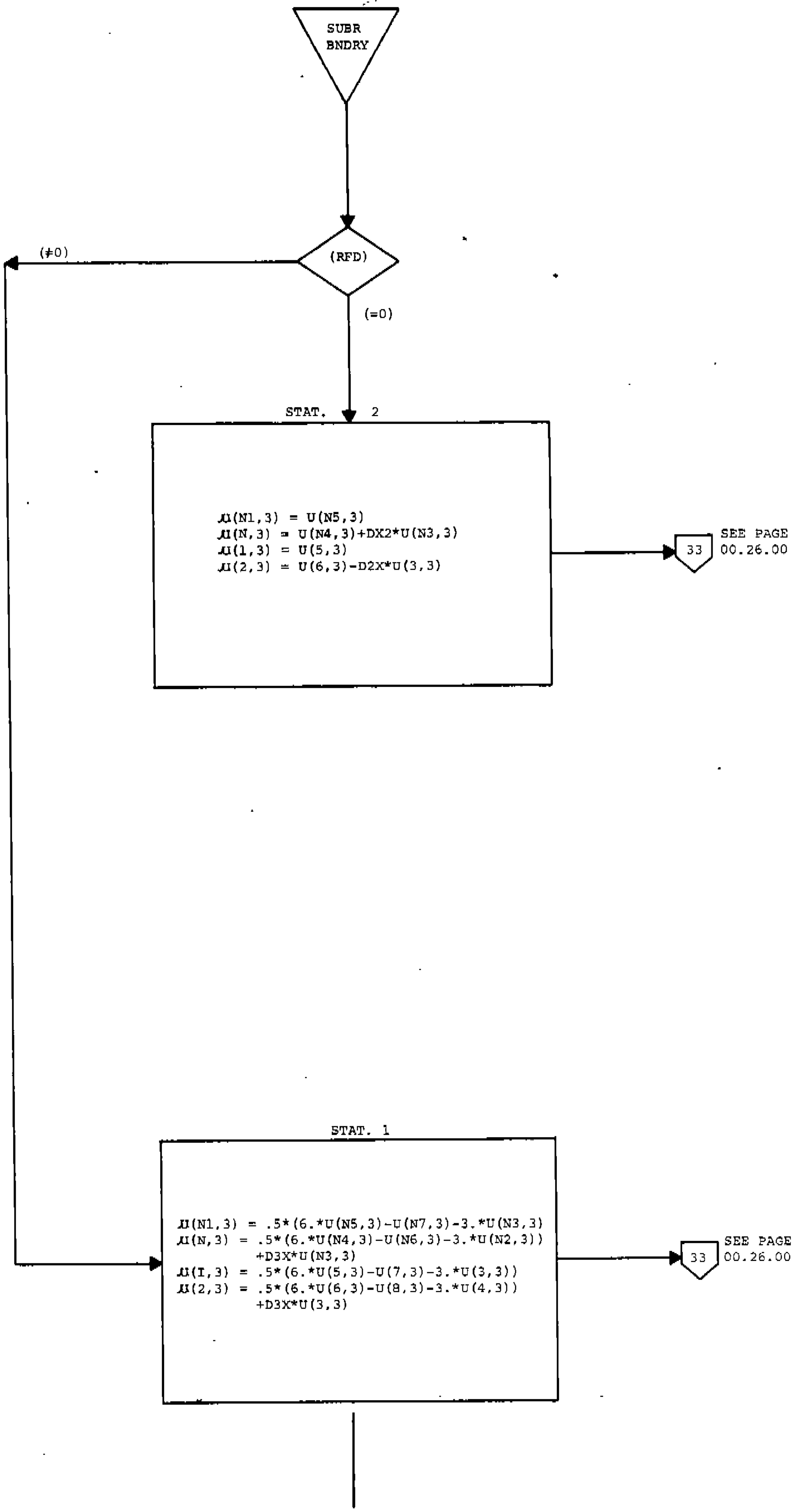


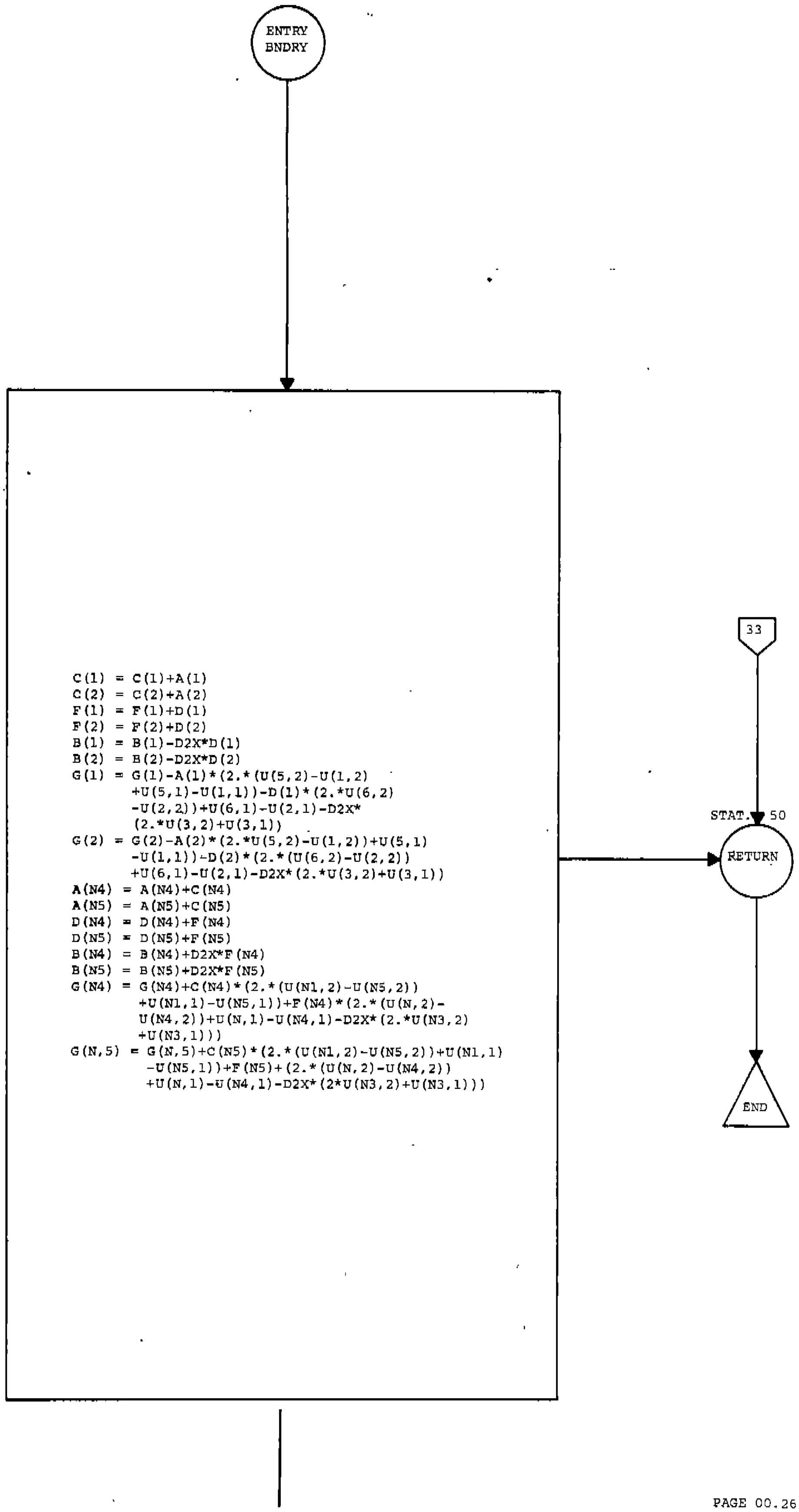
PAGE 00.21.00

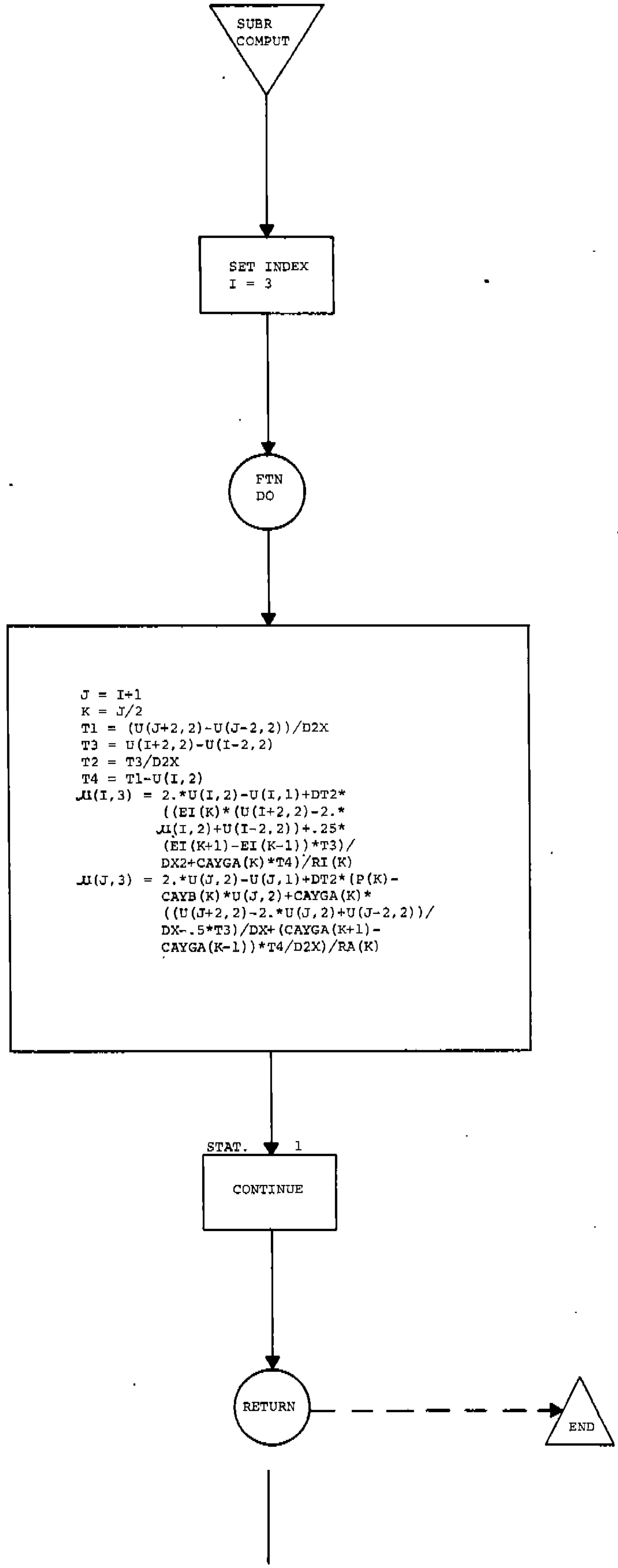

SUBR FORCE
PIE = 3.1416
SET INDEX I = 2
CF(I) > 0
FTN DO
(TIME-TAUC(I)
(≥0)
STAT. 5
STAT. 4
P(I) = XTDX(I)
(<0)
(TIME-XTAU(I) -TAUC(I))
CF(I) > 0
6
ARG = 2XPIE XCF(I)XTIME
STAT. 1
STAT. 9
P(I) = P(I) SIN(ARG)
CONTINUE
P(I) = 0.0
ADD 1 TO INDEX I
IS INDEX I > MM
NO
YES
RETURN
END

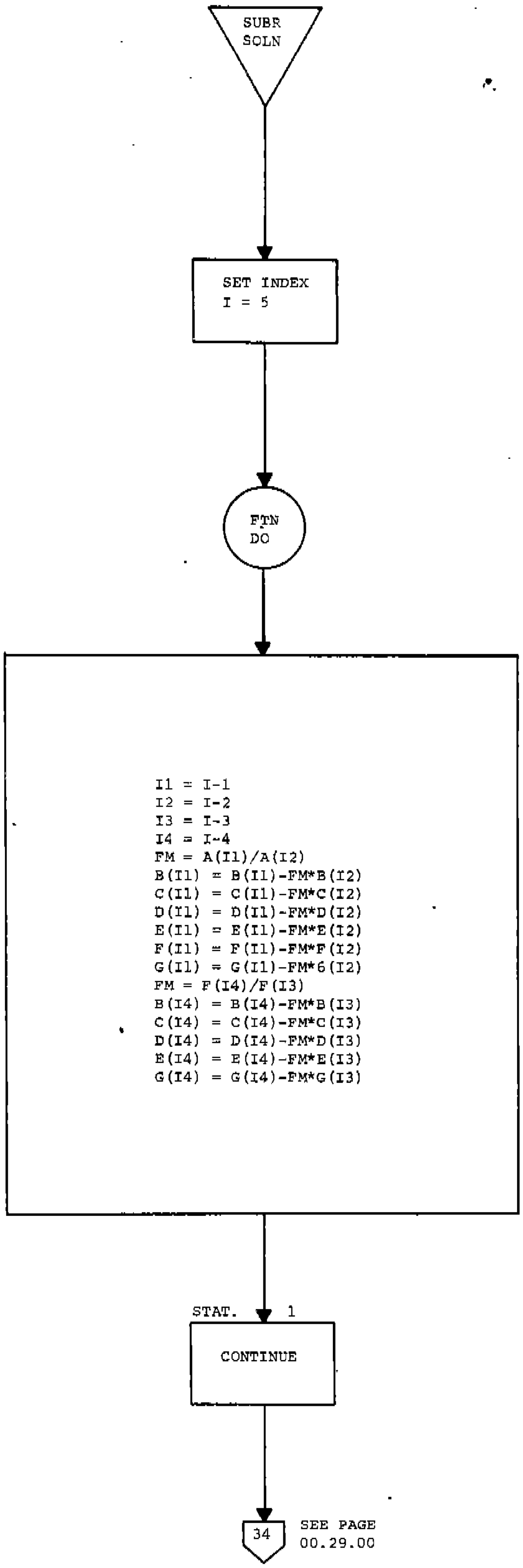

SUBR
GENER
SET INDEX
I = 3
FTN
DO
31
J = I+1
K = J/2
I1 = I-1
I2 = I-2
I4 = I+2
I5 = J+2
I3 = J-2
A(I2) = S1(K)
A(I1) = -S4(K)
B(I2) = S2(K)
B(I1) - S5(K)
C(I2) = S3(K)
C(I1) = -A(I1)
D(I2) = S4(K)
D(I1) = S6(K)
E(I2) = 0.0
E(I1) = -(S7(K)+.25*CAYB(K))
F(I2) = A(I1)
F(I1) = S8(K)
32

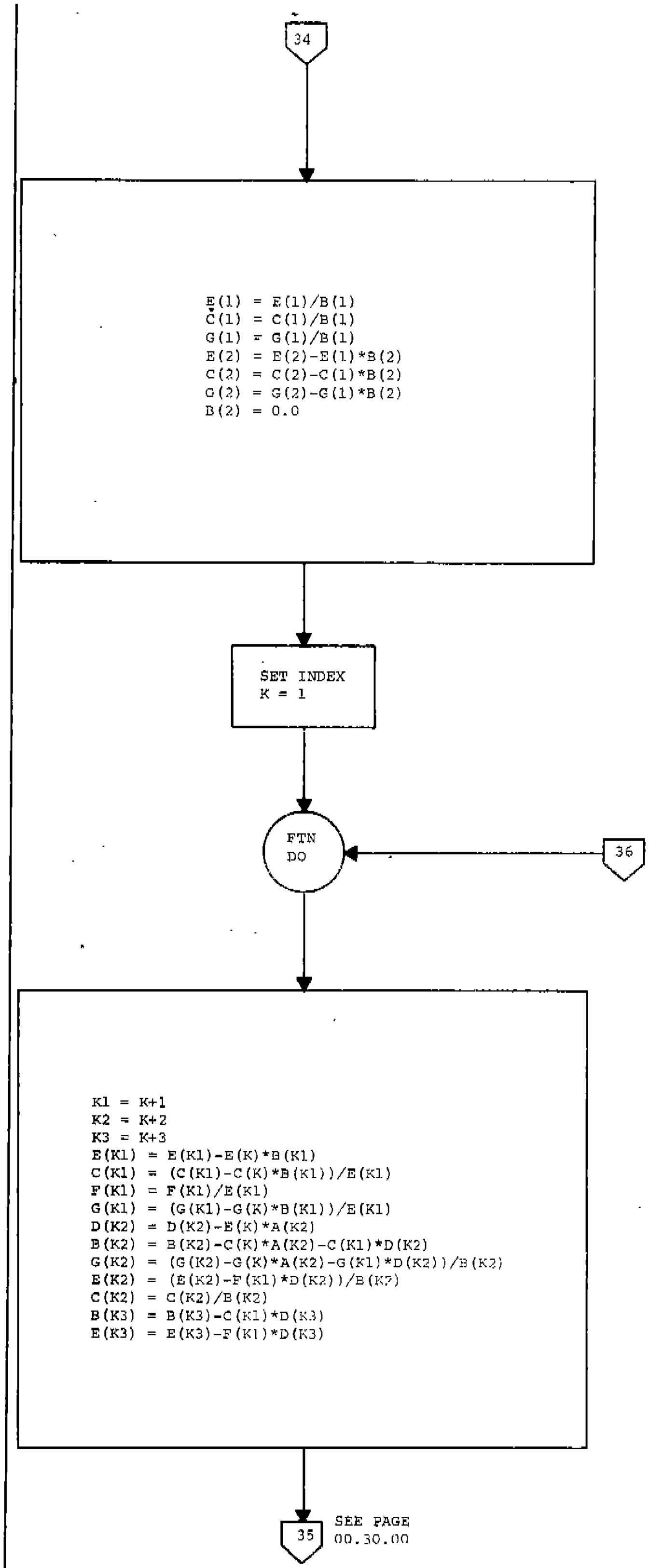

32
G(I2) = RI(K)*(U(I,1)-2.*U(I,2))/DT2-.25*EI(K)
(2.(U(I4,2)-2.*U(I,2)+U(I2,2))+U(I4,1)
-2.*U(I,1)+U(I2,1))/DX2-S9(K)*(2.*(U(I4,2)
-U(I2,2))+U(I4,1)-U(I2,1)+S4(K)*(2.*(U(I5,2)
-U(I3,2))+U(I5,1)-U(I3,1))+.25*CAYGA(K)
*(2.*U(I,2)+U(I,1))
T1 = (U(I5,2)-U(I3,2))/DX
G(I,1) = -P(K)+.25*CAYB(K)*(2.*U(J,2)+U(J,1))
+ RA(K)*(U(J,1)-2.*U(J,2))/DT2+S4(K)*
((2.*(U(I5,2)-2.*U(J,2)+U(I3,2))+U(I5,1)
-2.*U(J,1)+U(I3,1)*2./DX-(2.*(U(I4,2)
-U(I2,2))+U(I4,1)-U(I2,1))-S10(K)
*(2.*T1+(U(I5,1)-U(I3,1))/DX-(2.*U(I,2)
+U(I,1)))
STAT. 1
CONTINUE
ADD 2 TO
INDEX I
IS INDEX
I > N2
NO
31
SEE PAGE
00.23.00
YES
RETURN
END

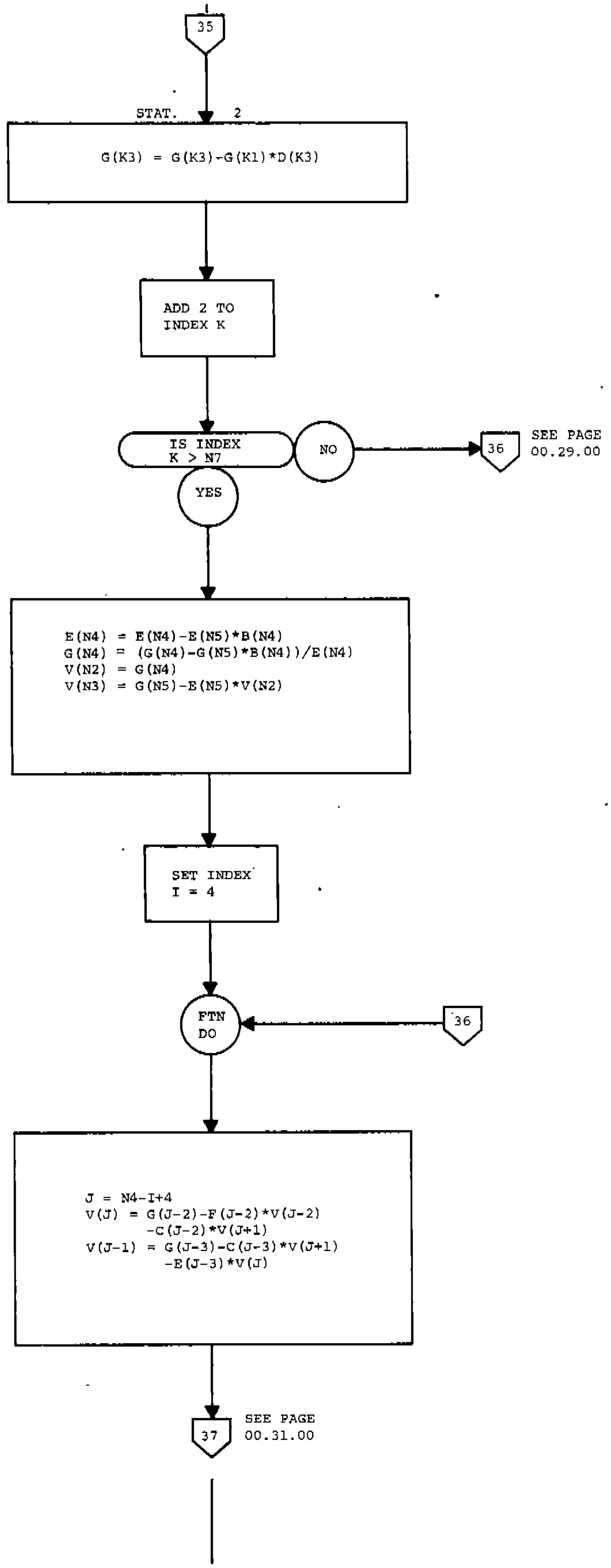

SUBR
BNDRY
(RFD)
(≠0)
(=0)
STAT. 2
U(N1,3) = U(N5,3)
U(N,3) = U(N4,3)+DX2*U(N3,3)
U(1,3) = U(5,3)
U(2,3) = U(6,3)-D2X*U(3,3)
33
SEE PAGE
00.26.00
STAT. 1
U(N1,3) = .5*(6.*U(N5,3)-U(N7,3)-3.*U(N3,3)
U(N,3) = .5*(6.*U(N4,3)-U(N6,3)-3.*U(N2,3))
+D3X*U(N3,3)
U(I,3) = .5*(6.*U(5,3)-U(7,3)-3.*U(3,3))
U(2,3) = .5*(6.*U(6,3)-U(8,3)-3.*U(4,3))
+D3X*U(3,3)
33
SEE PAGE
00.26.00

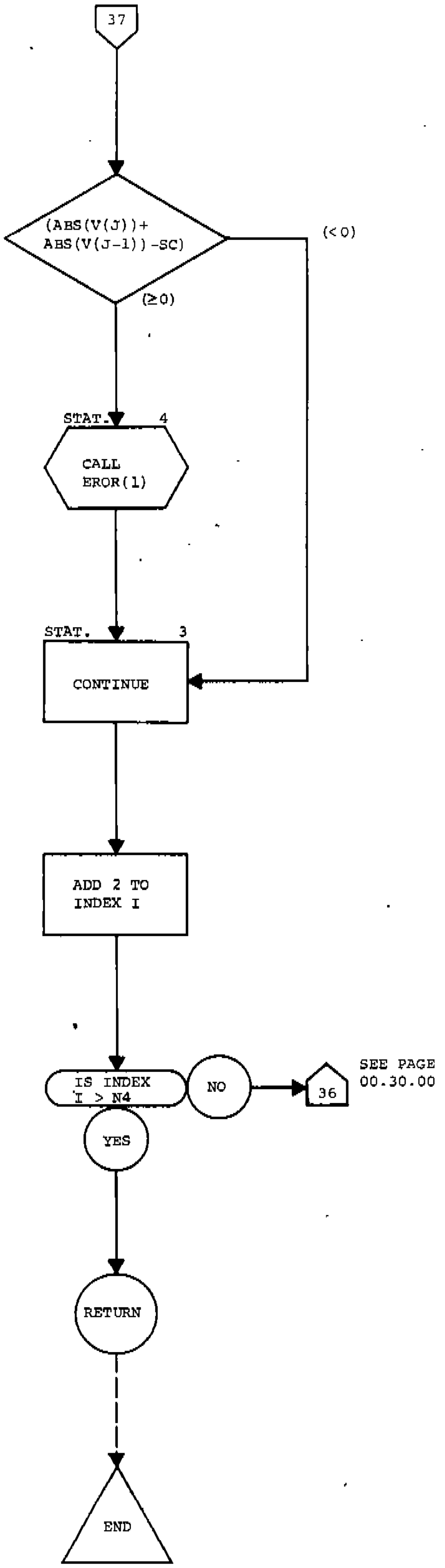

ENTRY
BNDRY
C(1) = C(1)+A(1)
C(2) = C(2)+A(2)
F(1) = F(1)+D(1)
F(2) = F(2)+D(2)
B(1) = B(1)-D2X*D(1)
B(2) = B(2)-D2X*D(2)
G(1) = G(1)-A(1)*(2.*(U(5,2)-U(1,2)
+U(5,1)-U(1,1))-D(1)*(2.*U(6,2)
-U(2,2))+U(6,1)-U(2,1)-D2X*
(2.*U(3,2)+U(3,1))
G(2) = G(2)-A(2)*(2.*U(5,2)-U(1,2))+U(5,1)
-U(1,1))-D(2)*(2.*(U(6,2)-U(2,2))
+U(6,1)-U(2,1)-D2X*(2.*U(3,2)+U(3,1))
A(N4) = A(N4)+C(N4)
A(N5) = A(N5)+C(N5)
D(N4) = D(N4)+F(N4)
D(N5) = D(N5)+F(N5)
B(N4) = B(N4)+D2X*F(N4)
B(N5) = B(N5)+D2X*F(N5)
G(N4) = G(N4)+C(N4)*(2.*(U(N1,2)-U(N5,2))
+U(N1,1)-U(N5,1))+F(N4)*(2.*(U(N,2)-
U(N4,2))+U(N,1)-U(N4,1)-D2X*(2.*U(N3,2)
+U(N3,1)))
G(N,5) = G(N,5)+C(N5)*(2.*(U(N1,2)-U(N5,2))+U(N1,1)
-U(N5,1))+F(N5)+(2.*(U(N,2)-U(N4,2))
+U(N,1)-U(N4,1)-D2X*(2*U(N3,2)+U(N3,1)))
33
STAT. 50
RETURN
END


SUBR
COMPUT
SET INDEX
I = 3
FTN
DO
J = I+1
K = J/2
T1 = (U(J+2,2)-U(J-2,2))/D2X
T3 = U(I+2,2)-U(I-2,2)
T2 = T3/D2X
T4 = T1-U(I,2)
U(I,3) = 2.*U(I,2)-U(I,1)+DT2*
((EI(K)*(U(I+2,2)-2.*
U(I,2)+U(I-2,2))+.25*
(EI(K+1)-EI(K-1))*T3)/
DX2+CAYGA(K)*T4)/RI(K)
U(J,3) = 2.*U(J,2)-U(J,1)+DT2*(P(K)-
CAYB(K)*U(J,2)+CAYGA(K)*
((U(J+2,2)-2.*U(J,2)+U(J-2,2))/
DX-.5*T3)/DX+(CAYGA(K+1)-
CAYGA(K-1))*T4/D2X)/RA(K)
STAT. 1
CONTINUE
RETURN
END

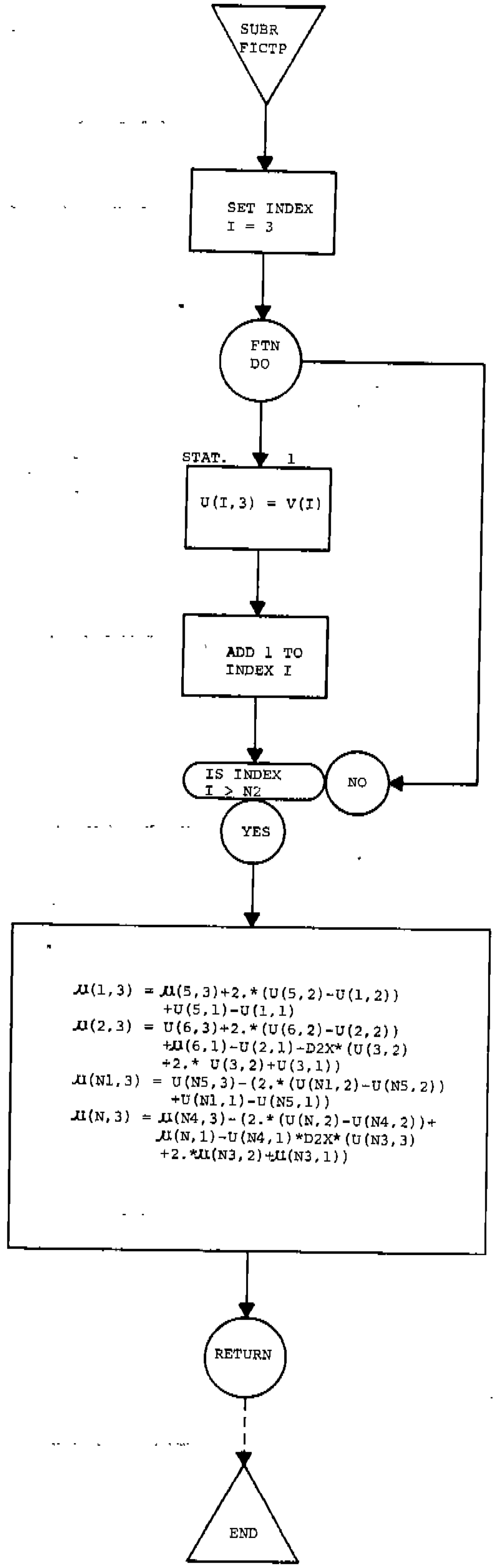

SUBR
SOLN
SET INDEX
I = 5
FTN
DO
I1 = I-1
I2 = I-2
I3 = I-3
I4 = I-4
FM = A(I1)/A(I2)
B(I1) = B(I1)-FM*B(I2)
C(I1) = C(I1)-FM*C(I2)
D(I1) = D(I1)-FM*D(I2)
E(I1) = E(I1)-FM*E(I2)
F(I1) = F(I1)-FM*F(I2)
G(I1) = G(I1)-FM*6(I2)
FM = F(I4)/F(I3)
B(I4) = B(I4)-FM*B(I3)
C(I4) = C(I4)-FM*C(I3)
D(I4) = D(I4)-FM*D(I3)
E(I4) = E(I4)-FM*E(I3)
G(I4) = G(I4)-FM*G(I3)
STAT. 1
CONTINUE
34
SEE PAGE
00.29.00


34
E(1) = E(1)/B(1)
C(1) = C(1)/B(1)
G(1) = G(1)/B(1)
E(2) = E(2)-E(1)*B(2)
C(2) = C(2)-C(1)*B(2)
G(2) = G(2)-G(1)*B(2)
B(2) = 0.0
SET INDEX
K = 1
FTN
DO
36
K1 = K+1
K2 = K+2
K3 = K+3
E(K1) = E(K1)-E(K)*B(K1)
C(K1) = (C(K1)-C(K)*B(K1))/E(K1)
F(K1) = F(K1)/E(K1)
G(K1) = (G(K1)-G(K)*B(K1))/E(K1)
D(K2) = D(K2)-E(K)*A(K2)
B(K2) = B(K2)-C(K)*A(K2)-C(K1)*D(K2)
G(K2) = (G(K2)-G(K)*A(K2)-G(K1)*D(K2))/B(K2)
E(K2) = (E(K2)-F(K1)*D(K2))/B(K2)
C(K2) = C(K2)/B(K2)
B(K3) = B(K3)-C(K1)*D(K3)
E(K3) = E(K3)-F(K1)*D(K3)
35
SEE PAGE
00.30.00

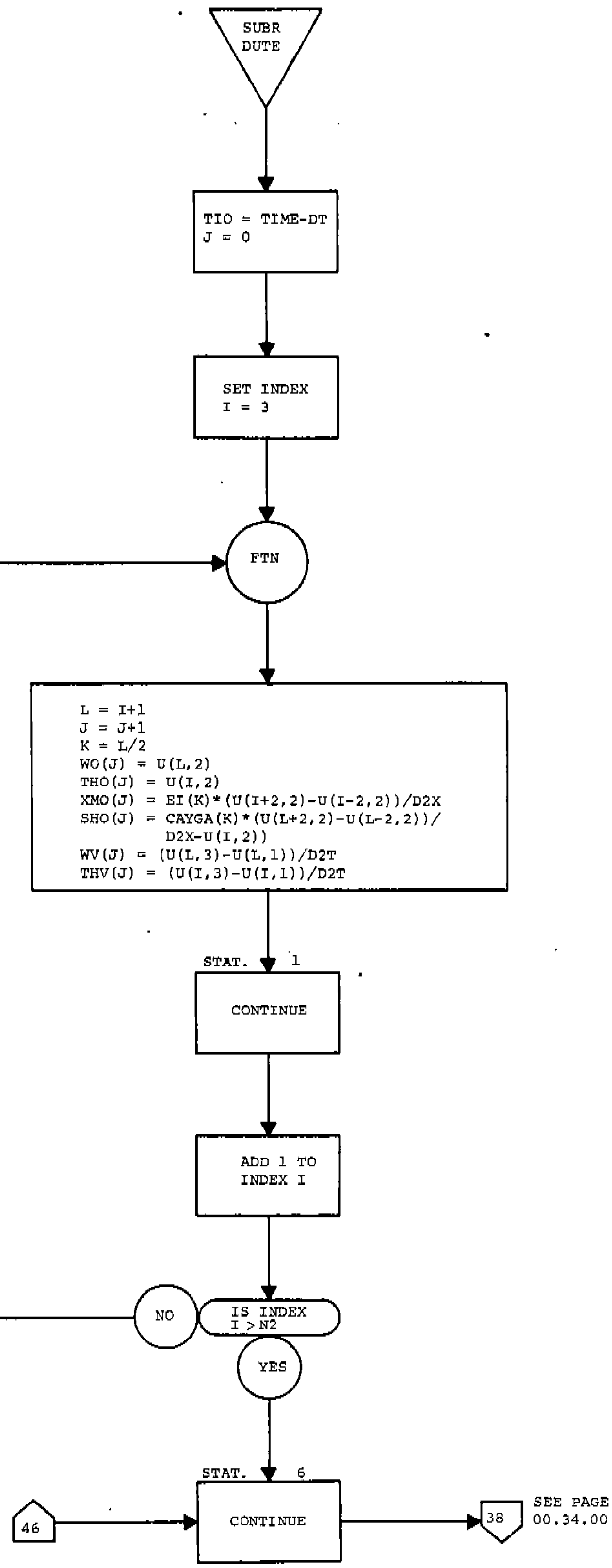

35
STAT. 2
G(K3) = G(K3)-G(K1)*D(K3)
ADD 2 TO INDEX K
IS INDEX K > N7
NO
36
SEE PAGE 00.29.00
YES
E(N4) = E(N4)-E(N5)*B(N4)
G(N4) = (G(N4)-G(N5)*B(N4))/E(N4)
V(N2) = G(N4)
V(N3) = G(N5)-E(N5)*V(N2)
SET INDEX I = 4
FTN DO
36
J = N4-I+4
V(J) = G(J-2)-F(J-2)*V(J-2)
-C(J-2)*V(J+1)
V(J-1) = G(J-3)-C(J-3)*V(J+1)
-E(J-3)*V(J)
37
SEE PAGE 00.31.00

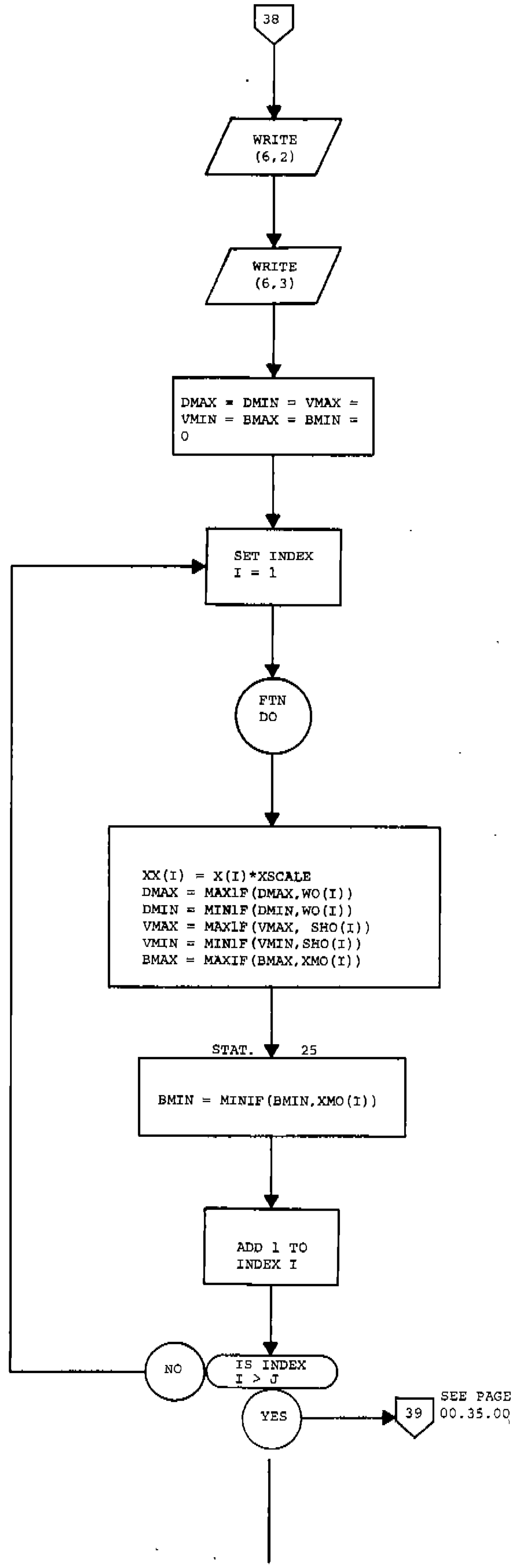


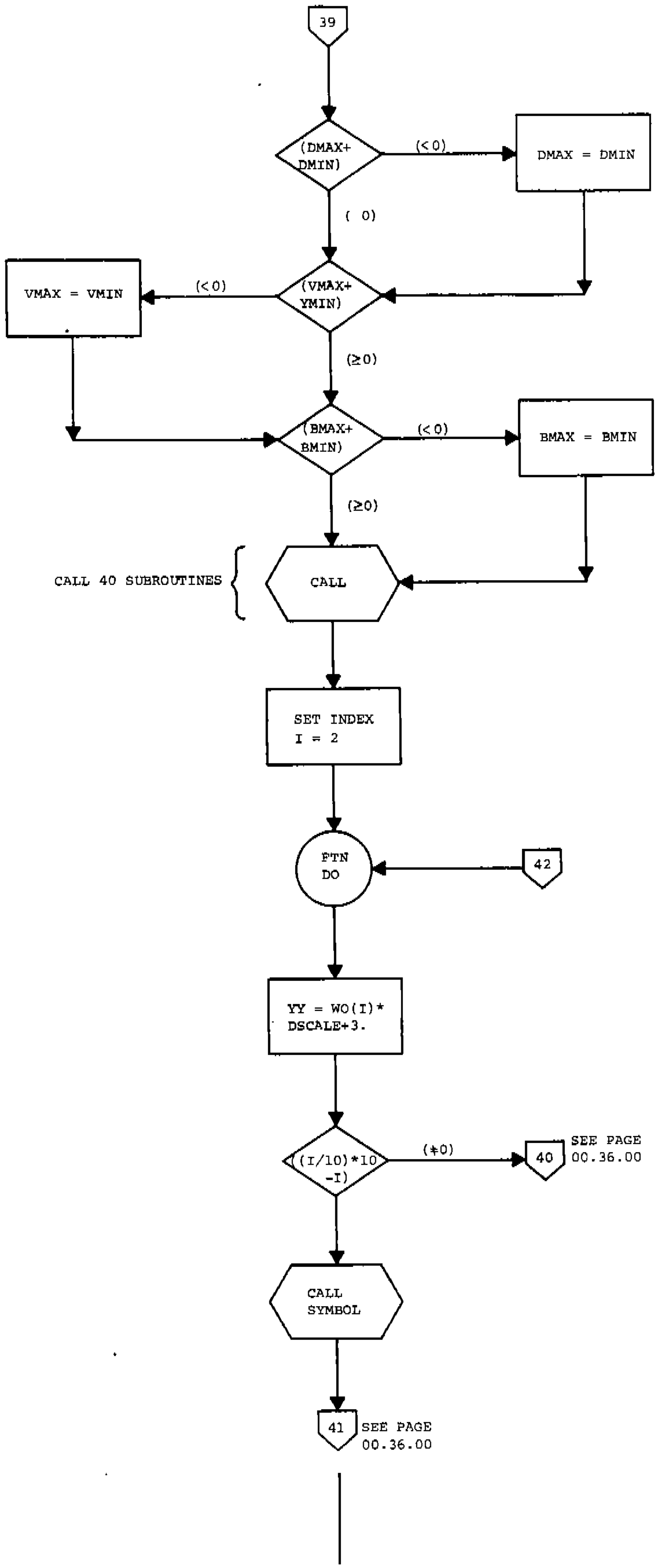
37
(ABS(V(J))+
ABS(V(J-1))-SC)
(<0)
(≥0)
STAT. 4
CALL
EROR(1)
STAT. 3
CONTINUE
ADD 2 TO
INDEX I
IS INDEX
I > N4
NO
YES
36
SEE PAGE
00.30.00
RETURN
END

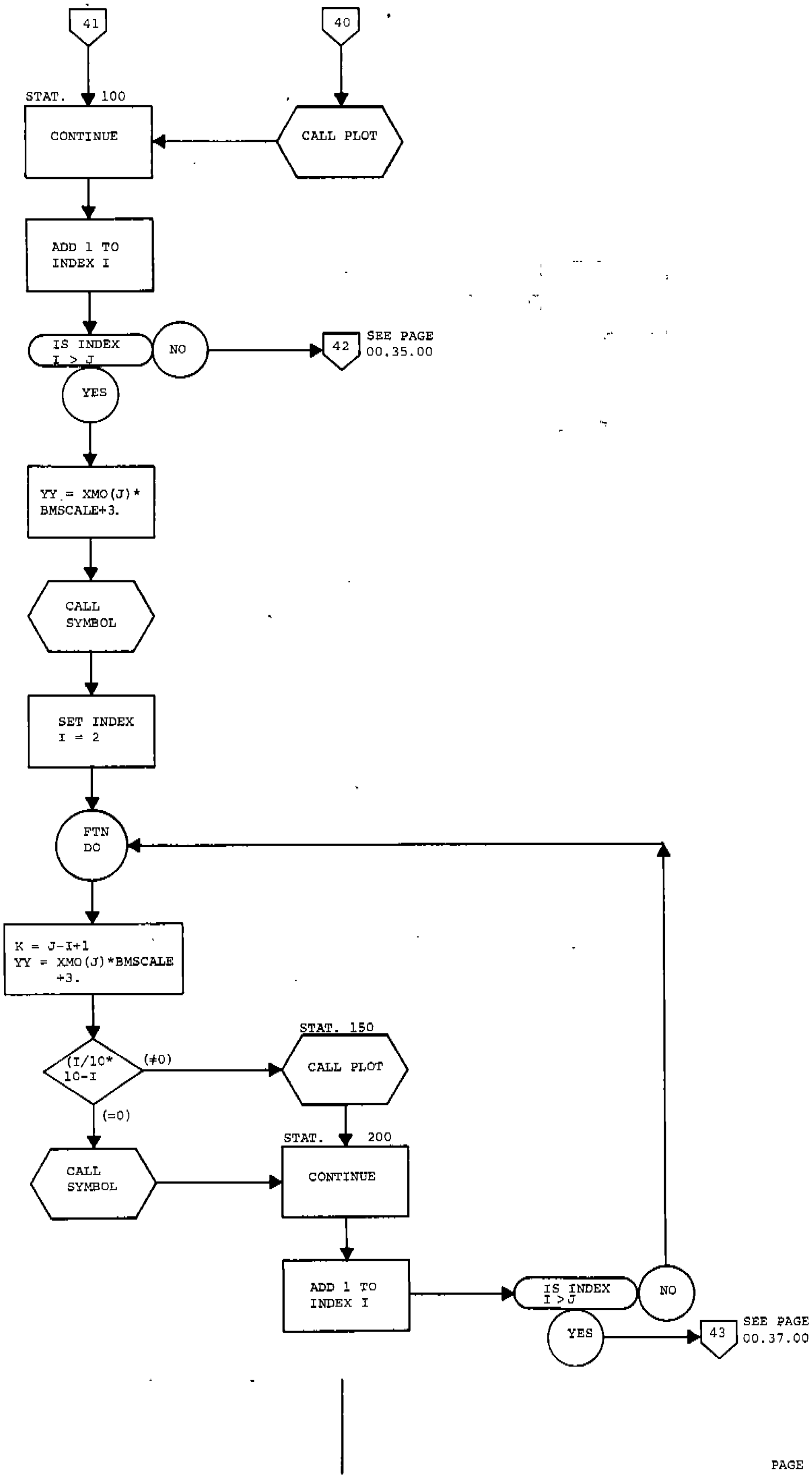


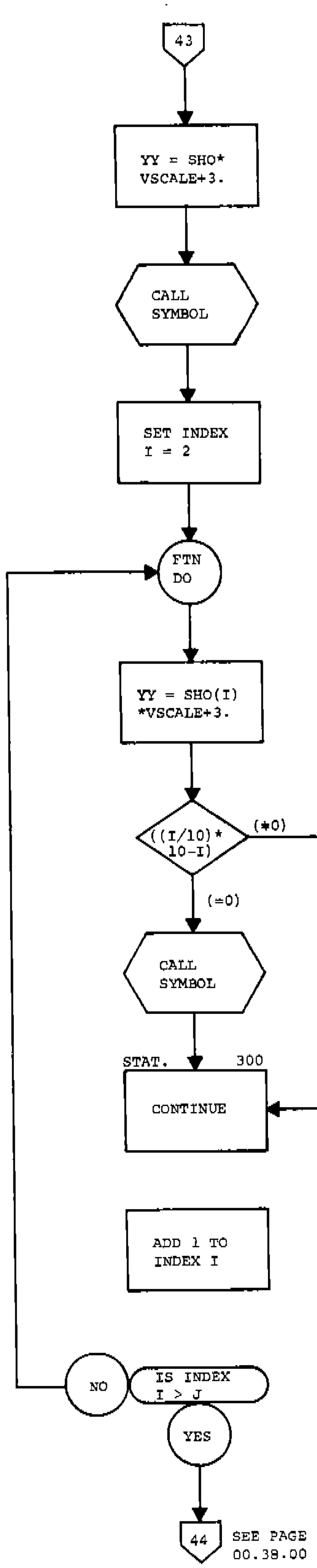
SUBR
FICTP
SET INDEX
I = 3
FTN
DO
STAT. 1
U(I,3) = V(I)
ADD 1 TO
INDEX I
IS INDEX
I > N2
NO
YES
U(1,3) = U(5,3)+2.*(U(5,2)-U(1,2))
+U(5,1)-U(1,1)
U(2,3) = U(6,3)+2.*(U(6,2)-U(2,2))
+U(6,1)-U(2,1)-D2X*(U(3,2)
+2.* U(3,2)+U(3,1))
U(N1,3) = U(N5,3)-(2.*(U(N1,2)-U(N5,2))
+U(N1,1)-U(N5,1))
U(N,3) = U(N4,3)-(2.*(U(N,2)-U(N4,2))+
U(N,1)-U(N4,1)*D2X*(U(N3,3)
+2.*U(N3,2)+U(N3,1))
RETURN
END

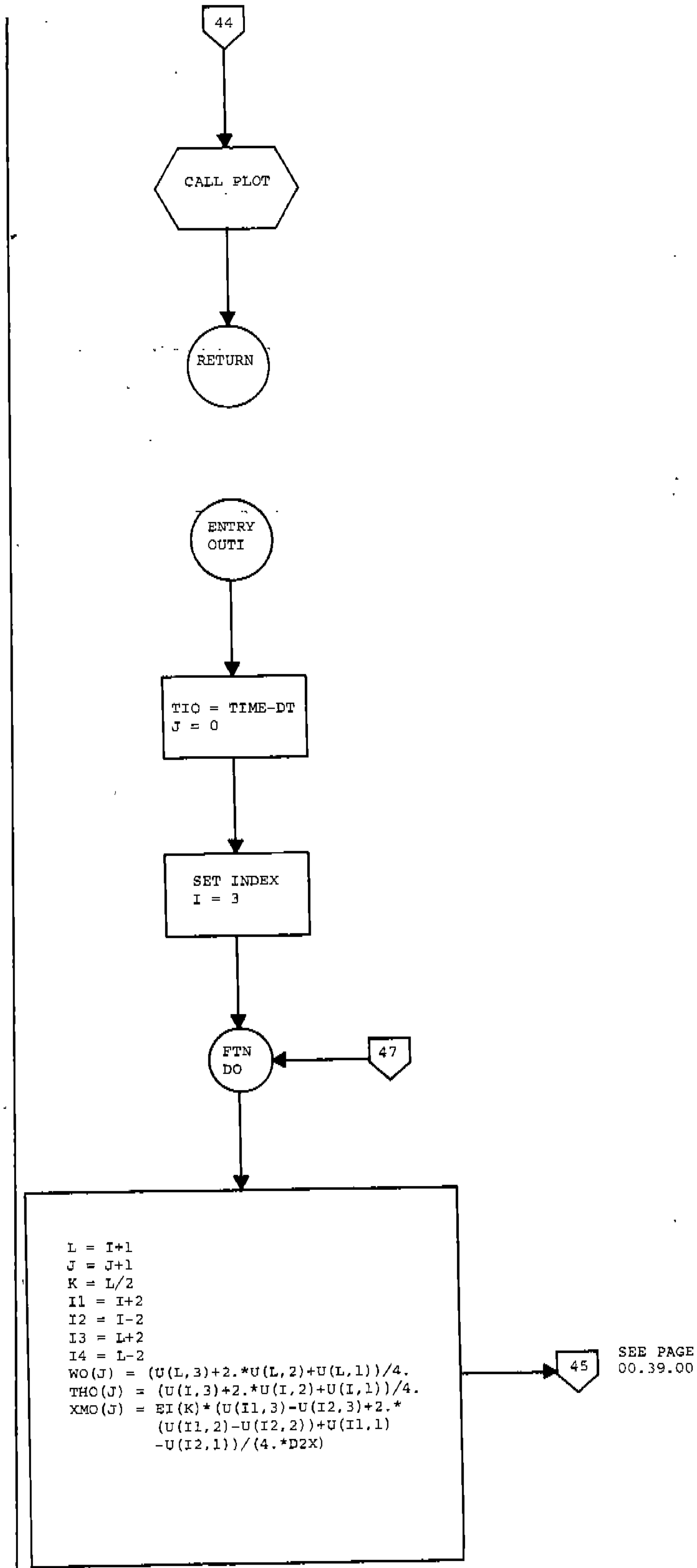

SUBR DUTE
TIO = TIME-DT
J = 0
SET INDEX
I = 3
FTN
L = I+1
J = J+1
K = L/2
WO(J) = U(L,2)
THO(J) = U(I,2)
XMO(J) = EI(K)*(U(I+2,2)-U(I-2,2))/D2X
SHO(J) = CAYGA(K)*(U(L+2,2)-U(L-2,2))/
D2X-U(I,2))
WV(J) = (U(L,3)-U(L,1))/D2T
THV(J) = (U(I,3)-U(I,1))/D2T
STAT. 1
CONTINUE
ADD 1 TO
INDEX I
NO
IS INDEX
I > N2
YES
STAT. 6
46
CONTINUE
38
SEE PAGE
00.34.00

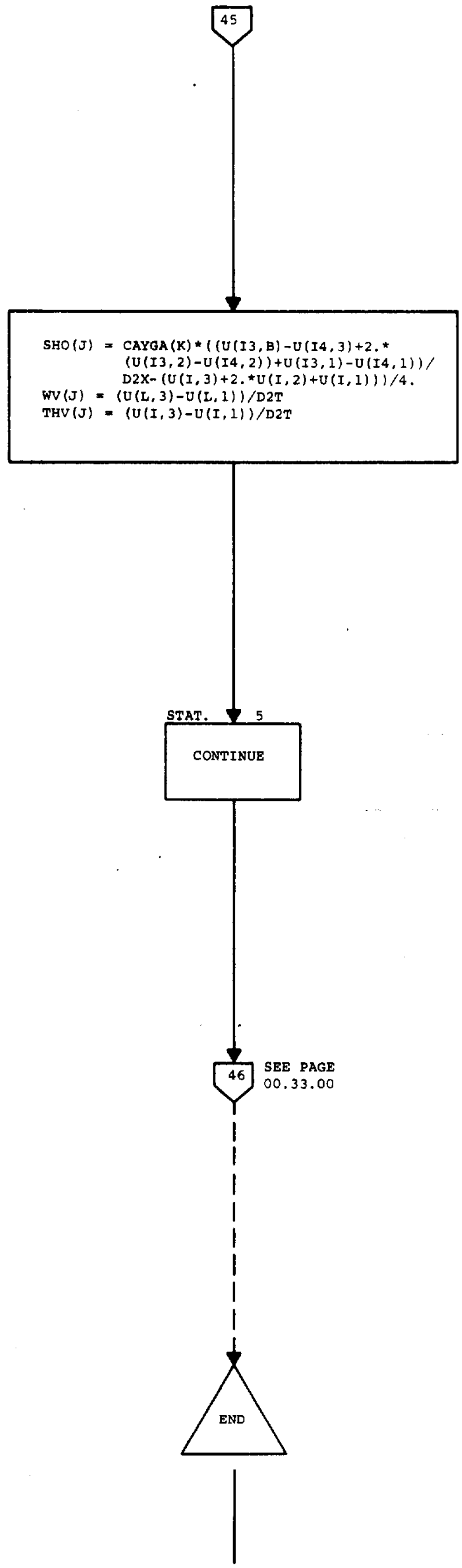

38
WRITE (6,2)
WRITE (6,3)
DMAX = DMIN = VMAX = VMIN = BMAX = BMIN = 0
SET INDEX I = 1
FTN DO
XX(I) = X(I)*XSCALE
DMAX = MAX1F(DMAX,WO(I))
DMIN = MIN1F(DMIN,WO(I))
VMAX = MAX1F(VMAX, SHO(I))
VMIN = MIN1F(VMIN,SHO(I))
BMAX = MAXIF(BMAX,XMO(I))
STAT. 25
BMIN = MINIF(BMIN,XMO(I))
ADD 1 TO INDEX I
IS INDEX I > J
NO
YES
39
SEE PAGE 00.35.00


39
(DMAX+
DMIN)
(< 0)
DMAX = DMIN
(0)
VMAX = VMIN
(< 0)
(VMAX+
YMIN)
(≥0)
(BMAX+
BMIN)
(< 0)
BMAX = BMIN
(≥0)
CALL 40 SUBROUTINES
CALL
SET INDEX
I = 2
FTN
DO
42
YY = WO(I)*
DSCALE+3.
(I/10)*10
-I)
(≠0)
40
SEE PAGE
00.36.00
CALL
SYMBOL
41
SEE PAGE
00.36.00


41
40
STAT. 100
CONTINUE
CALL PLOT
ADD 1 TO INDEX I
IS INDEX I > J
NO
42
SEE PAGE 00.35.00
YES
YY = XMO(J)* BMSCALE+3.
CALL SYMBOL
SET INDEX I = 2
FTN DO
K = J-I+1
YY = XMO(J)*BMSCALE +3.
(I/10* 10-I
(≠0)
(=0)
STAT. 150
CALL PLOT
STAT. 200
CALL SYMBOL
CONTINUE
ADD 1 TO INDEX I
IS INDEX I > J
NO
YES
43
SEE PAGE 00.37.00


43
YY = SHO*
VSCALE+3.
CALL
SYMBOL
SET INDEX
I = 2
FTN
DO
YY = SHO(I)
*VSCALE+3.
((I/10)*
10-I)
(≠0)
CALL PLOT
(=0)
CALL
SYMBOL
STAT. 300
CONTINUE
ADD 1 TO
INDEX I
NO
IS INDEX
I > J
YES
44
SEE PAGE
00.38.00


44
CALL PLOT
RETURN
ENTRY
OUTI
TIO = TIME-DT
J = 0
SET INDEX
I = 3
FTN
DO
47
L = I+1
J = J+1
K = L/2
I1 = I+2
I2 = I-2
I3 = L+2
I4 = L-2
WO(J) = (U(L,3)+2.*U(L,2)+U(L,1))/4.
THO(J) = (U(I,3)+2.*U(I,2)+U(I,1))/4.
XMO(J) = EI(K)*(U(I1,3)-U(I2,3)+2.*
(U(I1,2)-U(I2,2))+U(I1,1)
-U(I2,1))/(4.*D2X)
45
SEE PAGE
00.39.00


45
SHO(J) = CAYGA(K)*((U(I3,B)-U(I4,3)+2.*
(U(I3,2)-U(I4,2))+U(I3,1)-U(I4,1))/
D2X-(U(I,3)+2.*U(I,2)+U(I,1)))/4.
WV(J) = (U(L,3)-U(L,1))/D2T
THV(J) = (U(I,3)-U(I,1))/D2T
STAT. 5
CONTINUE
46
SEE PAGE
00.33.00
END

APPENDIX J

RESULTS

The distribution of vertical shear, bending moment and vertical deflection for the specific case of a hull rigidity corresponding to 75 percent of normal and of a unit impulse having a duration of 0.1 sec and applied at the forward quarter point are plotted Figs. J-1 thru J-44 for consecutive instants of time. The values shown on the ordinate scale are the maxima for the instant shown.

To inquire into maximum bending moments experienced regardless of time at which occurring, profiles of the envelopes of such maximum values were derived. The envelopes for the case corresponding to the figures to which reference has already been made are shown in Fig. 1 of the text.

To bring out the influence of hull rigidity cross plots of maximum values of bending moment at four important locations are presented in Fig. 2 of the text. The conclusion to be drawn from this figure is that an increase in hull flexibility tends to reduce bending moment. At the bow and amidships the reduction in maximum bending moment is close to the square root of the ratio of hull rigidities, but at the quarter points this reduction is considerably less.

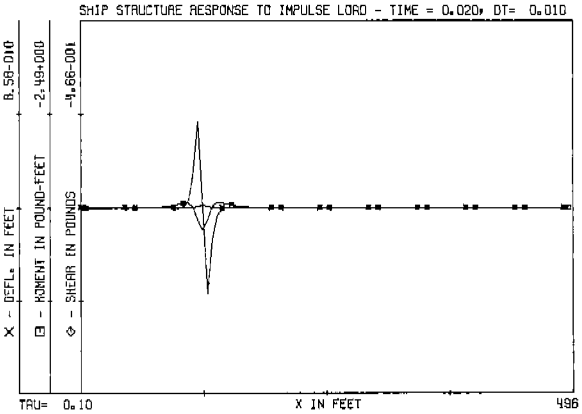


Fig. J-1

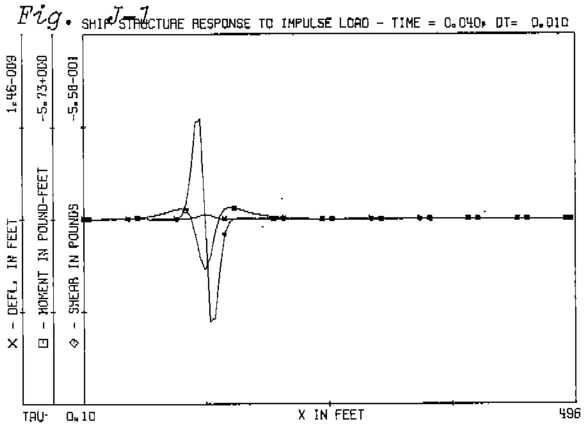


Fig. J-2

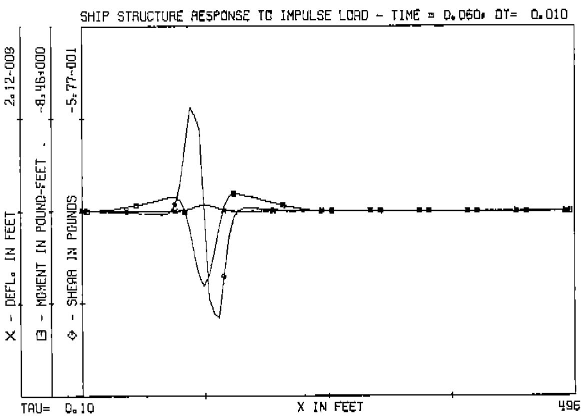


Fig. J-3

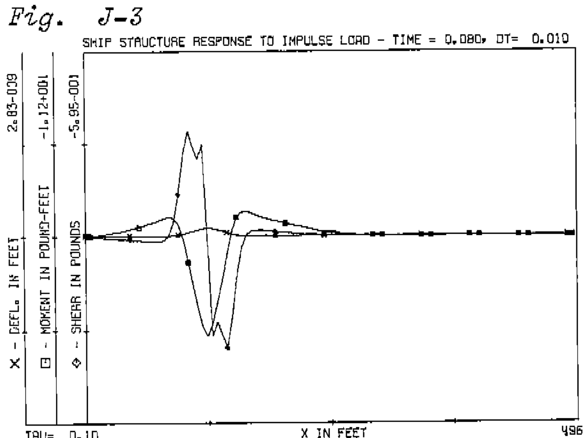


Fig. J-4

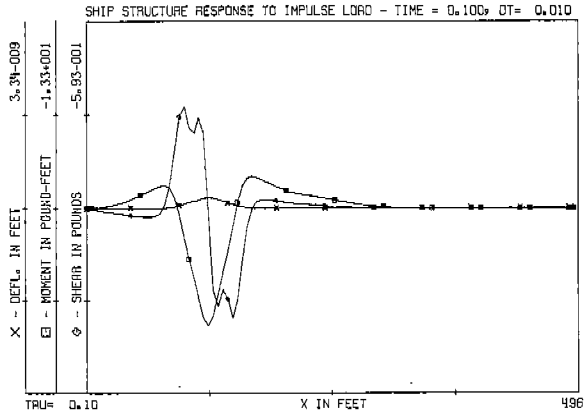


Fig. J-5

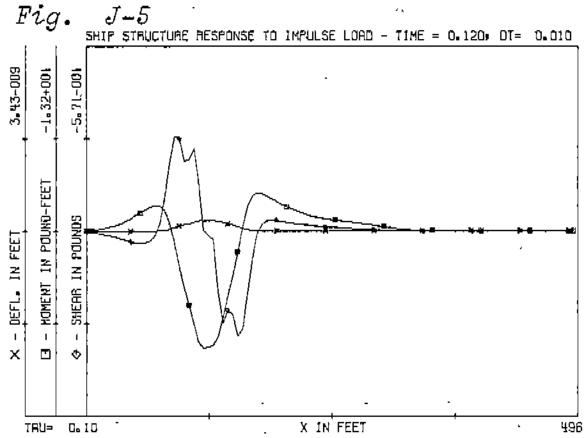


Fig. J-6

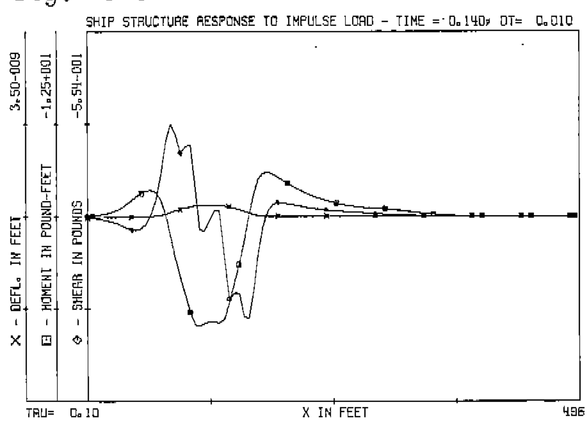


Fig. J-7

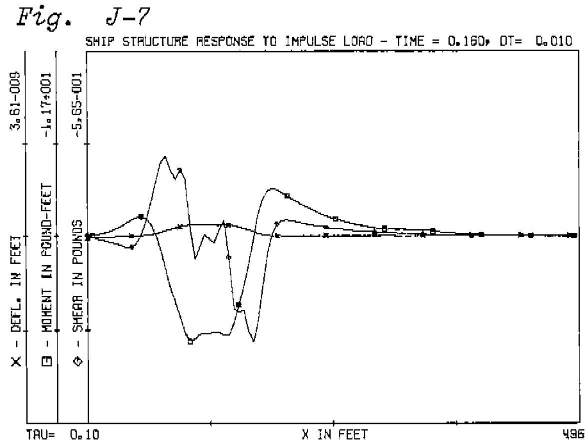


Fig. J-8

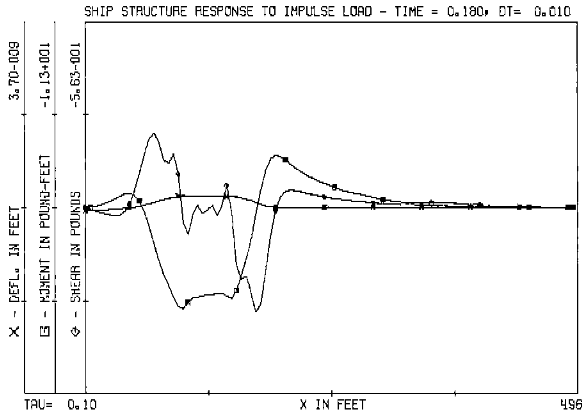


Fig. J-9

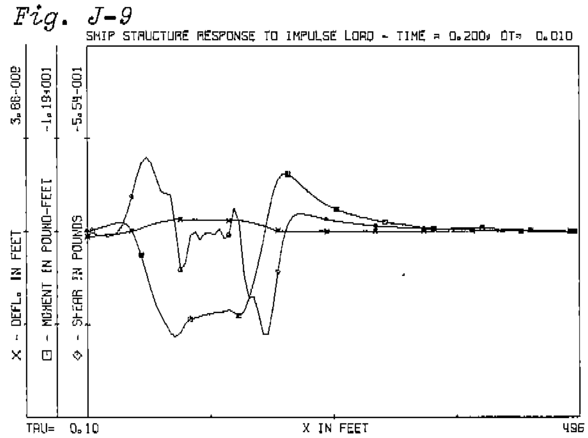


Fig. J-10

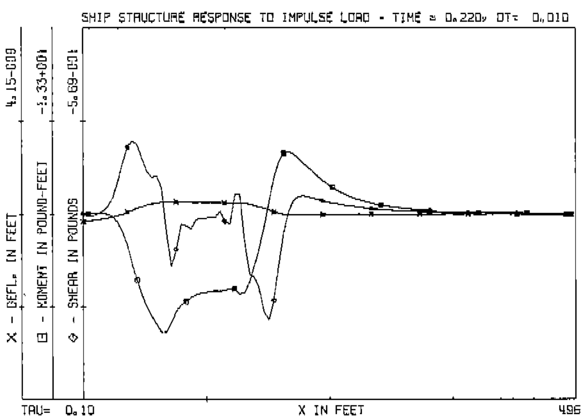


Fig. J-11

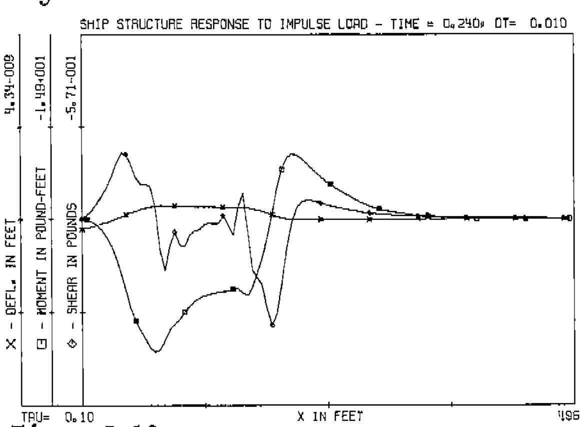


Fig. J-12

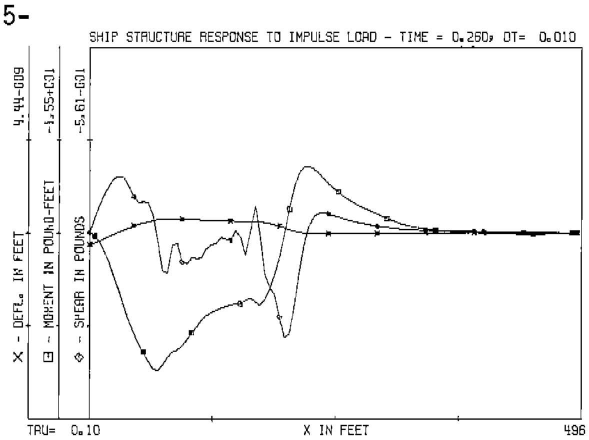


Fig. J-13

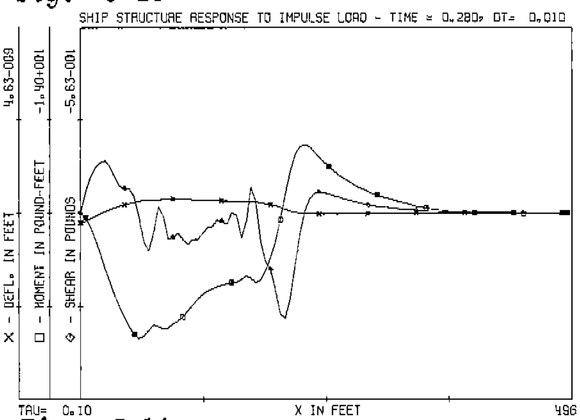


Fig. J-14

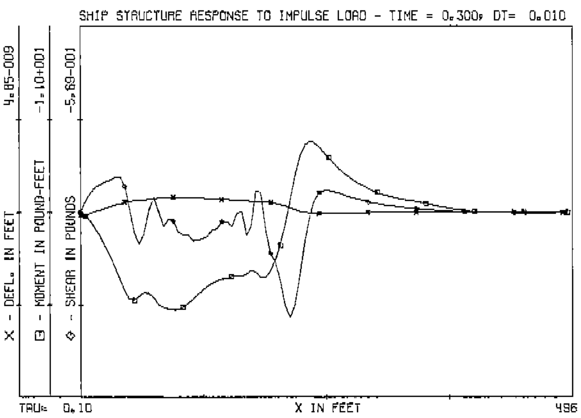


Fig. J-15

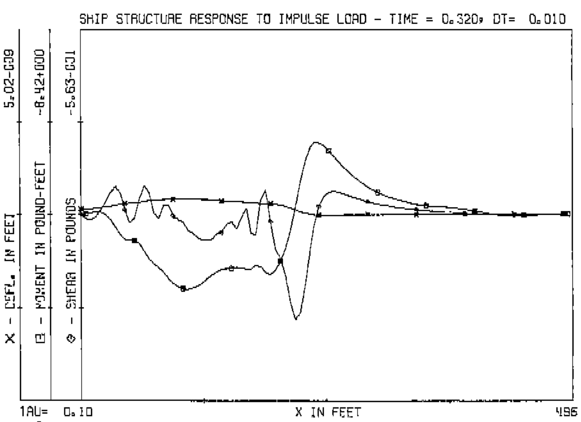


Fig. J-16

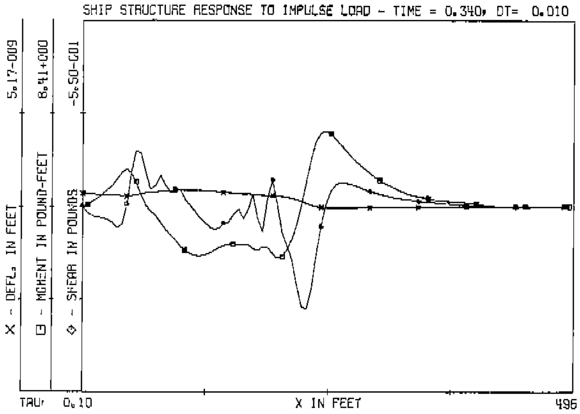


Fig. J-17

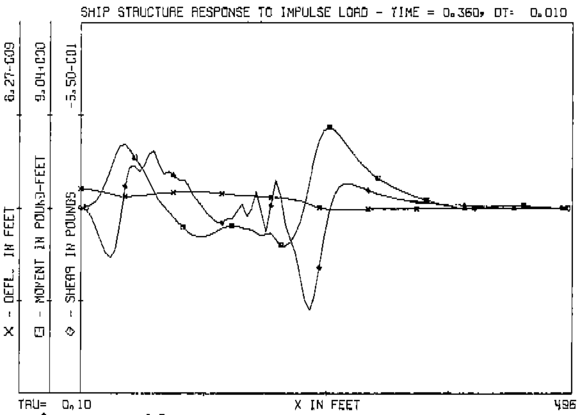


Fig. J-18

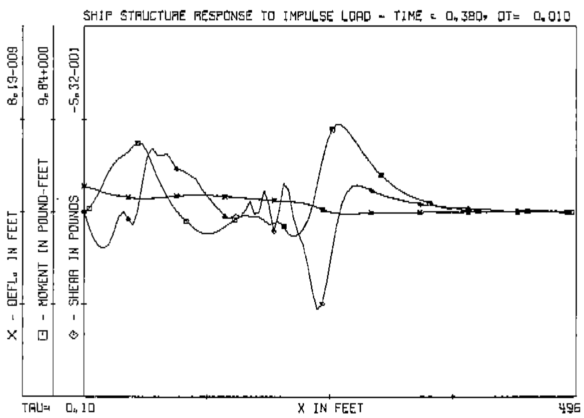


Fig. J-19

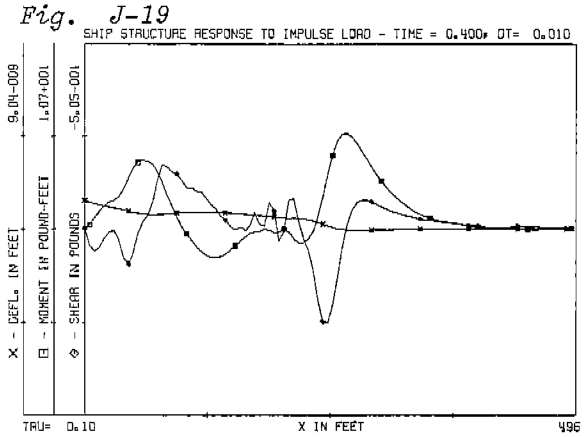


Fig. J-20

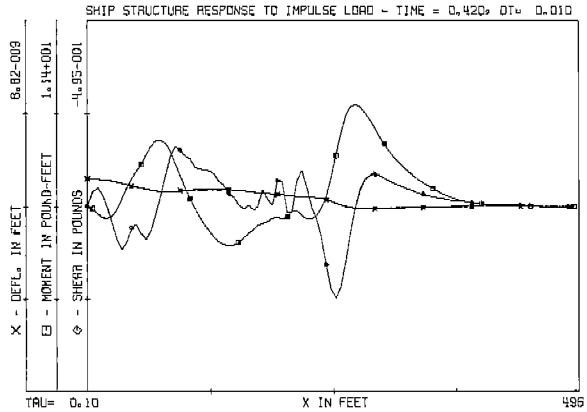


Fig. J-21

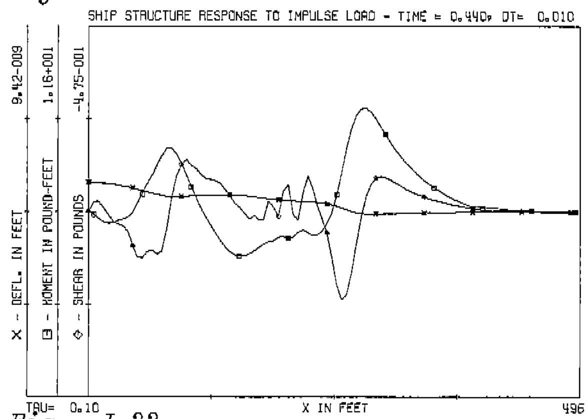


Fig. J-22

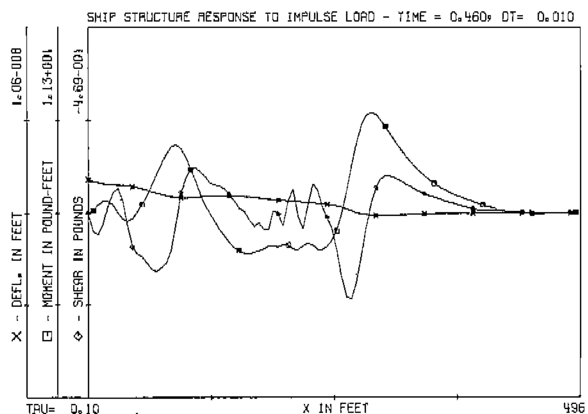


Fig. J-23

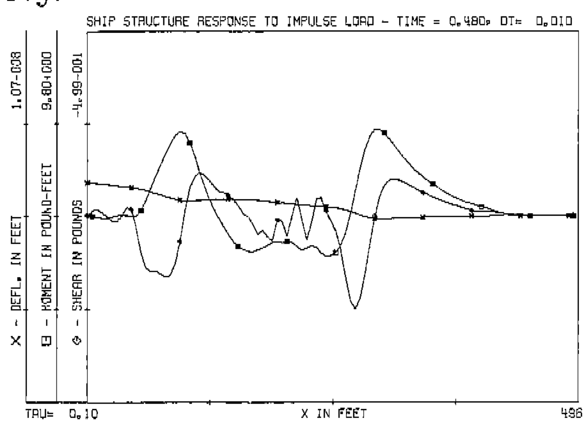


Fig. J-24

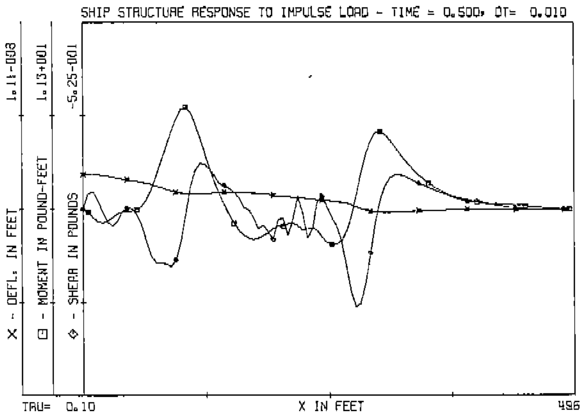
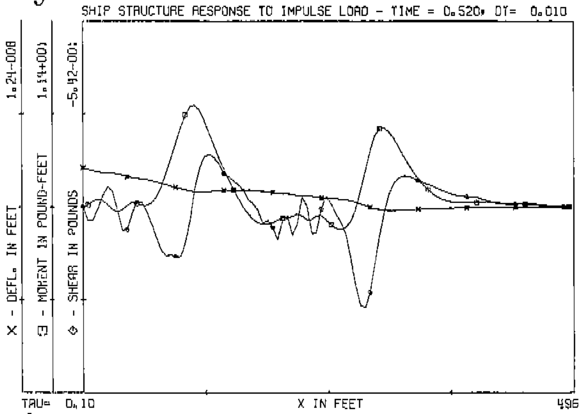


Fig. J-25



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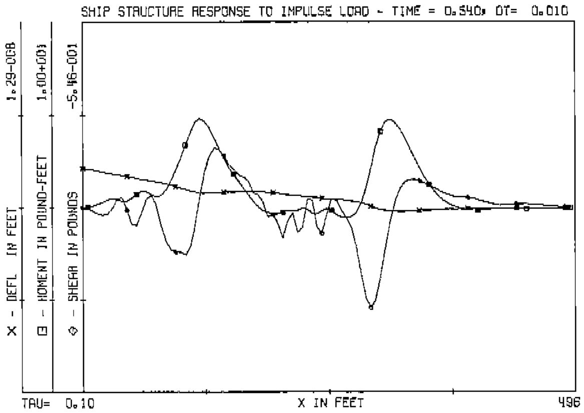

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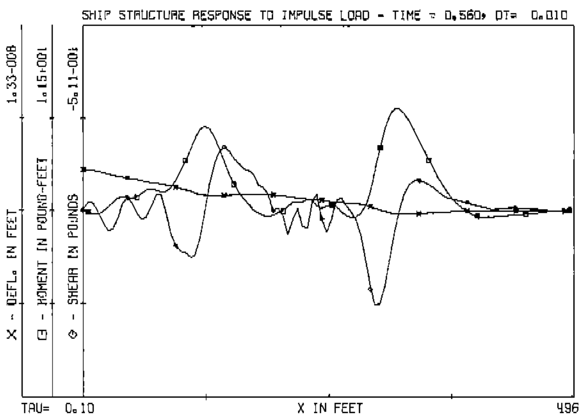

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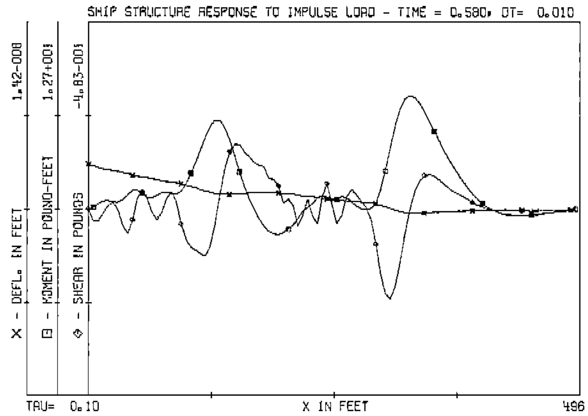

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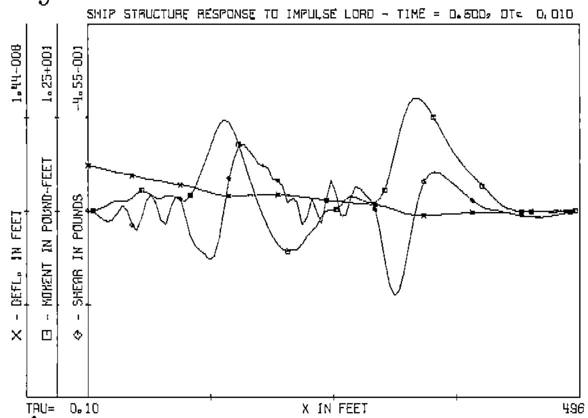

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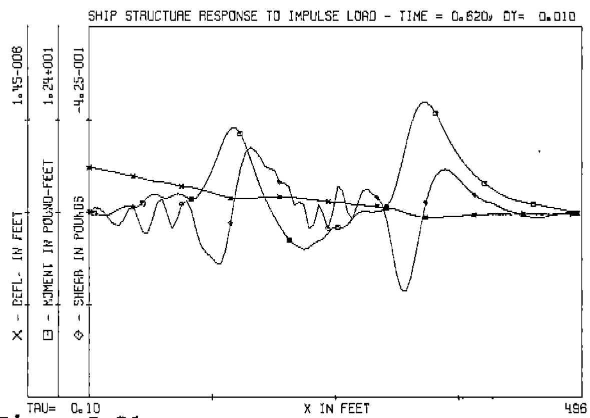

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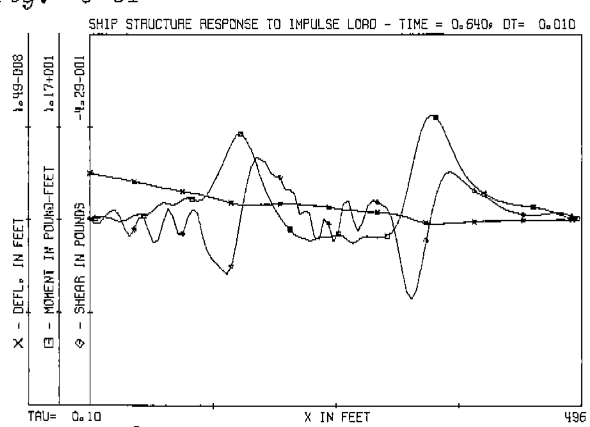

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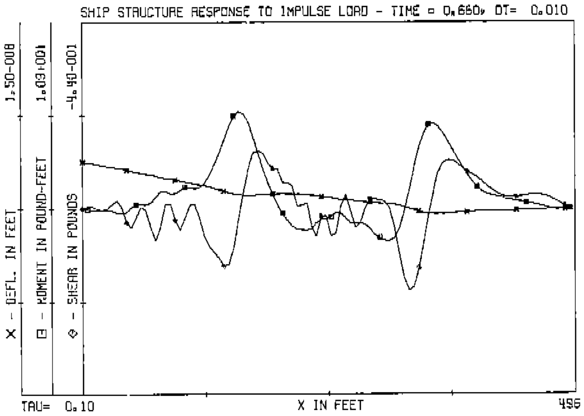

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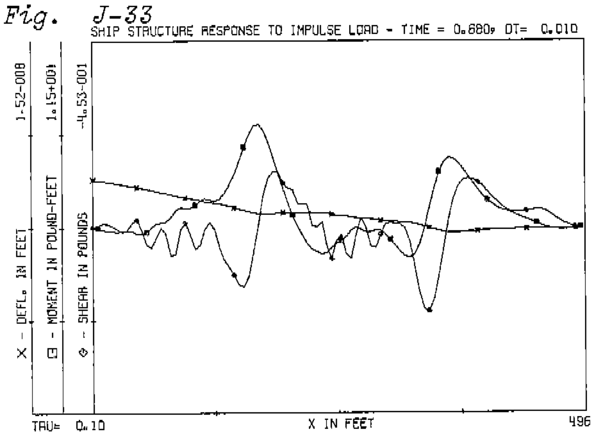

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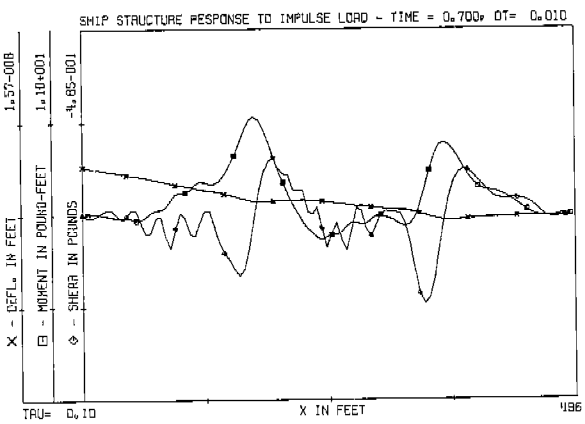

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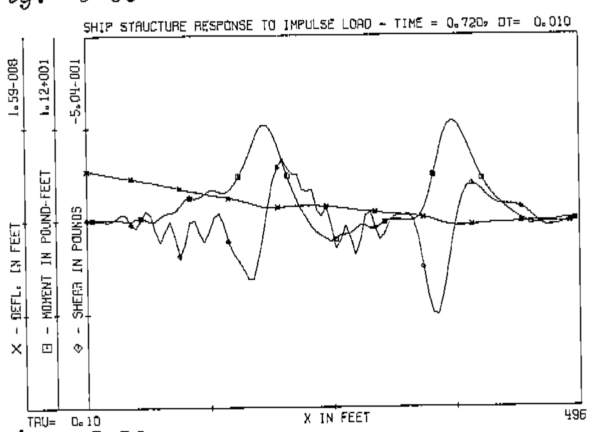

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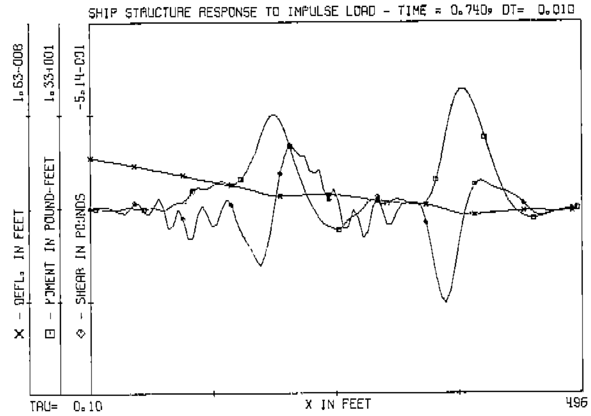

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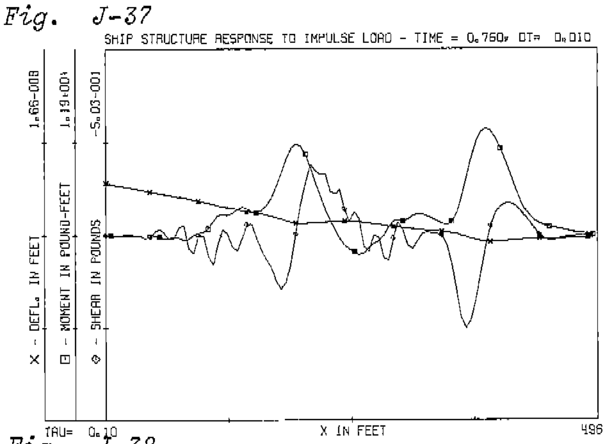

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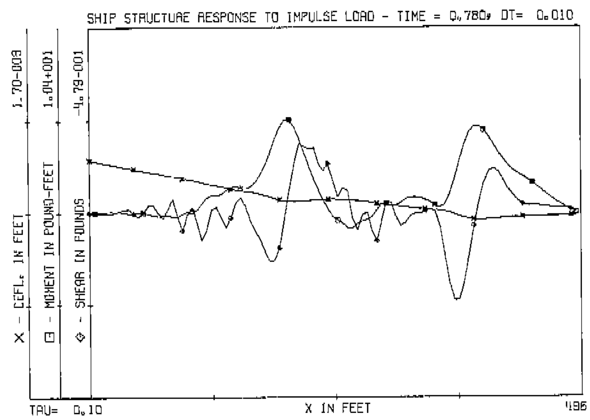

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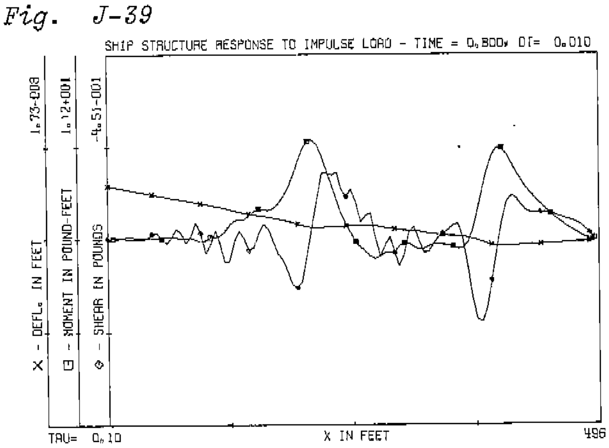

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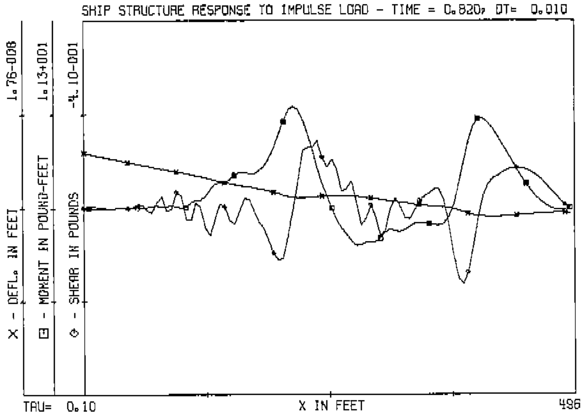

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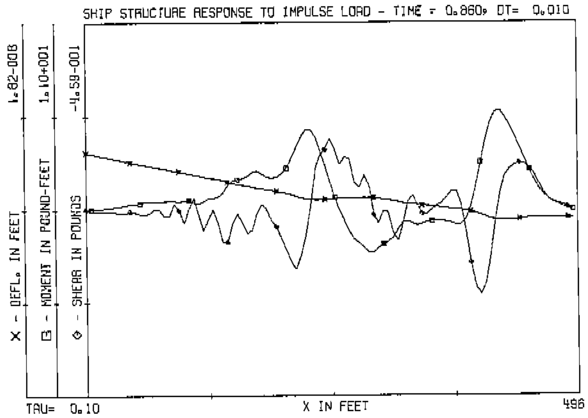

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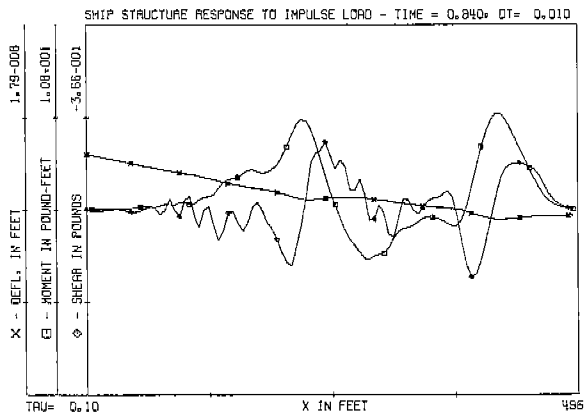

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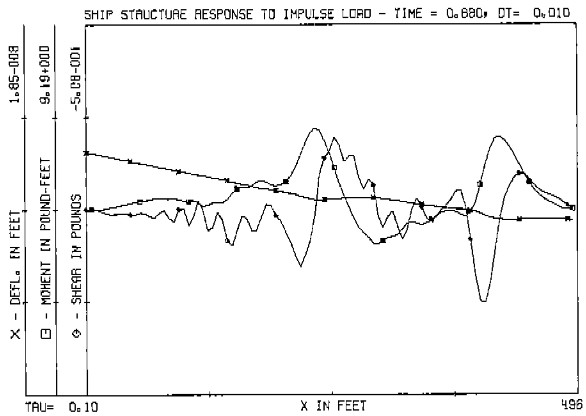

Fig. J-44

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13. ABSTRACT

The purpose of the study is to set up a computer program to investigate the dynamic effects resulting from an impulsive loading on a ship and to determine how these effects tend to vary with the stiffness of the hull girder. The hull is treated as a Timoshenko beam and the solution is obtained by finite difference technique. Two codes are written: an implicit one, which is more efficient for short durations, and an implicit one, which is superior for long durations of impulse. Application is made to a dry cargo ship. Limited analysis of her response to a unit impulse indicates that, in general, reduced hull rigidity tends to be beneficial.

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