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DEVELOPMENT OF MATHEMATICAL MODELS FOR DESCRIBING SHIP STRUCTURAL RESPONSE IN WAVES

SHIP STRUCTURE COMMITTEE

January 1969

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ADDRESS CORRESPONDENCE TO:
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January 1969

Dear Sir:

In order to improve the analytical tools for calculation of loads on a ship's hull, the Ship Structure Committee is sponsoring a project to develop a computer simulation of a ship in waves. Herewith is a first technical progress report entitled *Development of Mathematical Models For Describing Ship Structural Response In Waves* by Paul Kaplan of Oceanics, Inc.

This report is being distributed to individuals and groups associated with or interested in the work of the Ship Structure Committee. Comments concerning this report are solicited.

Sincerely,

D. B. Henderson

D. B. Henderson
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure Committee

SSC-193

Progress Report

on

Project 174

"Ship Computer Response"

to the

Ship Structure Committee

DEVELOPMENT OF MATHEMATICAL MODELS FOR DESCRIBING SHIP STRUCTURAL RESPONSE IN WAVES

by

Paul Kaplan
Oceanics, Inc.

under

Department of the Navy
Naval Ship Engineering Center
Contract NObs-94322

U. S. Coast Guard Headquarters
Washington, D. C.

January 1969

ABSTRACT

Available mathematical models that describe ship-wave interactions are evaluated in order to develop a technique for predicting ship structural response characteristics. Major consideration is given to the bending moment and slamming responses for an arbitrary ship form in any state of sea, at any relative heading and forward speed. The slowly varying vertical and lateral bending moments due to waves are obtained using a linearized model based on strip theory, where the effect of roll motion and its influence in the lateral plane are included, with the model sufficiently general to also allow extension to computation of torsional moments due to waves. Comparison of the results of a limited series of hand computations with available experimental data indicates a good degree of agreement, as well as an overall consistency, for the analysis of wave-induced bending moments. A mathematical representation of the bending moment due to slamming is also described, and computational procedures for obtaining an output compatible with the wave-induced bending moment are outlined. Methods of analysis in terms of power spectra as well as time histories are considered, and the utility of different types of computers for presentation of information on ship structural response is described.

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INTRODUCTION

The effective design of ship structures requires knowledge of the various loads that the ship will experience during its intended service. Accordingly great emphasis has been placed upon obtaining information on the bending moments acting on ships at sea. This large interest in bending moment evaluation for ships in various sea states is primarily due to its influence as the main hull girder structural response. The resulting stresses on a particular ship structure can be determined from knowledge of the bending moment and the nature of the basic structural characteristics of the ship itself. Model tests in waves have been carried out many times (e.g. [1], [2]) to determine the magnitudes of the vertical bending moment in head seas, which has been assumed (in most of these cases) to be the most critical condition. In addition, full scale tests of instrumented ships at sea have been carried out in order to measure the bending moments under various sea conditions (see [3] and [4]). Considering the level of effort and expense for such experimental procedures, with the various possible weight distributions for an operational ship, it can be seen that this is indeed an expensive and time-consuming procedure. In view of this prime aspect, it would be more efficient to evaluate the bending moment computationally, using modern high speed computers, if an appropriate set of equations were available for that purpose.

Analytical techniques already exist for determining the vertical bending moment acting on ships in regular head seas [5], and a computational technique for hand calculations has also been formulated [6]. These procedures are based upon the fundamental strip theory methods developed for determining the motions of heave and pitch in waves [7], and have the same general degree of validity as the motion calculations. Thus an available mathematical technique exists, which has demonstrated a fair degree of applicability, for evaluating the vertical bending moments in regular head sea waves.

With the more recent emphasis on motions at oblique headings to waves in model towing tanks, which tends to approach more realistic conditions at sea by taking account of ship heading relative to the waves, there has been some interest in lateral bending moments as well [8]. From the point of view of structural response, as measured by the stress level at various points on the ship, it appears that the combined effects of the vertical and lateral moments are most significant in representing the actual stress level on a ship. Thus a method for computation of lateral bending moments in regular oblique waves has been developed for this purpose ([9], [10]). Furthermore, the necessity of developing a mathematical procedure for computing the vertical bending moments on ships at oblique headings to regular waves is evident if proper representations of the actual loads under those conditions is required.

In addition to the direct bending moment developed by the wave action, in conjunction with inertia forces and hydrodynamic forces due to the ship rigid body responses (i.e. pitch, heave, sway, etc.), additional contributions to the bending moment are provided by "slamming" type motions of the bow, which result in "whipping" motion of the ship structure. The significant contribution to the bending moment and resulting stress from this latter effect is primarily due to the effect of the lowest natural mode of vibration of the hull-girder system [11]. Thus the direct wave-induced bending moments are slowly varying, essentially reflecting the frequency content (i.e. bandwidth) contained in the wave encounter spectra, while the bending moments due to the slamming are higher frequency phenomena that evidence the dynamic elastic response characteristics of the ship structure.

Questions of linearity or nonlinearity of the ship motions, hydrodynamic forces, and the resultant structural response characteristics are ever present in consideration of the many effects that result in a final measure of the bending moment response. Certain specific tests ([12], [13]) in varied wave systems of extreme proportions have shown that the direct wave-induced bending moments are linear with respect to wave height, while mathematical simulations [11] (as well as physical reasoning of the processes involved) have indicated that the effects of slamming impulses due to relative motion of the ship bow in a wave system will behave in a nonlinear fashion. These results therefore indicate that it is possible that different means of analysis can (or should) be applied to each of these constituent elements of a bending moment response, and hence the most useful final form of the representation of ship response (i.e. whether in spectral form, or time history, etc.) is not readily known for analytical purposes. However, the methods for determining response output, and from this output to determine extreme values, are the vital issues that have to be resolved in applying mathematical modeling and computer simulation for purposes of ship structural design.

In view of the importance of developing a mathematical model for simulation of ship structural response by use of computers, the present study is devoted to selection of appropriate equation systems that will adequately describe ship structural response in waves. The adequacy of the selected systems will be demonstrated by means of limited hand calculations for specific cases where experimental data is available for comparison purposes. Recognizing the level of effort that has already been expended in formulating certain mathematical models for particular restricted conditions (such as head seas, neglect of various degrees of freedom, etc.), these available methods will be extended, as necessary, in order to increase the range of applicability and validity of the final model. A description of the procedures and results obtained in this program is presented in the present report.

This work was carried out at Oceanics, Inc. for the Ship Structure Committee by means of Naval Ship Systems Command Contract NObs-94322, Project Serial No. SF013-03004, Task 2022, SR-174.

ASSUMPTIONS AND TECHNIQUES USED IN ANALYSIS

The problem of bending moment determination will be divided into an analysis of two separate processes; i.e. the bending moment directly induced by the waves, and the bending moment arising from slamming responses. The first quantity, that directly due to the waves, is essentially a steady state phenomenon which can be represented fairly accurately as a linear process, in accordance with previous experimental and analytical studies ([12], [6]). The slamming responses are primarily of a transient nature, and are significantly affected by nonlinearity.

Considering the direct bending moment effects due to motion in waves, it is necessary to determine the ship rigid body motion responses to a wave input, and all degrees of freedom except surge motion will be included since surge is not expected to have a significant effect on the bending moment value. The motions of heave and pitch are coupled to each other, and essentially only influence the vertical bending moment, as demonstrated in [6], while the motions of sway, yaw and roll are linearly coupled with each other and will be important for determination of the lateral bending moment ([9], [10]). Roll motion has been considered necessary in the formulation of representations of lateral motions, as well as lateral bending moment evaluations, but no analytical evaluation of its hydrodynamic elements or actual computation of its influence on lateral bending moments has been given in the previous literature. The importance of roll motion will be primarily in the region of roll resonance, since the large angular displacements in roll under those conditions will then have their greatest influence on the resultant lateral forces and the lateral bending moment.

The ship motions will be determined by computations based on the "strip theory" approach originally outlined in [7], which has been shown to yield a fairly accurate representation of vertical plane ship motions. Similarly lateral motions may be obtained in the same way by use of such a simplified set of linear equations of motion. It is only necessary to make use of available two-dimensional theoretical representations of the added mass and damping coefficients for appropriate cross-sections in this type of motion analysis. The hydrodynamic forces and moments due to waves for a ship at an arbitrary oblique angle to the waves can be evaluated by similar strip theory techniques, based on certain developments of slender body theory applied to surface ships, e.g. [14] and [15], with special modifications to provide more accurate values for short wavelengths. The local load at any section is determined by combining the hydrodynamic forces due to the rigid body motions together with the inertial forces and the wave induced loads, as shown in [6], [9], and [10]. The bending moment is then found by appropriate double integrations over the ship length. The vertical bending moment will involve force contributions due to the heave and pitch displacements, velocities and accelerations, and the vertical wave force distribution. The lateral bending moment will involve velocities and accelerations due to sway, yaw, and roll, together with the lateral wave force.

The vertical bending moment, and also the lateral bending moment, will be evaluated for a range of wavelengths, forward speeds, and headings in order to determine effective transfer functions for these bending moments relative to a wave input reference. This information can then be used to determine spectral representations of the effective total bending moment (allowing for the relative phase and amplitude of the vertical and lateral bending moments) due to waves by application of linear superposition techniques, such as that used in [16], where realistic wave spectra can be used as the wave input. Thus it is possible to arrive at spectral representations of the wave-induced bending moments, accounting for the contributions of all significant ship rigid body degrees of freedom, and thereby including the effects of both vertical and lateral bending moments. This linear representation for an arbitrary sea condition, with directional properties, is then available as one element in the total bending moment determination.

When considering the bending moment due to slamming-type impulses on an elastic structure (i.e. the ship), the output is best expressed as a time history. The occurrence of slamming is due to a relative motion between the ship bow region and the local wave at the bow, and the nonlinear force developed due to this action, so that the phase relations between ship motions and the local wave are extremely important. The equations of motion that account for the elastic characteristics of a ship can be formulated in a similar manner to those in [11] and [17], without any specific indication initially of whether the ship is to be considered as made up of "lumped" elements or in a continuous fashion. The effects of relative motion between the ship and the wave at the bow region, which is expected to lead to the slamming impulses, must be determined from the rigid body motion of the ship, and the nonlinear hydrodynamic forces developed by the varying buoyancy and added mass terms are established after accounting for the local geometry of the ship sections and including frequency-dependent transport delay terms for the wave structure. The equation systems that account for the ship structural responses will be based upon the representation of the ship as a beam, with appropriate elastic and inertial properties, as outlined in [17]. Solutions for different wave inputs, both regular and random, can be obtained directly from the local ship and wave geometry in this type of formulation and will result in a time history of the slamming impulse bending moment.

The slamming impulses described above are due to the effects of bow flare, as indicated in [11], and no consideration is given to the slamming forces that occur due to bottom impact when a ship section enters the water and changes its immersion suddenly. This neglect is based upon the difficulties in obtaining a proper analytical representation of the local force loads that occur due to such motions, and reflects the present state of the art concerning this particular problem. Furthermore, another restriction in regard to the slamming-type bending moment evaluation discussed above is the fact that only vertical plane motions and impulsive forces are considered, so that the results are only applicable to head seas. This restriction is based upon the fact that the local wave structure near the bow is difficult to represent for oblique headings and short-crested seas, and the sensitivity of these impulsive forces to relative heading orientations of the ship is also unknown. Similarly the mechanism for the occurrence of horizontal impulses is presently unknown, so that there is no means of representing such excitations within the mathematical model. However, it is known that the major effects due to slamming loads, including those due just to bow flare, are primarily due to the vertical plane motions of the ship. Accordingly only head sea conditions will be considered initially in the formulation of the equations representing elastic response of the ship, and this will be considered sufficient as a measure of the expected severe structural responses that will be experienced by a ship at sea.

Since the responses due to slamming loads are represented in terms of a time history output, and include the influence of nonlinearity, nonstationarity, etc., they cannot be readily combined with the output obtained for the slowly varying wave-induced bending moments described previously, which are generally represented in terms of a frequency domain power spectral form. The possibility of determining a spectral response representation for the bending moments due to slamming will be investigated, and similarly a means for representing the wave-induced vertical bending moment in time history form will also be sought. These two approaches will be considered, and the methods of combining the separate outputs will be studied so that the actual level of bending moment and resulting stress in a ship structure can be obtained in a way that reflects both contributions, viz. the slowly varying wave-induced bending moments and those due to slamming.

Selected hand computations will be carried out, and the results compared with experimental data in order to determine the expected degree of validity of the equations developed in this investigation. The experimental data to be used for comparison will be that obtained on a model of a dry cargo vessel that is presently being tested under a Ship Structure Committee project [18], and it is the same ship that is being used in full scale tests at sea. The data will be values of the wave-induced vertical and lateral bending moments that are obtained from tests in regular sinusoidal waves at various headings, ranging from head through following seas. Since the model tested was not dynamically scaled to represent the elastic properties of the prototype ship, and also since no significant bow flare effects are present, no bending moments due to slamming are considered or measured in the test program. Similarly no computations of such effects are included in the present report for illustration, especially since hand calculations for the elastic responses due to slamming impulses would be laborious and noninformative. The nature of the basic equations for an elastic ship structure (see [11] or [17]) requires some type of computer to obtain efficient solutions. Thus the particular type of computer simulation required for the overall problem of total bending moment response, including the direct wave-induced effects as well as those due to slamming, will be examined.

The form of the response output, i.e. whether in a power spectral form or as a time history, will have an important bearing on the type of computer system required for providing ship structural response information on an operational basis, since a particular output form can be obtained more simply with one type of computer than with another. It is possible that either a digital computer or analog computer system alone will not be sufficient for the simulation required in this program, or they may require excessive elements for such a special purpose, so that a hybrid computer system which combines elements of both analog and digital computers may be appropriate for the purpose of determining ship structural response characteristics. A recommendation of a final operational computer system will consider various aspects such as cost of initial installation, operating costs, and availability of equipment in view of the state of the art of computer development.

The input data for this program is envisioned to be ship lines, weight distribution, speed, heading, and information on the wave system (either spectral data, or time history, etc.). The output, in the form of bending moment information primarily, will have certain limits of applicability in view of the assumptions made in the equation formulation. These limits will be outlined, and the "gaps" in knowledge necessary for extending the range of applicability of such computational techniques will be indicated in this study. It is expected however, that the mathematical equation systems developed in this investigation will yield useful information on structural response that includes more physical factors than presently existing computational techniques, since they include lateral bending moments; more rigid body degrees of freedom; the effect of heading is considered; the wave-induced effects and those due to slamming are computed separately, using models appropriate only to each phenomenon, and the results are then combined later; etc. The details of the various procedures used in this investigation will be presented in the ensuing sections of this report.

EQUATIONS FOR WAVE-INDUCED BENDING MOMENTS

The wave-induced bending moments, both vertical and lateral, are determined from the loads distributed along the ship hull that arise from the local wave forces and the loads due to the rigid body motions of the ship. Thus it is necessary to determine the rigid body motions of a ship in regular waves in order to obtain the direct bending moment due to waves. The equations of motion of the ship are linear, as is the bending moment determination, in accordance with the assumptions outlined previously. Since the predominant technique used for determining ship motions in waves is by application of strip theory, where the local forces on different ship sections are evaluated independent of the influence of neighboring sections (i.e. no interactions between sections), that method is used for the development of the bending moment equations.

For the case of vertical plane motions, the equations of motion are formulated relative to an axis system whose origin is located at the CG of the ship. A right-handed cartesian coordinate system is selected with the axes fixed in the body, with the x-axis positive toward the bow (in the direction of forward motion), the y-axis positive to port, and the z-axis positive upward. These axes are defined to have a fixed orientation, i.e. they do not rotate with the body, but they can translate with the body. The ship angular motions can be considered to be small oscillations about a mean position given by the axes. The dynamic variables for this case are the heave displacement z along the z-axis, and the pitch angular displacement θ which is defined as positive in the direction of negative rotation about the y-axis (i.e. bow-up), in conformity with the system used in [5], [6] and [7].

The hydrodynamic forces and moments are composed of terms of inertial nature due to body dynamic motions; dissipative terms due to damping action; and exciting effects due to the oncoming waves. The effect of the free surface is accounted for in the inertial and wave forces by frequency dependent factors that modify the added masses, and all couplings of inertial and dissipative nature are included in the analysis. The previous results ([5], [6], and [7]) are easily extended to the general case of oblique waves by changing the form of the exciting wave force and moment to include the effects of heading. This change is effected by a new definition of the surface wave elevation which relates the position of the body with respect to the wave orientation, as shown in Figure 1. The waves propagate

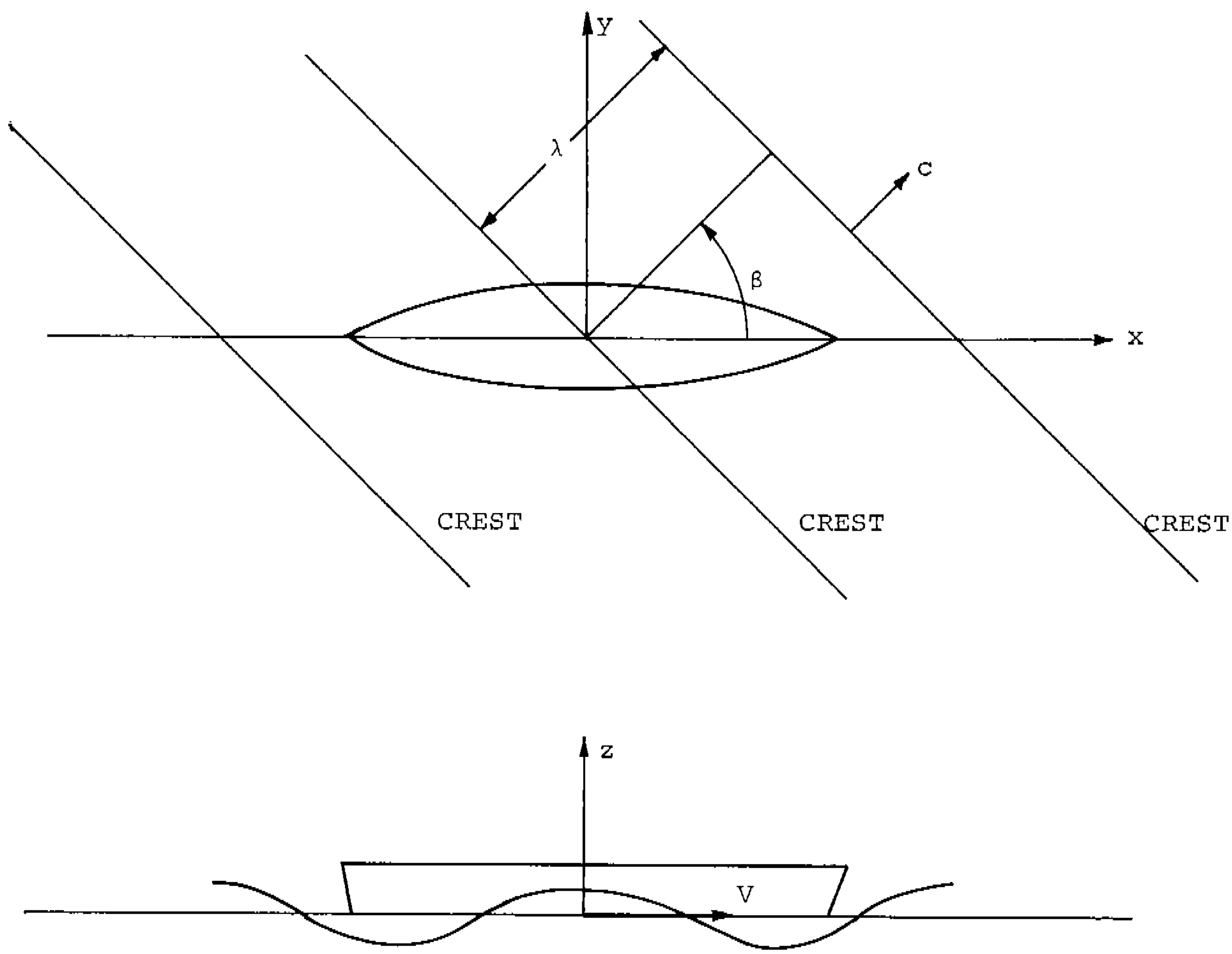

Fig. 1 Relation Of Body To Waves.

with speed c in a direction oblique to the forward motion of the ship, where the angle between the x-axis and the normal to the crests is denoted by β, where β lies in the range $-\pi/2 \leq \beta \leq \pi/2$. The wave propagation speed c is interpreted as $c > 0$ for following seas and $c < 0$ for head sea. The wave elevation $\eta(x,y,t)$ is represented by

$$\eta = a \sin \frac{2\pi}{\lambda} \left[x \cos\beta + y \sin\beta + (V \cos\beta - c)t\right] \tag{1}$$

where V is the ship forward speed, and for application to determine the wave forces this expression is only applied (in accord with the slender body theory assumptions) along the ship hull centerline $y = 0$. Thus the wave expression η used in the following development will be

$$\eta(x,t) = a \sin \frac{2\pi}{\lambda} \left[x \cos\beta + (V \cos\beta - c)t\right] \tag{2}$$

which, aside from its effect on the definition of the frequency of encounter, is equivalent to interpreting the wavelength to be an "equivalent" wavelength of $\lambda \sec\beta$.

The local vertical wave force acting on a ship section can be represented, according to [5] and [6], as

$$\frac{dZ_w}{dx} = \left\{ \rho g B^* \eta + \left[N'_z(x) - V \frac{dA'_{33}}{dx}\right]\dot{\eta} + A'_{33}\ddot{\eta} \right\} e^{-\frac{2\pi\bar{h}}{\lambda}} \tag{3}$$

where B^* is the local beam, $N'_z(x)$ is the local damping force coefficient, A'_{33} is the local section vertical added mass, and $\bar{h}$ is the mean draft of the section. The vertical velocity and acceleration of the water particles at the wave surface are

$$\dot{\eta} = \frac{D\eta}{Dt} = \left(\frac{\partial}{\partial t} - V \frac{\partial}{\partial x}\right) \eta(x,t)$$

$$= - \frac{2\pi ac}{\lambda} \cos \frac{2\pi}{\lambda} \left[x \cos\beta + (V \cos\beta - c)t\right] \tag{4}$$

and

$$\ddot{\eta} = \frac{D\dot{\eta}}{Dt} = - \frac{2\pi ag}{\lambda} \sin \frac{2\pi}{\lambda} \left[x \cos\beta + (V \cos\beta - c)t\right], \tag{5}$$

and the circular frequency of encounter is then

$$\omega_e = \frac{2\pi}{\lambda} (V \cos\beta - c) \tag{6}$$

which is used in representing the final sinusoidal form of responses to sinusoidal waves. The total vertical wave force is obtained by integrating the expression in Equation (3) over the ship hull, i.e.

$$Z_w = \int_{x_s}^{x_b} \frac{dZ_w}{dx}\, dx \tag{7}$$

and the pitch moment due to waves is

$$M_w = \int_{x_s}^{x_b} x\, \frac{dZ_w}{dx}\, dx \tag{8}$$

where x_s and x_b are the stern and bow x-coordinates, respectively.

The coupled equations of motion for heave (z) and pitch (θ) of a ship in regular waves are given by

$$a\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + g\theta = Z_w(\beta,\lambda,V)e^{i\omega_e t} \tag{9}$$

$$A\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{z} + E\dot{z} + Gz = M_w(\beta,\lambda,V)e^{i\omega_e t} \tag{10}$$

where the excitations are represented in complex form (for ease of solution) with the parametric dependence upon heading, forward speed etc. indicated. The coefficients of these equations are

$$\left.\begin{aligned}
&a = m + \int_{x_s}^{x_b} A'_{33}dx \quad ; \quad b = \int_{x_s}^{x_b} N'_z dx \quad ; \quad c = \rho g\int_{x_s}^{x_b} B^* dx \quad ; \\
&d = D = \int_{x_s}^{x_b} A'_{33}\, x\, dx \quad ; \quad e = \int_{x_s}^{x_b} N'_z\, x\, dx - 2V\int_{x_s}^{x_b} A'_{33}dx - V\int_{x_s}^{x_b} x\, d(A'_{33}) \quad ; \\
&g = \rho g\int_{x_s}^{x_b} B^*\, x\, dx - Vb \quad ; \quad A = Iy + \int_{x_s}^{x_b} A'_{33}\, x^2 dx \quad ; \\
&B = \int_{x_s}^{x_b} N'_z\, x^2\, dx - 2VD - V\int_{x_s}^{x_b} x^2\; d(A'_{33}) \quad ; \quad C = \rho g\int_{x_s}^{x_b} B^*\, x\, dx - VE \quad ; \\
&E = \int_{x_s}^{x_b} N'_z\, x\, dx - V\int_{x_s}^{x_b} x\, d(A'_{33}) \quad ; \quad G = \rho g\int_{x_s}^{x_b} B^*\, x\, dx
\end{aligned}\right\} \tag{11}$$

where it is only necessary to carry out integrations involving the local section geometry (i.e. local section beam) and the added mass and damping coefficient for vertical section oscillations. The added mass and damping coefficients for two-dimensional sections can be obtained from the results of a number of different investigations, but the simplest data is obtained from the results of Grim [19]. The ship sections are transformed by a two-parameter conformal transformation to be fitted by Lewis-form sections [20], and the added mass and damping coefficients are determined as functions of frequency for various values of the geometric parameters of beam-draft ratio (B^*/H) and section coefficient ($C_s = S/B^*H$, where S is the section area).

An examination of Equations (9) and (10), with the coefficients defined by Equation (11), indicates that these linear equations lack symmetry in the coefficient matrix. In addition the coefficients themselves are frequency-dependent, and as such Equations (9) and (10) do not represent a normal set of differential equations that would describe the ship motion. However what they do represent is a frequency domain description of the ship's response, valid for sinusoidal inputs. A discussion of the full meaning of these equations and their relation to a proper set of equations, which are actually coupled integro-differential equations, is presented in [21]. The major effect of asymmetry in the coefficients is due to the influence of forward speed, and this fact has already been demonstrated in [22] where the effects of the free surface produce these differences when considering hydrodynamic force components in the frequency domain. Nevertheless this set of equations, whose basic form was originally derived in [7], represents the present state of the art in ship motion response computations and has produced valid results for most ship forms. Thus it will be used as the basic element in establishing the procedures for computing bending moments as well.

The local loading at a section is made up of the loads due to the inertia forces of the ship mass and the added mass; the loads due to displacement or hydrostatic effects; loads due to the damping due to ship velocities; and the loads due to the direct wave effects. The total loading at a section, in equation form, is then

$$\frac{df_z}{dx} = -\delta m(\ddot{z}+x\ddot{\theta}) - A'_{33}(\ddot{z}+x\ddot{\theta}-2V\dot{\theta}) - \rho g B^*(z+x\theta)$$
$$- \left[N'_z(x) - V\frac{dA'_{33}}{dx}\right](\dot{z}+x\dot{\theta}-V\theta) + \frac{dZ_w}{dx} \qquad (12)$$

where δm is the local mass loading (slugs/ft.) at the section and the value of the local wave force $\frac{dZ_w}{dx}$ is given in Equation (3). Integrating the loading (lb./ft.) over the ship from one end up to a station gives the vertical shear at that station, and integrating the shear up to a station gives the vertical bending moment at that station. Alternatively, the vertical bending moment may be represented mathematically as

$$BM_z(x_o) = \int_{x_s}^{x_o} (x-x_o) \frac{df_z}{dx} dx \tag{13}$$

where x_o is the location of the station at which the bending moment is desired, and similarly by the relation

$$BM_z(x_o) = \int_{x_b}^{x_o} (x-x_o) \frac{df_z}{dx} dx \tag{14}$$

since the requirements for a body in equilibrium are that the total force on the body, and the total moment about any point, must equal zero (the "closing" conditions for shear and bending moment).

In the case of lateral bending moments, the available published literature and results are not as extensive as for vertical bending moments, and similarly in the case of lateral motions as compared to vertical plane motions. The only previous analytical work on lateral bending moments ([9] and [10]) provides a framework for development of an appropriate system of equations for use in the present investigation. The method for developing the equations is essentially the same as for the vertical motion case, and uses strip theory techniques that determine the forces in terms of the local force reactions (i.e. added mass, damping, etc.) at various two-dimensional sections.

The previous work did not include roll motion effects, and also used low frequency limits for the lateral damping and added mass together with a crude representation of the added mass frequency variation. In the present analysis, the local forces are determined from the results of Tasai [23], which provide information on the lateral force and roll moment acting on two-dimensional Lewis-form sections due to oscillatory sway and roll motions. The axis system used in the lateral motion analysis is changed somewhat in order to make direct use of the results in [23], and the change in axis system results in the z-axis to be positive downward, the y-axis to be positive to starboard, and the x-axis to remain positive in the direction of forward motion of the ship.

The local lateral force and roll moment are represented in the same manner as in [24], using the notation of [23], and generalizing to take the forward speed effects into account. Thus with the basic lateral displacement expressed as $(y+x\psi)$ with y the sway displacement and ψ the yaw angle, and using the total time derivative operator $\frac{D}{Dt} = \frac{\partial}{\partial t} - V \frac{\partial}{\partial x}$, the local lateral force is represented by

$$\frac{dY_o}{dx} = -\frac{D}{Dt}\left[M_s(\dot{y}+x\dot{\psi}-V\psi) - F_{rs}\dot{\phi}\right] - N_s(\dot{y}+x\dot{\psi}-V\psi) + N_{rs}\dot{\phi} \tag{15}$$

where this result is valid for roll motion about an origin on the free surface. Similarly the roll moment, for rolling about this same waterline reference position, is represented by

$$\frac{dK_o}{dx} = -\frac{D}{Dt}\left[I_r\dot{\phi} - M_{s\phi}(\dot{y}+x\dot{\psi}-V\psi)\right] - N_r\dot{\phi} + N_{s\phi}(\dot{y}+x\dot{\psi}-V\psi) \qquad (16)$$

In order to reference the force and moment to the ship CG position, roll motion about the CG is interpreted as equivalent to rolling about the waterline level together with a uniform lateral velocity $v = -|OG|\dot{\phi}$ where $|OG|$ is the vertical distance between the water surface and the vertical CG position. This lateral velocity term is added to the previous lateral velocity terms (i.e. $\dot{y}+x\dot{\psi}-V\psi$) in Equations (15) and (16), producing additional local force and moment contributions, viz.

$$\Delta\frac{dY_o}{dx} = |OG|\frac{D}{Dt}(M_s\dot{\phi}) + |OG|N_s\dot{\phi} \qquad (17)$$

$$\Delta\frac{dK_o}{dx} = -|OG|\frac{D}{Dt}(M_{s\phi}\dot{\phi}) - |OG|N_{s\phi}\dot{\phi} \qquad (18)$$

and the local lateral force and roll moment, with reference to motion about the CG position, are given by

$$\frac{dY}{dx} = \frac{dY_o}{dx} + \Delta\frac{dY_o}{dx} \qquad \frac{dK}{dx} = \frac{dK_o}{dx} + \Delta\frac{dK_o}{dx} - |OG|\frac{dY}{dx}\ . \qquad (19)$$

Thus the local lateral force due to dynamic motion (i.e. inertial) and damping effects may be expressed as

$$\frac{dY}{dx} = -M_s(\ddot{y}+x\ddot{\psi}-2V\dot{\psi}) + \left(V\frac{dM_s}{dx} - N_s\right)(\dot{y}+x\dot{\psi}-V\psi)$$
$$+ \left(F_{rs} + |OG|M_s\right)\ddot{\phi} + \left[N_{rs} + |OG|N_s - V\left(\frac{dF_{rs}}{dx} + |OG|\frac{dM_s}{dx}\right)\right]\dot{\phi} \qquad (20)$$

and the local roll moment at a section is

$$\frac{dK}{dx} = -\left[I_r + |OG|\left(M_{s\phi} + F_{rs} + |OG|M_s\right)\right]\ddot{\phi} + \left[V\left(\frac{dI_r}{dx} + |OG|\frac{dM_{s\phi}}{dx}\right)\right.$$
$$- |OG|\left(N_{rs} + |OG|N_s\right) + |OG|V\left(\frac{dF_{rs}}{dx} + |OG|\frac{dM_s}{dx}\right)$$
$$\left. - \left(N_r + |OG|N_{s\phi}\right)\right]\dot{\phi} + \left(M_{s\phi} + |OG|M_s\right)(\ddot{y}+x\ddot{\psi}-2V\dot{\psi})$$
$$+ \left[N_{s\phi} + |OG|N_s - V\left(\frac{dM_{s\phi}}{dx} + |OG|\frac{dM_s}{dx}\right)\right](\dot{y}+x\dot{\psi}-V\psi) \qquad (21)$$

The contribution to the yaw moment acting on a ship is obtained from the relation

$$\frac{dN}{dx} = x\,\frac{dY}{dx} \tag{22}$$

The wave excitation effects are obtained by similar procedures as used in [5], [15] and [24], leading to an expression for the lateral wave force at a section given by

$$\frac{dY_w}{dx} = (\rho S + M_s)\,\frac{Dv_w}{Dt} - Vv_w\,\frac{dM_s}{dx} + N_s v_w \tag{23}$$

which reflects both inertial and damping effects, where the lateral orbital velocity v_w is given by

$$v_w = -\,\frac{2\pi ac}{\lambda}\,e^{-\frac{2\pi\bar{h}}{\lambda}}\,\sin\beta\,\sin\frac{2\pi}{\lambda}\left[x\cos\beta + (V\cos\beta - c)t\right] \tag{24}$$

and

$$\frac{Dv_w}{Dt} = \frac{2\pi ag}{\lambda}\,e^{-\frac{2\pi\bar{h}}{\lambda}}\,\sin\beta\,\cos\frac{2\pi}{\lambda}\left[x\cos\beta + (V\cos\beta - c)t\right]\quad, \tag{25}$$

where $\bar{h}$ is an effective depth which is taken to be at the average half-draft for each section. The local roll moment due to waves, with reference to an origin at the surface, is expressed as

$$\frac{dK_{O_w}}{dx} = -\,\frac{D}{Dt}\,(M_{s\phi}v_w) + \rho\left(\frac{B^{*3}}{12} - S\bar{z}\right)\frac{Dv_w}{Dt} - N_{s\phi}v_w \tag{26}$$

where $\bar{z}$ is the vertical coordinate of the section center of buoyancy measured from the free surface level, following the methods in [15] and [24], and the roll moment (due to waves) referred to the CG of the ship is

$$\frac{dK_w}{dx} = \frac{dK_{O_w}}{dx} - |OG|\,\frac{dY_w}{dx} \tag{27}$$

The local roll moment at a section is then given by

$$\frac{dK_w}{dx} = \left[\rho\left(\frac{B^{*3}}{12} - S\bar{z}\right) - M_{s\phi} - |OG|\,(\rho S + M_s)\right]\frac{Dv_w}{Dt}$$

$$+ \left[V\left(\frac{dM_{s\phi}}{dx} + |OG|\,\frac{dM_s}{dx}\right) - N_{s\phi} - |OG|\,N_s\right]v_w \tag{28}$$

The total lateral force and roll moment due to waves is obtained by integrating the expressions in Equations (23) and (28) over the ship hull length, i.e.

$$Y_w = \int_{x_s}^{x_b} \frac{dY_w}{dx}\,dx \qquad , \qquad K_w = \int_{x_s}^{x_b} \frac{dK_w}{dx}\,dx \tag{29}$$

The total yaw moment due to waves is given by

$$N_w = \int_{x_s}^{x_b} x\,\frac{dY_w}{dx}\,dx \tag{30}$$

The expressions derived above are combined to produce the linear equations of lateral motion, with three degrees of freedom, in regular sinusoidal oblique waves, according to the relations

$$m\ddot{y} = \int_{x_s}^{x_b} \frac{dY}{dx}\,dx + Y_w \tag{31}$$

$$I_z\ddot{\psi} = \int_{x_s}^{x_b} x\,\frac{dY}{dx}\,dx + N_w \tag{32}$$

$$I_x\ddot{\phi} = \int_{x_s}^{x_b} \frac{dK}{dx}\,dx - W|GM|\phi + K_w \tag{33}$$

In Equation (33) W is the total ship displacement (weight) and $|GM|$ is the metacentric height, so that the hydrostatic restoring moment in roll is included in the dynamic equations of motion. For general representation purposes, the equations of motion are expressed as

$$a_{11}\ddot{y} + a_{12}\dot{y} \qquad + a_{14}\ddot{\psi} + a_{15}\dot{\psi} + a_{16}\psi + a_{17}\ddot{\phi} + a_{18}\dot{\phi} = Y_w(\beta,\lambda,V)e^{i\omega_e t} \tag{34}$$

$$a_{21}\ddot{y} + a_{22}\dot{y} \qquad + a_{24}\ddot{\psi} + a_{25}\dot{\psi} + a_{26}\psi + a_{27}\ddot{\phi} + a_{28}\dot{\phi} = N_w(\beta,\lambda,V)e^{i\omega_e t} \tag{35}$$

$$a_{31}\ddot{y} + a_{32}\dot{y} \qquad + a_{34}\psi + a_{35}\dot{\psi} + a_{36}\psi + a_{37}\ddot{\phi} + a_{38}\dot{\phi} + a_{39}\phi$$
$$= K_w(\beta,\lambda,V)e^{i\omega_e t} \tag{36}$$

The local lateral loading at any section is made up of the various contributions of inertial (ship mass and added mass) forces, damping forces, and wave exciting forces. It is given by

$$\frac{df_Y}{dx} = - \delta m\left[\ddot{y}+x\ddot{\psi}-\zeta(x)\ddot{\phi}\right] + \frac{dY}{dx} + \frac{dY_w}{dx} \tag{37}$$

where the expressions for $\frac{dY}{dx}$ and $\frac{dY_w}{dx}$ are given in Equations (20) and (23) respectively, and the term $\zeta(x)$ represents the vertical position of the local CG of each elemental ship section, relative to the overall ship CG position, with $\zeta(x)$ positive downward. The requirement on $\zeta(x)$ is

$$\int_{x_s}^{x_b} \delta m\,\zeta(x)\,dx = 0 \tag{38}$$

since $\zeta(x)$ is measured relative to the ship CG and all first moments about that point must sum to zero, by definition. The expression in Equation (37) is applicable to the case where the ship has three lateral degrees of freedom and it may be simplified for the case where roll is neglected, resulting in

$$\frac{df_Y}{dx} = - \delta m(\ddot{y}+x\ddot{\psi}) - M_s(\ddot{y}+x\ddot{\psi}-2V\dot{\psi}) + \left(V\frac{dM_s}{dx} - N_s\right)(\dot{y}+x\dot{\psi}-V\psi) + \frac{dY_w}{dx} \tag{39}$$

In the derivation of the wave excitation force, it has been assumed that the assumptions of slender body theory (e.g. [14]), are valid, i.e. that the wavelength of the waves is large compared to lateral dimensions of the ship. However conditions at oblique headings, for short wavelengths, may be important for determining lateral bending moments and in that case the slender body assumptions are not fully valid. Considerations for a similar situation were made in [24], and this involved two additional factors. The first factor is to account for the vertical gradient in the lateral orbital velocity, which adds a term of the form $\frac{2\pi}{\lambda}$ (the gradient effect) multiplied by the next higher inertia coefficient of the section, i.e. a term that is included in the roll moment due to waves. Thus the additional term in the local wave force is

$$\Delta\frac{dY_w}{dx} = \frac{2\pi}{\lambda}\left[-\frac{D}{Dt}(M_{s\phi}v_w)\right]$$
$$= \frac{2\pi}{\lambda}\left[-M_{s\phi}\frac{Dv_w}{Dt} + V\frac{dM_s}{dx}v_w\right] \tag{40}$$

and this term is added to all other expressions for the local wave force, the yaw moment due to waves, the total loading at a section, etc. The second factor will account for the fact that there is a variation of the wave and the associated velocities, accelerations, etc. across the width of the ship, which is illustrated by the variation with the coordinate y, as shown in Equation (1). Similar effects would occur in the case of the lateral orbital velocity v_w, which can be represented as

$$v_w = -\frac{2\pi ac}{\lambda} e^{-\frac{2\pi\bar{h}}{\lambda}} \sin\beta \sin\frac{2\pi}{\lambda}\left[x\cos\beta + y\sin\beta + (V\cos\beta - c)t\right] \tag{41}$$

where the variation over the two-dimensional surface is shown. In that case it is necessary to account for the variation of the force developed at different lateral points along a ship hull, and in essence the integration over the hull for determining the total forces will result in a factor given by

$$\frac{\sin\left(\frac{\pi B}{\lambda}\sin\beta\right)}{\frac{\pi B}{\lambda}\sin\beta}$$

where, for simplicity, this factor will be evaluated assuming the beam there is the ship maximum beam. This approximation will then be extended as applicable to the local section forces, which will then only vary with the coordinate x along the ship hull, thereby also affecting the loading distribution as well. It is expected that these two additional factors, which represent a modification to account for short wavelengths, will only have meaningful influence for wavelengths less than the ship length, and hence the effect should not be accounted for in computations for longer waves.

Returning to consideration of the local lateral loading, given by Equation (37) (or by Equation (39) for the simplified case where roll is neglected), this expression may be integrated to obtain the lateral shear force. Similarly the lateral bending moment can be obtained by integrating the shear, or alternatively by the mathematical representation

$$BM_y(x_o) = \int_{x_s}^{x_o} (x-x_o)\frac{df_y}{dx}\,dx = \int_{x_b}^{x_o} (x-x_o)\frac{df_y}{dx}\,dx \tag{42}$$

All of the results obtained in the preceding analyses have been appropriate to conditions of regular sinusoidal unidirectional waves, which only occur in model test tanks. In the open sea, the waves are random, and the motions and structural responses also have a random nature. Thus, in order to characterize the response of a ship in a random sea, the function known as the energy spectrum of the response must be determined. This spectrum is a measure of the variation of the squares of the amplitudes of the various sinusoidal components of the response, presented as a function of the frequency of encounter and the wave direction. The spectral technique for analyzing random irregular time histories of motion and structural response is applicable to linear systems only, since in that case a unique response amplitude operator is obtained. The spectral techniques evolve as a result of linear superposition of the responses to individual frequency components contained in the excitation (i.e. the waves in this case), and the final synthesis of the effects (in a mean-square-sense) is represented by the motion spectrum.

For an arbitrary motion or response, represented by the i-subscript, the energy spectrum of that motion due to the effects of irregular random waves is represented by

$$\Phi_i(\omega_e) = |T_{i\eta}(\omega_e)|^2 A^2(\omega_e) \tag{43}$$

for a particular fixed ship heading in a unidirectional sea, where $A^2(\omega_e)$ is the wave spectrum and $|T_{i\eta}|$ is the response amplitude operator for that heading (amplitude of motion per unit wave amplitude). Since the response of a ship is often represented as a function of the frequency of encounter ω_e, it is necessary to represent the wave spectrum also as a function of ω_e so that the total area of the spectrum (which is a necessary characteristic for determining statistical information on response characteristics) can be easily determined by integration with respect to ω_e. The wave spectrum is generally represented as a function of the frequency ω, which is a pure wave frequency related to the wavelength λ by

$$\omega = \sqrt{\frac{2\pi g}{\lambda}} \tag{44}$$

The frequency of encounter ω_e can be shown to be related to ω by

$$\omega_e = \omega - \frac{\omega^2 V}{g} \cos \beta \tag{45}$$

and it is necessary when representing the wave spectrum as a function of ω_e to present it in the form given by

$$A^2(\omega_e) = R^2\left[\omega(\omega_e)\right] J(\omega_e) \tag{46}$$

where $J(\omega_e)$ is the Jacobian given by

$$J(\omega_e) = \frac{1}{\sqrt{1 - \frac{4\omega_e V}{g} \cos \beta}} \tag{47}$$

(see [25] for details). It is also possible to represent the wave spectrum for a non-unidirectional sea, allowing for angular variation (a two-dimensional spectrum), which results in a modification to the basic frequency domain representation. The weighting function to account for the angular variation is given in terms of the angle β_w, which is measured from the direction toward which the wind is blowing (the predominant wave direction). For the case of a two-dimensional wave spectrum, the response spectrum that would occur for a particular ship heading measured relative to the wind direction can be obtained by integrating with respect to the angle β_w.

Under the assumption that the seaway is a Gaussian or normal stochastic process which is exciting a linear system (a ship in the present case), the responses of the system will in turn represent a Gaussian stochastic process. From the spectral density function $\Phi_i(\omega_e)$ for a particular response, there may be obtained, in principle, all of the statistical or probabilistic properties possessed by the particular Gaussian random process. For example, the total area E_i under the spectral density function curve, as defined above, given by

$$E_i = \int_o^\infty \Phi_i(\omega_e)d\omega_e \tag{48}$$

is equal to σ_i^2; i.e. the variance of the ordinates on the corresponding time-history curve. Here the ordinate dispersion, or standard deviation, has been denoted by σ_i, which is the root-mean-square value of deviations of the ordinates from the mean or average ordinate, which is assumed to be zero for consideration of all wave-induced effects. For the case of vertical bending moments, there is a non-zero mean value, which is the still water bending moment, but all appropriate corrections can be made to account for the influence of this effect when a detailed data analysis is being carried out.

Characteristics of the motion time history may be obtained in terms of the quantity E_i by relating the behavior of the envelope of the record (interpreted as the instantaneous amplitude of the time-history curve) to this quantity. Such relations are based upon an assumed narrow-band behavior of the energy spectrum (see [25]) and yield expressions for the mean amplitude of oscillation (half the distance between the trough and crest of an oscillation), the mean of the highest 1/3 of such amplitudes (known as the significant amplitude), and other related statistical parameters of interest for a specified sea condition, ship speed, and relative heading. These results are applicable to any linear response of a ship to waves, and hence the vertical and lateral bending moments can each be separately analyzed to determine their respective spectra. Appropriate combinations of the vertical and lateral bending moment spectra can be made to determine the effect of their combined influence, when proper account is taken of their phase relations, so that deck edge stresses and other effects can also be analyzed in the spectral domain.

EQUATIONS FOR SLAM-INDUCED BENDING MOMENTS

The equations that represent the phenomena associated with slamming-type impulses applied in the bow region of a ship include the elastic, as well as the dynamic and hydrodynamic forces arising during such interaction with oncoming waves. As discussed previously, only vertical plane motions and responses are considered, and the wave system orientation will be that of head seas. The basic equations are established on the basis of approximating the ship structure as an elastic beam with nonuniform mass and elastic properties distributed along its length. The elastic effects that are accounted for in the equations are bending and shear, while the influence of rotary inertia is also included.

The equations of motion governing the response characteristics of the ship are essentially the same as those presented in [11] and [17], with the main concern for the present study being the hydrodynamic forces acting as the excitation input. The basic equations are as follows:

$$\mu \frac{\partial^2 z_e}{\partial t^2} + c \frac{\partial z_e}{\partial t} + \frac{\partial V_s}{\partial x} = P(x,t) \tag{49}$$

where $\mu = \mu(x)$ is the sum of the ship mass and the added mass at a section; z_e represents the vertical elastic deflection; c is the damping coefficient; V_s is the shear force; and $P(x,t)$ is the local input force due to ship-wave interaction.

$$\frac{\partial M}{\partial x} = V_s + I_r \frac{\partial^2 \gamma}{\partial t^2} \tag{50}$$

where M is the bending moment, I_r is the mass moment of inertia of a section; and γ is a deformation angle, with the last term on the right in Equation (50) representing the rotary inertia.

$$M = EI \frac{\partial \gamma}{\partial x} \tag{51}$$

is the fundamental elastic equation, with EI the bending flexural rigidity.

$$\frac{\partial z_e}{\partial x} = - \frac{V_s}{KAG} + \gamma \tag{52}$$

relates the bending and shear effects, where KAG is the vertical shear rigidity.

All of the above equations are partial differential equations, with the independent variables being x and t, so that the fundamental quantities of interest (i.e. bending moment, deflection, etc.)

vary both temporally and spacially along the hull. The equations are appropriate to the same axes and coordinate system as was used in the analysis of vertical wave-induced bending moments, given by Equations (1)-(14), so that some of the expressions used there can be applied directly in the present case. The main emphasis in the consideration of the elastic response of the ship due to an impulsive force, arising from a slamming-like phenomenon associated with bow flare, is the determination of this force. The force input arises from interaction between the ship hull geometry and the wave, and the particular impulsive-type force must be distinguished from the ordinary wave-induced forces that cause the ship rigid body motions and the wave-induced bending moments. These latter forces are determined in accordance with linear theory, and they are found in terms of the ship geometry corresponding to an immersed portion defined by the still water equilibrium reference position, i.e. the mean value of the wave elevation. The difference in the actual immersed area, local form geometry, etc. due to the wave elevation and/or the resulting rigid body motions is not considered in the linear analysis that characterizes the work in [7], [14], [15], etc., as well as in the analytical development concerned with wave-induced bending moments in the present report, i.e. Equations (1)-(42). Thus the input force $P(x,t)$ represented in Equation (49) will have all linear wave effects separated out, since they have already been accounted for in determining the vertical wave-induced bending moments.

The input force $P(x,t)$ is made up of two terms, one of which is of inertial nature while the other is due to buoyancy, and is represented by

$$P(x,t) = P_1(x,t) + P_2(x,t) \tag{53}$$

The force $P_1(x,t)$ is of inertial nature, and is represented by

$$P_1(x,t) = -\frac{D}{Dt}(\overline{m}_{n\ell} w_r) \tag{54}$$

where the operator

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - V\frac{\partial}{\partial x}$$

$\overline{m}_{n\ell}$ is the additional added mass at a section that is determined from the instantaneous immersion geometry of the ship section, after subtracting out the added mass determined from the still water (linear theory) reference geometry, and w_r is the relative velocity at the section, given by

$$w_r = \dot{z} + x\dot{\theta} - V\theta - w_o(x,t) \tag{55}$$

where the rigid body motions z and θ (and their derivatives) are determined from linear theory solutions (from Equations (9)-(11)), and $w_o(x,t)$ is the wave orbital velocity given by

$$w_o(x,t) = \frac{D\eta}{Dt} = -\frac{2\pi ac}{\lambda}\cos\frac{2\pi}{\lambda}\left[x + (V+c)t\right] \tag{56}$$

for the present head sea case (illustrated for sinusoidal waves). The force $P_2(x,t)$, which is due to buoyancy, is represented by

$$P_2(x,t) = \rho g \bar{A}_{n\ell}(z_r;x) \tag{57}$$

where $\bar{A}_{n\ell}$ is the additional cross-sectional area at a section due to the difference between the area corresponding to the instantaneous submerged portion of the ship section and that corresponding to the still waterline, thereby eliminating the linear buoyancy force terms. The quantity $\bar{A}_{n\ell}$ is determined, for a particular ship section, as a function of the relative immersion change

$$z_r = z + x\theta - \eta(x,t) \tag{58}$$

The equations given above in this section include representations appropriate to values at a ship sectional element, and for solution to obtain response characteristics the ship must be represented in lumped parameter form. The total force at any lumped segment of the ship is represented by an integrated average over the segment length, and is similar in concept to the methods applied to the computation of wave-induced bending moments (e.g. see [5] and [6]). Solutions of the equations can be obtained, when they are structured in this form, by either a digital or analog computer, within the limits of accurate representation possible using a particular type of computer.

In the development of the above equations certain assumptions were made that differed from procedures used in other studies, and similarly more discussion of limits of applicability of certain expressions, sources of applicable data, etc. is necessary before continuing with considerations of solution per se. One of the important aspects of separation of the wave-induced bending moments from those including elastic ship characteristics is the ability to represent the appropriate physical elements with the proper phenomena. As an example, the added mass values that are used in determining the slowly varying wave-induced bending moments must be frequency-dependent, as required in the basic ship motion analysis procedures (e.g. [7]), while those used in the representation of the higher frequency vibratory responses due to slamming will be frequency-independent. A review of the methods that can be applied to determine the added mass for vibratory analyses of ships is presented in [26]. Another important element in this type of analysis is the resulting distinction between the damping associated with the wave-induced bending moment and rigid body motion analyses, as compared to that associated with the elastic response. For the wave-induced effects the damping is almost completely due to wave generation on the free surface due to body oscillations, and this effect has a very strong and significant influence on the resulting heave and pitch motions. This wave dissipation damping is very frequency-dependent, and it decays extremely fast for high frequencies so that it cannot be expected to play an important role in determining the damping associated with the higher frequency elastic responses. In that case the damping is primarily structural in nature, and the use of Rayleigh damping (a viscous damping proportional to mass) has been suggested ([17], [27]) as being appropriate for this case. The estimates of the values of c/μ to be used in a computer study must be determined from available empirical data in that case.

The representation of the nonlinear inertial and buoyancy forces can be ideally carried out for each and every section of a ship, but from practical considerations it is only necessary to obtain such detail in regions where significant changes can take place, viz. in the bow region where the bow flare effect can manifest itself strongly. In that case only a few ship stations have to be analyzed, and the number of sections chosen for study can be selected in order to reduce the element spacings and obtain greater detailed representations, as desired in a particular application. The methods to be used in arriving at values of these quantities, as well as the techniques to be used in obtaining solutions of the equations in order to find the bending moment time history for particular operating conditions such as regular waves, irregular random waves with a particular spectrum, etc. are subjects to be covered in a later section of this report, when discussion of computational techniques will be presented.

Without actually carrying out specific computations, it is still possible to consider certain output information and approximations that may be made in order to obtain further insight into the predominant effects that influence the bending moment responses due to slamming. The basic set of equations of motion given by Equations (49)-(52) can be solved for the case of no wave influence, at zero forward speed, with simple harmonic driving force inputs at different frequencies in order to observe the normal modes and frequencies. When considering the representation of a particular response in terms of normal modes, it is possible to achieve a simplification of the equations if rotary inertia is neglected. In that case the responses can be represented (see [17]) in a product form as

$$z_e(x,t) = \sum_{i=1}^{\infty} q_i(t) X_i(x) \quad , \qquad M(x,t) = \sum_{i=1}^{\infty} q_i(t) M_i(x) \tag{59}$$

where $X_i(x)$ is the nondimensional mode shape of the i^{th} normal mode and $q_i(t)$ is a time-varying function with the dimension of length. The term $M_i(x)$ is given by

$$M_i = \int_0^x \int_0^x \mu(x) \omega_i^2 X_i(x) dx \, dx \tag{60}$$

where ω_i is the circular natural frequency (in rad./sec.) of the i^{th} mode. Assuming Rayleigh damping with $c \sim \mu$, the equations can be reduced to the form

$$\bar{\mu}_i \ddot{q}_i + C_i \dot{q}_i + K_i q_i = Q_i(t) \tag{61}$$

where

$$\bar{\mu}_i = \int_0^L \mu(x) X_i^2(x)\,dx \quad , \quad C_i = \int_0^L c(x) X_i^2(x)\,dx \quad , \; K_i = \omega_i^2 \bar{\mu}_i \tag{62}$$

and

$$Q_i(t) = \int_0^L P(x,t) X_i(x)\,dx \quad . \tag{63}$$

Equation (61) can be easily solved for any input, i.e. sinusoidal, transient, etc., and the question that arises is how well the solutions of this simplified set of equations will duplicate those that can be obtained from the more complicated set in Equations (49)-(52). Since the neglect of rotary inertia has only a small effect on the natural frequencies and mode shapes for the first mode of motion, which is the predominant response expected due to slamming impulses [11], this simplified approach may have some merit. Similarly only the first, and possibly the second mode of response, are the only contributions that may be necessary in this case, thereby reducing the computational effort even further in this approach. In view of this approximation, and the neglect of higher modes of vibration, there may be a question as to the applicability of a modal approach to represent this type of response properly, since there is no propagation of a flexural wave along the hull. However the response to various oscillatory forcing functions at different frequencies can be obtained by such a modal analysis, and by Fourier integral superposition the response to a transient readily follows. With a response due to slamming that is predominantly due to the first mode, there is a possibility of this particular approach yielding a useful answer. It remains to investigate this particular approach during computational investigations of the utility of the various equations and techniques described in this report, which is beyond the scope of the present investigation.

RESULTS OF COMPUTATIONS

As a means of checking the capability of the equations described herein to predict the bending moments acting on ships in waves, some limited hand calculations of the wave-induced bending moments acting on a ship model in regular oblique waves were carried out. Calculations for both vertical and lateral wave-induced bending moments were made, and no consideration was given to bending moments due to slamming, for reasons described earlier in this report. The computations were carried out on the basis of a complete theoretical representation, where all elements, such as the ship motions themselves as well as the bending moments, were determined in this manner. The experimental data was obtained from

[18], where results of tests on a 1/96 scale model of the SS WOLVERINE STATE at a mean draft of 19.3 ft. were presented. The ship has a 496 ft. LWL and a maximum beam of 71.5 ft., and copies of the ship lines were obtained from Davidson Laboratory, together with information on the ship speed, heading, and other test conditions so that computations could proceed.

The indicated displacement was 11,770 tons, corresponding to 29.8 lb. in model scale, and the indicated speed was 17.5 knots. Unfortunately, due to an error of interpretation, the 17.5 knot speed was only to be interpreted as a calm water speed, with the actual test data obtained at the more realistic condition equivalent to 16 knots, while the computations were carried out for the 17.5 knot speed. However some data was obtained at higher speeds, for certain heading conditions, and proper inference can be made of the ability of the theoretical equations to predict wave-induced bending moment characteristics.

The tests were made at headings extending from 0° (following seas) to 180° (head seas), at 30° heading increments, and measurements made of the midship vertical and lateral bending moments. The weight curve of the model is shown in Figure 2, and from this infor-

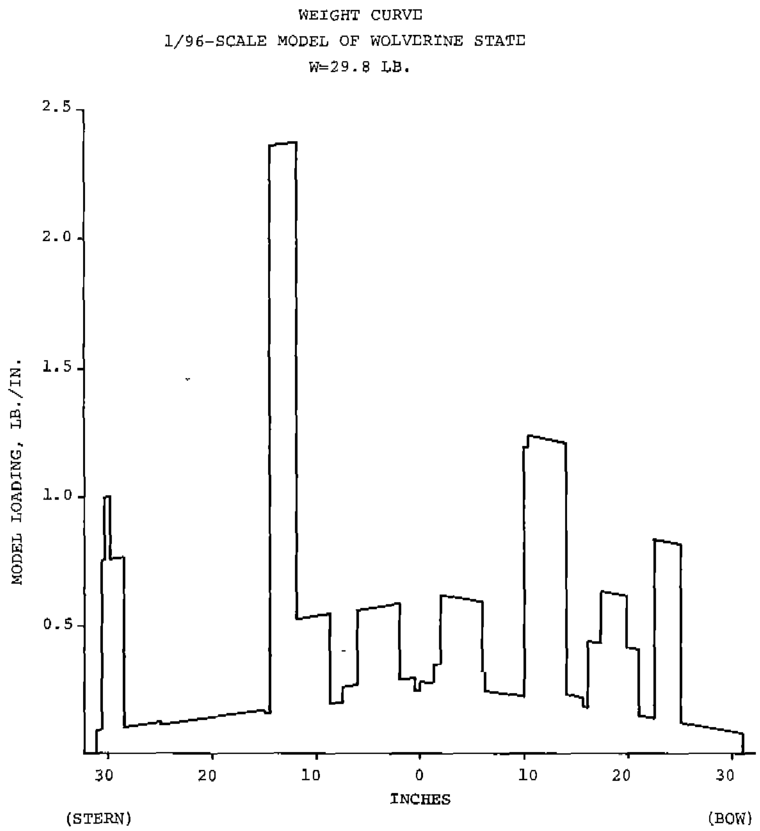

Fig. 2 Weight Curve of Wolverine State Model.

mation an average loading curve was obtained for a 20 station lumped parameter representation (shown in Figure 3) and for a 10 station representation (Figure 4). An examination of Figure 3 shows large differences in the average loading curve from station to station, and computational difficulties were experienced initially due to this effect. In addition the number of computations was twice that for a 10 station subdivision, and since the accuracy of bending moment computations in [5] and [6] was not altered significantly by such a selection of the number of stations (i.e. 10 stations), all of the computations were carried out on the basis of 10 stations.

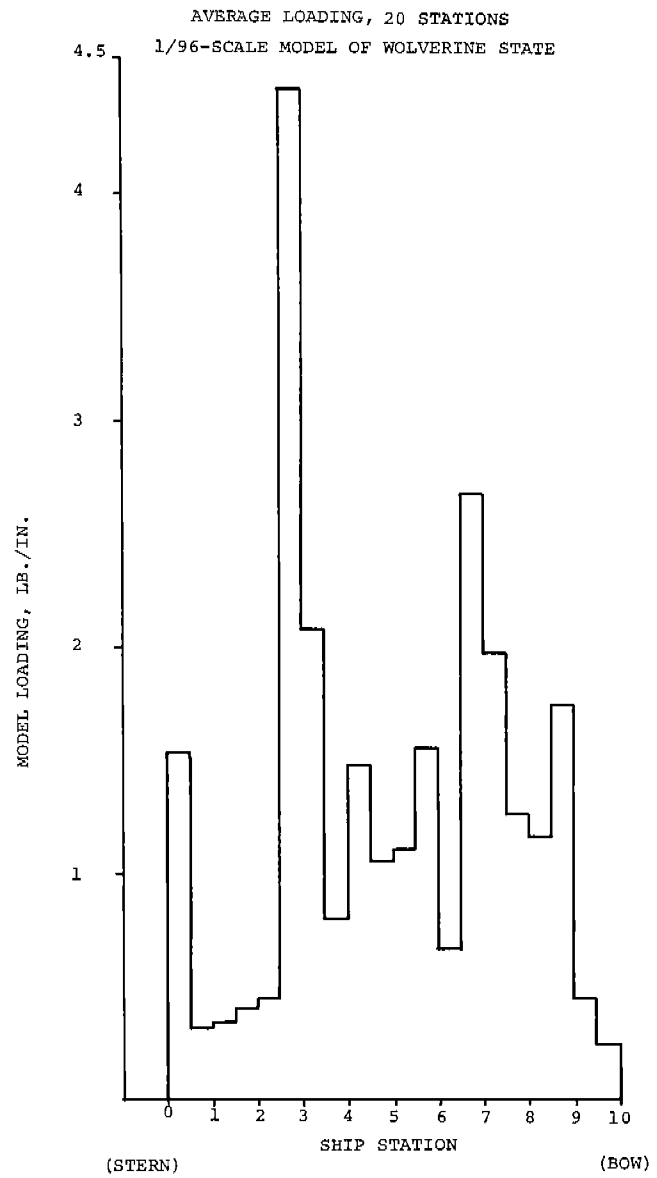

Fig. 3 Average Loading Of Wolverine State Model, 20 Stations.

The midship bending moment, both vertical and lateral, was determined by pure summation operations on the local loading determined from all contributing sources, as given by Equations (12) and (39), and this is an appropriate procedure in view of the lumped parameter approach. The value of the midship bending moment in each case was obtained by averaging the values computed from summations on either side of midships (i.e. summations over stations 0-5 and over stations 5-10).

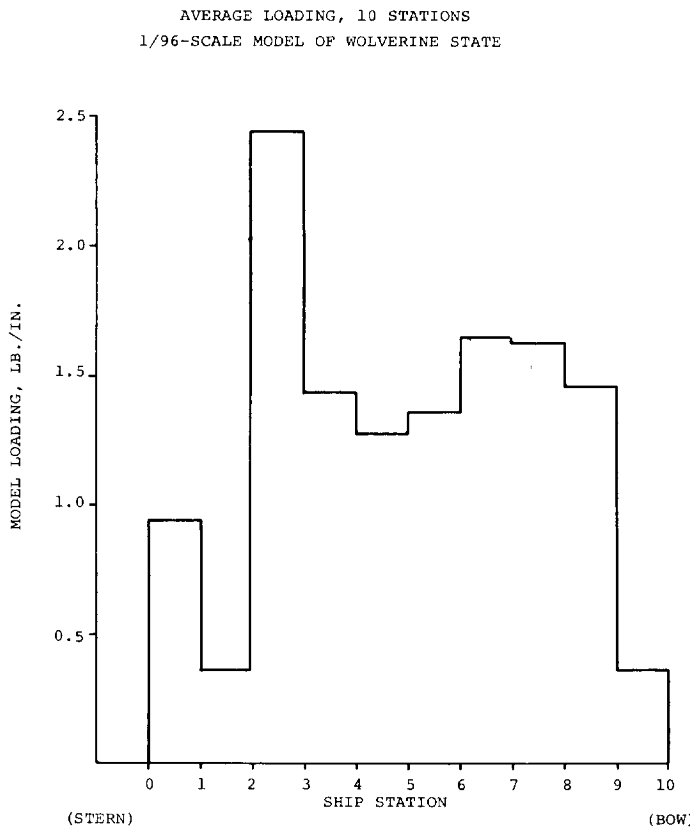

Fig. 4 Average Loading Of Wolverine State Model, 10 Stations.

The results of the computations carried out by hand calculations are shown in Figures 5-8, for the vertical bending moments due to waves, where the experimental data obtained from [18], at the forward speed of 16 knots, is also shown. The results are presented in the form of the midship vertical bending moment amplitude per unit wave amplitude, as a function of the ratio of the wavelength to the ship length. While there is general agreement for all of the cases, at the various headings, the lack of precise agreement is due to the fact that the forward speed for the experimental conditions was less than that used in the theoretical computations. An examination of some of the experimental data (in [18]) obtained for the 60° heading case shows that the vertical bending moment will increase as the speed increases for wavelengths less than the ship length, so that agreement will be better if the proper experimental data is used. Similar

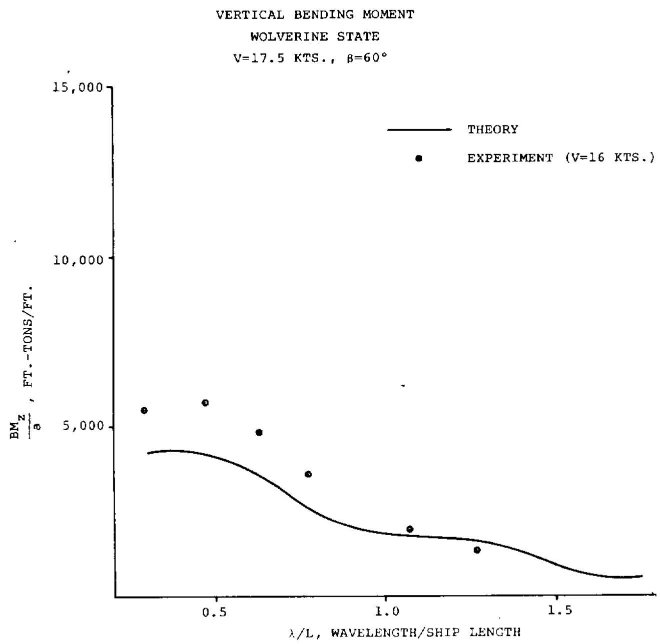

Fig. 5 Comparison Of Theory And Experiment, Vertical Bending Moment, β= 60°.

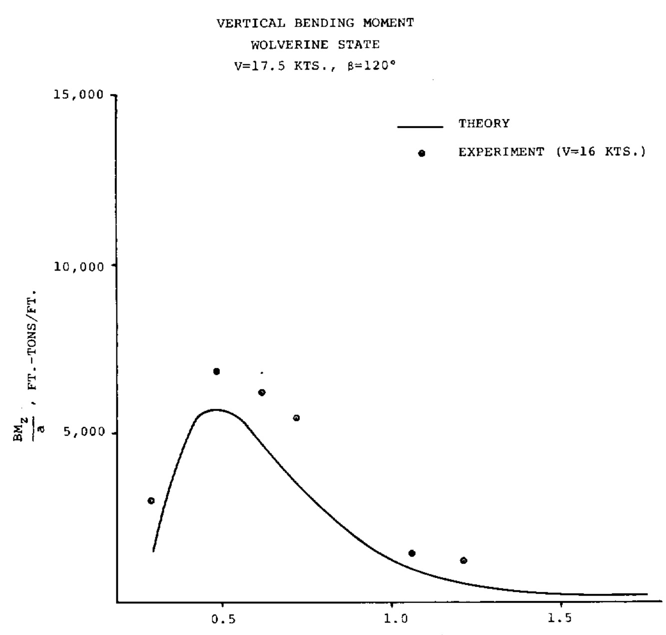

Fig. 6 Comparison of Theory And Experiment, Vertical Bending Moment, β=120°.

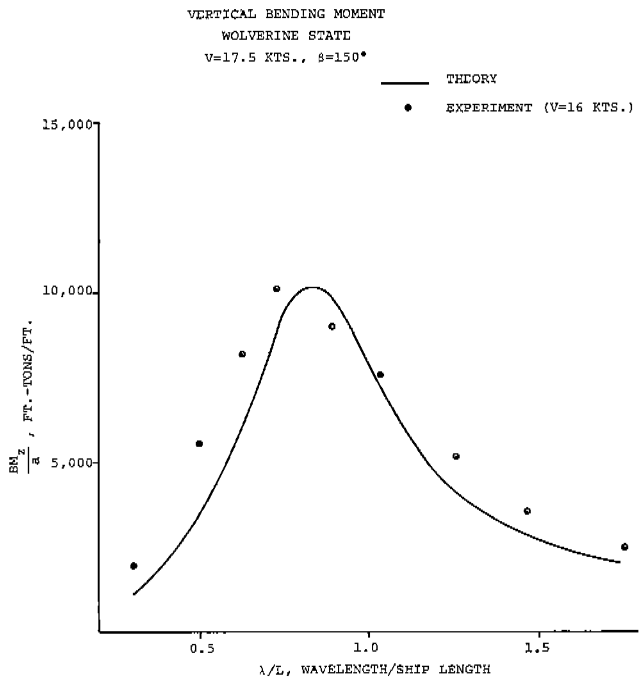

Fig. 7 Comparison of Theory And Experiment, Vertical Bending Moment, β=150°.

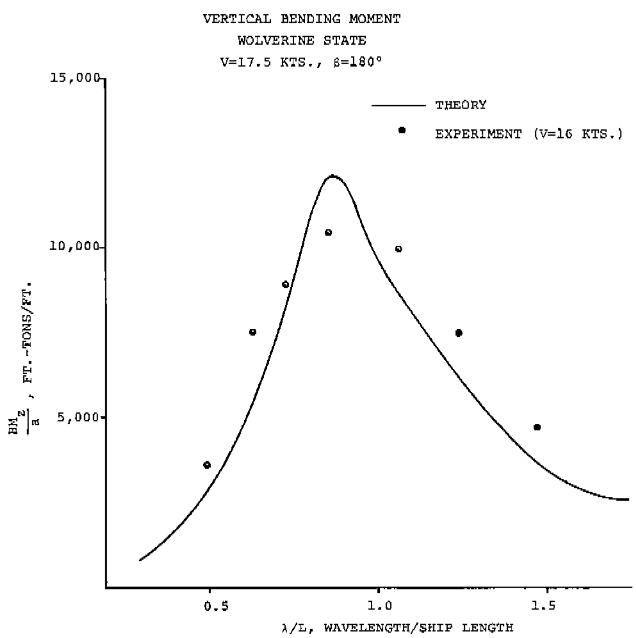

Fig. 8 Comparison Of Theory And Experiment, Vertical Bending Moment, β=180°.

results hold for the 120° heading case (Figure 6), while the degree of agreement is reduced somewhat for some of the cases at the 150° and 180° heading conditions. As a general conclusion however, it can be seen that the theoretical results, as obtained by hand calculations, have a good degree of agreement with the available experimental data.

The results were checked in order to determine the consistency in the analysis and computations by examining the requirement of satisfying the closing conditions, i.e. zero (or near zero) values for the shear and bending moment at the bow and stern extremities. The present results were obtained by means of a computational procedure similar to that employed in [6], with the exception of the treatment of the local mass loading terms given as the first term in Equation (12). This was brought about by the problems associated with using the average mass loading at each station (as given by Figure 4), since computational errors accumulated when carrying out the operations. This difficulty was overcome by obtaining the original data on the precise locations of the mass elements on the ship model hull, together with separate measurements of the radius of gyration of each separate half of the model. The required operations were applied to the known mass loadings in order to obtain a precise measure of the mass loading terms that enter into the determination of the shear and bending moment, using the precise definitions of the center of gravity location, the value of the radius of gyration, etc. This procedure resulted in satisfying the closing conditions, and due to the general smooth variation of the hydrodynamic forces entering into the local loading equation, Equation (12), the net result was proper determination of the desired values and satisfaction of all associated physical boundary conditions and requirements.

For the case of lateral bending moments, an investigation was initially carried out to determine the influence of the roll degree of freedom on the calculations. The original model data gave a roll natural frequency of $\omega_\phi = 5.07$ rad./sec., and it was then necessary to find the value of $|GM|$ that would give the desired natural frequency value. Assuming a roll radius of gyration given by $k_x = 0.35B$ leads to a value of the roll moment of inertia, in model scale, given by $I_x = 0.0628$ slug-ft.^{2}, and a value of the quantity $W|GM| = 2.0$ lb.-ft. was chosen as a proper value for use in the following computations. The vertical CG location was 2.81 in. above the keel, thereby resulting in $|OG| = 0.4$ in. in model scale.

The conditions chosen for the computations in this study were those corresponding to headings of 60° and 120°, which resulted in the largest lateral bending moment values in [18]. The various elements required in determining the local lateral force, yaw moment, and roll moment, as given by Equations (15)-(30), were evaluated for a series of wavelengths covered in the experiments. The final equations of motion, Equations (34)-(36), were solved for these different wavelengths, at the two selected headings, for the three lateral degrees of freedom. Similarly solutions were obtained for Equations (34) and (35), with all roll terms neglected, thereby obtaining the rigid body lateral motions without the roll influence. The lateral bending moment corresponding to the two degrees of

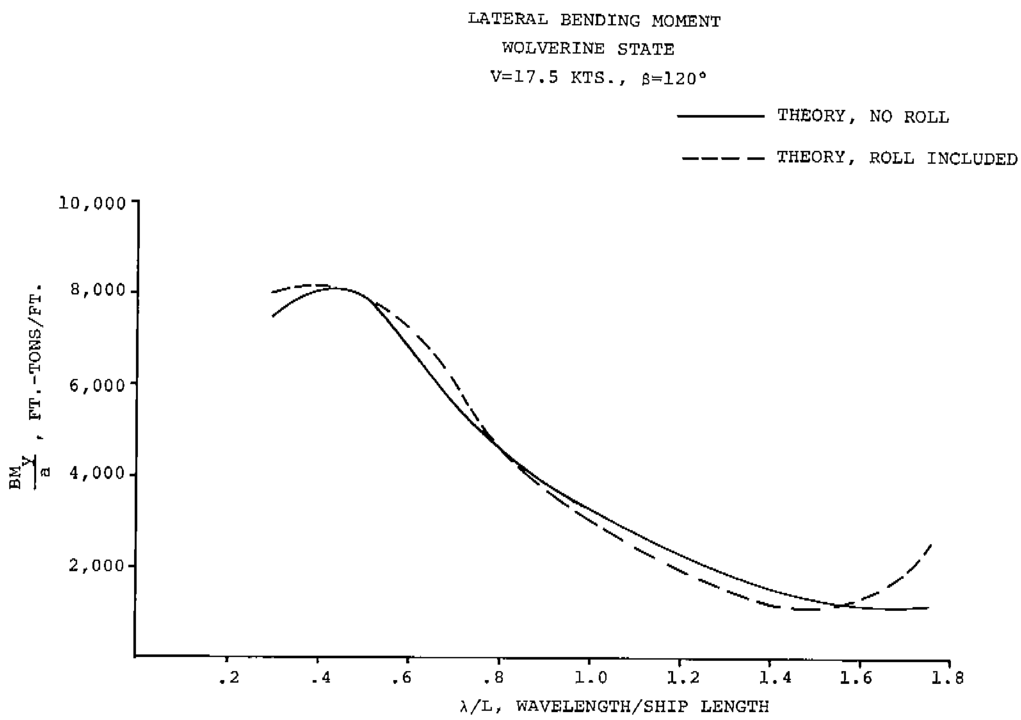

Fig. 9 Effect Of Roll Motion On Theoretical Lateral Bending Moment, β=120°.

freedom was obtained using the expression for local lateral load in Equation (39), and similarly an evaluation of the lateral bending moment using the expression for the local lateral loading given in Equation (37) was also carried out. The lateral bending moments were evaluated in accordance with the summation procedures that have been used for the vertical bending moment computations described previously, and the same procedure for evaluating the local mass loading effects was applied as in the case of the vertical bending moment evaluation. Since no information was given concerning the quantity $\zeta(x)$, which is the vertical position of the local CG of each elemental ship section, this contribution of rolling motion was neglected in the determination of the local loading according to Equation (37), and subsequently in the evaluation of the lateral bending moment.

Figure 9 illustrates the effect of the inclusion of roll motion on the theoretical results computed from the equations developed herein, for the 120° heading case. It can be seen that there is very little difference in the lateral bending moment, according to this theory, when roll is included, except in the case of the longest wavelength considered, where the frequency starts to approach the roll natural frequency. Thus, for this condition, the theoretical results indicate a small influence of roll motion. A comparison of the theoretical and experimental lateral bending moments for the

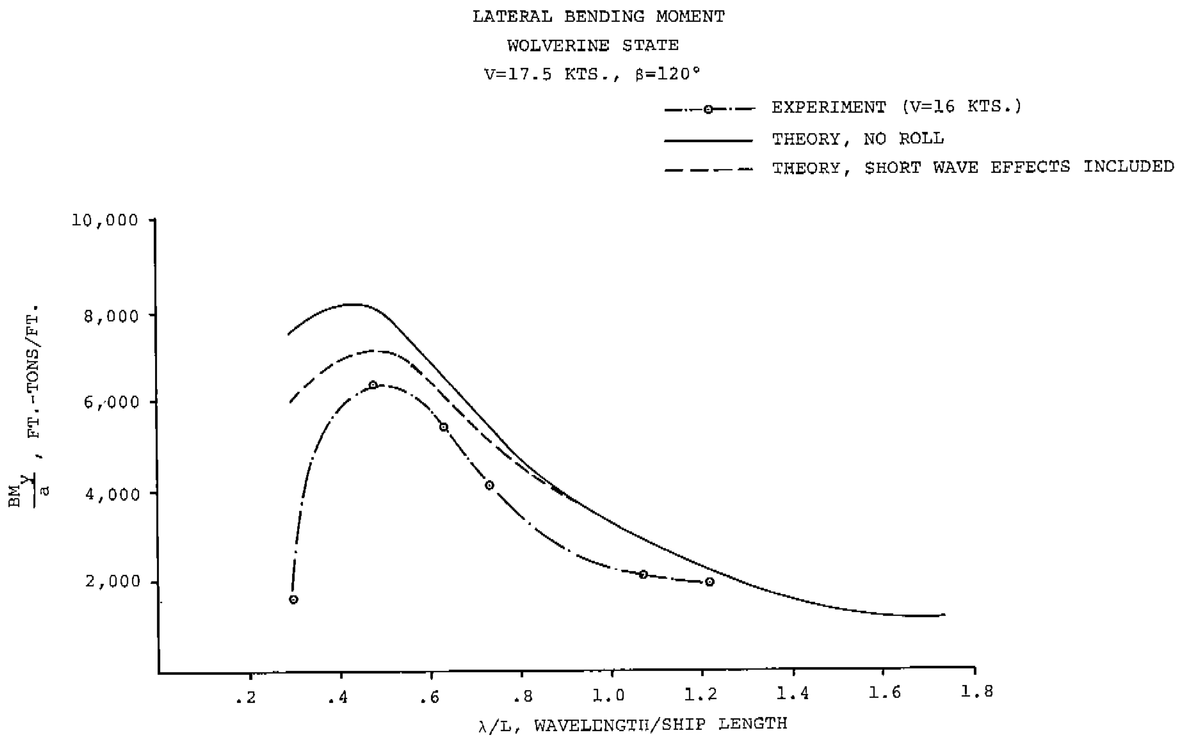

Fig. 10 Comparison Of Lateral Bending Moment Theory And Experiment, β=120°.

case of 120° heading is shown in Figure 10, where the basic theory with no roll (for V = 17.5 knots) is illustrated in comparison with experimental values obtained at a 16 knot forward speed. Another theoretical curve is shown in Figure 10, which illustrates the influence of the additional terms that account for short wave effects, viz. the expression in Equation (40) and its subsequent influence throughout the computations, together with the modification for lateral extent of the orbital velocity field given by the expression following after Equation (41). In that case it can be seen that the theoretical values reduce for the shorter wavelengths, as expected, and a better agreement with the experimental values is indicated. One particular point where there is very poor agreement is at the shortest wavelength λ/L = .3, where the lateral bending moment obtained in the experiments is much lower than any of the theoretical results. This illustrates the inability of the basic theory to properly account for short waves (and higher frequencies) in a complete sense. Since the fundamental methods applied to ship motion analysis are deficient in this range, this difficulty is not unexpected, and it may possibly have only a small influence in the spectral domain due to the general high frequency range to which it applies.

The effect of roll motion on the theoretical lateral bending moment for the case of a 60° heading angle is shown in Figure 11, and there is a significant difference between the results of the theory with and without roll, for wavelengths corresponding to the condition $\lambda/L \leq .6$. That particular condition is quite close to roll resonance, and deviations of this nature might be expected in that region. However the computations of lateral bending moment were based upon the neglect of a term in the local lateral loading given by $\delta m\ \zeta(x)\ddot{\phi}$, which was deleted due to a lack of knowledge of the values of the quantity $\zeta(x)$. This lack of information did not affect the dynamic motions given by the solutions of Equations (34)-(36), but only affected the local lateral loading at a section and hence the resulting lateral bending moment computation. In view

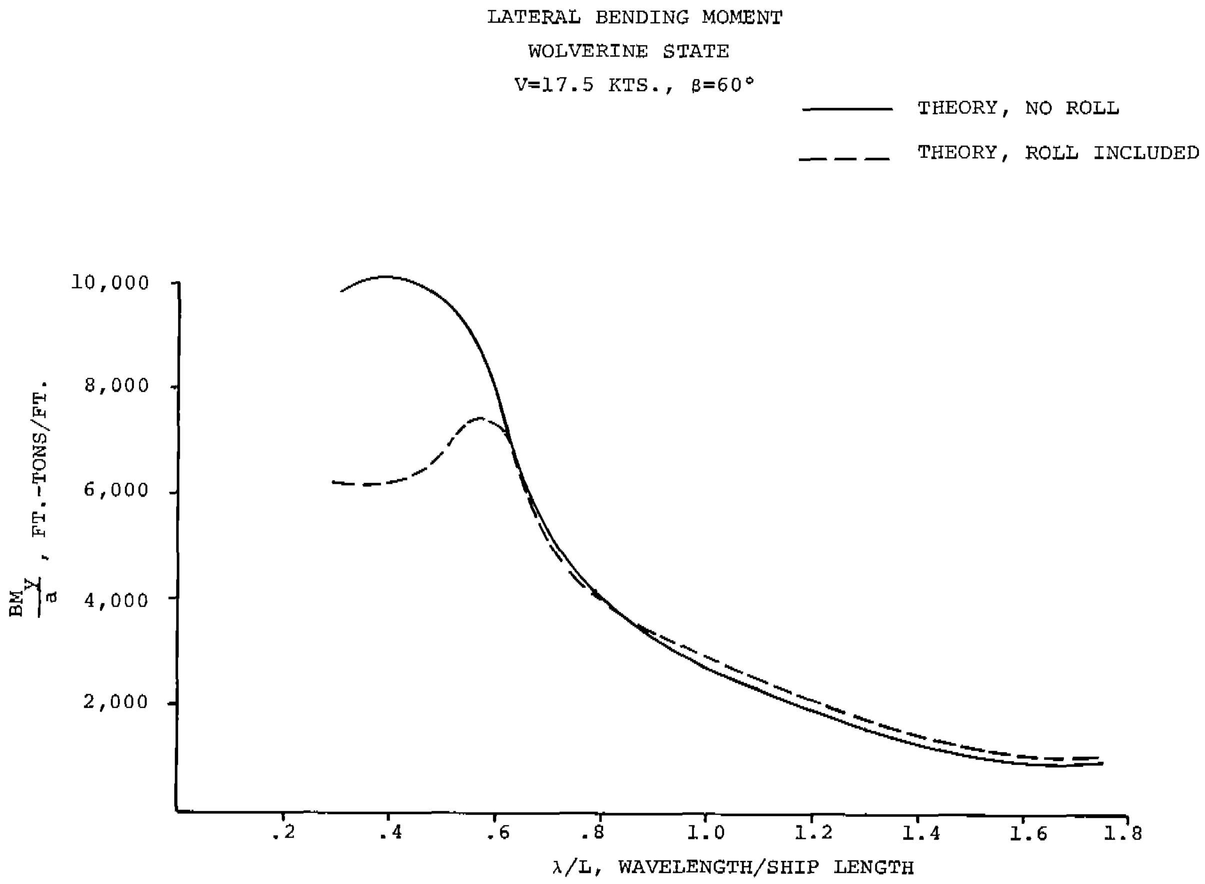

Fig. 11 Effect Of Roll Motion On Theoretical Lateral Bending Moment, β=60°.

of the inability to obtain information on this term, further theoretical computations were made without the influence of roll, i.e. a two degree of freedom representation. A comparison of the theoretical results without roll, together with the theory modification that accounts for short wave effects, is made with the experimental data available from [18] in Figure 12. The experimental data was obtained for the 16 knot condition, while the theoretical results are found for a 17.5 knot forward speed. It can be seen from Figure 12 that inclusion of the short wave effects produces better agreement between theory and the available experiments, but there is a relatively large difference for the longer wavelengths, (lower frequency conditions). The experimental lateral bending moment appears to decay more rapidly with increase in wavelength than the theory indicates, and at present there is no definite explanation for this occurrence except for the possibility that at such low frequencies the lateral motions become large and the ability of the present theory to predict the motions, and hence the lateral bending moments, under these conditions is somewhat limited (see [10] for further discussion of this effect).

Examination of the effect of forward speed on the experimental lateral bending moment (from [18]) does not produce any better agreement between theory and experiment, and hence other possibilities

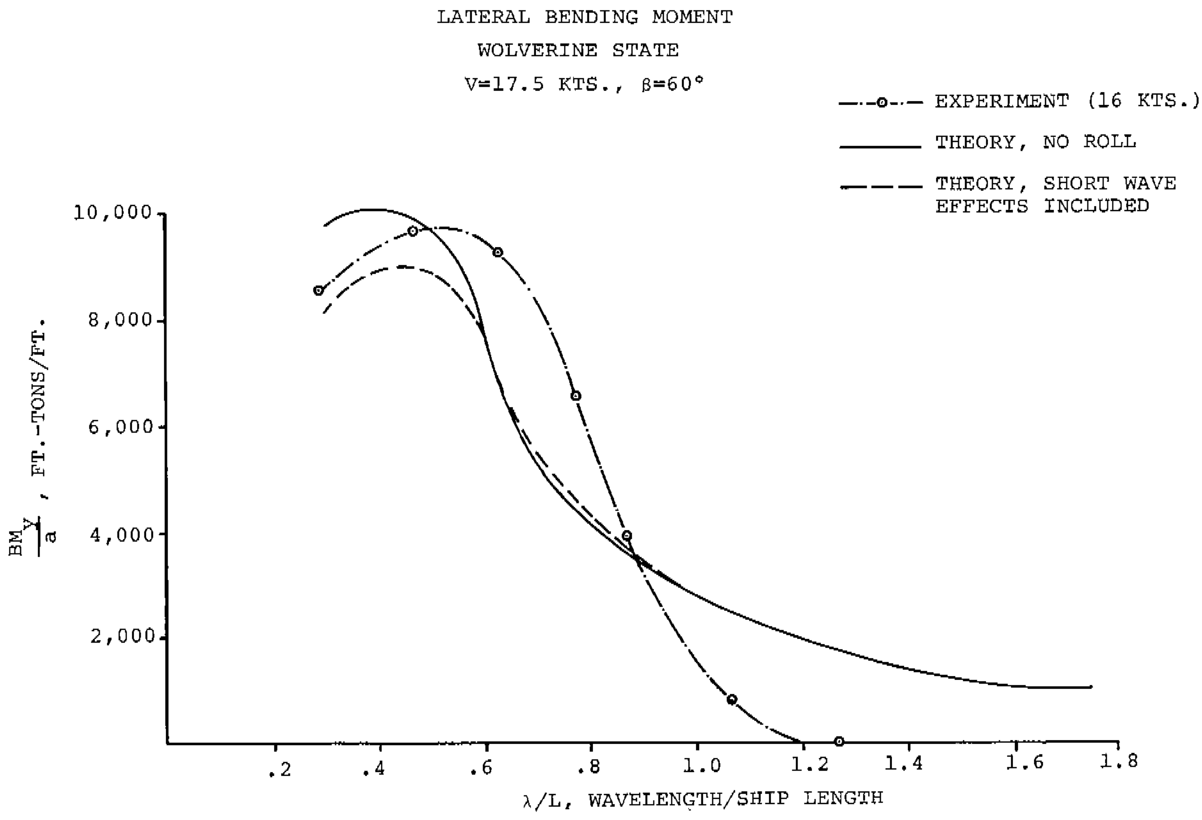

Fig. 12 Comparison Of Lateral Bending Moment Theory And Experiment, β=60°.

concerning the relation between theory and experiment must be examined. Since no detailed description of measurement methods interpretations, etc. are given in [18], there are some unanswered questions with regard to the experiments themselves. It is generally difficult to obtain measurements of lateral bending moments, and in addition there is no indication as to whether the final data presented in [18] represents a weather or leeward moment, or an average of those two. Data obtained in [8] indicates some differences between these two lateral bending moments, and in addition the importance of the "leeway" angle is also illustrated in [8]. No information on this angle, which determines the effective orientation of the ship hull with respect to the waves, is presented in [18]. Hence there are definite questions concerning the nature of the available experiments which are being compared with the present theory, but that is unfortunately a condition that exists in almost all considerations of lateral bending moments (see [10]).

As a result of the comparisons presented herein, it can be seen that there exists a useful tool for the theoretical prediction of vertical bending moments in oblique regular waves. A fair capability exists for the case of lateral bending moments, outside the range where roll effects are important. The inability to include another inertial reaction due to roll in the local lateral loading expression did not allow complete evaluation of the influence of roll, but on the basis of the limited computations carried out herein it does not appear that the inclusion of this additional degree of freedom, with its attendant complications in fundamental analysis and computation, is necessary for useful prediction of lateral bending moments. Further evaluation of other data, where the vertical CG location of the various mass elements of the ship are known will allow a more detailed determination of the influence of roll on lateral bending moments. Similarly with such data available it would be possible to extend the methods developed in this study to evaluate the torsional moment due to waves, since a fairly complete analysis of roll motion and its coupling with the lateral motions of yaw and sway is available as a result of the present investigation.

COMPUTATIONAL CONSIDERATIONS AND PRESENTATION OF OUTPUT RESULTS

Since the bending moment due to "whipping" of the ship, resulting from slamming in the bow region due to nonlinear forces arising from bow flare, is best expressed as a time history output, the basic problem remains as to the best method of presentation of the total bending moment output resulting from ship-wave interaction. The most useful method of presenting information on the characteristics of the wave-induced bending moments, both vertical and lateral, is by means of a spectral representation since relations to the input wave spectrum are easily obtained; the phenomena are characterized by a single parameter, viz. the variance, as a result of the Gaussian properties of these responses, allowing generalization and design applicability; and the general manner of representation of results in spectral form has become an accepted procedure over the years to which many naval architects are accustomed. It would certainly be beneficial if it were possible to determine the statistical characteristics of the bending moments due to slamming in conjunction with those of the wave-induced bending moments. A number of problems exist in attempting to achieve this union, and some of them are discussed below.

Aside from the fact that the hydrodynamic slamming impulse forces are nonlinear, the responses are primarily of a transient nature, since the bending moment due to slamming is not a "continuous" process as is the case of the bending moment directly induced by the waves. The bending moment response of the ship due to the sequences of random impulses arising from slamming thus has nonstationary characteristics, and its spectral interpretation will be different from that appropriate to the slowly varying bending moment directly induced by waves. Similarly, the bending moment induced by slamming is of a higher frequency than the direct wave-induced effects, and this is primarily due to exciting the ship in the lowest natural mode of vibration of the hull-girder system, and possibly higher modes. No simple spectral technique for representing the bending moment due to slamming is readily available, and even if such a method were developed it would be difficult to combine it with the spectral representation of the wave-induced bending moments so that a uniform spectral treatment of these two effects would be obtained. A nonstationary spectral analysis produces results that are fundamentally different from those of a stationary process. An examination of available literature, e.g. [28], shows that the covariance of a nonstationary process depends upon two time variables, not the time difference as for a stationary process. Fourier transform techniques applied to this form of covariance function produce a generalized power spectrum function that is defined in terms of two different frequency variables, and the variance of the process (a measure of the mean square) is determined as a double integral operation over the domain defined by the two frequency variables. The variance of a nonstationary process is also a time-varying quantity, and different values would probably be found following each series of slamming impulses. To complicate the matter further, it would be difficult to represent the force input properly, since it arises from nonlinear effects, and the previously outlined discussion of nonstationary processes, complicated as it is, is only valid for a linear dynamic system with linear inputs. Thus it is not fruitful to pursue a representation of the bending moment due to slamming in a spectral form for combining with the wave-induced bending moments.

In order to combine the two distinct types of bending moment outputs, a representation of the wave-induced effects as a time history appears to be the most direct and correct procedure since it can then be directly combined with the time history output due to slamming. Since it is difficult to relate the bending moment due to slamming to oblique waves, only the head sea case will be considered, as indicated in a previous section of this report. The basic element required for this procedure is knowledge of the vertical bending moment response operator, in the frequency (i.e. frequency of encounter) domain, which can be constructed from the data in Figure 8, for example. With a random wave time history input, representing the wave as measured at a point ahead of the bow, the vertical bending moment time history is given by a convolution integral in the time domain [29] where the weighting function on the wave input is the Fourier transform of the bending moment frequency response operator. These operations are expressed mathematically by the following:

$$T_{M\eta}(\omega_e;x_1) = \left|\frac{BM_z}{\eta}\right| e^{i\left[\phi_M - F(\omega_e)x_1\right]} \tag{64}$$

which defines the frequency response of vertical bending moment to a unit sinusoidal wave as measured at a point x_1 ahead of the origin of coordinates;

$$K_M(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} T_{M\eta}(\omega_e;x_1)e^{i\omega_e t}\,d\omega_e \tag{65}$$

which defines the weighting kernel function;.and

$$BM_z(t) = \int_{-\infty}^{\infty} K_M(t-\tau)\eta_m(\tau)d\tau \tag{66}$$

which defines the convolution integral operation producing a time history output of the vertical bending moment due to waves. Similar procedures can be used to represent any linear motion of the ship in response to waves, as well as the instantaneous relative motion of any point in the ship with respect to the wave record, such as the term defined by Equation (58).

The determination of the bending moment frequency response operator, as defined in Equation (64), is most appropriately carried out on a digital computer. Similarly the determination of the weighting function defined in Equation (65) must also be carried out digitally, and the same is true for the convolution integral operation defined in Equation (66). In order to obtain a time history output, this must be accomplished by a digital computer which is complemented by a hybrid linkage system containing A-D (analog to digital) and D-A (digital to analog) converters. The random wave input can be generated by a white noise generator, which is suitably filtered to produce the desired spectral form of the wave in the frequency of encounter domain. Thus the necessity for a time history output dictates the need of a particular type of computer facility, viz. a hybrid computer made up of a digital computer and an appropriate linkage system for transforming data from the analog to digital domain and vice versa.

The vertical bending moment due to slamming will be determined by solution of the coupled partial differential equations given by Equations (49)-(52). These equations represent the elastic response of the ship, with the effect of hydrodynamic forces manifested by added mass in the inertial term and the nonlinear portion of added mass and buoyancy forces in the bow region as the input excitation. For purposes of computer simulation, the hull will be assumed to be made up of "lumped" elements between the different ship stations (e.g. using 20 stations), with the appropriate elastic, inertial, etc. properties representative of each element. The partial differential equations will be converted to finite difference equation form (with respect to the axial space variable) and initially programmed for a hybrid computer (i.e. a combined system of analog and digital computer with an interface linkage of multiplexers, A-D converters, D-A converters, etc. that are compatible). This computer system has greater versatility than a pure analog system, as it makes use of the accuracy in function generation (such as evaluation of nonlinear buoyancy and added mass variation at each section) of the digital computer in conjunction with the integration and summing capability of the analog computer as a simulator (real time or in terms of a compressed time scale, i.e. many times faster than real time). The relative vertical velocity and displacement (relative to the wave surface) at different sections in the bow region can be generated in time history form as a convolution-type integral in accordance with the hybrid technique described previously, for particular input wave spectra, and the resulting nonlinear added mass and buoyancy forces obtained (using this data) in continuous time history form. The final output of this equation system will be the time history of vertical bending moment due to the nonlinear slamming forces, and this can be combined directly with the wave-induced vertical bending moment time history, which is generated as a response to the same wave input.

The preceding discussions have demonstrated the advantages of a hybrid computer, where the best suited computational capability and advantages of both analog and digital computers are efficiently applied to obtain a solution of a complicated dynamic simulation problem. The speed advantage of a hybrid computer is due to the parallel operation of the analog portion, thereby eliminating the serial calculations inherent in digital computation that usually result in relatively long solution times for digital simulations. This speed advantage increases as the highest frequency of interest in the physical phenomenon increases, and as long as there are no large requirements for storage capacity the hybrid computer has significant advantages for application to the present problem of ship structural response simulation. This particular advantage is evident if the output is obtained in a time history form, since the record can then be analyzed further to obtain information on various properties such as rms amplitude, frequency of occurrence of particular levels, etc. just as present analyses are carried out on records from instrumented full scale ships. The time compression capability results in obtaining this information in a relatively short time as compared to full scale or model test evaluation, and the attendant savings in cost are very evident.

The particular choice of digital computer and analog computer that make up the hybrid unit will be determined after some computational practice for particular cases that will illustrate the effectiveness of the procedures described above. The required capacities, speeds, etc. of the processors are also particular "hardware" questions that will be answered as a result of computer experimentation, which is another phase in the overall study associated with the Ship Computer Response program.

All of the considerations that led to selection of a hybrid computer for purposes of ship structural response simulation were dictated by the requirements of including the bending moments due to slamming. These effects are considered sufficiently important to require the program to provide this information, but there may be cases where slamming effects are not required to be studied. In those cases a spectral measure of response would be most appropriate. For that situation the digital computer can be applied to produce final spectral measures for different sea state conditions. A digital technique seems to be most appropriate for determining the linear rigid body ship motions and bending moments directly induced by the waves since the computational technique is basically arithmetic; the digital computer technique has great flexibility when altering conditions; analog representation of frequency-dependent coefficients is relatively difficult; and the digital method can be directly applied to determination of spectral representations, especially when considering an input wave spectrum with directional characteristics. All of these digital computer operations are aimed at final representation of bending moment response in a spectral form. The relative importance of slamming effects will determine the final computer system required for simulation of ship structural response.

CONCLUSIONS

As a result of the development of the equations in the main body of this report, and the comparison of selected hand calculations with available experimental data, there is available a useful tool for predicting ship structural response characteristics. The effects of oblique heading, manifested by the presence of lateral bending moments in addition to modified vertical bending moments, are accounted for in the present mathematical model. Similarly roll motion and its influence are included, and with the availability of additional information on inertial properties of a ship it will be possible to easily extend the present model to allow computation of torsional moments due to waves.

A method of representing the bending moment due to slamming is discussed, and computational procedures for obtaining the output in a form compatible with the wave-induced bending moment are considered. Spectral techniques for this nonlinear and nonstationary effect are not practicable, and a time domain representation of both types of bending moment is considered as the best means of final output presentation. Methods of representing time histories of the wave-induced bending moment, by convolution integral operations, are outlined and the benefits of a hybrid computer simulation demonstrated for that case. The applicability of digital computers for representing spectral responses of wave-induced bending moments, when slamming effects are not required, is also shown and the mathematical model developed in the present study has the capability of presenting the effects of combined vertical and lateral bending moments, including the effects of directionality in the input wave spectrum.

The choice of a particular computer system depends upon the relative importance of including slamming effects. This selection will depend upon the particular class of problems being investigated. Similarly the choice of particular computers and their detailed characteristics will be based upon results obtained in computer experiments on certain particular illustrative problems, which will be the subject of another phase of the Ship Computer Response study.

REFERENCES

1. Lewis, E. V.: "Ship Model Tests to Determine Bending Moments in Waves", Trans. SNAME, 1954.

2. Dalzell, J. F.: "Effect of Speed and Fullness on Hull Bending Moments in Waves", Stevens Inst. of Tech., Davidson Lab. Report No. 707, Feb. 1959.

3. Fritch, D. J., Bailey, F. C., and Wise, N. S.: "Preliminary Analysis of Bending Moment Data from Ships at Sea", Report SSC-153, (AD436 877).

4. Fritch, D. J., Bailey, F. C., and Wise, N. S.: "Results from Full Scale Measurements of Midship Bending Stresses on Two C4-S-B5 Dry Cargo Ships Operating in North Atlantic Service", Report SSC-164, (AD605 535).

5. Jacobs, W. R.: "The Analytical Calculation of Ship Bending Moments in Regular Waves", Journal of Ship Research, Vol. 2, No. 1, June 1958.

6. Jacobs, W. R., Dalzell, J., and Lalangas, P.: "Guide to Computational Procedure for Analytical Evaluation of Ship Bending-Moments in Regular Waves", Stevens Inst. of Tech., Davidson Lab. Report No. 791, October 1960.

7. Korvin-Kroukovsky, B. V. and Jacobs, W. R.: "Pitching and Heaving Motions of a Ship in Regular Waves", Trans. SNAME, 1957.

8. Numata, E.: "Longitudinal Bending and Torsional Moments Acting on a Ship Model at Oblique Headings to Waves", Journal of Ship Research, Vol. 4, No. 1, June 1960.

9. Kaplan, P. and Ulc, S.: "A Dynamical Method for Calculating Lateral Bending Moments on Ships in Oblique Waves", TRG, Inc. Report No. 147-SR-1, Nov. 1961.

10. Raff, A. I.: "The Dynamic Calculation of Lateral Bending Moments on Ships in Oblique Waves", TRG, Inc. Report No. 147-FR, March 1964.

11. Andrews, John N. and Chuang, Sheng-Lun: "Seaworthiness Analog Computer", DTMB Report No. 1829, August 1965.

12. Dalzell, J. F.: "An Investigation of Midship Bending Moments Experienced in Extreme Regular Waves by Models of the Mariner Type Ship and Three Variants", Stevens Inst. of Tech., Report SSC-155, January 1964.

13. Dalzell, J. F.: "An Investigation of Midship Bending Moments Experienced in Extreme Regular Waves by Models of a Tanker and a Destroyer", Stevens Inst. of Tech., Report SSC-156, February 1964.

14. Kaplan, Paul and Hu, Pung Nien: "Virtual Mass and Slender Body Theory for Bodies in Waves", Proc. of the Sixth Midwest Conference on Fluid and Solid Mechanics, Univ. of Texas, September 1959.

15. Kaplan, Paul and Sargent, Theodore P.: "Theoretical Study of the Motions of an Aircraft Carrier at Sea", Oceanics, Inc. Report No. 65-22, January 1965.

16. Wachnik, Z. G. and Zarnick, E. E.: "Ship Motions Prediction in Realistic Short-Crested Seas", Trans. SNAME, 1965.

17. McGoldrick, R. T.: "Ship Vibration", DTMB Report No. 1451, December 1960.

18. Progress Report on SR-165, Bending Moment Determination, with data from model tests of a ship model in oblique waves; Davidson Lab., Stevens Inst. of Tech., March 1967.

19. Grim, O.: "Die Schwingungen von schwimmenden, zweidimensionalen Korpern", HSVA Rpt. No. 1171, September 1959.

20. Lewis, F. M.: "The Inertia of Water Surrounding a Vibrating Ship", Trans. SNAME, 1929.

21. Ogilvie, T. Francis: "Recent Progress Toward the Understanding and Prediction of Ship Motions", Proc. of Fifth Symposium on Naval Hydrodynamics, Bergen, Norway, September 1964.

22. Timman, R. and Newman, J. N.: "The Coupled Damping Coefficients of a Symmetric Ship", Journal of Ship Research, Vol. 5, No. 4, March 1962.

23. Tasai, F.: "Hydrodynamic Force and Moment Produced by Swaying and Rolling Oscillation of Cylinders on the Free Surface", Rpt. of Research Inst. for Applied Mechanics, Japan, Vol. 9, No. 35, 1961.

24. Kaplan, Paul and Putz, Robert R.: "The Motions of a Moored Construction-Type Barge in Irregular Waves and their Influence on Construction Operation", Rpt. for U. S. Naval Civil Eng. Lab., August 1962.

25. St. Denis, M. and Pierson, W. J., Jr.: "On the Motions of Ships in Confused Seas", Trans. SNAME, 1953.

26. Kaplan, Paul: "A Study of the Virtual Mass Associated with the Vertical Vibration of Ships in Water", Davidson Lab., Stevens Inst. of Tech., Rpt. No. 734, December 1959.

27. McGoldrick, Raymond T. and Russo, Vito L.: "Hull Vibration Investigation on SS Gopher Mariner", Trans. SNAME, 1955.

28. Caughey, Thomas K.: "Nonstationary Random Inputs and Responses", published in random vibration, volume 2, edited by Stephen H. Crandall, MIT Press, 1963.

29. Kaplan, Paul: "A Study of Prediction Techniques for Aircraft Carrier Motions at Sea", Oceanics, Inc. Report No. 65-23, September 1965.

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DEVELOPMENT OF MATHEMATICAL MODELS FOR DESCRIBING SHIP STRUCTURAL RESPONSE IN WAVES

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13. ABSTRACT Available mathematical models that describe ship-wave interactions are evaluated in order to develop a technique for predicting ship structural response characteristics. Major consideration is given to the bending moment and slamming responses for an arbitrary ship form in any state of sea, at any relative heading and forward speed. The slowly varying vertical and lateral bending moments due to waves are obtained using a linearized model based on strip theory, where the effect of roll motion and its influence in the lateral plane are included, with the model sufficiently general to also allow extension to computation of torsional moments due to waves. Comparison of the results of a limited series of hand computations with available experimental data indicates a good degree of agreement, as well as an overall consistency, for the analysis of wave-induced bending moments. A mathematical representation of the bending moment due to slamming is also described, and computational procedures for obtaining an output compatible with the wave-induced bending moment are outlined. Methods of analysis in terms of power spectra as well as time histories are considered, and the utility of different types of computers for presentation of information on ship structural response is described.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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