

SSC-196

**ANALYSIS AND INTERPRETATION OF  
FULL-SCALE DATA ON MIDSHIP BENDING  
STRESSES OF DRY CARGO SHIPS**

**This document has been approved  
for public release and sale; its  
distribution is unlimited.**

**SHIP STRUCTURE COMMITTEE**

**JUNE 1969**

# SHIP STRUCTURE COMMITTEE

**MEMBER AGENCIES:**

UNITED STATES COAST GUARD  
NAVAL SHIP SYSTEMS COMMAND  
MILITARY SEA TRANSPORTATION SERVICE  
MARITIME ADMINISTRATION  
AMERICAN BUREAU OF SHIPPING

**ADDRESS CORRESPONDENCE TO:**

SECRETARY  
SHIP STRUCTURE COMMITTEE  
U.S. COAST GUARD HEADQUARTERS  
WASHINGTON, D.C. 20591

June 1969

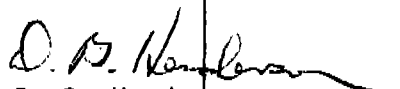
Dear Sir:

This report contains a statistical technique analysis of primary bending moment data measured on vessels at sea. The collection of measured values were extrapolated to predict the type and level of bending moment a vessel will experience throughout its life.

The enclosed report presents the analysis made to date, together with a description of the methods used in the prediction.

This report is being distributed to individuals and groups associated with or interested in the work of the Ship Structure Committee. Comments concerning this report are solicited.

Sincerely,



D. B. Henderson  
Rear Admiral, U. S. Coast Guard  
Chairman, Ship Structure Committee

Report  
from

Project SR-171 "Ship Statistics Analysis"

to the  
Ship Structure Committee

SSC-196

ANALYSIS AND INTERPRETATION OF FULL-SCALE DATA  
ON MIDSHIP BENDING STRESSES OF DRY CARGO SHIPS

by

D. Hoffman and E. V. Lewis  
Webb Institute of Naval Architecture  
Glen Cove, New York

under

Department of the Navy  
NAVSEC Contract #NObs 92384

*This document has been approved for public release and sale;  
its distribution is unlimited.*

U. S. Coast Guard Headquarters  
Washington, D. C.

June 1969

## ABSTRACT

Results of the analysis of stress data from full-scale measurements on two C-4 type cargo vessels, the *S. S. Wolverine State* and *S. S. Hoosier State*, are presented in the form of histograms and cumulative distributions, which together with previously analyzed full-scale data cover a total of five years of normal ship operation in the North Atlantic. In addition, results of analysis of full-scale data are given for two additional ships, the *Mormacscan* and the *California Bear*. The latter two ships represent higher speed types than the first two, and results cover several different trade routes.

Two rational techniques are given for the extrapolation of full-scale data to longer periods of time, in order to predict extreme bending stresses (or bending moments) in service. One of the techniques employs the integration of rms stress data from individual stress records; the other makes use of the highest stresses obtained in each record (extreme values). Both techniques involve the classification of data by severity of weather in order to obtain greater generality of results. It is shown that extrapolated trends from the two methods are similar but reveal differences that warrant further investigation.

Recommendations are made for more data collection from different ships on different routes, for investigation of other statistical techniques, and for development of methods for model predictions of long-term trends.

## CONTENTS

	<u>Page</u>
INTRODUCTION .....	1
REDUCTION OF STRESS DATA .....	7
HISTOGRAM ANALYSIS OF STRESSES .....	10
RELATIONSHIP BETWEEN STRESSES AND BENDING MOMENTS .....	17
EXTRAPOLATION BASED ON RMS VALUES .....	20
EXTRAPOLATION BASED ON EXTREME VALUES .....	34
COMPARISON OF RMS AND EXTREME VALUE EXTRAPOLATIONS .....	39
CONCLUSIONS AND RECOMMENDATIONS .....	41
ACKNOWLEDGEMENTS .....	42
REFERENCES .....	43
APPENDIX A - STATISTICAL TECHNIQUES .....	45
by O. J. Karst	
APPENDIX B - STATISTICAL TESTS .....	52
BY Dan Hoffman	

## SHIP STRUCTURE COMMITTEE

The SHIP STRUCTURE COMMITTEE is constituted to prosecute a research program to improve the hull structures of ships by an extension of knowledge pertaining to design, materials and methods of fabrication.

RADM D. B. Henderson, USCG - Chairman  
Chief, Office of Engineering  
U. S. Coast Guard Headquarters

Captain W. R. Riblett, USN  
Head, Ship Engineering Division  
Naval Ship Engineering Center

Mr. E. S. Dillon  
Chief, Division of Ship Design  
Office of Ship Construction  
Maritime Administration

Captain T. J. Banvard, USN  
Maintenance and Repair Officer  
Military Sea Transportation Service

Mr. D. B. Bannerman, Jr.  
Vice President - Technical  
American Bureau of Shipping

## SHIP STRUCTURE SUBCOMMITTEE

The SHIP STRUCTURE SUBCOMMITTEE acts for the Ship Structure Committee on technical matters by providing technical coordination for the determination of goals and objectives of the program, and by evaluating and interpreting the results in terms of ship structural design, construction and operation.

### NAVAL SHIP ENGINEERING CENTER

Mr. J. J. Nachtsheim - Chairman  
Mr. J. B. O'Brien - Contract Administrator  
Mr. G. Sorkin - Member  
Mr. H. S. Sayre - Alternate  
Mr. I. Fioriti - Alternate

### MARITIME ADMINISTRATION

Mr. F. Dashnaw - Member  
Mr. A. Maillar - Member  
Mr. R. Falls - Alternate  
Mr. W. G. Frederick - Alternate

### AMERICAN BUREAU OF SHIPPING

Mr. G. F. Casey - Member  
Mr. F. J. Crum - Member

### OFFICE OF NAVAL RESEARCH

Mr. J. M. Crowley - Member  
Dr. W. G. Rauch - Alternate

### MILITARY SEA TRANSPORTATION SERVICE

LCDR R. T. Clark, USN - Member  
Mr. R. R. Askren - Member

### U. S. COAST GUARD

CDR C. R. Thompson, USCG - Member  
CDR J. L. Howard, USCG - Member  
LCDR L. C. Melberg, USCG - Alternate  
LCDR R. L. Brown, USCG - Alternate

### NAVAL SHIP RESEARCH & DEVELOPMENT CENTER

Mr. A. B. Stavovy - Alternate

### NATIONAL ACADEMY OF SCIENCES

Mr. A. R. Lytle, Liaison  
Mr. R. W. Rumke, Liaison

### AMERICAN IRON AND STEEL INSTITUTE

Mr. J. R. LeCron, Liaison

### BRITISH NAVY STAFF

Mr. H. E. Hogben, Liaison  
Mr. D. Faulkner, Liaison

### WELDING RESEARCH COUNCIL

Mr. K. H. Koopman, Liaison  
Mr. C. Larson, Liaison

## INTRODUCTION

### Background

The purpose of the Ship Structure Committee project SR-171 has been stated to be (1)\*to "analyze the data on bending moment versus sea state obtained on both full-scale ships in service and on ship models with the objective of predicting the type and level of bending-moment history that a ship will undergo throughout its life. This can then serve as an important guide for ship design."

The work on this project is not complete, but it is the object of this report to provide a progress report on work done to date toward the above objective. Although direct assistance to the ship designer is not yet provided, it is hoped that the completion of the project will yield results that are indeed useful in ship design.

There has been a remarkable trend in recent years toward larger tankers and bulk cargo carriers, as well as a steady increase in the speed of general cargo ships. Questions have arisen as to the applicability of the old empirical standards of longitudinal strength to these new ships, and a need has arisen for a more fundamental approach to the design of ships for adequate longitudinal strength.

Longitudinal hull girder stresses arise primarily as the result of the differences in fore and aft distribution of buoyancy and weights. In many ships the bending moments can reach large values even in still water, but such girder loadings can be readily calculated by classical beam theory. A more elusive loading on the hull is that induced by the waves encountered by the ship at sea.

In this report we shall consider only one of the many factors involved in longitudinal strength -- wave-induced bending moment -- with the recognition that other factors, such as still water loads, slamming stresses, temperature effects, and combined loads must not be neglected. The wave bending moment is not a static quantity, and it depends on the response of the ship to particular seas. Since the seaway is constantly changing in a completely random and unpredictable way, and since it has been shown by previous investigators that response is affected by ship speed, heading, weight distribution, etc., it is obvious that a simple deterministic solution is not possible.

---

\*Numbers in parentheses refer to References listed at the end of this report.

Full-scale strain measurements on ships in service have been made in various countries (2) (3) (4). Actual stress records have been found to be very irregular, both within a single record and from one record to another, because of the fluctuations in the waves encountered. Such records reveal that ships on rough weather routes occasionally experience extremely high stress values. It will be shown how records can be analyzed in the same way as the records of the irregular surface of the sea and the frequency of occurrence of extreme stresses predicted by the use of statistics and probability theory.

### Probability Model

One of the fundamental philosophical problems in a statistical approach to wave-induced bending moments is that one can never hope to obtain a complete life's history of bending moment experienced by even one ship. One is forced therefore to work with samples and then to devise a probability model that fits the data satisfactorily and can be extrapolated to much longer periods of time. As stated in (5), this means that the appropriate philosophy of probability theory is that predictions may be made of what is likely to happen in the future on the basis of statistical analysis of the past, provided that conditions remain unchanged.

The difficulty resulting from limited data has been partly overcome by a decision of the Ship Research Committee to continue data collection on the Wolverine State for a much longer period of time than has been customary in other such data collection projects. (Total period of time covered by records of the Wolverine State and Hoosier State so far analyzed in this report was 5 years.) Thus, the records obtained by Teledyne represent a particularly comprehensive source of ship stress data. Furthermore, not only do they cover a long period of time but they consist of actual magnetic tape records that can be reduced in various ways. Earlier work of this type is demonstrated in several reports in which time histories of stress on several ship types have been analyzed (6) (7) (8).

Another principle adopted very early by all concerned with this project, and the related data-collection project SR-153, was the identification of physical factors affecting bending moment that were not random in nature. This permitted the data to be subdivided and the statistical analysis applied to the random factors only. Some physical factors known to affect the wave bending moment are as follows:

1. Ship loading condition -- cargo distribution and drafts.
2. Ship speed.
3. Ship heading.
4. Sea conditions encountered.

It was recognized that item 4 was a factor of basic importance, since



the bending moment statistics must depend greatly on the sea conditions actually experienced by the ship during the sampling of bending moment (stress) data. Furthermore, if different ships are to be compared -- on the same or different routes -- any difference in weather encountered by each would affect the comparison. (This was the difficulty experienced by Johnson and Larkin (2) in comparing extensive stress data on different ships.)

#### Classification by Weather

Accordingly, from the beginning of project SR-153 a separation of data on the basis of weather was introduced. Ideally this classification of data should have been based on wave heights, but it was felt that the observed heights recorded in the logbooks could not be considered as reliable as the observed Beaufort Numbers representing wind velocity. Hence, the classification was based on the Beaufort Scale. The relationship between wind velocity and wave height must then be considered on a statistical basis, since the wave build-up will lag behind wind velocity as a storm approaches, and the wave decline will also lag when the storm moves away. Furthermore, swell from previous or distant storms will be independent of the local wind, and the background swell will vary seasonally, being more pronounced in winter than in summer. Distant shores will also provide local sheltering effect, as between the U.S. east coast and the Grand Banks on the North Atlantic route. Ocean currents and relative sea-air temperatures affect storm wave build-up, and shoaling water over continental shelves -- as at the approaches to the English Channel -- increases wave steepness. So far all these factors have been lumped together in the statistical treatment of the data based on Beaufort Number or wind speed. Some more detailed study of logbooks and meteorological data over selected periods might be enlightening.

Sample analyses of Wolverine State data indicated that the other three physical factors mentioned above were of relatively minor importance. First, the ship's loading showed surprisingly small variations from voyage to voyage, whereas model tests show that relatively large changes are required to produce significant change in bending moment. Ship speed showed a consistent variation with weather severity, and again model tests showed small effects on bending moment to result from large speed changes. Finally, over a period of 11 voyages of the Wolverine State, ship headings were found to be almost equally divided around the compass (9). Initially it was decided not to attempt to classify the data by heading, on the assumption that the resulting variation would be random and that the statistical analysis would give a satisfactory picture. However, it is felt that a limited analysis of data classified by ship heading relative to the sea should be carried out before the project is completed.

It has been pointed out that in addition to the familiar seasonal variation in the severity of wind speed and wave height, there is also a longer-term variation from year to year. Data on the frequency of winds exceeding 33 knots in the North Atlantic (10) suggest the possi-

bility of a 12-year cycle. At any rate, it appears that the weather conditions were more severe in 1959-1962 than in 1951-1954. It is fortunate that the data collection on the Wolverine State and Hoosier State included the years of severe weather.

The above weather variability is another reason for classifying stress data by weather in the analysis. When long-term trends are corrected to average weather taken over many years, a direct comparison is possible between similar ships having data collection in different years.

### Generalization of Results

Another basic philosophical problem of ship bending moment data collection is that no matter how good the results may be for the ship or ships investigated, they can provide guidance only for the design of other very similar ships. It was for this reason that the Ship Research Committee decided to carry out comprehensive model tests of several of the ships in the data collection program. (The work was done at the Davidson Laboratory under project SR-165 for the Wolverine State and more recently for the California Bear.) The hope has been that some coordination between model and ship data would permit generalization of results that would be useful to the ship designer.

Since then methods have been developed for predicting long-term distributions of bending moment from model test results and ocean wave spectra, indicating what may be expected in a ship's lifetime. If it can be shown that predictions made from model test data can be correlated with analysis and extrapolation of full-scale ship data, then it should be possible to provide a general answer to the problem of predicting wave loads for any ship for any period of time on any sea route. Thus the collection of ship stress data is now viewed primarily as a basis for evaluating methods of long-term prediction. Work on methods of predicting long-term trends from model tests is also under way at Webb Institute under Project SR-171. Comparisons of predicted trends for the Wolverine State with those obtained from statistically analyzed ship stress data showed excellent agreement, as previously reported (11). A new report including more recent results for several ships is planned as a sequel to the present report (12).

Furthermore, considering the advance made in recent years in computing ship response in regular waves theoretically (13) (14) (15), it is hoped that such computational techniques will become sufficiently satisfactory to reduce the number of required model tests in the future, since model and full-scale data collection are time-consuming and economically not always feasible.

This report is intended then to be a progress report on the ship statistics analysis aspects of project SR-171. It will review the manner in which the ship strain data are obtained by others and reduced for analysis, classified by weather severity. Then the histogram type analysis of data will be described and results presented, followed by the description of two methods of extrapolation of data

to give long-term trends. After comparing the results by these methods, tentative conclusions will be drawn and recommendations made for further work.

The work has been carried out at Webb Institute of Naval Architecture under the sponsorship of the Ship Structure Committee, through the Ship Research Committee. The project has been designated SR-171 "Ship Statistics Analysis."

### Full-Scale Analyses

Results obtained on board several ships under normal service conditions have been gathered by the Teledyne Materials Research Company and results over the past five years analyzed at Webb Institute. Presently data from four ships representing three types operating in three different sea areas have been collected and analyzed, and analysis is continuing as further data become available. Previous publications (3) (5) (11) have partially covered the method of analysis and some of the results obtained. The present report extends the work to include alternative techniques of analysis and to give additional results, all under Ship Structure Committee project SR-171. Previously reported work is reviewed where necessary for clarity.

All the data collection has been done on magnetic tape by the Teledyne Materials Research Company of Waltham, Massachusetts, who install, maintain and service the recording equipment as well as perform the initial reduction of the taped data. (SSC project SR-153.)

The most comprehensive data were obtained from the sister ships, Hoosier State and Wolverine State, C4-5-B5 type machinery aft dry cargo vessels, and the data have been analyzed for the years 1960-1965 in the North Atlantic route. Data collection is still continuing on the Wolverine State. Records were also taken on the S.S. Mormacscan, a machinery amidships dry cargo vessel, operating in both the North Atlantic and the South American services. Data analyzed for about three years (up to early 1967) allow a comparison to be made of stresses experienced in the same sea area by different types of ships, and on the same ship in different sea areas. The fourth ship for which data have been collected is the California Bear, a Mariner type in the North Pacific route. Data for only five voyages (up to early 1967) have been analyzed. Collection of data on both the Mormacscan and the California Bear terminated in mid-1968 and analysis is continuing.

For all the above ships strain data were recorded mainly as the sum of port and starboard transducers, from which the average can be obtained. These strain data can be related to the stress or to vertical bending moment experienced at the location of the gages. Records made on board the Wolverine State since January 9, 1964 were in the form of separate records of port and starboard stresses in order to indicate lateral bending moment effects and were later combined in the Teledyne laboratory to give the combined average port and star-

board stresses. Particulars of the above three types of ships are given in Table I.

TABLE I SHIP PARTICULARS.

	<u>SS Hoosier State &amp; SS Wolverine State</u>	<u>SS Mormacscan</u>	<u>SS California Bear</u>
Type	C4-S-B5 Dry Cargo	C3-S-33A Dry Cargo	C4-S-1a Mariner Dry Cargo
Machinery location	Aft	Amidships	Amidships
Builder	Sun Shipbuilding & Dry Dock Co.	Sun Shipbuilding & Dry Dock Co.	Bethlehem Steel Co., San Francisco Yard
Date	September, 1945	October, 1960	1954
Hull Number	359	622	
Length Overall	520' - 0"	483' - 3"	563' - 7 3/4"
Length between Perp.	496' - 0"	458' - 0"	528' - 6"
Beam, Molded	71' - 6"	68' - 0"	76' - 0"
Depth, Molded	54' - 0"	41' - 6"	44' - 6"
Load Draft, Keel	32' - 9 7/8"	31' - 5"	29' - 10 1/16"
Waterplane Coefficient	.752 (30' draft) .685 (18' draft)	.730	.724
Gross Tonnage	10,747	9,315	9,216
Net Tonnage	6,657	5,609	5,366
Midship Section Modulus (to Upper Deck)	45,631 in. <sup>2</sup> -ft.	30,464 in. <sup>2</sup> -ft.	43,900 in. <sup>2</sup> -ft.
Dead Weight at Load Draft	15,348 L.T.	12,483 L.T.	13,418 L.T.
Shaft Horsepower, Normal	9,000	11,000	17,500
Shaft Horsepower, Maximum	9,900	12,100	19,250

L.T. = long tons.

## REDUCTION OF STRESS DATA

### Data Recording

Signals generated by the strain transducers were automatically recorded on a magnetic tape system aboard ship for a half-hour period at the beginning of each four-hour watch. The recorded tapes were reduced in the Teledyne laboratory, using a magnetic tape playback unit and a direct recording oscillograph which accepts either: (a) the direct output of the playback unit, thus tracing the original recorded information, or (b) the output of a special-purpose probability analyzer. The technical details of the above instrumentation are given in Ref. (3). The two types of output mentioned above were studied and analyzed in a different manner.

### Direct Print-Out

The reconstructed records obtained from the magnetic tape playback system are referred to as "quick looks," in Teledyne terminology, since they are compressed representations of the actual tape record. They were used by Teledyne merely to assess roughly the quality of the data and the order of magnitude involved. Between each adjacent record interval was a calibration signal for the following record.

### Probability Analyzer

The stress histogram of each recording interval was obtained as an output of the probability analyzer, which uses as input the output of the tape playback system and filters it to remove high frequency slamming signals. Calibration signals at the beginning of each recording interval were superimposed on the record, thus triggering the probability analyzer during analysis.

The analyzer makes use of digital peak detectors whereby counts at given signal levels are stored in a series of sixteen counters. The output is a graphical histogram on paper tape of sixteen levels for which the number of peak-to-trough (or trough-to-peak) occurrences (hog plus sag) is plotted as an ordinate. Thus if the maximum stress expected is predetermined from the "quick looks" to be 8 KPSI or 12 KPSI each of the stress levels will cover a range of .5 or .75 KPSI, respectively. One of the advantages of the Teledyne system over one in which counters are located aboard ship is this feature that permits the selection of ranges to suit the individual records.

Other information obtained from the digital registers of the probability analyzer is the total number of peak-to-trough stress cycles analyzed and the maximum peak-to-trough stresses ( $X_m$ ) encountered in individual intervals.

Calculated Statistics

The root mean square (rms) values of peak-to-trough stresses were calculated by Teledyne for each record interval from the output of the stress analyzer. These rms values are designated  $\sqrt{E}$ , as in Band's work (5) and elsewhere, to distinguish them from the rms of record  $\sigma$  (i.e., rms value of equally spaced points on a record). Hence, in general,

$$\sqrt{E} = \sqrt{\frac{\sum X^2}{n}} \quad [1]$$

where X = magnitude of peak-to-trough (or trough-to-peak) stress variation, and n = number of stress reversals or half-cycles in the sample record, i.e., variations of stress from peak (maximum) to trough (minimum), or from trough to peak, with a zero crossing between.\*

In this case, where the stress data have been classified by stress ranges,

$$\sqrt{E} = \sqrt{\frac{\sum n_i X_i^2}{n}} \quad [2]$$

where  $X_i$  = mean value of the  $i$ th range  
 $n_i$  = the number of reversals which fall within the  $i$ th range  
 $n$  = total number of reversals =  $\sum n_i$

In Teledyne notation (3),

$$\sqrt{E} = \frac{\sqrt{Q^2 K}}{\sqrt{4 \sum n_i}} = 1/2 \sqrt{\frac{Q^2 K}{\sum n_i}} \quad [3]$$

where  $K = \frac{4}{Q^2} \sum n_i X_i^2$

Q = a calibration factor determined by the overall range required.

Sometimes  $X_i/Q$  is replaced by  $l_i$ .

All of the above data were then accumulated by Teledyne on punch cards, each representing one record interval and including the log book data such as: Latitude, longitude, course, ship speed, wind speed,

---

\*In Band's work (5), the word "cycle" is used to mean such a variation of stress from peak to trough or from trough to peak, although "half-cycle" or stress reversal would have been a more appropriate term.

wind direction, Beaufort number, wave direction, significant wave height, average wave period, average wave length and ship heading.

The individual results on cards for all intervals can be summed up by computer to yield the mean and standard deviation over a complete voyage for each Beaufort number or weather group required. This has been done at Webb Institute, but in the future will be done by Teledyne as part of their data reduction.

It has been the practice of Teledyne to present the rms  $\sqrt{E}$  and maximum stress data in relation to Beaufort number in graphical form for a number of voyages. See Figs. 1-2 and 4-5 of (3) and Fig. 7 of this report. The mean value for each Beaufort number is also shown.

It has been pointed out by Teledyne (3) that the peak-to-trough histograms seem to be approximated by the Rayleigh distribution,

$$p(X) = \frac{2X}{E} e^{-X^2/E} \quad X \geq 0 \quad [4]$$

where  $p(X)$  = probability density of  $X$ , i.e., a function that indicates the percentage of times that different values of  $X$  occur.

This distribution is a convenient one to use, since it has only a single parameter, the mean square value of  $X$ , or  $E$ . It has been found to apply quite well to ocean wave records (16), to ship motions (16) (17), and to stresses and bending moments (17) (18).

Figure 1(a) shows excellent agreement between actual peak-to-trough stress data from one typical record with an ideal Rayleigh curve. Teledyne has also compared the actual maximum stress reversals in individual records,  $X_m$ , with the values predicted from the Rayleigh distribution and found large individual differences but good agreement on the average (19). Band made an overall comparison of average values and found good agreement (5). However, it is interesting to note that when a histogram of stresses summarized over several voyages and many records is compared with a Rayleigh curve (Fig. 1(b)), the agreement is not very good, because the sum of many Rayleigh distributions does not yield a new average Rayleigh curve.

Using the histogram data from Teledyne, three methods of analysis were employed at Webb, two of them providing smooth cumulative stress distributions that can be extrapolated to longer periods of time. These approaches will be discussed below and results presented in the following order:

- a) Actual distribution of recorded data based on the histograms.
- b) Idealized cumulative distribution based on the rms  $\sqrt{E}$  values and their standard deviations, assuming each record to be a Rayleigh distribution.

- c) Idealized distribution based on extreme values and their standard deviations, assuming a normal distribution of extreme values within each weather group.

Other possibilities which may be tried in future will also be discussed briefly.

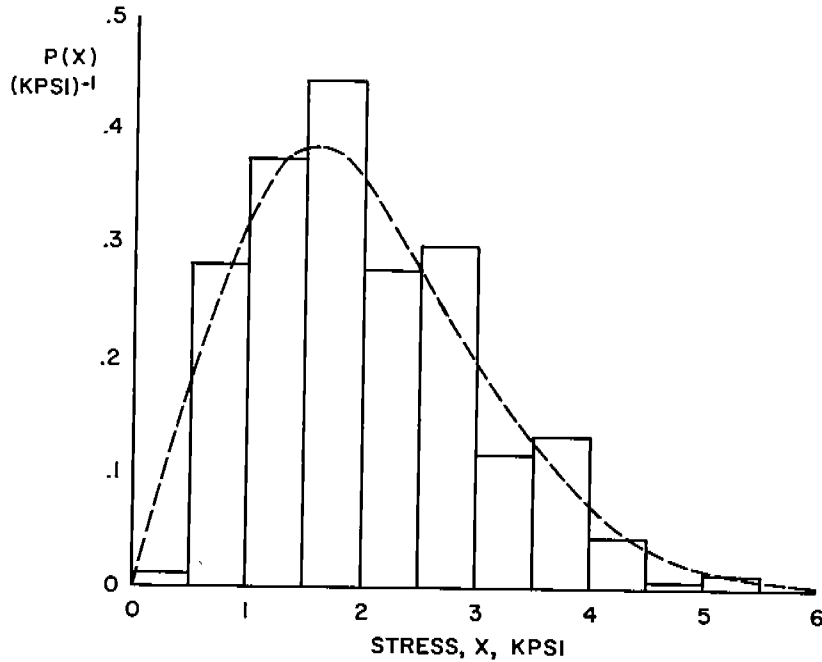


Fig. 1a Comparison of Stress Histogram for one Typical Record with Ideal Rayleigh Curve, S.S. Wolverine State (9).

### HISTOGRAM ANALYSIS OF STRESSES

#### Basic Principles

The data in the histograms of individual stress records obtained from Teledyne were tabulated and combined to give the total number of reversals or counts that exceeded certain prearranged stress range levels on each reel of tape for the instrumented ships. For each ship the tabulations for individual tapes were then combined (see Table II, for example) and the results plotted. Using a semi-logarithmic plot such as Fig. 2, the points represent a cumulative distribution of peak-to-trough stress variation which indicates the probability of exceeding a given range in any one reversal. This concept of probability of exceedance per cycle may also be interpreted in terms of a large real or imagined data sample. A probability of exceedance of  $10^{-6}$ , for



example, means that in a data sample of  $10^6$  reversals of stress ( $n = 10^6$ ) we would expect that one value would exceed the indicated level of stress (or bending moment).

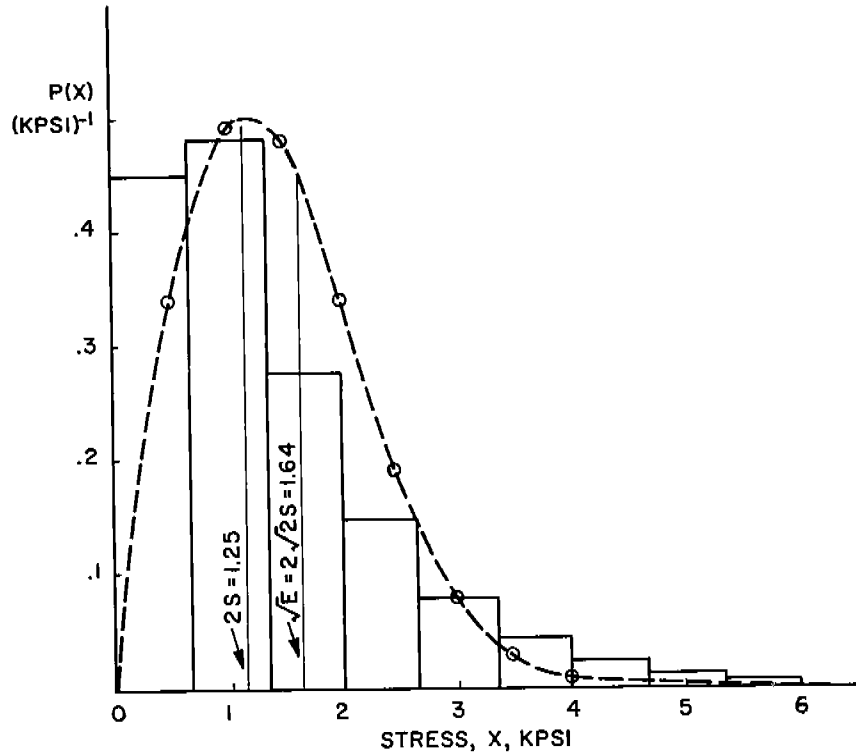


Fig. 1(b) Comparison of Actual Combined Histogram of 941 Record Intervals with Ideal Rayleigh Distribution, S. S. Wolverine State.

However, this graph only indicates the probable number of exceedances. If we had data for  $n = 10^6$  cycles for each of ten sister ships in the same service we would expect that some of them would have no exceedances, some would have one and a few might have two or more. The expected exceedance of 1 means that the average for all ten ships should be close to 1; the average exceedance for 100 ships should be even closer to 1. Or one could say that the value of stress that would be exceeded once in  $n$  reversals would vary among ten similar ships in the same service, but the average stress for one exceedance should agree with the curve. On the basis of the above interpretation of probability a second scale has been added to Fig. 2 so that the graph shows the wave bending stress that is expected to be exceeded once in the indicated number of stress reversals  $n$ . Or if the average number of reversals per hour is estimated, the scale can be expressed in days or months at sea, or -- allowing for time spent in port -- years in service.

It has sometimes been stated that a cumulative probability curve shows the highest expected value of stress in n reversals. Although this is approximately true, a more rigorous statement -- on the basis of the above discussion -- would be that the curve shows values of stress that we expect to be exceeded only by the highest stress in n reversals. (See Appendix A.) A distribution curve obtained in this manner from stress histograms can be considered to be a "limited" long-term distribution, since it is limited by the length of time over which data have been collected.

TABLE II TYPICAL TABULATION OF STRESS COUNTS, PORT AND STARBOARD AVERAGED.

<u>S.S. Wolverine State</u>	
Voyages No. 219-241	
<u>Stress Range (KPSI)</u>	<u>Number of Occurrences</u> <u>(Stress Reversals)</u>
0 - 0.65	86483
0.66 - 1.32	92916
1.33 - 1.99	52883
2.00 - 2.65	28401
2.66 - 3.32	15497
3.33 - 3.99	8497
4.00 - 4.65	4511
4.66 - 5.32	2301
5.33 - 5.99	1054
6.00 - 6.65	538
6.66 - 7.32	211
7.33 - 7.99	97
8.00 - 8.65	42
8.66 - 9.32	15
9.33 - 9.99	3
10.00 - 10.65	0
$\Sigma n_i = 293449$	

The limited cumulative or long-term stress distributions for the Wolverine State, Mormacscan in two different services, and the California Bear are given in Fig. 3 as series of points obtained from the data tabulations. The maximum recorded stress in a stated number of reversals is illustrated for each of the four distributions plotted.

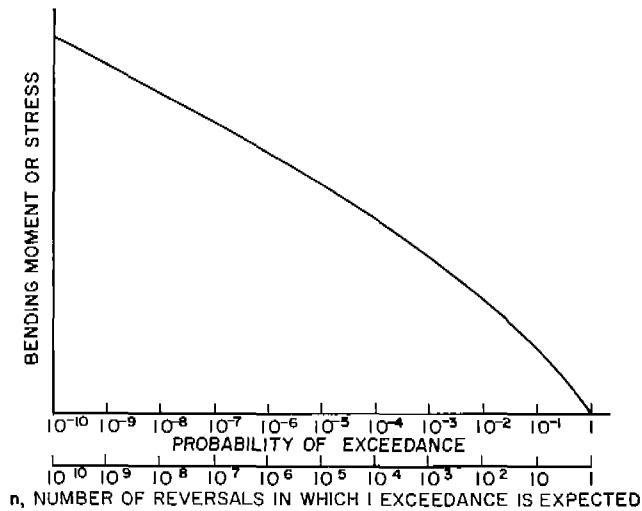


Fig. 2 Typical Cumulative Probability Curve to Illustrate Long-Term Trends of Bending Moment or Stress.

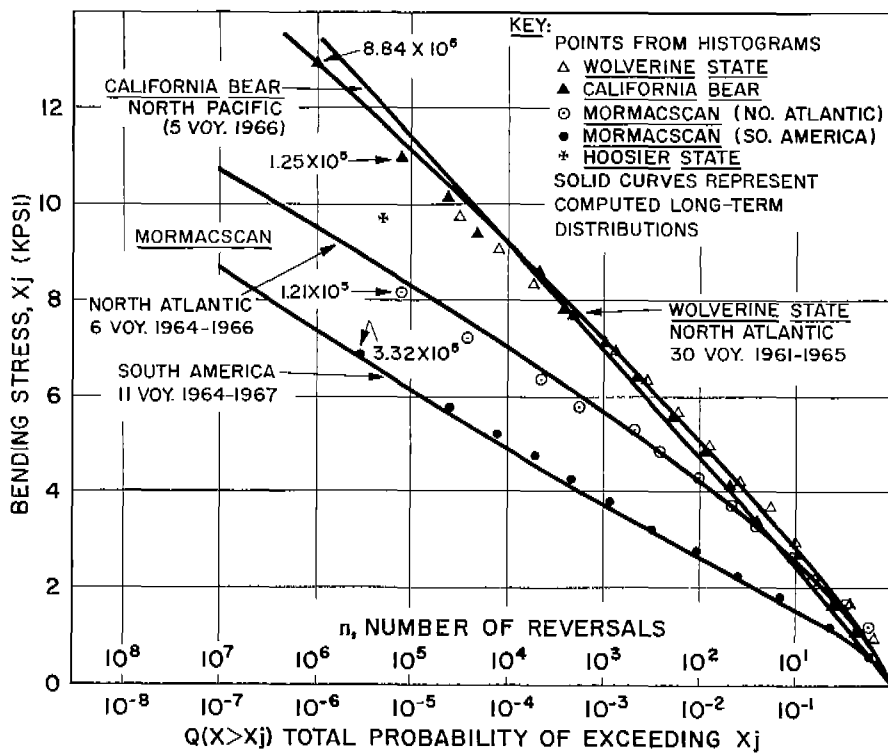


Fig. 3 Long-Term Trends of Stress Obtained from Histograms for Four Ships and Computed from rms Values.

(No analysis has been made of the Hoosier State histograms, but the highest value is shown as a single point.)

Also given in Fig. 3 are the ideal long-term distributions (solid curves) obtained from the rms  $\sqrt{E}$  values of the records. These will be discussed later and presently are only intended for comparison with the histogram analysis results. Generally good agreement is shown except for the single maximum value shown for the Hoosier State which falls below the combined results obtained for both the CG-S-B5 ships in the North Atlantic. It will be shown later that this discrepancy is the result of differences in the weather actually experienced by the two ships. This suggests the desirability of taking weather into account in the analysis, as mentioned before and discussed later on.

The data for the various ships in Fig. 3 differ considerably, and one cannot tell whether this is because of differences in the ships' services, in their structural designs, or in their characteristic responses to the sea. If both weather and structural differences can be allowed for in some way, as discussed later in the report, then more meaningful comparisons of different ships can be made.

It should be pointed out that the actual number of reversals in each record interval may differ slightly from the figure obtained from the probability analyzer due to the fact that at the lowest stress range of 0 - .65 KPSI it is difficult to distinguish cycles of small magnitude. It is expected that no substantial change in the plots shown in Fig. 1 will be experienced as a result of the above omissions, however.

### Analysis Details

Further discussion and explanation of the results for each individual ship will now be given. The Wolverine State data are the most comprehensive, covering 30 voyages from 12/19/61 to 3/29/65, and including 2651 record intervals. Data for the first 25 voyages are averages of port and starboard gages, as previously noted. For the other 5 voyages, separate port and starboard records were obtained; the individual port and starboard signals were added electrically ("combined") in the Teledyne laboratory at half amplitude to simulate the "average" or single channel signal from the two transducers formerly in one bridge circuit on board ship. The resulting signal was a new instantaneous average of the signals from both sides, and such electrical averaged results were used in the tabulations and plots.

The main reason for the separate recording of the port and starboard signals was to obtain an insight into lateral bending, which had not been possible in the case of the combined signals. For ideally the average of port and starboard readings will be a function of the vertical component of wave bending (i.e., bending in the center-line plane of the ship), while the difference will be a measure of the lateral component (i.e., bending in a plane parallel to the decks). Vertical and lateral bending will generally not be in phase, and there-

fore if both are present the average peak-to-trough values in a continuous record of the sum of port and starboard gages will be less than the sum of average values from separate records. However, an important fact was found by Teledyne in the course of the investigation that led to a reassessment of all previously published data: consistently different average results were obtained from the port and starboard gages. The cause of the differences was revealed to be a significant unfairness of the shear strake plating on the starboard side between the frames at the location of the gage, which resulted in lower stress values at the starboard side. These findings led to the introduction of correction factors. On the basis of tentative recommendations by Teledyne (20), the following multiplication factors were adopted:

Port Side	1.20
Starboard	1.45
Average	1.325

Hence, an average calibration correction factor of 1.325 was used in preparing Fig. 3.

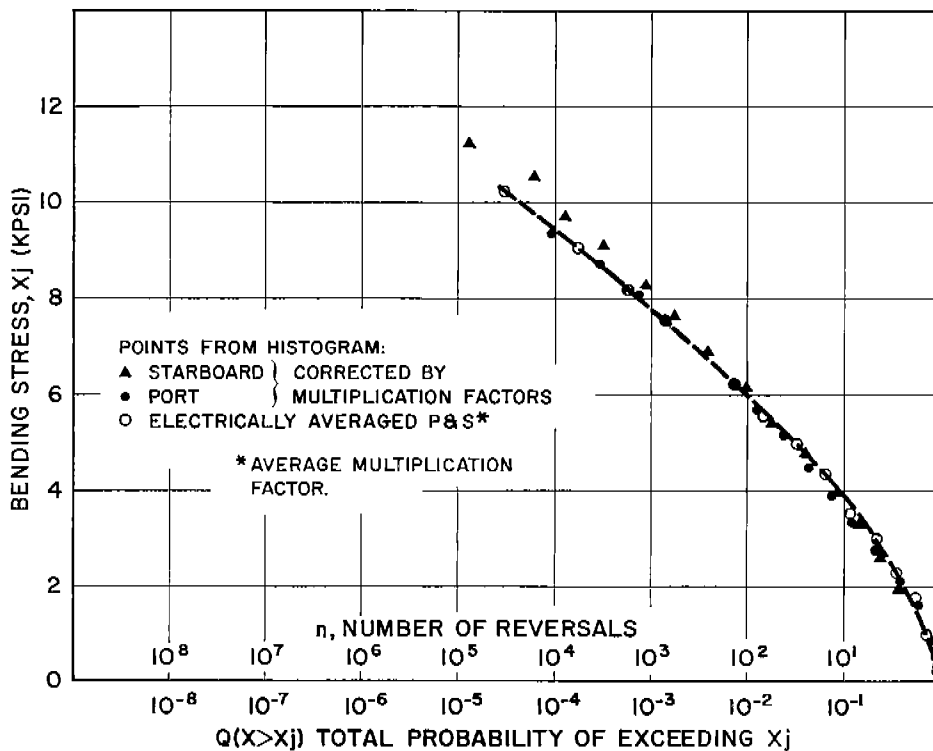


Fig. 4 Comparison of Long-Term Trends of Stress, Separate and Combined Port and Starboard Data, S.S. Wolverine State. (Voyages 219-241).

Due to the fact that 555 intervals were measured separately, port and starboard, as shown in Fig. 4, for the Wolverine State, it was

possible to plot two separate sets of data points and to compare them. The data shown here are those corrected by the individual calibration factors for port and starboard. It may be seen that the results for port and starboard gages are in good agreement, up to a probability level of  $10^{-4}$ , for which sufficient data were available. Before applying the calibration factors a distinct difference existed between the port and starboard data, as shown in Ref. (21).

Also shown in Fig. 4 are the electrically averaged port and starboard data, using the averaged multiplication factor. It may be seen that at the lower range of  $n$  (cycles) the average curve coincides with the separate port and starboard data. However, as  $n$  increases the averaged curve falls below the mean of the separate port and starboard data, indicating the effect of lateral bending. This observation is also illustrated in Fig. 11 for the limited long-term distribution, to be discussed later.

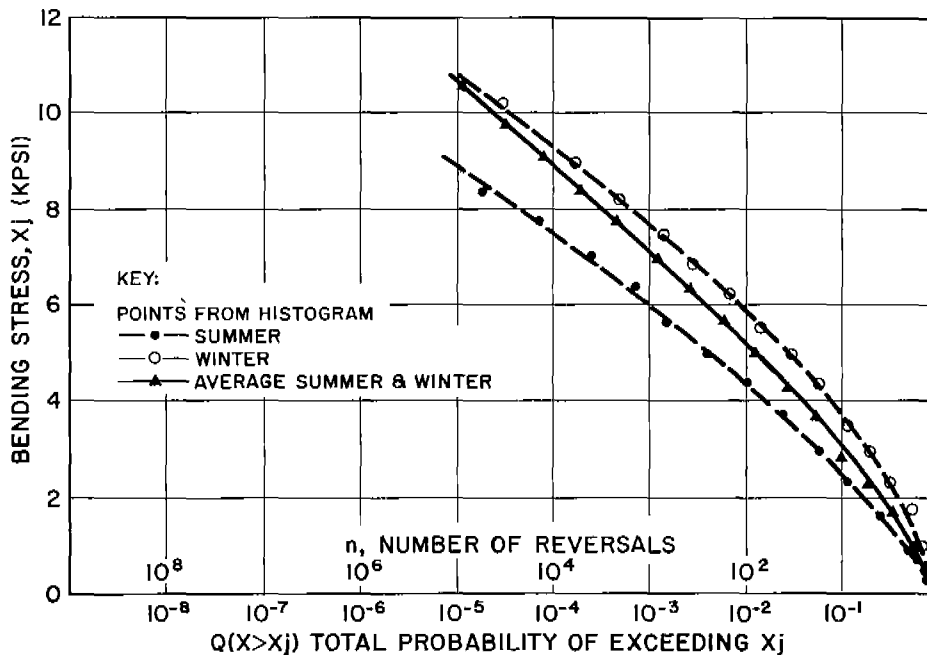


Fig. 5 Comparison of Long-Term Trends of Stress, Separate and Combined Summer and Winter Data, S.S. Wolverine State. (Voyages 219-241).

Another different grouping of voyages was carried out to distinguish between winter and summer periods, and results are plotted in Fig. 5 for about one year. It was found, as expected, that in voyages between November and May the ship experienced much higher stresses compared to those recorded between June and October. It can be generally concluded from about a year of operation that the maximum expected stress in the winter will be about 20-25% higher than that

expected during the summer months for the same number of stress cycles. The summer curve was obtained from the combined signal as recorded on the magnetic tape. The winter curve represents the electrical average of the separately recorded port and starboard signals, and therefore should be comparable. Also given in Fig. 5 is the combined winter and summer stress distribution covering a total period of ten voyages. It should be noted that the combined curve is a fair average of the winter and summer data at the lower range of  $n$ . But with increase in  $n$  the combined curve seems to agree with the winter data, as expected, since the cumulative stress curve over a long period of time is determined by the winter weather.

From the data recorded on board the S.S. Mormacscan, seventeen voyages have been analyzed so far, out of which five were in the North Atlantic, totaling 407 record intervals, and 12 on the U.S. to South America route covering 1234 recording intervals, i.e., about half a ship year in the North Atlantic and well over a year in the South American run. All data represent averages of port and starboard gages. There is a possibility that a recent full-scale stress vs. bending moment calibration may require a small calibration correction to be made later. The distributions are given in Fig. 1 for the above two routes and indicate maximum stresses expected in the North Atlantic about 50% higher than on the South American run over a period of half a year of operation.

Stress data analyzed on the California Bear, a C4-S-1a Mariner, in the North Pacific route to Japan covered voyages from 15 Jan. 1966 to 9 Feb. 1967, totaling 420 record intervals to date. Again there is the possibility of a calibration factor to be introduced later before final plotting. Results shown in Fig. 3 indicate a trend of stresses comparable to the Wolverine State.

The amount of data accumulated so far from the California Bear is insufficient for any conclusive remarks. It is indicated, however, from the preliminary results that stresses of equal magnitude to that encountered by the Wolverine State in the North Atlantic were recorded on the North Pacific route. However, the California Bear represents a different type of ship, the "Mariner" class, and as previously noted, comparison of the different ships on the basis of stress is not valid because of possible differences in structural design. Accordingly, the next step in the analysis was to transform all the data of Fig. 3 to a bending moment coefficient basis.

#### RELATIONSHIP BETWEEN STRESSES AND BENDING MOMENTS

The wave bending moment can be expressed in terms of the effective wave height,  $h_e$ , defined as the height of a trochoidal wave whose length is equal to that of the ship, which by conventional static bending moment calculation (Smith effect excluded) gives a bending moment (hog or sag) equal to that experienced by the ship in an irregular sea. Thus, if  $h$  is the wave height used in a static calculation,

$$\frac{\text{Static Wave B.M.}}{\bar{h}} = \frac{\text{Irregular Wave B.M.}}{h_e}$$

or

$$\frac{h_e}{\bar{h}} = \frac{\text{Irregular Wave B.M.}}{\text{Static Wave B.M.}}$$

Representing the static wave bending moment amplitude (hog or sag) by an equation,

$$c = \rho g \bar{h} B L^2 c_w$$

the coefficient  $c$  depends on the trochoidal wave form and the hull form of the ship. Hence,  $c$  has a convenient physical interpretation in terms of conventional wave bending moment calculations made by naval architects.  $L$  is length,  $B$  is breadth,  $c_w$  is waterplane coefficient,  $\rho$  is mass density and  $g$  is the acceleration of gravity.

Substituting the above expression for static wave bending moment,  $\bar{h}$  cancels out, and

$$h_e = \frac{\text{Irregular Wave B.M.}}{c \rho g B L^2 c_w}$$

Since the wave bending moment is continually varying in irregular waves, the value used here must be defined as one-half of a peak-to-trough value -- average, highest expected value in 10,000 cycles, or any other similar statistical measure.

The effective wave height is convenient to use in plotting. But a useful nondimensional coefficient is obtained by dividing by  $L$ ,

$$\frac{h_e}{L} = \frac{M}{2c \rho g B L^3 c_w}$$

where  $M/2$  is the irregular bending moment amplitude.

Values of static bending moments were calculated by Swaan (22) as a function of the waterplane coefficient, and these values can be used for convenience to determine  $c$  to a good approximation. In Swaan's notation,

$$M_w = \rho g \bar{h} B L^2 m_w$$

and therefore the nondimensional coefficient

$$M_w = 2c c_w$$

Thus  $M$  can be selected from Swaan's curves for the particular  $c_w$  and  $c$  is evaluated from

$$c = M_w / 2c_w$$



The waterplane coefficients at the load waterline for the above three ships are given in Table I. Accordingly, the c values were obtained for  $\lambda = L$  as follows:

<u>Wolverine State</u>	.01955
<u>Mormacscan</u>	.01900
<u>California Bear</u>	.01899

Thus, since

Bending Moment = Stress x Section Modulus, or

$$M = XZ$$

$$h_e/L = XZ/2c\rho gBL^3c_w$$

where X = peak-to-trough bending stress,  
Z = section modulus at strain gage section.

Ideally the measured stresses should be translated into bending moments on the basis of a full-scale calibration of the ship. That is, a known bending moment should be applied in calm water and the corresponding change in stress (strain) recorded. In practice it is very difficult to obtain a good calibration, particularly for a general cargo ship, because of insufficient tank capacity to provide a sufficiently large change in moment. One calibration was obtained on the Hoosier State in November, 1960, with a small bending moment variation. The measured stresses were reported to be within 5 percent of the value calculated from the section modulus (5), and therefore the calculated relationship was used. A calibration of the Wolverine State was attempted in August, 1965, with inconclusive results. It was therefore decided to use the calculated section modulus of each ship as a basis for comparison. The following results were obtained (X in KPSI):

<u>Wolverine State</u>	$h_e/L = .0028X$
<u>Mormacscan</u>	$h_e/L = .0026X$
<u>California Bear</u>	$h_e/L = .0022X$

The above relationships are based on geometrical particulars of the ship at the load waterline. However, it is known that these ships were often operating at a much reduced draft. In order to estimate the effect of the reduced draft on the  $h_e/L$  to stress relationship the appropriate value of  $h_e/L$  was also computed for the Mormacscan based on mean operating draft of 22'-6". At this draft  $c_w = .700$ . Thus  $c = .01855$  and  $h_e/L = .0028X$ , a difference of 8% from full load. All the results quoted in this report are on the basis of assumed loaded draft for the relationship between  $h_e/L$  and X.

It is of interest to note that plotting on the basis of bending moment (Fig. 6) instead of stress (Fig. 3) results in a distinct separa-

tion of the Wolverine State and California Bear. The relative positions of the Mormacscan on two routes is not changed significantly.

The differences among the ships shown in Fig. 6 must be due in part to differences in the ships themselves and partly due to the different weather conditions encountered. This suggests the desirability of bringing weather conditions into the analysis, and this will be dealt with in the following section.

This direct histogram approach to obtaining a limited long-term stress distribution and hence the bending moment distribution is simple and accurate and can therefore be used as a basis for certain comparisons. However, as previously noted, the determination of the maximum stress expected is limited by the length of time over which records were obtained for the particular ship and the particular weather experienced by the ship. For application to design problems the trends must be extrapolated to much longer periods of time, and an adjustment made for differences in weather. Two methods of extrapolation will be discussed in the next two sections.

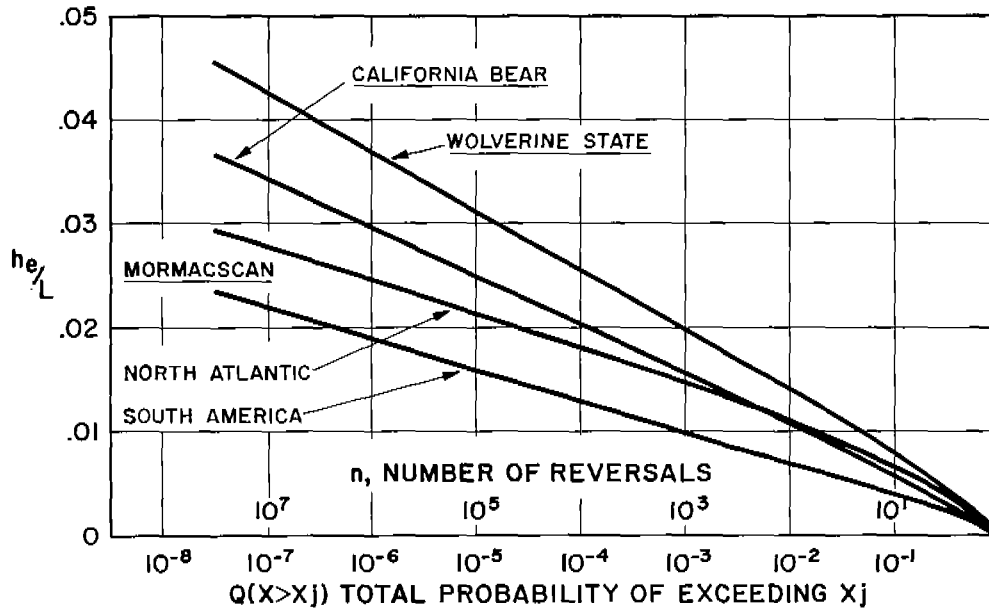


Fig. 6 Long-Term Trends of Bending Moment Coefficient for Three Ships in Actual Weather Conditions.

#### EXTRAPOLATION BASED ON RMS VALUES

##### Theory

Applicable theory will now be reviewed (5) (11). The method of analysis and extrapolation of ship stress data adopted here was that of Bennet (4), as elaborated by Band (5). This approach, which is

now widely used by researchers in Japan (22), Scandinavia (24), and Britain (25), relates the observed data to the physical cause -- the sea conditions -- rather than relying on the adoption of a particular distribution function that happens to match the data at low n values. It is believed that not only does this method result in reliable extrapolation of the data to large values of n, but it permits comparison of ships on different services by reducing results to the same "standard" or typical weather conditions.

The detailed analysis of 30 voyages of the Wolverine State and Hoosier State data was made by Band (5). As indicated in the previous section, peak-to-trough stresses in the individual record intervals were found to fit the so-called Rayleigh distribution quite closely (3), as given by the equation,

$$p(X) = (2X/E)e^{-X^2/E} \quad X \geq 0 \quad [5]$$

where  $p(X)$  is the probability of a stress value  $X$ . If one considers an increment of stress,  $dX$ , the probability of  $X$  lying between the stress values  $X$  and  $(X + dX)$  is  $p(X) dX$ .  $E$ , the parameter of the Rayleigh distribution, is the mean square value of all the peak-to-trough stress variations in the record. Since the data in individual records -- including extreme values, on the average -- were found to fit the Rayleigh distribution, each record can be adequately described by the appropriate value of  $E$ , or the root-mean-square value,  $\sqrt{E}$ . It is, of course, much easier to work with these  $\sqrt{E}$  values than with the many  $X$  values. (It should be noted here that the rms peak-to-trough value  $\sqrt{E}$  is related to the rms value of the record  $\sigma$  by a constant factor; hence  $8\sigma^2 = E$ .)

At this point it would be desirable to convert the  $\sqrt{E}$  stress values to bending moment coefficients for greater generality. However, because of the present lack of full-scale calibrations of some of the ships with known applied bending moments, the analysis was done in terms of stress with the idea that conversion to bending moment could be made later.

It was necessary next to relate these rms stresses to sea conditions. Ideally one should have simultaneous records of the sea surface which could be analyzed in the same way as stresses to give the mean square wave height. This is possible only in rare cases of very complete ship trials. In general it is necessary to characterize the seaway by observed significant wave heights or simply by observed wind velocities or Beaufort numbers. Rms stresses can then be classified and plotted as shown in Fig. 7 (from reference (3)) for the S.S. Wolverine State, using Beaufort number as the most reliable basis in this case. However, it should be pointed out that this figure covers only the first 20 voyages.

It will be noted in Fig. 5 that the average values of rms stress ( $\sqrt{E}$ ) at various Beaufort numbers, indicated by crosses, show a smooth

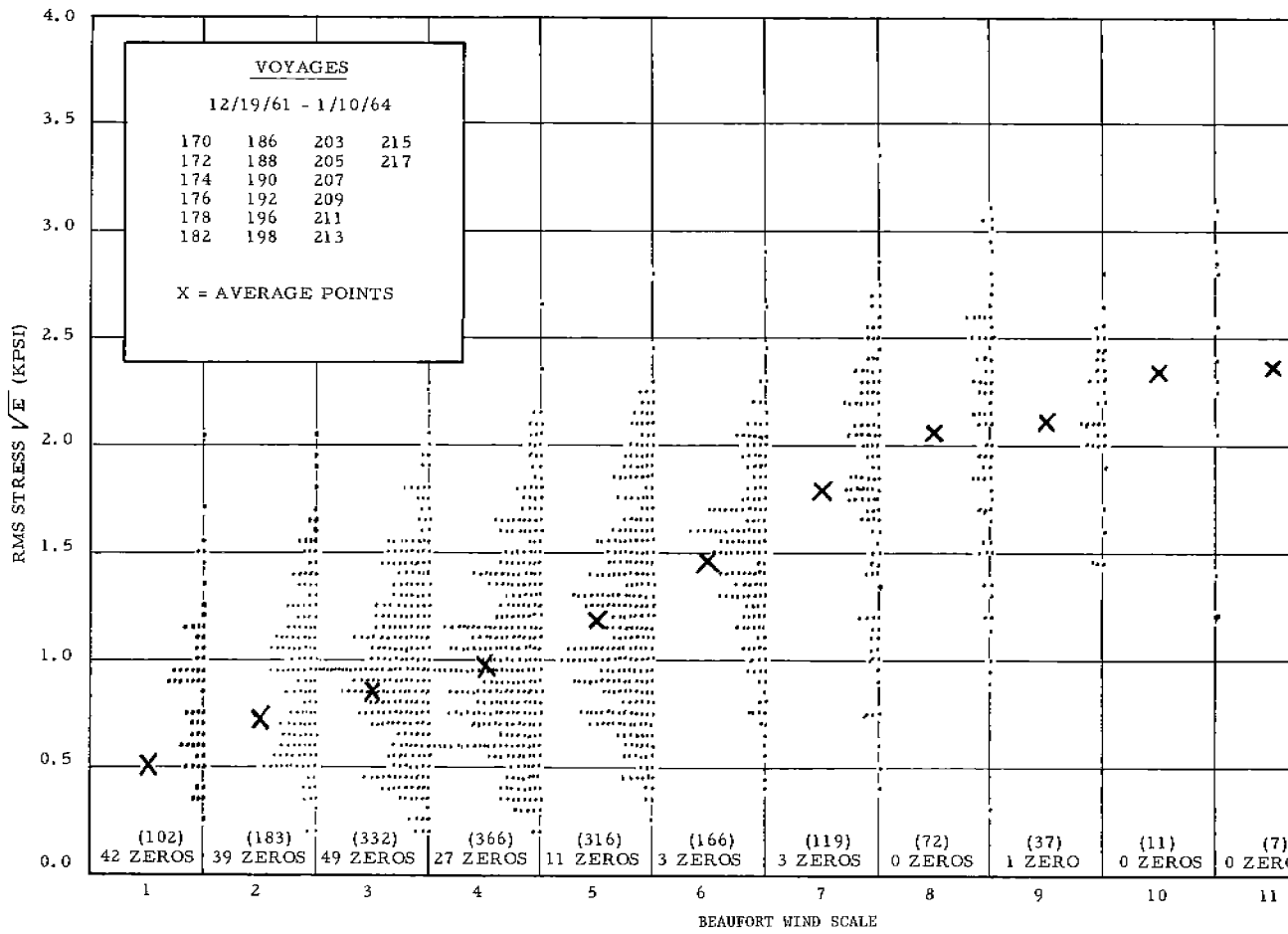


Fig. 7 Typical plot of r.m.s. Stress Values from Short-Term Records vs. Beaufort Wind Scale  
S. S. Wolverine State (3).

upward trend, becoming erratic only at Beauforts 11 and 12 where the number of data points is small. Since many factors, such as presence of swell, duration of wind, fetch, speed and heading of the ship, condition of loading, and so forth, have an effect on the mean square bending moment, it is not surprising that considerable scatter of stress vs. wind speed is shown in Fig. 7. This scatter can be more conveniently studied by grouping together the data for a number of different Beaufort numbers, so that the number of data points in each "weather group" is increased. Band (5) made use of five weather groups as shown in the following table.

<u>Weather Group</u>	<u>Beaufort No.</u>	<u>Wind Velocity, Knots</u>
I	0 to 3	1 to 10
II	4 to 5	11 to 21
III	6 to 7	22 to 33
IV	8 to 9	34 to 47
V	10 to 12	48 to 71

Plotting the Wolverine State data on probability paper (5) showed good agreement with a normal distribution in weather groups I to III, but only fair agreement in IV and V where the data were scarce. However, experience with the above weather grouping so far in the investigation has suggested that a slightly different classification might be more satisfactory for future use.

The more recent analysis at Webb Institute of additional data for the Wolverine State, with certain calibration corrections, along with data for the sister ship Hoosier State (5), shows excellent agreement between the two ships throughout groups I to IV (see Fig. 8). There also appears to be a distinct tendency for the lines to be parallel, which suggests a constant standard deviation of rms stress ( $\sqrt{E}$ ) in each weather group. Considering the differences between the two ships in Groups I and V, it is felt that the former is due to an error in the number of "zeros" reported for the Hoosier State. However, the difference between lines in the figure has no significant effect on the long-term distribution. Differences in Group V appear to result from the small amount of data recorded.

Hence, it seemed desirable to combine data for the two ships from this point on, in order to provide a larger statistical sample. The resulting combined plot similar to Fig. 8 (see Fig. 6 of Ref. 11) showed better agreement than before between the data points and the normal lines throughout Weather Groups I - IV, and fair agreement for V. All of the lines appeared to be parallel.

On the basis of the above findings the best way to utilize the observed stress data obtained over a limited period of time (Fig. 7) to extrapolate to a long-term distribution appears to be to make two arbitrary assumptions. These assumptions seem reasonable, but their validity cannot be absolutely proved. Indications are, however, that if they err, they do so on the safe side. The assumptions:

1. The trend of mean stress or bending moment vs. Beaufort No. in weather groups I to IV can be extrapolated by means of a straight line to higher winds, neglecting the few points in group V (which has only 38 points compared to 210 in group IV).
2. The standard deviation found in groups I to IV can be assumed to remain the same in higher weather groups.

Actually there are indications of less scatter at high Beaufort Numbers than at low, but the above assumption seems reasonable and on the safe side.

The first step in the extrapolation then is to adopt a probability model or idealization of the statistical data that can be assumed to apply to a much larger "population" (or quantity of data). We then need to determine the probability distribution of all peak-to-trough stresses in each weather group. On the basis of the previous discussion, our probability model can be based on the following idealizations:

1. The actual stress (or bending moment) values, X (peak-to-trough and trough-to-peak), in any sample record are Rayleigh distributed.
2. In each weather group, the mean square values of stress (or bending moment coefficient),  $\sqrt{E}$ , from many records are normally distributed.

Item 1 is expressed mathematically by equation [5]. Item 2 leads to a probability density function  $f(\sqrt{E})$  for the assumed normal distribution of  $\sqrt{E}$  values in a particular weather group given (5) by

$$f(\sqrt{E}) = \frac{1}{\sqrt{2\pi}s^2} e^{-\frac{(\sqrt{E} - m)^2}{2s^2}} \quad [6]$$

where the parameters are m, the mean value of  $\sqrt{E}$ , and s, the standard deviation of  $\sqrt{E}$  values about m.

The combined probability distribution is then the product of equations [5] and [6], representing the Rayleigh distribution of X for each value of  $\sqrt{E}$  and the normal probability distribution of  $\sqrt{E}$ :

$$P(X) = p(X) \cdot f(\sqrt{E}) \quad [7]$$

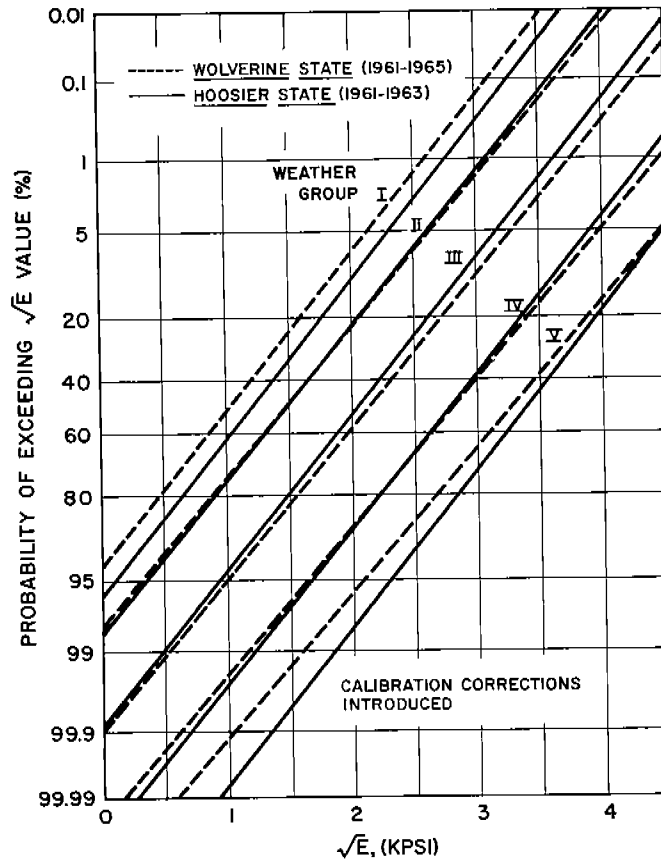


Fig. 8 Probability of Exceeding r.m.s. Stress Values in Different Weather Groups, S. S. Wolverine State, all Available Voyages 1961-1965.

However, of particular interest in the present problem is the probability of exceeding different values of peak-to-trough stress  $X$ , or bending moment. This information is given by the cumulative distribution which is obtained by integrating the previously combined probability. That is,

$$Q_i(X > X_j) = \int_{-\infty}^{\infty} \int_{X_j}^{\infty} p(X)f(\sqrt{E})dXd\sqrt{E} \quad [8]$$

The meaning of  $Q_i(X > X_j)$  is the probability that  $X$  will exceed any specified value  $X_j$  in weather group  $i$ . The first integration of the Rayleigh distribution with respect to  $X$  is easily accomplished, since

$$\int_{X_j}^{\infty} p(X)dX = e^{-(X_j^2/E)} \quad [9]$$

This is the cumulative form of the Rayleigh distribution. Equation [8] then becomes

$$Q_i(X > X_j) = \int e^{-(X_j^2/E)} f \sqrt{E} d \sqrt{E} \quad [10]$$

This can be evaluated numerically by computer or with the help of a family of derived curves given by Nordenstrom (see (26)). Since there are no negative values of  $\sqrt{E}$ , the lower limit of integration is actually zero. A finite upper limit of  $\sqrt{E}$  must be specified in order to obtain a solution. However, Band adopted a value of 5s for the limit, which he has shown to be the minimum value to insure sufficient accuracy in the final result. It will be noted from Equation 10 and long-term curves such as Figs. 3-6 that the higher the value of  $X_j$  the lower the probability that it will be exceeded. Conversely, the greater the number of stress cycles -- or the longer the period of data collection -- the higher the stress that is expected to be exceeded. Therefore, when data are separated into weather groups, the stress to be exceeded once depends on both the severity of the weather and the duration of the ship's exposure to it.

Typical results for the Wolverine State and Hoosier State combined are plotted in Fig. 9, which shows clearly that the highest expected bending moment for a typical cargo ship in 20 years of North Atlantic service is more likely to be caused by Beaufort 8 to 9 storms than by Beaufort 10 to 12, since the latter occur so rarely. This removes the urgency from the search for an elusive "worst possible storm."

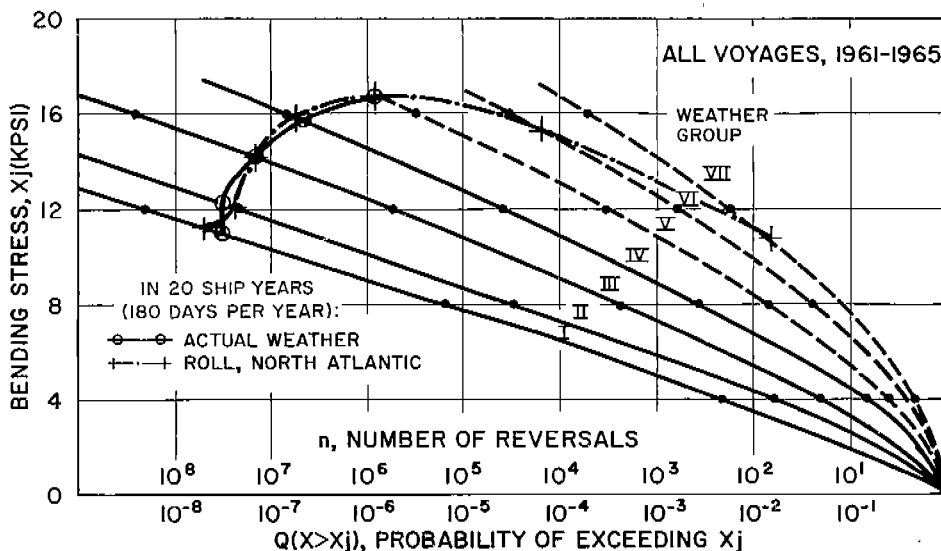


Fig. 9 Long-Term Probability of Exceeding peak-to-mean Stress Values in Different Weather Groups, S.S. Wolverine State and Hoosier State, all Available Voyages 1961-1965.



Band also felt that to provide satisfactory accuracy in the final result it was necessary to assume the existence of two more hypothetical weather groups of very low probability of occurrence designated VI and VII. This is believed to be a doubtful and unnecessary assumption.

Finally, taking into account the frequency of occurrence of all different weather conditions during the period of data taking, the total probability of exceeding  $X_j$  in all sea states will be,

$$Q(X > X_j) = \sum_{i=1}^V P_i Q_i(X > X_j) \quad [11]$$

where  $P_i$  is probability of meeting the  $i$ th weather group.

The result is a single curve shown in Fig. 10\* for each of the following assumed weather distributions (tabulated in the figure):

- (a) Overall actual weather experienced in total of 44 voyages of Wolverine State and Hoosier State.
- (b) Typical average North Atlantic weather as given by Bennet (8, 11).

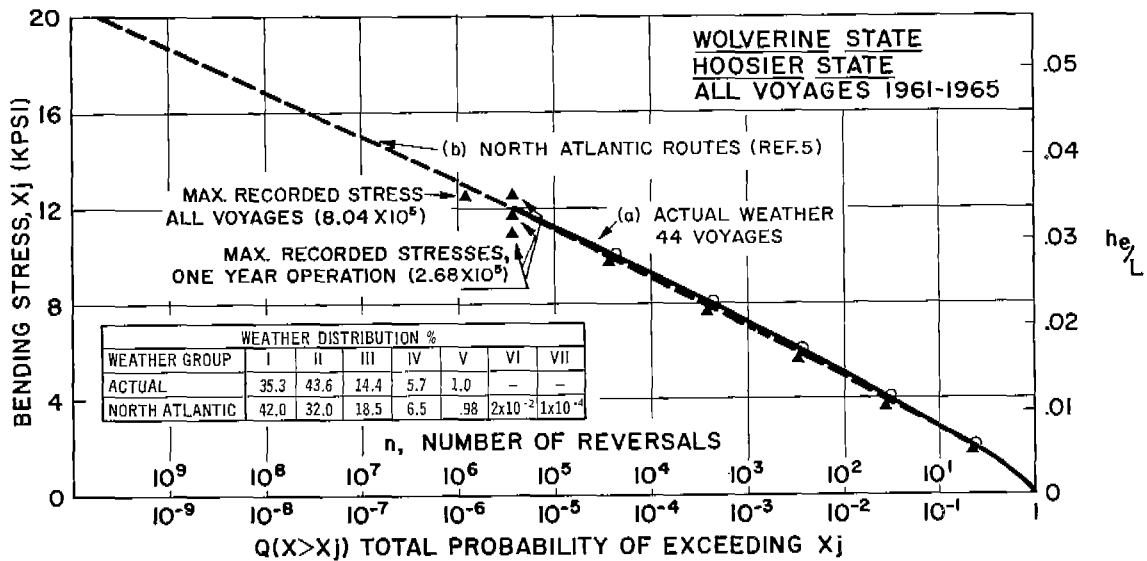


Fig. 10 Long-Term Probability of Exceeding Peak-to-Trough Stress Values in Different Assumed Weather Distributions, S.S. Wolverine State and Hoosier State based on Fig. 9.

\*Reproduced from Fig. 8 of (11).

The highest stress in each of three full years of operation was found from the records and plotted at the corresponding average number of stress reversals  $n = 2.68 \times 10^5$ . As expected, they show some scatter above and below the ideal curve (a). But the highest of all of the records, when plotted at its value of  $n = 8.04 \times 10^5$ , shows excellent agreement with the ideal curve. This result further confirms the validity of the procedure when applied over the period of actual ship stress observations and gives confidence in using it for extrapolation to longer periods of time.

Accordingly, the predicted long-term curve (b) for typical North Atlantic weather has been drawn. It happens to coincide with the "actual weather" curve (up to  $n = 10^6$ ) and has been continued on to  $n = 10^{10}$ . The result should be a reliable indication of the wave-induced stresses expected on C4-S-B5 type cargo ships in North Atlantic service, exclusive of the effects of slamming. Similar long-term curves for the Mormacscan and California Bear have been plotted in tentative form but are not included here since data collection was not complete enough.

#### Details of Analysis

A total of 44 voyages of the Hoosier State and the Wolverine State has now been analyzed, covering 1226 hours of continuous recording up to May 1965 (Fig. 10). All 3677 records were taken in the North Atlantic and are representative of about  $1 \times 10^6$  reversals.

In contrast to earlier data analyzed by Band (5), where there was no distinction made between the port and starboard transducers and the stress reported was the combined port and starboard signal, part of the later data were recorded on separate channels for the port and starboard gages, as discussed under Histogram Analysis. These data were later combined electrically in the Teledyne laboratory to simulate average port and starboard stress as obtained for the directly recorded stresses.

In order to further study the effect of gage location in determining the calibration factors, two additional gages were installed by Teledyne, one on each side, at a slightly higher position on the shear strake closer to the stringer plate. However, the data obtained so far by simultaneous recordings from all four gages are insufficient for reaching any decisive conclusions. Furthermore, no still water calibration has yet been carried out on the ship with the two new gages in use.

Rather good agreement was obtained between port and starboard results after the application of separate correction factors, as shown in Fig. 4. It was found, however, in plotting rms data from the probability analyzer that the electrically averaged line fell below the arithmetical average of the separate corrected port and starboard results, indicating roughly a 10% difference, as shown in Fig. 11. This phenomenon can be explained by the fact that lateral bending

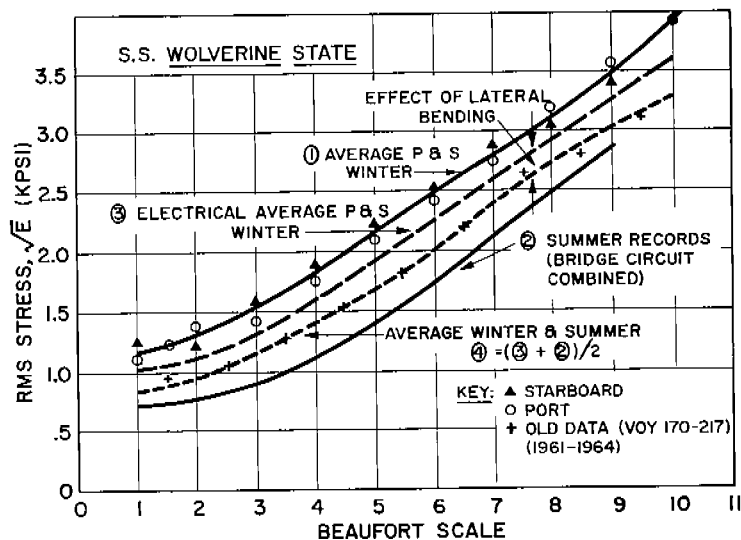


Fig. 11 Trends of Average r.m.s. Stress Values vs. Beaufort Wind Scale, Showing Differences Between Port and Starboard, Winter and Summer Data.

effects are eliminated in the electrically combined results but not in the average of separate port and starboard results. It may therefore be assumed that the difference between the average port and starboard line and the electrical average is an indication of the rms stress due to lateral bending. However, judgment should be withheld until this aspect is studied separately in the future, making use of the fact that records are available from six voyages both in the form of single channel output of the port and starboard transducers and as electrical averages of both signals recombined before input to the probability analyzer.

Some details regarding the technique used for electrical averaging are given in a Technical Memo by Teledyne (20).

Another aspect of the analysis of the later data was the separate winter and summer results. Some recent work by Walden (27) tabulates the frequency of "high" and "very high" waves in each season and in the whole year in percentages and illustrates some distinct differences between observations during the winter and summer periods. It was therefore of particular interest to arrange the stress results from each voyage in two groups, representing summer and winter respectively. Fig. 11 illustrates the mean lines of stress obtained by seasonal grouping. The difference between summer and winter, which amounts to 35-40% is in agreement with Walden's observations of the frequency of "high" waves in each season (27) defining winter-spring as the "winter" group and summer-autumn as the "summer" group. This difference can be partially attributed to the effect of swell on the bending moments induced. As mentioned previously, the Beaufort number estimations are based on observations of wind rather than sea or swell,

and at any wind speed the amplitude of swell is bound to be greater in the winter, and can therefore be the cause for higher stresses recorded for the same wind conditions. It should also be noted that the number of "zeroes" is considerably larger in the summer months. Regardless of the explanation of the differences between winter and summer data, the data can be directly averaged to obtain year-round figures, due to the fact that the number of records in each period is roughly the same.

The actual comparison of the old and new year-round data is given in Fig. 12 and very good agreement can be seen to exist. The final distribution for the 44 voyages of the two ships is given in Table III and Fig. 10.

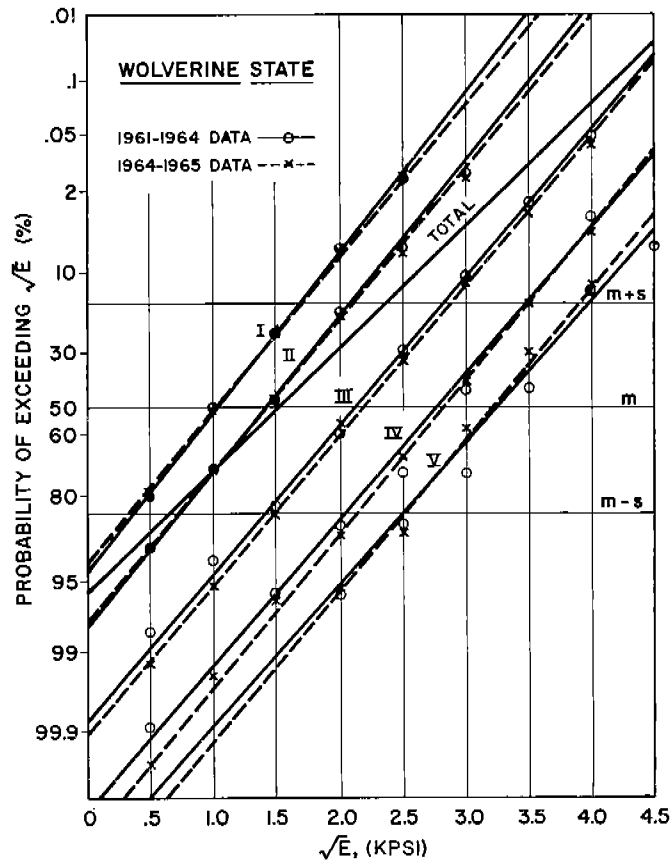


Fig. 12 Probability of Exceeding r.m.s. Stress Values in Different Weather Groups, S.S. Mormacscan. (No Calibration Factor Applied).

TABLE III ACTUAL DISTRIBUTION OF SHORT TERM RECORDS CLASSIFIED BY  $\sqrt{E}$  VALUE AND WEATHER GROUP SS WOLVERINE STATE AND SS HOOSIER STATE, C4-S-B5 CARGO VESSELS. (NORTH ATLANTIC 44 VOYAGES).

Weather Group	I	II	III	IV	V	Total
Beaufort No.	1, 2, 3	4, 5	6, 7	8, 9	10, 11, 12	
$\sqrt{E}$ Range						
K P S I						
Mean Value						
4.5-4.95 4.75					1	1
4.0-4.45 4.25			1	8	3	12
3.5-3.95 3.75		3	11	23	13	50
3.0-3.45 3.25	4	28	58	56	11	157
2.5-2.95 2.75	33	98	116	70	6	323
2.0-2.45 2.25	57	186	135	21	2	401
1.5-1.95 1.75	207	402	112	23	1	745
1.0-1.45 1.25	356	467	58	5	1	887
.5- .95 .75	374	298	32			704
.0- .45 .25	261	49	7			317
<b>Total</b>	<b>1292</b>	<b>1531</b>	<b>530</b>	<b>206</b>	<b>38</b>	<b>3597</b>

Some statistical tests such as the  $\chi^2$  test were carried out for the above data to check the validity of the normal distribution assumption, and the confidence limit lines were calculated and drawn for some of the cases discussed. A discussion and illustration of the above is given in Appendix B, and the techniques used are discussed in Appendix A. Table IV summarizes the results obtained from the four ships discussed above giving mean, standard deviation and number of records upon which the data are based. Where extrapolation of the mean and standard deviation was required, both actual and estimated values are given.

The rms or  $\sqrt{E}$  data concerning the Mormacscan and the California Bear are given in Figs. 13 and 14. Results may be considered provisional, due to the fact that no still water calibrations were yet available. The mean rms stresses from the short-term records as plotted in Figs. 13 and 14 for the Mormacscan in the North Atlantic and South American route and the California Bear in North Pacific are based on a limited number of recording intervals. In particular,

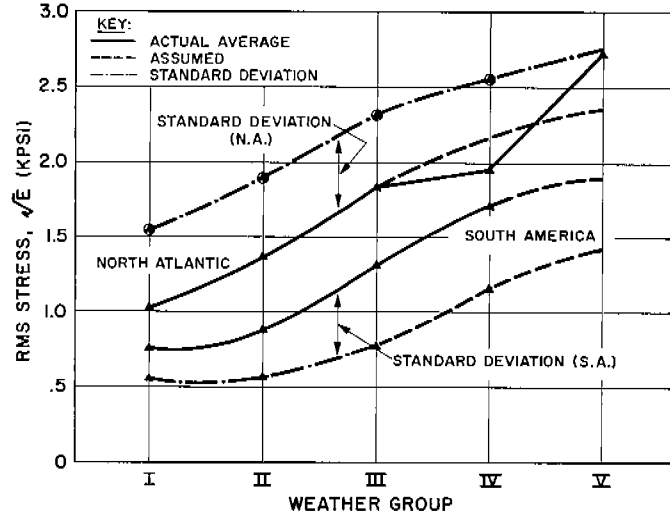


Fig. 13 Trends of r.m.s. Stress Values vs. Weather Group for Two Different Routes, S.S. Mormacscan. (No Calibration factor applied).

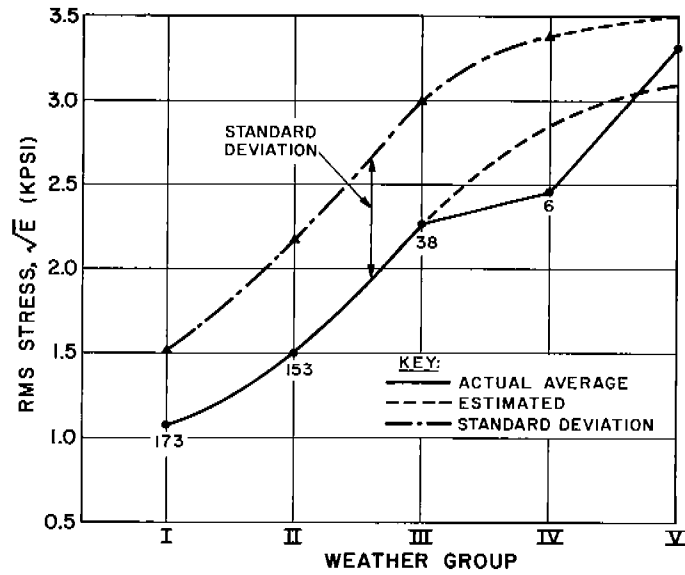


Fig. 14 Trends of r.m.s. Stress Values vs. Weather Group S.S. California Bear. (No Calibration factor Applied).

only one case was recorded for the two higher weather groups IV and V. Thus there is some uncertainty regarding the upper portion of the curve. However, the effect of such an error on the prediction of long-term trends was tested numerically and found to be small because of the infrequency of the more severe weather. The long-term curves, as predicted for North Atlantic weather for all ships, are given in Fig. 15 based on  $h_e/L$ .

As previously noted, the curves for the Mormacscan and California Bear must be considered tentative, since all data had not been collected and analyzed, and calibration factors were not yet available. However, in this presentation the effect of weather differences has been eliminated by giving results for the same typical North Atlantic weather, and the effect of individual structural differences has been eliminated tentatively by converting to bending moment coefficient. It thus appears that significant differences remain which can be attributed to differences in ship size, ship characteristics, and mode of operation.

Table I shows that the California Bear is a much bigger ship than the Wolverine State, and this can explain its lower level of bending moment trend. But for the Mormacscan the values in Fig. 15 appear to be unexpectedly low. It is hoped that including additional data from the Mormacscan and California Bear -- plus model comparisons of the Wolverine State and California Bear -- will lead to a plausible explanation of the differences shown in future.

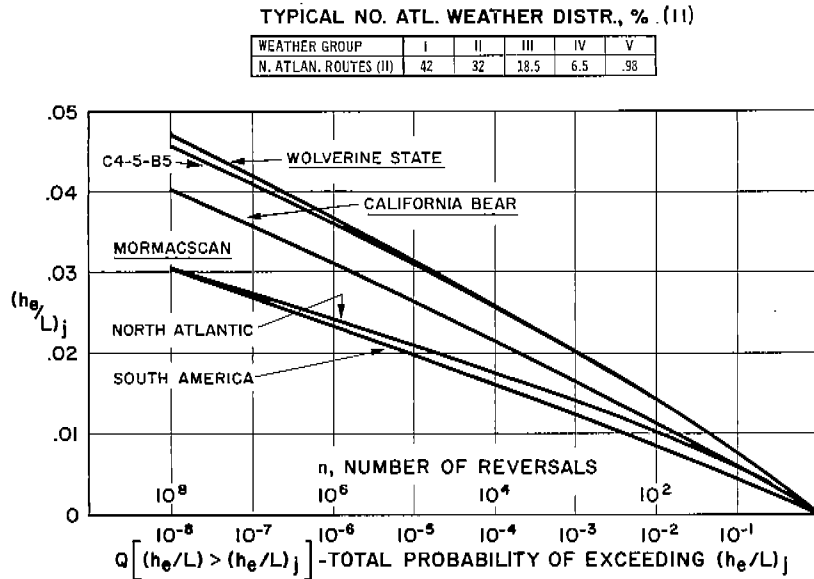


Fig. 15 Long-Term Trends of Bending Moment Coefficient Computed From r.m.s. Values for Three Ships in Typical North Atlantic Weather.

**TABLE IV** SUMMARY OF MEAN RMS AND STANDARD DEVIATION VALUES, NO. OF RECORDS AND FREQUENCY OF OCCURRENCE.

<u>Wolverine State</u>	I	II	III	IV	V	Total
m - mean	1.02	1.48	2.18	2.82	3.28	1.52
s - Stand. Dev.	.72	.72	.72	.72	.72	--
N <sub>i</sub> -No. of Records	1069	960	422	117	33	2651
P <sub>i</sub> = N <sub>i</sub> / $\sum_{i=1}^V N_i$	.403	.362	.159	.067	.012	1.0
<u>Hoosier State</u>						
m	1.24	1.48	2.08	2.79	3.15	1.47
s	.72	.70	.70	.70	.67	--
N <sub>i</sub>	230	647	108	36	5	1026
P <sub>i</sub>	.224	.631	.105	.035	.005	1.0
<u>Mormacscan (South America)</u>						
m	.76	.86	1.32	1.70	1.85	1.16
s	.25	.30	.56	.60	.55	--
N <sub>i</sub>	488	441	69	1	--	999
P <sub>i</sub>	.488	.441	.069	.001	--	.999
<u>Mormacscan (North Atlantic)</u>						
m	1.02	1.37	1.83	1.95/2.17	2.70/2.39	1.76
s	.54	.54	.50	.40	.40	--
N <sub>i</sub>	82	179	84	22	8	375
P <sub>i</sub>	.219	.477	.224	.059	.021	1.00
<u>California Bear (North Pacific)</u>						
m	1.09	1.49	2.26	2.45/2.80	3.30/3.10	2.12
s	.49	.68	.83	.60	1.60	--
N <sub>i</sub>	173	153	38	6	1	371
P <sub>i</sub>	.467	.412	.102	.016	.003	1.00

EXTRAPOLATION BASED ON EXTREME VALUES

Theory

The main purpose of the long-term stress distribution is to allow the prediction of the maximum stress expected within any period of operation. Therefore, another approach to this problem is to deal directly with the "extreme values," instead of rms values, in all the records. By "extreme value" is meant the actual highest peak-to-trough stress in a particular 20-minute record. A theoretical advantage of this approach is that it permits a prediction to be made of the highest expected extreme value, rather than merely the value



expected to be exceeded only once. This approach also offers potentially less work in data collection and, as shown below, somewhat simpler statistical analysis. On the other hand, it may be less reliable because of the fact that it makes use of less data (one value per record instead of the rms of the entire record). This question is still under study.

It has been found that the maximum peak-to-trough stresses are approximately normally distributed within specified groups of Beaufort numbers (28), as shown in Fig. 16 for the case of twenty voyages of the Wolverine State, in the same manner as the rms values (Fig. 8). This is the consequence of the fact that all records are the same length and therefore have approximately the same number of peak-to-trough cycles of stress and that peak-to-trough stresses follow a Rayleigh distribution. Consequently, Band showed (5) that the mean line through the average rms ( $\sqrt{E}$ ) values plotted against weather and the mean line through the average extreme values differed by the appropriate Rayleigh factor.

Thus it is possible to predict a long-term distribution of highest expected stress or bending moment directly, without the introduction of assumed Rayleigh distributions or assuming any arbitrary

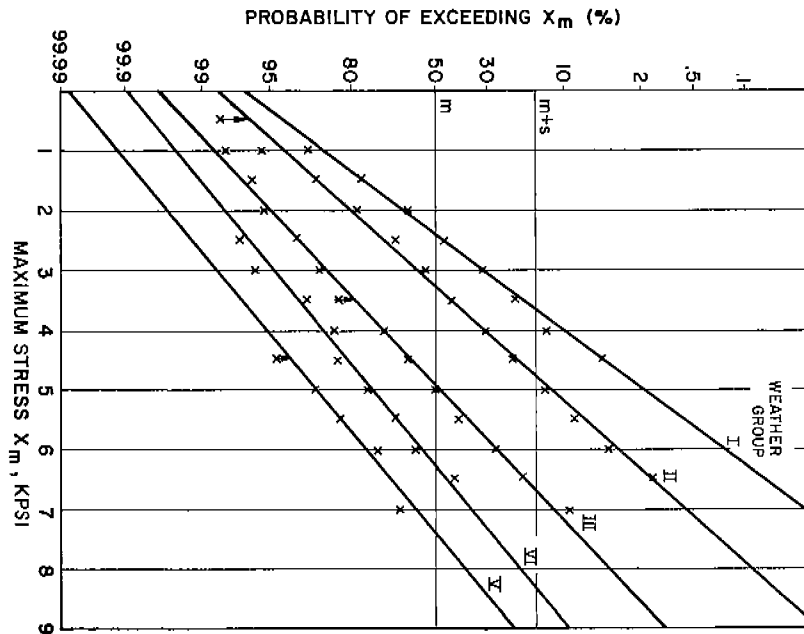


Fig. 16 Normal Distributions of Extreme Stress Values in Different Weather Groups, S. S. Wolverine State, 20 Voyages, 1961-1964.

form for the long-term distribution.

The procedure to be followed will now be described. For each weather group, the mean extreme value (M) and the standard deviation (S) can be determined either numerically or graphically,

$$M_i = \frac{\sum_{m=1}^N X_m}{N}$$

$$S_i = \sqrt{\frac{\sum_{m=1}^N (X_m - M_i)^2}{N}} \quad [12]$$

where  $X_m$  is the highest value recorded in each twenty-minute record, and N is the number of records. The normal distribution of  $X_m$  in each weather group is described by the probability density function as follows:

$$f_i(X_m) = \left( \frac{1}{\sqrt{2\pi S_i^2}} \right) e^{-(X_m - M_i)^2 / 2S_i^2} \quad [13]$$

We wish to determine the probability  $Q_i$  of a stress  $X_m$  exceeding  $X_j$  in a particular weather group. Then, we may write first,

$$Q_i(X_m < X_j) = \int_{-\infty}^{X_j} \frac{1}{S \sqrt{2\pi}} e^{-(X_m - M_i)^2 / 2S^2} dX_m \quad [14]$$

As is customary in statistical work, the above equation may be simplified by letting  $Z = (X_m - M_i)/S$ . In order to evaluate  $Q_i(X_m < X_j)$  in terms of Z, where  $X_m$  has a normal distribution, we can write,

$$Q_i(X_m < X_j) = Q_i[(X_m - M_i) < (X_j - M_i)] = Q_i\left[\left(\frac{X_m - M_i}{S}\right) < \left(\frac{X_j - M_i}{S}\right)\right]$$

$$= Q_i\left[Z < \left(\frac{X_j - M_i}{S}\right)\right] \quad [15]$$

Thus, the probability that  $X_m < X_j$  is the same as the probability that  $\frac{X_j - M_i}{S} > Z$ . Then, making substitutions, equation [14] becomes,

$$Q_i(X_m < X_j) = \int_{-\infty}^{(X_j - M_i)/S} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = \phi_i\left(\frac{X_j - M_i}{S}\right) \quad [16]$$

Thus  $Q_i$  can be evaluated in terms of Z, where Z is the standardized normal variable having a probability density function,

$$\phi_i(Z) = \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} \quad -\infty < Z < \infty \quad [17]$$

and the relationship between the X-axis and the Z-axis given as follows:

X	m	m + s	m + 2s	.....	m - s	m - 2s
Z	0	1	2	.....	-1	-2

Equation [16], which represents the area under the normal curve, can be expressed, due to symmetry,

$$Q_i (X_m < X_j) = .5 + \int_0^{(X_j - M_i)/S} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \quad [18]$$

The upper limit of integration is determined by the practical range of interest of the long-term probability curve extended to between 100 to 1000 ship years, i.e., up to about  $N = 3 \cdot 10^6$ . To assure accuracy to this level it is necessary to have the high value of  $X_m = M_i + 6S_i$  as the upper limit when computing the above, i.e., in that case

$$Q_i (X_m < X_j) = .5 + \int_0^6 \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \quad [19]$$

By setting the upper limit as  $X_{max} = M_i + 6S$ , the probability of exceeding that value is in the vicinity of  $3 \cdot 10^{-7}$  and the reciprocal N is equivalent to the number of records expected in over a thousand ship years. This limit is higher than assumed by Band (5), and is felt to be satisfactory.

It is also shown in Figs. 14 and 15 of Ref. (5) that the line  $M_i + 6S$ , as the upper limit of integration, practically coincides with the  $\infty$  line just above it.

Values of the integral on the right-hand side of equation [19] which represent the standardized normal distribution function can be found in statistical tables. However, most references do not exceed  $X_{max} = M_i + 4S$ . An attempt was therefore made to solve the above by expansion of the exponential series as follows,

$$Q_i (X_m < X_j) = .5 + \frac{1}{\sqrt{2\pi}} \int_0^{(X_j - M_i)/S} e^{-z^2/2} dz$$

After expansion,

$$Q_i (X_m < X_j) = .5 + \frac{1}{\sqrt{2\pi}} \left[ z + \sum_{N=2}^M \frac{(-1)^{N+1} z^{2N+1} / 2^N}{N! (2N + 1)} \right]_0^z \quad [20]$$

The above equation was programmed to yield the individual probabilities of exceedance of a certain  $X_{max}$  for each weather group. It should be noted, however, that for large values of  $Z$  ( $Z > 4$ ) the summation of the above series expansion breaks down because of the truncation error of most computers. Thus, due to the difficulties in solving the integral numerically using a series expansion, a numerical integration was used instead, which lends itself to simple analysis by digital computer. Reducing the integral form to an approximately equivalent summation between suitable finite limits,

$$Q_i(X_m < X_j) = .5 + \frac{1}{\sqrt{2\pi}} \int_0^{(X_j - M_i)/S} e^{-Z^2/2} dZ = .5 \pm \frac{1}{\sqrt{2\pi}} \sum_0^{(X_j - M_i)/S} e^{-Z^2/2} dZ \quad [21]$$

where the minus sign accounts for those cases where  $Z$  is negative. The above summation can be carried out satisfactorily up to  $Z = 6$ , as required. The solution given by equation [20] might be preferable if a sufficiently large computer were available.

The total probability  $Q(X_m < X_j)$  of exceeding each bending stress  $X_j$  in a distribution of weather conditions experienced or expected during the operational life of the ship is then:

$$Q(X_m < X_j) = \sum_{i=1}^V P_i Q_i(X_m < X_j) \quad [22]$$

where  $\sum P_i = 1.0$

and  $P_i$  is the probability of occurrence of weather group  $i$ , having  $i$  values  $I$  to  $V$ , inclusive.

In all the above analyses the effect of ship heading was not considered. However, it is possible by the initial grouping of the information to calculate the mean and standard deviation separately for various headings, e.g., head, bow, beam, quartering, and following seas, and the mean and standard deviation for each weather group could then be weighted in accordance with the time spent at each heading. This refinement was not felt to be necessary at this stage.

## Results

Long-term distribution curves were computed for all four ships by the extreme value approach. Figure 17 shows the results for two of the ships; i.e., the C4-S-B5 class cargo ship and the Mormacscan, both in North Atlantic service. In order to compare the results with those obtained previously using the rms values, the abscissa scale in Figure 17 is also given in terms of stress reversals,  $n$ , rather than number of records,  $N$ , and an average figure of 300 bending stress reversals per 20-minute recording was used as a conversion factor between the two scales, i.e.,  $n = 300 N$ .

The above assumption regarding the abscissa scale is believed to be reasonable enough in the range of  $10^4$  to  $10^8$  stress reversals. However, in the lower range, i.e., up to roughly 3000 reversals or 10 recording intervals, the two curves cannot be compared.

COMPARISON OF RMS AND EXTREME VALUE EXTRAPOLATIONS

It is evident from Figure 17 that the curves obtained by extreme value data tend to converge to the curve obtained by the rms data. Hence, in the practical range of full-scale stress measurements covering about 1 ship-year to 100 ship-years, there seems to be good agreement between the results obtained by the two methods and the actual histogram results.

Figure 17 also shows a tendency for the extreme value extrapolation to level off at very large values of  $n$ , while the rms extrapolation continues to rise. Further investigation is required to determine whether this difference in trends is real, and if so which method is a more valid basis of extrapolating the observed data.

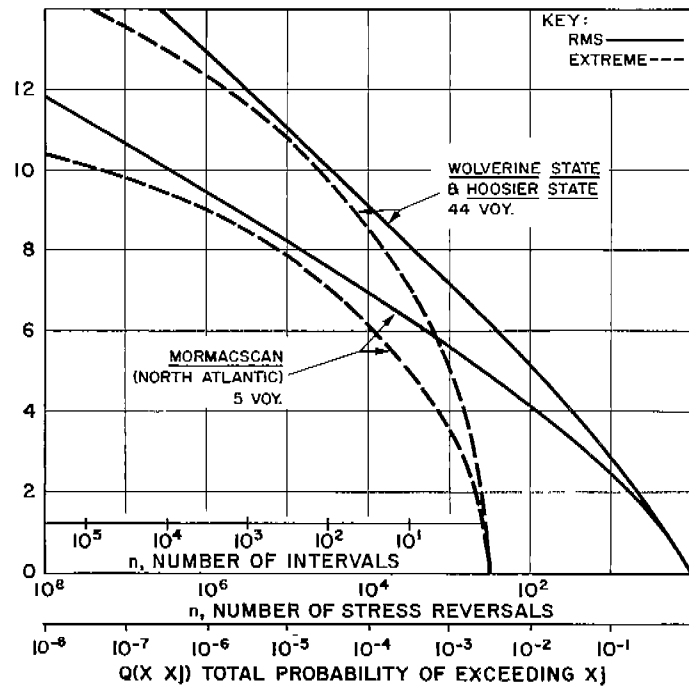


Fig. 17 Comparison of Long-Term Trends of Stress Computed from r.m.s. Values and from Extreme Values for Two Ships.

It should be noted that the definition of ship-year is rather flexible and dependent on the type of ship and its service. In the case of the above ships, assuming perfect operation of the recording equipment, it would be expected that each ship would spend 180 days in the open sea, and the number of stress reversals would then be:

$$n = 300 \times 6 \times 180 = 3.24 \times 10^5,$$

assuming 300 stress reversals per 20-minute record and 6 records per day.

Alternatively, in terms of records,

$$N = 6 \times 180 = 1.08 \times 10^3$$

Thus for 100 ship-years,  $n = 3.24 \times 10^7$  and  $N$  is roughly  $10^5$  record intervals. The present sample for the C4-S-B5 class was drawn from roughly 3650 records, and for the Mormacscan from only 400 records.

It should, however, be noted that the actual number of stress reversals experienced in a ship-year would be greater by a factor of 12 than the number recorded in 20-minute samples every 4 hours, i.e.,

$$n = 900 \times 24 \times 180 = 3.9 \times 10^6, \text{ instead of } 3.24 \times 10^5.$$

Care should be taken when interpreting the cumulative probability curves in terms of ship-years for application to design.

The question arises as to the preference between the extreme value method and the procedure using rms values previously described. The relative merits of the two approaches depend ultimately on the level of reliability with which the short-term data fit a Rayleigh distribution, particularly in the "tails". If for the sake of argument the data were found to fit such a distribution very closely then the rms value for each record interval would provide a precise prediction of the expected highest value in such an interval. The actual highest value in each interval, on the other hand, would represent only one realization of the Rayleigh distribution and would therefore be less suitable for use in further analysis.

But if the data fit to the Rayleigh distribution is poor, particularly in the "tails", then it might very well be that the use of the actual extreme values will give a better prediction.

Looking at it another way, we can consider that each 20-minute record is a sample of a 4-hour period during which conditions may be expected to remain essentially stationary. The question is, can we characterize best the extremes during a 4-hour period by use of the rms value obtained in 20-minutes or of the highest value in 20 minutes? The highest value in 4 hours may be greater than the highest recorded in 20 minutes -- by an unknown amount. But if the Rayleigh distribution accurately describes the data, then the expected highest value in 4 hours based on the 20-minute rms value should not only be more reliable but can yield a measure of confidence, as well. Alternatively,

other assumptions regarding the distribution of the extreme values are now being studied, such as Gumbel's first and third as asymptotes. It is expected that the results obtained from this further study will shed more light on the problem of extrapolating statistical data.

## CONCLUSIONS AND RECOMMENDATIONS

### Conclusions

1) It has been shown that classifying ship stress data in respect to wind force provides a basis for analysis of long-term trends that takes into account the different weather conditions encountered by different ships in service.

2) Using the two methods of stress data analysis presented here, data obtained from several different ships on the same and on different trade routes can be compared on the basis of non-dimensional wave bending moment coefficients in the same "standard" weather distributions, extrapolated to long periods of time.

3) Both methods of analysis were found to yield long-term distributions of stress that agreed very well with histogram data over the limited period covered by the data (maximum of 3 ship-years); the two methods that could be extrapolated to longer periods of time also showed good agreement within the range of interest ( $10^4$  to  $10^8$  stress reversals).

### Recommendations

1) In order to provide a rational basis for a quantitative determination of wave bending moment requirements in the design stage, it is believed that further refinement and verification of the above procedure are required. Possible sources of error in the results presented in this report are:

- (a) Low level of bending moment attainable in full-scale, stress-bending moment calibrations suggests a possible ± 5% error in calibration factors.
- (b) The form coefficient,  $c$ , used in estimating the conventional static wave bending moment is an approximation and varies considerably with draft, indicating a possible ± 5% deviation in results.

Further study of the data classified by season and by ship heading should provide useful additional information.

2) Other types of ship should be investigated, such as the all-match container ship, the Great Lakes type ship, and floating structures. Refinements should include better oceanographic data, in particular for severe weather conditions on different routes, as well

as a better wind-wave relationship.

3) The possibility of experimenting with other types of probability distributions, such as Gumbel's, appears promising, and future work should be conducted along this line with special emphasis on the leveling trend of the long-term distribution for long periods of observation.

4) The use of model test results to predict long-term trends should be pursued further, along with two alternatives:

- (a) The use of standard model series which have been tested extensively, such as Vossers', so long as the actual hull characteristics are not too different from those given by the series.
- (b) The use of theoretical ship motion and bending moment calculations based on strip theory principles. It is feasible with present-day knowledge to compute the response amplitude operators for most hulls with a satisfactory degree of accuracy for that purpose, and thereby to make the entire long-term prediction by calculation alone.

#### ACKNOWLEDGEMENTS

This project is sponsored by the Ship Structure Committee with guidance from the Ship Research Committee, and its Advisory Subgroup I, of the Maritime Transportation Research Board, National Research Council.

The authors are particularly indebted to Dr. James Williamson of Belfast Institute of Technology (formerly Webb Institute) for assistance in the early stages of the project, and to Professor Otto J. Karst of Webb Institute for his advice on statistical theory and for preparing Appendix A. Acknowledgement is also given to Mr. Rutger Bennet and Mr. E. G. U. Band, who initiated much of the work during their times at Webb, to the Teledyne Materials Research Co. (formerly Lessells and Associates), under the directorship of Mr. Fred Bailey, who furnished the full-scale data, and to members of the Ship Research Committee and Advisory Subgroup I for their helpful comments.

Also acknowledged is the assistance of Cadets E. Nathanson and A. Fiori of the State Maritime College S.U.N.Y. in the analysis of the data and the preparation of the computer program.



REFERENCES

- (1) "Review and Recommendations for Ship Structure Committee Research Program, 1966-1968," National Academy of Sciences - National Research Council, April 1967.
- (2) JOHNSON, A. J. and E. Larken, "Stresses in Ships in Service." RINA Transactions, 1964.
- (3) FRITCH, D. J., Bailey, F. C. and Wise, N. S., "Results from Full-Scale Measurements of Midship Bending Stresses on two C4-S-B5 Dry Cargo Ships Operating on North Atlantic Service." Ship Structure Committee Report SSC-164, Sept. 1964.
- (4) BENNET, R., Ivarson, A. and Nordenstrom, N., "Results from Full-Scale Measurements and Predictions of Wave Bending Moments Acting on Ships." Report No. 32 of the Swedish Shipbuilding Research Foundation, October 1962.
- (5) BAND, E. G. U., "Long-Term Trends of Hull Bending Moments." American Bureau of Shipping Report, 1966.
- (6) JASPER, N. H., "Statistical Presentation of Motions and Hull Bending Moments of Essex-Class Aircraft Carriers." DTMB Report 1251, June 1960.
- (7) BIRMINGHAM, J. T., Brooks, R. L. and Jasper, N. H., "Statistical Presentation of Motions and Hull Bending Moments of Destroyers." DTMB Report 1198, September 1960.
- (8) JASPER, N. H., Brooks, R. L. and Birmingham, J. T., "Statistical Presentation of Motions and Hull Bending Moments of Essex-Class Aircraft Carriers." DTMB Report 1251, February 1959.
- (9) "Results from Full-Scale Measurements of Midship Bending Stresses on Two C4-S-B5 Dry Cargo Ships Operating in North Atlantic Service." Fourth Progress Report of Project SR-153, Lessells and Associates, 1 February 1964.
- (10) Proceedings of International Ship Structures Congress, September 1967, Report of Committee No. 1, Environmental Conditions, pp. 42-47.
- (11) LEWIS, E. V., "Predicting Long-Term Distributions of Wave-Induced Bending Moment on Ship Hulls." SNAME Spring Meeting, July 1967.
- (12) HOFFMAN, D. and Williamson, J., "Correlation of Full-Scale and Model Results in Predicting Ship Wave Bending Moment Trends" (in preparation).
- (13) GRIM, O., "A Method of More Precise Computation of Heaving and Pitching Motions Both in Smooth Water and in Waves." Third Symposium on Naval Hydrodynamics, Sept. 1960 Scheveningen, Netherlands. ONR Publication ACR-65.

- (14) GERRITSMA, J., "Distribution of Hydrodynamic Forces Along the Length of a Ship Model in Waves." Laboratorium Voor Scheepsbouwkunde, Report 144, March 1966.
- (15) HOFFMAN, D., "Distribution of Wave Caused Hydrodynamic Pressures and Forces Along a Ship Hull." Skipsmodelltanken publication No. 94, October 1966, Trondheim, Norway.
- (16) ST. DENIS, M. and Pierson, W. J., "On the Motions of Ships in Confused Seas." Trans. SNAME, 1953.
- (17) Discussion of (18) by E. V. Lewis, pp. 415-417.
- (18) JASPER, N., "Statistical Distribution Patterns of Ocean Waves and of Wave-Induced Ship Stresses and Motions, with Engineering Applications." SNAME, Vol. 64, 1956, pp. 375-432.
- (19) FRITCH, D. J., Bailey, F. C. and Wise, N. S., "Preliminary Analysis of Bending Moment Data from Ships at Sea." Ship Structure Committee Report SSC-153, December 27, 1963.
- (20) "Summary Port and Starboard Data with Correction Factors." Technical memo, Teledyne Materials Research Co., July 1966.
- (21) FRITCH, D. J., and Bailey, F. C., "An Unmanned System for Recording Stresses and Accelerations on Ships at Sea." Ship Structure Committee Report No. SSC-150, 3 June 1963.
- (22) SWAAN, W. A., "Amidship Bending Moments for Ship in Waves." International Shipbuilding Progress, Vol. 6, N. 60, August 1959.
- (23) FUKUDA, J., "Theoretical Determination of Design Wave Bending Moments." Japan Shipbuilding and Marine Engineering, Vol. 2, No. 3, May 1957.
- (24) NORDENSTRÖM, N., "On Estimation of Long-Term Distribution of Wave Induced Bending Moments on Ships." Chalmers Tekniska Högskola, August 1963.
- (25) GOODRICH, G. J., "The Prediction of Long-Term Distribution of Ship Bending Moments from Model Tanks." Transaction of North East Coast Institution of Engineers and Shipbuilders, Vol. 82, 1966.
- (26) NORDENSTRÖM, N., "On Estimation of Long-Term Distribution of Wave Induced Bending Moments on Ships." Chalmers Tekniska Högskola, August 1963.
- (27) "Die Eigenschafter der Meereswellen in Nord Atlantischen Ozean," Deutsch. Wetterd. Seewetteramt, Pub. No. 41, 1964.
- (28) LUNDGREN, J. and Hoffman, D., "Analysis of Extreme Value Data to Predict Long-Term Ship Stress Probability." Written contribution to ISSC 1967, Oslo.

APPENDIX A

Statistical Techniques

by

O. J. Karst

Introduction

It is the purpose of this Appendix to clarify the meaning of the long-term probability curves presented in the report (Figs. 2, 3, 6, 10, 12, 15), to develop the statistical inference that may be made about exceedances of a given stress (or bending moment) in one ship's lifetime, or in the individual lifetimes of a fleet of ships, and to discuss the confidence limits applicable to the long-term curves.

In the first place, it should be noted that the long-term curves appear to be stable from a statistical viewpoint. When curves were developed for similar ships in the same service, or for the same ship in different years, the curves were found to be very much alike.

Meaning of Scales on the Curves

The probability curves in this report are cumulative curves that give the probability that a given stress  $X_j$  will be exceeded in one half oscillation (reversal) of stress.

It will be useful to clarify the meaning of the scales on the figures. The scale  $Q(X > X_j)$  is clearly a probability ranging from 1 on the right and decreasing toward zero logarithmically to the left. Thus, for example from Fig. 10 we see that the probability that any single observation of  $X_n$  will exceed 13 KPSI is  $10^{-6}$ . This should be interpreted to mean that if we observe  $X$  many times, the ratio of exceedances at  $X = 13.0$  KPSI to the total number of observations will approach  $10^{-6}$  as a limit. It specifically does not mean that we will have exactly one such exceedance in every  $10^6$  observations.

Just above this probability scale we see a scale of  $n$ , titled "Number of Reversals," and in Fig. 2 it is labeled "Number of Reversals in which 1 Exceedance is Expected." We note that this scale is the reciprocal of the probability scale. Its meaning is the number of reversals for which the ratio of the number of exceedances of  $X_j$  to the observed average number of exceedances will equal 1. This implies that if many distinct blocs of  $10^6$  cycles were observed and the number of times that the observed  $X$  exceeds  $X_j = 13.0$  KPSI in each bloc recorded (note that this might range from 0 exceedances to  $10^6$  exceedances in any particular bloc of  $10^6$  observations), then the average of the number of exceedances per bloc of  $10^6$  observations would approach 1 as a limit as more and more blocs were observed. Again, it should be noted that while in this sense the expected

number of exceedances in  $10^6$  observations is 1, the probability of this happening in any one bloc may not be great.

With this understanding of the Q and n scales, one could add a third scale of probability of exceedance per year, taking into account the number of reversals expected in the number of days the ship will be at sea per year.

### Statistical Inference from Probability Curves

The curves under consideration are actually misnamed in calling them "Long-Term Probability." Actually they represent the probability that  $X_j$  will be exceeded on one half-cycle or reversal. From a practical viewpoint this is of little interest. The attempt to extend this to many cycles by means of the reciprocal scale (as discussed above) is valid if correctly interpreted in the light of expected values, but does not answer the basic questions:

1. What is the probability that  $X_j$  will be exceeded at least once by a ship in its lifetime?
2. Out of  $\bar{N}$  similar ships what is the probability that a specified number of them will exceed  $X_j$  in their lifetimes?

We now proceed to a consideration of question 1.

Let  $Q$  = probability that a specified  $X_j$  will be exceeded in one cycle. This  $Q$  is read directly off the curves for whatever ship or weather condition is under investigation. Then  $p = 1 - Q$  is the probability that the specified  $X_j$  will not be exceeded on one "cycle," i.e., half-cycle or reversal.

Let  $n$  be the number of reversals under consideration, e.g., for one ship's lifetime  $n = 10^8$ . Then  $p^n$  = probability of not exceeding a given  $X_j$  in  $n$  reversals, and  $1 - p^n$  = probability of at least one exceedance of  $X_j$  in  $n$  reversals.

Let  $P = 1 - p^n$ . Thus,  $P$  is the answer to question 1. Note the difference in meaning between  $P$  and  $Q$ .

We now consider question 2. Let  $\bar{N}$  = number of ships under consideration. This set of ships must be all of one type and operating under similar conditions to those which gave rise to the curve from which  $Q$  is originally obtained.

Let  $y$  be a random variable defined as the number of ships of the set  $\bar{N}$  that experience at least one exceedance of  $X_j$  in each of their lifetimes. Clearly  $y$  is a non-negative integer such that  $0 \leq y \leq \bar{N}$ . The entire problem is seen then to fit into the theory of the binomial frequency function. Any one ship either does or does not have at least one exceedance of  $X_j$ . The probability of at least one exceed-

ance is P as seen above. Hence, the frequency function of y is:

$$f(y) = \binom{\bar{N}}{y} P^y (1-P)^{\bar{N}-y}; y = 0, 1, 2, \dots, \bar{N}$$

If  $\bar{N}$  is large (as it is here if  $\bar{N} = 100$ ) and if  $\bar{N}P$  is small ( $\bar{N}P < 5$ ) then the binomial frequency function is approximated by the Poisson frequency function, and

$$f(y) = \frac{e^{-\bar{N}P} (\bar{N}P)^y}{y!} \quad y = 0, 1, 2, \dots, \bar{N}$$

Hence, the answer to question 2 is given as follows: The probability that exactly r of the  $\bar{N}$  ships will experience an exceedance of  $X_j$  in their lifetimes is  $f(r)$ .

### The Binomial Model

Let us now consider the case of one ship's lifetime in greater detail in order to determine a stress or bending moment that will have a very small probability of exceedance.

If the probability of a certain basic event is Q, and if this event is repeated n times, the random variable x, which is the number of occurrences of the basic event in the n trials, has the sample space  $U = (0, 1, 2, \dots, n)$ . The density function of x is again given by the binomial probability model and is

$$f(x) = \binom{n}{x} Q^x (1-Q)^{n-x}$$

In our problem, Q is the probability of exceedance of  $X_j$ , and n is the number of half-oscillations in the lifetime of a ship. We shall take  $n = 10^8$ . Hence, we have

$$f(x) = \binom{10^8}{x} Q^x (1-Q)^{10^8-x}$$

Due to the large value of n this expression would be unwieldy to use. However, in our applications the following conditions will always hold:

$$\begin{aligned} Q &\ll 1 \\ n &\gg 1 \\ nQ &< 5 \end{aligned}$$

Under these conditions the binomial model is closely approximated by the Poisson density function,\* which we now consider.

### The Poisson Model

The Poisson density function is a one parameter, denumerably infinite, discrete function. It is stated as:

$$f(x) = \frac{m^x e^{-m}}{x!}$$

It can be shown that the parameter  $m$  is both the population mean and the population variance. Under the conditions stated, the Poisson density function approximates the binomial density function with

$$m = nQ$$

Hence, in our case we have

$$f(x) = \frac{(nQ)^x e^{-nQ}}{x!}$$

as the density function for  $x$ , the number of exceedances of a given  $X_j$ . Note the clear distinction between  $x$ , the random variable, and  $X_j$ , the stress whose exceedances we are discussing. It is only a coincidence that similar symbols are used.

### One Ship's Lifetime

We now formulate our problem. In one ship's lifetime, i.e.,  $n = 10^8$  oscillations, what is the stress  $X_j$ , such that the probability that the ship will not exceed it is equal to .99?

This implies that in the above equation we have  $f(0) = .99$  since  $x = 0$  is the condition for no exceedances in  $n$  trials. Hence

$$f(0) = \frac{(nQ)^0 e^{-nQ}}{0!} = .99,$$

or, since  $n = 10^8$

$$e^{-Q \cdot 10^8} = .99.$$

Using a table of exponential functions it is found that

---

\*Dixon and Massey, Introduction to Statistical Analysis, McGraw-Hill, p. 194.

$$e^{-.01} = .990050, \text{ or with reasonable accuracy}$$

$$e^{-.01} = .99.$$

Hence  $-Q \cdot 10^8 = .01, \text{ or}$

$$Q = 10^{-10}$$

This is the value of  $Q$  with which we enter a probability curve such as Fig. 2 or Fig. 10, and read off on the left vertical scale the desired  $X_j$ , which is the solution of the problem stated at the beginning of this section. In other words, if we design a ship with this  $X_j$ , the probability is .99 that the ship will never exceed this  $X_j$  in its lifetime.

Due to the fortuitous circumstance that  $e^{-.01} = .99$ , it is possible to use the long-term curves in a simple nomographic procedure, to find the design stress  $X_j$  for any given number of oscillations  $n$  (so long as the conditions stated on p. 47 are met). Note that if we follow the procedure above with a general  $n$ , we have,

$$e^{-nQ} = .99,$$

$$-nQ = .01,$$

$$Q = 10^{-2n-1}.$$

Hence, we may enter the curve with the desired  $n$  in the reciprocal  $Q$  scale, along the bottom, go two units of 10 to the left, go up to the curve, and then read off the desired  $X_j$  on the left scale. This means, referring to Fig. 2, that for a ship's lifetime of  $n = 10^8$ , the stress that one expects to be exceeded once is the value read from the curve at  $n = 10^8$ . But if we wish to know the stress for which there is a probability of 0.99 that it will not be exceeded in the ship's lifetime, we must read the value corresponding to  $n = 10^{10}$ .

### Fleet of Ships

Let us consider a fleet of  $N$  similar ships operating in the same weather conditions which gave rise to the particular curve used. The expected number of ships that will not exceed the design stress  $X_j$  in their individual lifetimes is  $.99\bar{N}$ . The probabilistic nature of this statement must be kept clearly in mind. For example if  $\bar{N} = 200$ , then we would expect that 198 of them would have no exceedances of  $X_j$  in their lifetimes. However, in any particular bloc of 200 ships, the number of them that have no exceedances is a random variable which has a sample space 0, 1, 2.....198, 199, 200. If we took many, many blocs of 200 ships, the average of the numbers in each bloc that have no exceedances would approach 198 as a limit.

### Confidence Interval of Points on the Curves

Although the curve itself may be very stable statistically, the individual points on the curve may not be. The theory for the variation of such points is outlined in the Band Report (5), and in Jasper's paper (18). The mathematical treatment may be found in detail in Cramer, Mathematical Theory of Statistics, page 369. By means of this theory, the desired confidence interval of the various points on the curve may be determined. According to Jasper, the quantile under analysis should not be too extreme. Since we are interested in extreme values, caution should be used. If we calculated the confidence intervals at the lower quantiles, it may be possible to extrapolate the curves to the higher values. Cramer in his development of the theory makes no such restriction, but since the method is at best an approximation, Jasper's warning should be heeded.

The curves of Fig. 9 of the report are calculated from data obtained from individual ships. If one of the ships, e.g., S.S. Wolverine State were to collect another set of data, another set of curves similar to but not exactly like those would eventuate. If the same ship were to collect still another and another such set, we would have a statistical sampling of each of the seven curves of Fig. 9. Curve III, for example, would vary with each set of data. It is the purpose of this discussion to establish so-called "confidence" limits on this variation of each of the curves, so that we can say that 67% or 90% or  $\alpha$ % of all such curves will lie within these confidence limits.

We consider Table III which gives the tabulation of  $\sqrt{E}$  for five weather groups. In each group,  $\sqrt{E}$  is normally distributed,\* and from these data the curves of Fig. 9 are calculated following the theoretical development in Reference (5). If we can analyze how the data of Table III would vary statistically with repeated sampling then it should be possible to ascertain the desired confidence limits on the curves of Fig. 9. We know that the distribution of  $\sqrt{E}$  is given by

$$f(\sqrt{E}) = \frac{1}{\sqrt{2\pi} s_i} e^{-\frac{(\sqrt{E} - m_i)^2}{2 s_i^2}}$$

where  $m_i$  and  $s_i$  are the sample mean and standard deviation of the  $i$ th weather group. Now, as we repeat the basic experiment (i.e., the collection of data for the Wolverine State), we should expect  $m_i$  and  $s_i$  to vary randomly, and thus establish a new  $f(\sqrt{E})$  for each sample. There is strong evidence that  $s_i$  does not vary significantly, at least from sea state to sea state in any one sample. It would seem reason-

---

\*See Appendix B.



able therefore to assume that  $s_i$  would not vary significantly from one sample to another. This assumption will therefore be made.

We then consider the variation of  $m_i$  from sample to sample in any one sea state. It is well known that the sample mean  $\bar{x}$  of a normal distribution is also normally distributed with the same mean  $\mu$  as the original distribution and standard deviation given by  $\sigma/\sqrt{n}$ , where  $\sigma$  is the standard deviation of the original distribution and  $n$  is the sample size.

Applying this theory to the case in hand, we then know that  $m_i$ , the sample mean, is normally distributed about  $\mu_i$  with a standard deviation  $\sigma_i$ , where  $\mu_i$  and  $\sigma_i$  are the unknown normal parameters of the  $i$ th weather group. Therefore (if we want, for example, the 90% confidence limit),

$$\Pr \left[ -1.65 < \frac{m_i - \mu_i}{\frac{\sigma_i}{\sqrt{n}}} < 1.65 \right] = .90$$

Since in each weather group  $n$  is large we can replace the unknown  $\sigma_i$  by the known  $s_i$ . Hence:

$$\Pr \left[ -1.65 \frac{s_i}{\sqrt{n}} < m_i - \mu_i < 1.65 \frac{s_i}{\sqrt{n}} \right] = .90$$

$$\Pr \left[ m_i - 1.65 \frac{s_i}{\sqrt{n}} < \mu_i < m_i + 1.65 \frac{s_i}{\sqrt{n}} \right] = .90$$

This establishes the confidence limit on  $\mu_i$ , the theoretical mean of  $\sqrt{E}$  for each weather group.

Connecting the end points of these confidence intervals for each weather group will result in the desired confidence limit curves.

APPENDIX B

Statistical Tests

by

Dan Hoffman

The Chi-squared Test

Goodness of fit tests arise when we wish to test the compatibility of a set of observed frequencies with their expected (or theoretical) frequencies.

The  $\chi^2$  distribution may be used to test how well a sample distribution agrees with a theoretical distribution, the latter being deduced from the sample. The comparison is made on the basis of the observed and the theoretical frequencies for a suitable set of class intervals of the variable of the distribution. Thus the  $\chi^2$  procedure examines the whole sample distribution at once in relation to the theoretical distribution, and is in this sense more general than examination of a sample mean, sample variance, etc.

Let  $X_1, X_2, \dots, X_n$  be a sample of values of  $X$ , and let the range of  $X$  be divided into  $r$  class intervals  $X_1 \leq X \leq X_2, X_2 \leq X \leq X_3, \dots, X_r \leq X \leq X_{r+1}$ . Suppose the number of values  $X_i$  from the sample falling in each interval is  $f_1, f_2, \dots, f_r$ , respectively. Suppose that the relative frequencies in these same intervals expected in the theoretical distribution are  $g_1, g_2, \dots, g_r$  so that the numbers of the values expected in the class intervals from a sample of  $n$  are:  $f'_1 = ng_1, f'_2 = ng_2, \dots, f'_r = ng_r$ , respectively. The  $\chi^2$  test is concerned with the difference between  $f_i$  and  $f'_i$  for all classes of intervals. Thus:

$$\chi^2 = \sum_{i=1}^r [(f_i - f'_i)^2 / f_i] = \sum_{i=1}^r (f_i^2 / ng_i) - n$$

If  $f_i$  are exactly equal to  $f'_i$ , we have a "perfect fit" and  $\chi^2 = 0$ . Thus, large values of  $\chi^2$  will tend to discredit the hypothesis that the data fit the theoretical distribution, and smaller values of  $\chi^2$  tend to confirm the hypothesis.

For moderately large values of  $n$ , the distribution of the test statistic given in the formula above is approximately the chi-square distribution having  $r-1$  degrees of freedom. In practice, it is desirable that  $ng_i \geq 5$  for each  $i$ , and this can be achieved by regrouping, if necessary, into other classes.

Test of fit of sample to normal distribution

To test the hypothesis, that sample  $X_1, X_2, \dots, X_m$ , has been drawn from a normal population of variable  $X$  with unknown parameters  $m$  and  $\sigma$ , the following tests were taken:

$r$  class intervals for  $X$  were selected. The sample frequency in class interval  $i$  is  $f_i$ . Let  $\xi_i$  be the mid-point of the  $i$ th interval,  $h$  the length of the intervals. The number of values of  $X$  from a sample size  $n$  which we expected to fall in the  $i$ th interval  $I$  is:

$$f_i = n/\sigma \sqrt{2\pi} \int_I \exp [-(X-m)^2/2\sigma^2] dX$$

where the integral extends over the  $i$ th interval.

$$\chi^2 = \sum_{i=1}^r (f_i - f'_i)^2/f'_i$$

If  $\chi^2$  as obtained above appear to be too large, indicating a poor fit, when using the actual  $m$  and  $\sigma$  of the sample, then estimates of  $m$  and  $\sigma$  which minimize  $\chi^2$  can be obtained as follows:

$$m^* = 1/n \sum_{i=1}^r f_i \xi_i$$

$$\sigma^{*2} = 1/n \sum_{i=1}^r f_i (\xi_i - m^*)^2 - h^2/12$$

Thus  $m^*$  and  $\sigma^*$  are the new parameters defining the normal distribution representing the sample. The last term, involving  $h^2$ , is Sheppard's correction for using the midpoint of the  $X$  interval. The limiting  $\chi^2$  distribution has  $r-3$  degrees of freedom (D.F.). It should be noted that one degree of freedom was deducted for each parameter estimated. Since D.F. =  $r - 1$ , when there are  $r$  cells and the cell probabilities known, it follows that D.F. =  $r - 1 - b$  when the cell probabilities depend on  $b$  parameters. In the above case,  $b = 2$ , and thus D.F. =  $r - 3$ .

In the tables below are five Weather Groups representing 30 voyages of the Wolverine State tested separately, as well as the combination of all Weather Groups. The number of cells in the group ( $r$ ) varied between 5 and 8, and the degrees of freedom from 2 to 5. It may be seen that  $\chi^2$  varies from .095 to 11.75, whereas for a perfect fit  $\chi^2$  should equal 0. The significance of the actual values can best be judged by making use of the  $\epsilon$  values that are also given in the tables.  $\epsilon$  is defined as "level of significance" and it is equal to the probability that a random sample of data could deviate from the expected distribution, and

CHI SQUARE TESTS

S.S. WOLVERINE STATE (30 VOYAGES)

WEATHER GROUP I

$\sqrt{E}$	$f_i$	$f'_i$	$(f_i - f'_i)^2$	$\frac{(f_i - f'_i)^2}{f'_i}$
2.5-2.95	21	19.05	3.80	.18
2.0-2.45	49	57.73	7.81	.14
1.5-1.95	183	175.23	6.80	.04
1.0-1.45	274	271.59	5.81	.02
.5- .95	310	299.32	115.00	.34
0 - .45	<u>232</u>	244.87	166.00	<u>.65</u>
	$\Sigma f_i = 1069$		$\chi^2 =$	1.37

number of group = 6

degrees of freedom =  $(n-1)-2 = 3$

$\epsilon = .713$

WEATHER GROUP II

$\sqrt{E}$	$f_i$	$f'_i$	$(f_i - f'_i)^2$	$\frac{(f_i - f'_i)^2}{f'_i}$
3.0-3.45	22	21.85	.02	-
2.5-2.95	61	64.32	7.20	.11
2.0-2.45	127	159.36	104.00	.65
1.5-1.95	252	253.44	2.07	.01
1.0-1.45	246	235.20	116.30	.50
.5- .95	177	158.40	275.00	1.74
0 - .45	<u>77</u>	67.20	96.00	<u>1.42</u>
	$\Sigma f_i = 962$		$\chi^2 =$	4.51

$n = 7$

$\bar{s} = 4$

$\epsilon = .348$

WEATHER GROUP III

$\sqrt{E}$	$f_i$	$f'_i$	$(f_i - f'_i)^2$	$\frac{(f_i - f'_i)^2}{f'_i}$
3.5-3.95	15	12.46	6.41	.51
3.0-3.45	42	42.20	.04	-
2.5-2.95	85	78.91	37.00	.47
2.0-2.45	123	113.10	98.00	.87
1.5-1.95	95	101.28	39.40	.38
1.0-1.45	35	50.64	205.20	4.07
.5- .95	21	18.57	5.92	.32
0 - .45	<u>6</u>	4.64	1.72	<u>.37</u>
	$\Sigma f_i =$ 422			$\chi^2 =$ 6.98

n = 8  
 $\bar{s} = 5$   
 $\epsilon = .225$

WEATHER GROUP IV

$\sqrt{E}$	$f_i$	$f'_i$	$(f_i - f'_i)^2$	$\frac{(f_i - f'_i)^2}{f'_i}$
3.5-3.95	8	7.33	.45	.06
3.0-3.45	21	21.77	.59	.03
2.5-2.95	53	44.08	119.50	2.71
2.0-2.45	53	43.54	89.60	2.04
1.5-1.95	21	36.28	233.50	6.45
1.0-1.45	15	17.88	8.30	.46
.5- .95	<u>6</u>	6.04	-	<u>-</u>
	$\Sigma f_i =$ 177			$\chi^2 =$ 11.75

n = 8  
 $\bar{s} = 5$   
 $\epsilon = .048$

WEATHER GROUP V

$\sqrt{E}$	$f_i$	$f'_i$	$(f_i - f'_i)^2$	$\frac{(f_i - f'_i)^2}{f'_i}$
4.0-4.45	4	5.12	1.25	.24
3.5-3.95	11	8.62	5.66	.65
3.0-3.45	10	9.74	.07	-
2.5-2.95	4	5.18	1.39	.27
2.0-2.45	<u>4</u>	4.19	0.04	<u>.01</u>
	$\Sigma f_i = 33$			$\chi^2 = 1.16$

n = 5  
 $\bar{s} = 2$   
 $\epsilon = .60$

SUMMATION OF ALL WEATHER GROUPS

$\sqrt{E}$	$f_i$	$f'_i$	$(f_i - f'_i)^2$	$\frac{(f_i - f'_i)^2}{f'_i}$
4.0-4.45	13	14.01	1.02	.07
3.5-3.95	49	43.74	27.70	.63
3.0-3.45	127	117.59	88.50	.75
2.5-2.95	224	208.73	233.50	1.12
2.0-2.45	321	369.7	2367.00	6.43
1.5-1.95	548	548.71	.51	-
1.0-1.45	559	562.37	11.70	.02
0.5- .95	506	477.34	821.50	1.72
0 - .45	<u>317</u>	318.21	1.46	<u>-</u>
	$\Sigma f_i = 2654$			$\chi^2 = 10.75$

n = 9  
 $\bar{s} = 6$   
 $\epsilon = .095$

it is given in standard tables as a function of  $\chi^2$  and degrees of freedom. In the ideal case of a perfect fit, its value should be 1.0. Standard texts (1B) suggest that a value of 0.05 is sometimes arbitrarily assumed to be the limit below which there is doubt that the sample really comes from the assumed distribution. On this basis, 5 out of the 6 cases tested can be accepted without reservation. In the case of Weather Group IV,  $\epsilon = .048$ . However the degree of fit is not as good as in the other cases. Hence, the hypothesis that the experimental points fit a normal distribution can be given only marginal acceptance.

It should be further noted, as pointed out by previous investigators, (2B) (18), that for the type of record in which there is a considerable probability of error in the measurement, and where the records taken every four hours are not completely independent, the hypothesis should not be rejected for  $\epsilon > .0001$  (2B). On the basis of this criterion there would be no doubt that all cases are acceptable.

#### References

- 1B. "Handbook of Probability and Statistics with Tables," by Burlington and May, Handbook Publishers, Inc., Sandusky, Ohio, Reprint 1958.
- 2B. "Stresses and Motion Measurements on Ships at Sea," Parts I, II and III, Swedish Shipbuilding Research Foundation, Reports No. 13 and 15, 1958, 1959.

DOCUMENT CONTROL DATA - R&D		
<i>(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)</i>		
1. ORIGINATING ACTIVITY (Corporate author)  Webb Institute of Naval Architecture		2a. REPORT SECURITY CLASSIFICATION  Unclassified
		2b. GROUP
3. REPORT TITLE  Analysis and Interpretation of Full-Scale Data on Midship Bending Stresses of Dry Cargo Ships		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)  Technical report		
5. AUTHOR(S) (Last name, first name, initial)  Hoffman, Dan Lewis, Edward V.		
6. REPORT DATE  June 1969	7a. TOTAL NO. OF PAGES  57	7b. NO. OF REFS  28
8a. CONTRACT OR GRANT NO.  NObs-92384	9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.  S-F013-04 Task 2022		
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.	SSC-196	
10. AVAILABILITY/LIMITATION NOTICES  The distribution of this document is unlimited.		
11. SUPPLEMENTARY NOTES  Ship Structure Committee designation SR-171	12. SPONSORING MILITARY ACTIVITY  Ship Structure Committee through Naval Ship Systems Command	
13. ABSTRACT  Results of the analysis of stress data from full-scale measurements on two C-4 type cargo vessels, the <u>S. S. Wolverine State</u> and <u>S. S. Hoosier State</u> , are presented in the form of histograms and cumulative distributions, which together with previously analyzed full-scale data cover a total of five years of normal ship operation in the North Atlantic. In addition, results of analysis of full-scale data are given for two additional ships, the <u>Mormacscan</u> and the <u>California Bear</u> .  Two rational techniques are given for the extrapolation of full-scale data to longer periods of time, in order to predict extreme bending stresses (or bending moments) in service. One of the techniques employs the integration of rms stress data from individual stress records; the other makes use of the highest stresses obtained in each record (extreme values). It is shown that extrapolated trends from the two methods are similar but reveal differences that warrant further investigation.  Recommendations are made for more data collection from different ships on different routes, for investigation of other statistical techniques, and for development of methods for model predictions of long-term trends.		



14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Stresses on ship hulls Bending moments Wave loads Extreme values of stress Statistical analysis of stresses						

**INSTRUCTIONS**

1. **ORIGINATING ACTIVITY:** Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (*corporate author*) issuing the report.
- 2a. **REPORT SECURITY CLASSIFICATION:** Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.
- 2b. **GROUP:** Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.
3. **REPORT TITLE:** Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.
4. **DESCRIPTIVE NOTES:** If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.
5. **AUTHOR(S):** Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.
6. **REPORT DATE:** Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.
- 7a. **TOTAL NUMBER OF PAGES:** The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.
- 7b. **NUMBER OF REFERENCES:** Enter the total number of references cited in the report.
- 8a. **CONTRACT OR GRANT NUMBER:** If appropriate, enter the applicable number of the contract or grant under which the report was written.
- 8b, 8c, & 8d. **PROJECT NUMBER:** Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.
- 9a. **ORIGINATOR'S REPORT NUMBER(S):** Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.
- 9b. **OTHER REPORT NUMBER(S):** If the report has been assigned any other report numbers (*either by the originator or by the sponsor*), also enter this number(s).
10. **AVAILABILITY/LIMITATION NOTICES:** Enter any limitations on further dissemination of the report, other than those

imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through \_\_\_\_\_."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through \_\_\_\_\_."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through \_\_\_\_\_."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. **SUPPLEMENTARY NOTES:** Use for additional explanatory notes.

12. **SPONSORING MILITARY ACTIVITY:** Enter the name of the departmental project office or laboratory sponsoring (*paying for*) the research and development. Include address.

13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. **KEY WORDS:** Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

NATIONAL ACADEMY OF SCIENCES-NATIONAL RESEARCH COUNCIL

DIVISION OF ENGINEERING

*This project has been conducted under the guidance of Advisory Group I, Ship Research Committee. This is a committee of Maritime Transportation Research Board, National Academy of Sciences-National Research Council. The Committee has cognizance of Ship Structure Committee projects in materials, design and fabrication as relating to improved ship structures. In addition, this committee recommends research objectives and projects; provides liaison and technical guidance to such studies; reviews project reports; and stimulates productive avenues of research.*

SHIP RESEARCH COMMITTEE

*Chairman:* M. L. Sellers, (I, II, III)  
Naval Architect  
Newport News Shipbuilding and Drydock Co.

*Vice Chairman:* J. M. Frankland (I, II, III)  
(Retired) Mechanics Division  
National Bureau of Standards

*Members*

W. H. Buckley (I, II) Chief, Structural Criteria and Loads Bell Aerosystems Co.	J. E. Goldberg (I, II) School of Civil Engineering Purdue University
B. B. Burbank (III) (Retired) Chief Metallurgist and Chemist Bath Iron Works Corp.	J. E. Herz (I, II) Chief Structural Design Engineer Sun Shipbuilding & Drydock Co.
D. P. Clausing (III) Senior Scientist U. S. Steel Corporation	G. E. Kampschaefer, Jr. (III) Manager, Application Engineering ARMCO Steel Corp.
D. P. Courtsal (II, III) Assistant Chief Engineer Dravo Corporation	B. R. Noton (II, III) Visiting Professor Dept. of Aeronautics and Astronautics Stanford University
A. E. Cox (I, II) General Program Manager Newport News Shipbuilding and Drydock Co.	W. W. Offner (III) Consulting Engineer
F. V. Daly (III) Manager of Welding Newport News Shipbuilding and Drydock Co.	S. T. Rolfe (III), <i>Coordinator</i> Division Chief Mechanical Behavior of Metals
J. F. Dalzell (I) Senior Research Scientist Hydronautics Incorporated	M. Willis (I), <i>Coordinator</i> Assistant Naval Architect Sun Shipbuilding and Drydock Co.
	R. A. Yagle (II), <i>Coordinator</i> Department of Naval Architecture and Marine Engineering University of Michigan

(I) - *Advisory Group I, Ship Strain Measurement & Analysis*  
(II) - *Advisory Group II, Ship Structural Design*  
(III) - *Advisory Group III, Metallurgical Studies*

R. W. Rumke  
*Executive Secretary*

## SHIP STRUCTURE COMMITTEE PUBLICATIONS

*These documents are distributed by the Clearinghouse, Springfield, Va. 22151. These documents have been announced in the Technical Abstract Bulletin (TAB) of the Defense Documentation Center (DDC), Cameron Station, Alexandria, Va. 22314, under the indicated AD numbers.*

- SSC-182, *Twenty Years of Research under the Ship Structure Committee* by A. R. Lytle, S. R. Heller, R. Nielsen, Jr., and John Vasta. December 1967. AD 663677.
- SSC-183, *Metallurgical Structure and the Brittle Behavior of Steel* by Morris Cohen. May 1968. AD 670574.
- SSC-184, *Exhaustion of Ductility in Compressed Bars with Holes* by S. Kobayashi and C. Mylonas. June 1968. AD 670487.
- SSC-185, *Effect of Surface Condition on the Exhaustion of Ductility by Cold or Hot Straining* by J. Dvorak and C. Mylonas. July 1968. AD 672897.
- SSC-186, *Effect of Ship Stiffness upon the Structural Response of a Cargo Ship to an Impulsive Load.* by Manley St. Denis and Samuel N. Fersht. September 1968. AD 675639.
- SSC-187, *Biennial Report of Ship Structure Committee.* September 1968. AD 675022.
- SSC-188, *Effect of Repeated Loads on the Low Temperature Fracture Behavior of Notched and Welded Plates* by W. H. Munse, J. P. Cannon and J. F. Kiefner. October 1968. AD 676722.
- SSC-189, *The Video Tape Recording of Ultrasonic Test Information* by Robert A. Youshaw, Charles H. Dyer and Edward L. Criscuolo. October 1968. AD 677894.
- SSC-190, *Bending Moment Distribution in a Mariner Cargo Ship Model in Regular and Irregular Waves of Extreme Steepness* by Nareesh M. Maniar and Edward Numata. November 1968. AD 689187.
- SSC-191, *Plastic Flow in the Local on Notches and Cracks in Fe-3Si Steel Under Conditions Approaching Plane Strain* by G. T. Hahn and A. R. Rosenfield. November 1968. AD 680123.
- SSC-192, *Notch Brittleness after Fracture* by C. Mylonas and S. Kobayashi. January 1969. AD 681051.
- SSC-193, *Development of Mathematical Models for Describing Ship Structural Response in Waves* by Paul Kaplan. January 1969. AD 682591.
- SSC-194, *Feasibility Study of Model Test on Ship Hull Girder* by Herbert Becker. May 1969. AD 687220.
- SSC-195, *Recommended Emergency Welding Procedure for Temporary Repairs of Ship Steels* by Dan Hoffman and Edward V. Lewis. May 1969.