SSC-197

AN INVESTIGATION OF THE UTILITY OF COMPUTER SIMULATION TO PREDICT SHIP STRUCTURAL RESPONSE IN WAVES

SHIP STRUCTURE COMMITTEE

JUNE 1969

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SECRETARY
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U.S. COAST GUARD HEADQUARTERS
WASHINGTON, D.C. 20591

June 1969

Dear Sir:

This report covers the work accomplished in the second phase of a three-part Ship Structure Committee sponsored project on computer simulation of ship structural response to waves. In this phase the investigator converted the equations developed during Phase I into computer language, selected the analog, digital and hybrid computer solution techniques and made pilot runs to verify the program.

Herewith is the second technical progress report entitled, *An Investigation of the Utility of Computer Simulation to Predict Ship Structural Response in Waves*. This report is being distributed to individuals and groups associated with or interested in the work of the Ship Structure Committee. Comments concerning this report are solicited.

Sincerely,

D. B. Henderson

D. B. Henderson
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure Committee

SSC-197

Second Technical Progress Report

from

Project SR-174, "Ship Computer Response"

to the

Ship Structure Committee

AN INVESTIGATION OF THE UTILITY OF COMPUTER SIMULATION TO PREDICT SHIP STRUCTURAL RESPONSE IN WAVES

by

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under

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NAVSEC Contract #N00024-67-C-5254

U. S. Coast Guard Headquarters
Washington. D. C.

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ABSTRACT

Methods of computer simulation of ship structural response in waves are described, with emphasis given to the slowly varying bending moments due to waves and to slamming responses. Analog, digital, and hybrid computer systems are analyzed, and results obtained by use of the most efficient computational procedures for each type of structural response. The vertical and lateral bending moments due to waves are determined by use of a digital computer, and sample computations illustrated for determining frequency domain outputs. Time history outputs of vertical bending moments due to nonlinear slamming are obtained using a modal model of the ship structural dynamic representation, together with time histories of the wave-induced vertical bending moment due to the same wave system. The capabilities of various computer systems to obtain the required responses, the form of the mathematical model appropriate for computational means, and the time requirements for carrying out the operations are also presented. The rapid assessment of spectral responses and their related statistical properties by means of digital computation, together with time history responses at rates faster than real time, provides a useful tool for determining many aspects of ship structural response characteristics by means of computer simulation.

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INTRODUCTION

A number of studies have been carried out, by use of both experimental and analytical techniques, to determine the bending moment on a ship in waves. The time and cost limits inherent in full scale testing of ships at sea preclude obtaining by this means the required design data that would represent the expected service loads for a new ship design. Similarly, difficulties in structural modeling, coupled with the cost factors required in a large scale model testing program, also make the model test approach difficult to carry out for every new ship design. As a result, a program was instituted under the sponsorship of the Ship Structure Committee, with the aid of an advisory panel appointed by the National Academy of Sciences, in order to investigate the utility of a computer simulation approach for determining ship bending moment response in waves.

The original program was considered to be made up of three separate phases of work which include:

1. An assembly of a system of equations that would adequately describe ship structural responses due to the effect of waves.

2. The conversion of these equations to a computer program or to the design of a computer analog.

3. Computer evaluation of the ship response mathematical model with the verification of the entire procedure provided by such evaluation.

The first phase of this work has been completed and a report [1] describing the mathematical models suitable for representing ship structural response has been distributed. The emphasis in this first phase of work was mainly devoted to determining the slowly varying bending moments due to waves, with some consideration given to the effects of slamming as well. The bending moments (i.e. vertical and lateral) due to waves were obtained on the basis of a linearized ship theory mathematical model, where the distributed local loads acting on the rigid ship hull were used to determine the bending moment and shear force at a particular station on the ship. A set of hand computations of the wave-induced bending moments were compared with model test data obtained under support of a separate project (SR-165) being carried out under cognizance of the Ship Structure Committee, and good agreement was obtained.

The report [1] also outlined a mathematical model for determining the bending moment due to slamming, and various computational procedures for obtaining and analyzing such an output were outlined. The particular type of slamming treated in [1] is that due to bow flare, where sudden increases of wave loading occur due to the generation of nonlinear forces associated with rapid immersion of the ship's bow. The form of final output representation, i.e. in terms of power spectra in the frequency domain and associated statistical properties obtained from such spectra, or as a time history from which averages, extremes, etc. are obtained was considered, and the utility of different types of computers for determining a particular desired form of final output was also discussed. The final form of bending moment output will depend upon the degree of importance of the effects of slamming, and this in turn will determine the nature of the computer system most suitable for simulation of ship structural response.

The second phase of the overall computer simulation project (SR-174) is concerned with converting the equations in the mathematical model presented in [1] into a computer program and/or analog system, and that is the subject of the present study. Considering the various aspects of the required bending moment information, the linear strip theory mathematical model provides valid results for the slowly varying wave-induced bending moments, based upon the success demonstrated in [1]. A digital computer program for determining these wave-induced bending moments has been formulated to duplicate the computational procedures applied in [1], which was a particular task in the present investigation. Techniques for evaluating the bending moment due to slamming are investigated, based on the mathematical models in [1]. Analog, digital, and hybrid computer methods are considered for this particular aspect, and their effectiveness analyzed in terms of their hardware and software requirements. The detailed procedures that are applied in carrying out these various tasks are described in the following sections of this report.

The work was carried out at Oceanics, Inc. for the Ship Structure Committee by means of Naval Ship Systems Command Contract No. N00024-67-C-5254, Project Serial No. SF013-03-04, Task 2022, SR-174.

ANALYSIS AND SIMULATION PROCEDURES

The bending moment determination is based upon separate treatment of two distinct processes, viz. the slowly varying wave-induced bending moment and the bending moment due to slamming responses. The wave-induced bending moments are steady state effects that can be considered as linear with respect to wave amplitude, and they are determined for a rigid ship using only interactions between inertial and hydrodynamic forces. The slam-induced bending moments are of transient nature, they evolve from nonlinear hydrodynamic force effects, and they represent an interaction between inertial, hydrodynamic, and elastic forces. Thus a fundamental difference between these two basic elements exists, and separate treatments of each quantity have been carried out in order to obtain the greatest degree of information on their basic characteristics as well as to select the most effective computer simulation technique.

Considering the vertical plane initially, the equations of motion for the rigid body heave and pitch degrees of freedom of a ship are solved simply for the case of regular waves by converting the resulting linear differential equations into linear algebraic equations. The vertical wave-induced bending moment arising from the inertial and hydrodynamic forces is then determined from these results by simple algebraic operations, as demonstrated in [1]. Similarly the lateral motions of sway, yaw, and roll, and the lateral bending moment induced by the waves (and the wave-induced lateral motions) are also found in this manner, as shown in [1]. These formulations have been programmed on a digital computer, following the basic computational methods outlined in [1] and other prior work. Various subprograms necessary to evaluate the sectional values of added mass, damping, wave excitation force, etc. have also been formulated or adapted, as the case may be, to prepare the various elements required for determination of the ship motions and bending moments. The basic input data for this computation are the ship lines (i.e. offsets at different stations) and loadings, and the environmental conditions will be ship speed, wavelength, and heading (for regular waves). Response amplitude operators for bending moment (per unit wave amplitude) are constructed from the output, and these serve as the basic tools for determining spectral measures of ship structural response in different sea state conditions.

A digital technique has been most appropriate for determining the linear rigid body ship motions and bending moments directly induced by the waves since the computational technique is basically arithmetic; the digital computer technique has great flexibility when altering conditions; analog representation of frequency-dependent coefficients is relatively difficult; and the digital method is directly applied to determination of spectral representations, especially when considering an input wave spectrum with directional characteristics. All of these digital computer operations have been aimed at final representation of bending moment response in a spectral form, so that measures of rms amplitudes, significant amplitudes, etc. can be directly determined as an output of this digital computer program for an arbitrary sea spectrum input.

The vertical bending moments due to slamming are determined from solution of coupled partial differential equations that include elastic effects represented by bending flexure, shear deformation, and rotary inertia, as well as the dynamic effects of inertial and hydrodynamic forces. These equations only represent the elastic response of the ship, with the effect of the hydrodynamic forces manifested by added mass in the inertial term and the <u>nonlinear</u> portion of added mass and buoyancy forces in the bow region (due to bow flare) as the input excitation.

For purposes of computer simulation a <u>nodal</u> model will be established with the hull assumed to be made up of "lumped" elements between the different ship stations (e.g. using 20 stations), with the appropriate elastic, inertial, etc. properties representative of each element. The various features of either analog, digital, or hybrid computer simulation for this model are delineated after the partial differential equations are reduced to a set of simultaneous differential-difference equations. Consideration also has been given to the methods of representing the hydrodynamic input forces for each of these different types of computers, as well as the manner of differencing the equations, i.e. the selection of the continuous variable as either space or time, leading to the choice of either a serial or parallel method of solution (for the hybrid computer system).

Another model considered for the study of the bending moment due to slamming is a modal model, where the solution is represented in terms of a series of normal modes. The equation variables are expressed as a product of two functions, each a function of only one of the two independent variables, i.e. space and time. This method requires a separate solution for the normal modes, which are determined from solving an eigenvalue problem for the natural frequencies and the mode shapes (eigenfunctions). The mathematical model underlying the modal method is developed and the recommended computer techniques for solution of this model are also presented. The assumptions inherent in the development of the equations and their computer solution are presented and the basic features of such an approach contrasted with the nodal model discussed previously.

As a result of the analyses described above, different computer simulation methods are applied to evaluate their suitability and accuracy by comparing computer output results with available model test data. The particular ship for comparison purposes has been the aircraft carrier USS ESSEX, for which an articulated model has been constructed and tested at the David Taylor Model Basin of the Naval Ship Research and Development Center (NSRDC), as described in a series of reports (e.g. [2], [3]). The bending moment response from those experiments contains information on the wave-induced bending moments as well as the sum of wave-induced and slamming responses, which is the actual form of measured total response in the full scale case also. The model tests were conducted in head seas only, so that vertical bending moments (both wave-induced as well as those including "whipping" responses due to slamming) are required by computer simulation in order to carry out the comparison. The results of such a comparison provide the means of verification of the computer model at this stage, as well as providing data on the degree of accuracy of the results. In addition to the comparison with the ESSEX, additional computations of the wave-induced bending moment at various oblique headings for another ship, the SS WOLVERINE STATE, are also presented. Comparison with the experimental data obtained in [4], which was originally shown in [1] for hand computations at a slightly different forward speed, provides a further source for judging the accuracy and degree of verification of this digital computer technique.

On the basis of these comparisons and the judgment resulting from them which provides support for a particular computer simulation technique, the necessary time for providing the required output data has been determined. The time requirement is determined initially as a result of these limited computer experiments, but certain projections are made for advanced computer hardware systems that may more adequately represent the state of the art in computer hardware development. The computational effort and time requirements for determining time histories of ship structural response, which are necessary if the slamming-type whipping responses have to be included in the simulation procedure, represent the upper bounds on these requirements since spectral information can be used to find statistical output properties in less time. Thus the importance of slamming responses in determining the design limits of ship structures (i.e. maximum bending moment) has been ascertained in order to decide on the final form of computer system, and its cost, both operationally and/or on the basis of capital investment requirements. The number of ships for which bow-flare slamming can occur, the present inability to represent bottom impact slamming due to the lack of an adequate hydrodynamic theory for its calculation, and the degree of control of slamming available to a ship's master (due to his ability to reduce speed and/or change course) are the factors that influence the decision as to the necessity of including the slamming responses represented in the present computer simulation study.

COMPUTER REPRESENTATION OF WAVE-INDUCED BENDING MOMENTS

The wave-induced bending moments, both vertical and lateral, are determined from the loads distributed along the ship hull that arise from the local wave forces and the loads due to the rigid body motions of the ship. Thus the rigid body motions of a ship in regular waves must be known in order to obtain the direct bending moment due to waves. The equations of motion of the ship are linear, as is the bending moment determination, in accordance with the results in [1] and the basic assumptions of the present study. The technique used for determining ship motions in waves is by application of strip theory, where the local forces on different ship sections are evaluated independent of the influence of neighboring sections (i.e. no interactions between sections), and that method is used for finding the bending moment.

For the case of vertical plane motions, the equations of motion are formulated relative to an axis system whose origin is located at the CG of the ship. A right-handed cartesian coordinate system is selected with the axes fixed in the body, with the x-axis positive toward the bow (in the direction of forward motion), the y-axis positive to port, and the z-axis positive upward. These axes are defined to have a fixed orientation, i.e. they do not rotate with the body, but they can translate with the body. The ship angular motions are considered to be small oscillations about a mean position given by the axes. The dynamic variables for this case are the heave displacement z along the z-axis, and the pitch angular displacement θ which is defined as positive in the direction of negative rotation about the y-axis (i.e. bow-up).

The hydrodynamic forces and moments are composed of terms of inertial nature due to body dynamic motions; dissipative terms due to damping action; and exciting effects due to the oncoming waves. The effect of the free surface is accounted for in the inertial and wave forces by frequency dependent factors that modify the added masses, and all couplings of inertial and dissipative nature are included in the analysis. Previous results for head seas are extended to the general case of oblique waves by changing the form of the exciting wave force and moment to include the effects of heading. This change is effected by a new definition of the surface wave elevation which relates the position of the body with respect to the wave orientation, as shown in Figure 1. The waves propagate with speed c in a direction oblique to the forward motion of the ship, where the angle between the x-axis and the direction of propagation of the waves is denoted by β, where β lies in the range $0° \leq \beta \leq 180°$. This angle is measured counterclockwise from the x-axis to the wave direction, so that $\beta = 0°$ for following seas and $\beta = 180°$ for head seas. The wave elevation $\eta(x,y,t)$ is represented by

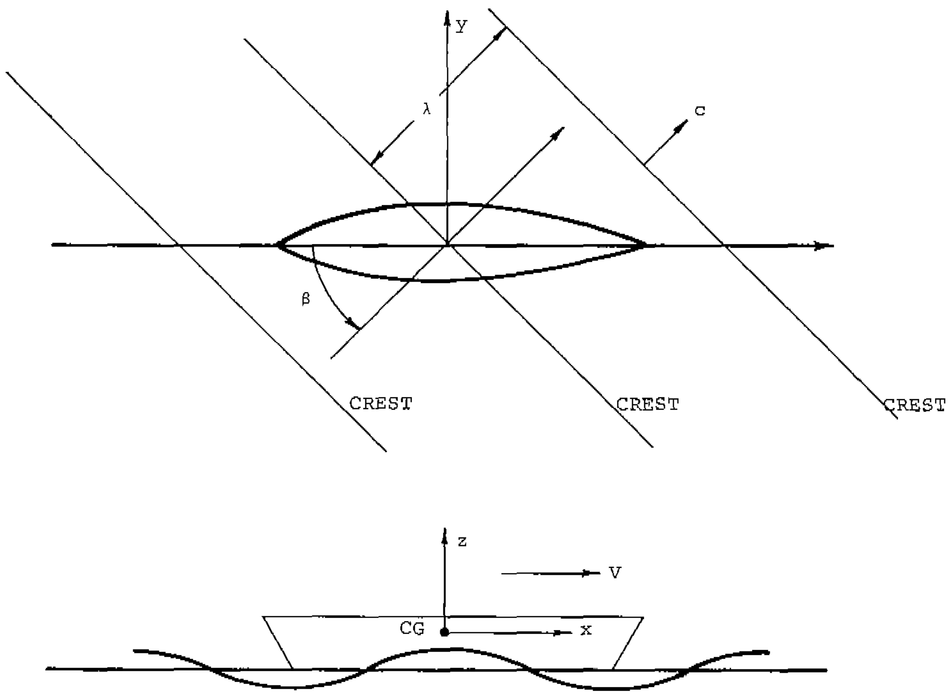

Fig. 1 Relation of Body to Waves.

$$\eta = a \sin \frac{2\pi}{\lambda} \left[x \cos \beta + y \sin \beta + (V \cos \beta - c) t \right] \quad , \quad (1)$$

where V is the ship forward speed, and for application to determine the wave forces this expression is only applied along the ship hull centerline $y = 0$ for the vertical plane motions. Thus the wave expression η used in the following development will be

$$\eta(x,t) = a \sin \frac{2\pi}{\lambda} \left[x \cos \beta + (V \cos \beta - c) t \right] \quad (2)$$

which, aside from its effect on the definition of the frequency of encounter, is equivalent to interpreting the wavelength to be an "equivalent" wavelength of $\lambda \sec \beta$.

The local vertical wave force acting on a ship section is given by

$$\frac{dZ_w}{dx} = \left\{ \rho g B^* \eta + \left[N'_z(x) - V \frac{dA'_{33}}{dx} \right] \dot{\eta} + A'_{33} \ddot{\eta} \right\} e^{-\frac{2\pi \bar{h}}{\lambda}} \quad (3)$$

where B^* is the local beam, $N'_z(x)$ is the local damping force coefficient, A'_{33} is the local section vertical added mass, and $\bar{h}$ is the mean draft of the section. The vertical velocity and acceleration of the water particles at the wave surface are

$$\dot{\eta} = \frac{D\eta}{Dt} = \left(\frac{\partial}{\partial t} - V \frac{\partial}{\partial x} \right) \eta(x,t)$$

$$= - \frac{2\pi ac}{\lambda} \cos \frac{2\pi}{\lambda} \left[x \cos \beta + (V \cos \beta - c) t \right] \quad (4)$$

and

$$\ddot{\eta} = \frac{D\dot{\eta}}{Dt} = - \frac{2\pi ag}{\lambda} \sin \frac{2\pi}{\lambda} \left[x \cos \beta + (V \cos \beta - c) t \right] \quad , \quad (5)$$

and the circular frequency of encounter is then

$$\omega_e = \frac{2\pi}{\lambda} (V \cos \beta - c) \quad (6)$$

which is used in representing the final sinusoidal form of responses to sinusoidal waves. The total vertical wave force is obtained by integrating the expression in Equation (3) over the ship hull, i.e.

$$Z_w = \int_{x_s}^{x_b} \frac{dZ_w}{dx}\, dx \tag{7}$$

and the pitch moment due to waves is

$$M_w = \int_{x_s}^{x_b} x \frac{dZ_w}{dx}\, dx \tag{8}$$

where x_s and x_b are the stern and bow x-coordinates, respectively.

The coupled equations of motion for heave (z) and pitch (θ) of a ship in regular waves are given by

$$a\ddot{z} + b\dot{z} + cz + d\ddot{\theta} + e\dot{\theta} + g\theta = Z_w(\beta,\lambda,V) \tag{9}$$

$$A\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{z} + E\dot{z} + Gz = M_w(\beta,\lambda,V) \tag{10}$$

where the coefficients are

$$a = m + \int_{x_s}^{x_b} A'_{33}\, dx \quad ; \quad b = \int_{x_s}^{x_b} N'_z dx - V \int_{x_s}^{x_b} \frac{dA'_{33}}{dx}\, dx \quad ;$$

$$c = \rho g \int_{x_s}^{x_b} B^* dx \quad ; \quad d = D = \int_{x_s}^{x_b} A'_{33}\, x\, dx \quad ;$$

$$e = \int_{x_s}^{x_b} N'_z\, x\, dx - 2V \int_{x_s}^{x_b} A'_{33} dx - V \int_{x_s}^{x_b} x\, d(A'_{33}) \quad ;$$

$$\left.\begin{aligned}
g &= \rho g \int_{x_s}^{x_b} B^* x\,dx - Vb \quad ; \quad A = Iy + \int_{x_s}^{x_b} A'_{33} x^2\,dx \quad ; \\
B &= \int_{x_s}^{x_b} N'_z x^2\,dx - 2VD - V \int_{x_s}^{x_b} x^2\,d(A'_{33}) \quad ; \\
C &= \rho g \int_{x_s}^{x_b} B^* x\,dx - VE \quad ; \quad E = \int_{x_s}^{x_b} N'_z\,x\,dx - V \int_{x_s}^{x_b} x\,d(A'_{33}) \quad ; \\
G &= \rho g \int_{x_s}^{x_b} B^* x\,dx
\end{aligned}\right\} \quad (11)$$

It is necessary to carry out integrations involving the local section geometry (i.e. local section beam) and the added mass and damping coefficient for vertical section oscillations to obtain these coefficients. The added mass and damping coefficients for two-dimensional sections are obtained from the results of Grim [5], using a new program that extends the frequency range to higher values than his earlier results (see [6] for a discussion of the limits of programs for calculating added mass and damping of two-dimensional sections for vertical oscillations). The ship sections are fitted by Lewis-form sections [7], a two-parameter family obtained from conformal transformation, and the added mass and damping coefficients are found as functions of frequency for various values of the geometric parameters beam-draft ratio (B^*/H) and section coefficient ($C_s = S/B^*H$, where S is the section area).

The local loading at a section is made up of the loads due to the inertia forces of the ship mass and the added mass; the loads due to displacement or hydrostatic effects; loads due to the damping arising from ship velocities; and the loads due to the direct wave effects. The total loading at a section, in equation form, is then

$$\frac{df_z}{dx} = - \delta m(\ddot{z}+x\ddot{\theta}) - A'_{33}(\ddot{z}+x\ddot{\theta}-2V\dot{\theta}) - \rho gB^*(z+x\theta)$$

$$- \left[N'_z(x) - V\frac{dA'_{33}}{dx}\right](\dot{z}+x\dot{\theta}-V\theta) + \frac{dZ_w}{dx} \qquad , \qquad (12)$$

where δm is the local mass loading (slugs/ft.) at the section and the value of the local wave force $\frac{dZ_w}{dx}$ is expressed by Equation (3). For use in the computer program the local wave force is expanded and expressed as

$$\frac{dZ_w}{dx} = \left[F_1 \cos \omega_e t + F_2 \sin \omega_e t\right] F_3 \qquad (13)$$

where

$$F_3 = ae^{-\frac{2\pi\bar{h}}{\lambda}} \qquad (14)$$

$$F_1 = \left(\rho gB^* - \frac{2\pi g}{\lambda} A'_{33}\right) \sin\left(\frac{2\pi}{\lambda} x \cos \beta\right)$$

$$- \frac{2\pi c}{\lambda}\left(N'_z - V\frac{dA'_{33}}{dx}\right) \cos\left(\frac{2\pi}{\lambda} x \cos \beta\right) \qquad (15)$$

$$F_2 = \left(\rho gB^* - \frac{2\pi g}{\lambda} A'_{33}\right) \cos\left(\frac{2\pi}{\lambda} x \cos \beta\right)$$

$$+ \frac{2\pi c}{\lambda}\left(N'_z - V\frac{dA'_{33}}{dx}\right) \sin\left(\frac{2\pi}{\lambda} x \cos \beta\right) \qquad (16)$$

Integrating the loading (lb./ft.) over the ship from one end up to a particular station gives the vertical shear at that station, and integrating the shear up to a station gives the vertical bending moment at that station. Alternatively, the vertical bending moment is represented mathematically as

$$BM_z(x_o) = \int_{x_s}^{x_o} (x-x_o) \frac{df_z}{dx} dx \qquad , \qquad (17)$$

where x_o is the location of the station at which the bending moment is desired, and similarly by the relation

$$BM_z(x_o) = \int_{x_b}^{x_o} (x-x_o) \frac{df_z}{dx} dx \tag{18}$$

since the requirements for a body in equilibrium are that the total force on the body, and the total moment about any point, must equal zero (the "closing" conditions for shear and bending moment).

The digital computer program for carrying out the calculation of vertical plane ship motions and vertical wave-induced bending moments is thereby similar to the program developed at MIT, which is presented in [8] and is restricted there to head seas. The essential change in the present program to allow for different headings is in the evaluation of the wave excitation, given by Equations (13)-(16), and the definition of the frequency of encounter (Equation (6)), where the sign of ω_e is examined and applied to the evaluation of the quantity F_2 appearing in Equation (13). The present program also allows an additional input, viz. wave heading angle, as well as minor input-output improvements.

For lateral wave-induced bending moments the mathematical model developed in [1] is the basic formulation that is used to establish the digital computer program, where the effect of roll motion is neglected, for simplicity. The ship dynamics representation thus includes only coupled sway and yaw motions, by application of strip theory techniques, where the local forces are determined in terms of the added mass and damping of two-dimensional sections oscillating laterally on the free surface. This information is available for Lewis-form sections in the work of Tasai ([9], [10]), where the necessary numerical procedures for calculation are presented together with typical results. In order to make direct use of the results in [9], the axis system is altered to have the y-axis positive to starboard and the z-axis positive downward, with the x-axis still positive in the direction of forward motion of the ship.

The equations of motion in yaw and sway in regular waves are given by

$$a_{11}\ddot{y} + a_{12}\dot{y} + a_{14}\ddot{\psi} + a_{15}\dot{\psi} + a_{16}\psi = Y_w(\beta,\lambda,V) \tag{19}$$

$$a_{21}\ddot{y} + a_{22}\dot{y} + a_{24}\ddot{\psi} + a_{25}\dot{\psi} + a_{26}\psi = N_w(\beta,\lambda,V) \tag{20}$$

where the coefficients are defined by

$$
\left.
\begin{aligned}
& a_{11} = m + \int_{x_s}^{x_b} M_s \, dx \quad ; \quad a_{12} = \int_{x_s}^{x_b} N_s \, dx - V \int_{x_s}^{x_b} \frac{dM_s}{dx} \, dx \quad ; \\
& a_{14} = a_{21} = \int_{x_s}^{x_b} M_s \, x \, dx \quad ; \quad a_{15} = \int_{x_s}^{x_b} N_s \, x \, dx - V \int_{x_s}^{x_b} \frac{dM_s}{dx} \, x \, dx \\
& - 2V \int_{x_s}^{x_b} M_s \, dx \quad ; \quad a_{16} = - V a_{12} \quad ; \\
& a_{22} = \int_{x_s}^{x_b} N_s \, x \, dx - V \int_{x_s}^{x_b} \frac{dM_s}{dx} \, x \, dx \quad ; \\
& a_{24} = I_z + \int_{x_s}^{x_b} M_s \, x^2 \, dx \quad ; \\
& a_{25} = \int_{x_s}^{x_b} N_s \, x^2 \, dx - V \int_{x_s}^{x_b} \frac{dM_s}{dx} \, x^2 \, dx - 2V \int_{x_s}^{x_b} M_s \, x \, dx \quad ; \\
& a_{26} = - V a_{22}
\end{aligned}
\right\} \quad (21)
$$

with M_s the local lateral added mass, N_s the local damping force coefficient, m the ship mass, and I_z the ship moment of inertia about the z-axis. The local lateral wave excitation force at a section is represented by

$$\frac{dY_w}{dx} = \left[G_1 \cos \omega_e t + G_2 \sin \omega_e t\right] G_3 \tag{22}$$

with

$$G_3 = \frac{2\pi a}{\lambda} e^{-\frac{2\pi \bar{h}}{\lambda}} \sin \beta \left[\frac{\sin\left(\frac{\pi B^*}{\lambda} \sin \beta\right)}{\frac{\pi B^*}{\lambda} \sin \beta}\right] \tag{23}$$

$$G_1 = g(\rho S + M_s) \cos\left(\frac{2\pi}{\lambda} x \cos \beta\right) - c\left(N_s - V \frac{dM_s}{dx}\right) \sin\left(\frac{2\pi}{\lambda} x \cos \beta\right) \tag{24}$$

$$G_2 = g(\rho S + M_s) \sin\left(\frac{2\pi}{\lambda} x \cos \beta\right) - c\left(N_s - V \frac{dM_s}{dx}\right) \cos\left(\frac{2\pi}{\lambda} x \cos \beta\right) \tag{25}$$

where the term within the $\left[\quad\right]$ symbol in Equation (23) represents the effect of variation of the wave properties across the width of the ship, as well as a means to account for the influence of short waves. The total wave force is obtained by integration over the ship length,

$$Y_w = \int_{x_s}^{x_b} \frac{dY_w}{dx}\, dx \tag{26}$$

and the total yaw moment due to waves is given by

$$N_w = \int_{x_s}^{x_b} x \frac{dY_w}{dx}\, dx \tag{27}$$

Expressing the solution of the equations of motion in regular waves (Equations (19) and (20)) in the form

$$y = y_r \cos \omega_e t + y_i \sin \omega_e t \tag{28}$$

$$\psi = \psi_r \cos \omega_e t + \psi_i \sin \omega_e t \tag{29}$$

the expression for local lateral loading is given as

$$\frac{df_y}{dx} = g_1 \cos \omega_e t + g_2 \sin \omega_e t \quad , \tag{30}$$

with

$$\begin{aligned} g_1 = {} & (\delta m + M_s)(\omega_e^2 y_r + \omega_e^2 x \psi_r) + 2VM_s \omega_e \psi_i \\ & - \left(N_s - V \frac{dM_s}{dx}\right) (\omega_e y_i + \omega_e x \psi_i - V\psi_r) + G_1 G_3 \end{aligned} \tag{31}$$

$$\begin{aligned} g_2 = {} & (\delta m + M_s)(\omega_e^2 y_i + \omega_e^2 x \psi_i) - 2VM_s \omega_e \psi_r \\ & + \left(N_s - V \frac{dM_s}{dx}\right) (\omega_e y_r + \omega_e x \psi_r + V\psi_i) + G_2 G_3 \end{aligned} \tag{32}$$

The lateral wave-induced bending moment at station x_o is then calculated from

$$BM_y(x_o) = \int_{x_s}^{x_o} (x - x_o) \frac{df_y}{dx} dx = \int_{x_b}^{x_o} (x - x_o) \frac{df_y}{dx} dx \tag{33}$$

by averaging the results from the bow and stern integrations.

The computer program for the lateral wave-induced bending moments, just as the program for the vertical wave-induced bending moments, was written in FORTRAN V for use with the EXEC II operating system on the UNIVAC 1108 digital computer. The lateral program is essentially the same as the vertical bending moment program, when structured in the manner described above, with the rigid body motion degrees of freedom reinterpreted and the equation coefficients and excitation revised in accordance with Equations (19)-(32). It is only necessary to have a new subroutine to determine the two-dimensional lateral added mass and wave damping terms, which replaces the subroutine for computing the analogous vertical force terms provided in the work of Grim [5]. This particular subroutine can reflect the results of Tasai [9] for Lewis forms or any other suitable formulation of these lateral force terms, such as that provided in [11] for arbitrary cylindrical sections.

All of the preceding developments for lateral bending moments, described by Equations (19)-(33), are completely programmed for digital computer evaluation. However the subroutine for mathematical evaluation of the two-dimensional hydrodynamic added mass and damping for lateral motion has not been prepared as a digital computer program. Numerical values for these quantities are available in graphical form, thereby allowing their use in computations, but a complete computer solution requires the programming of this particular subroutine.

The results obtained from the digital computer programs for wave-induced bending moments are expressed in the form of amplitude and phase of the rigid body motions, shear force, and bending moment as functions of ω_e, from which the quantity known as the response amplitude operator (RAO) is obtained. The RAO of a particular motion or response is the amplitude of response per unit wave amplitude and the general form of response from these equations may be represented, e.g. for heave motion, as

$$\frac{z}{a} = T_{z\eta}(\omega_e)e^{i\phi_z} \tag{34}$$

where ϕ_z is the phase of the motion relative to the wave at the ship CG position, and

$$\left|T_{z\eta}(\omega_e)\right| = (RAO)_z \tag{35}$$

According to linear superposition theory, as originally developed in [12], the power spectral density of an arbitrary response, represented by the i-subscript, is represented in a unidirectional random seaway by

$$\Phi_i(\omega_e) = \left|T_{i\eta}(\omega_e)\right|^2 A^2(\omega_e) \tag{36}$$

where $A^2(\omega_e)$ is the wave spectrum representation in terms of the frequency of encounter ω_e. The wave spectrum is generally represented as a function of the frequency ω, which is a pure wave frequency related to the wavelength λ by

$$\omega = \sqrt{\frac{2\pi g}{\lambda}} \tag{37}$$

The frequency of encounter ω_e is related to ω by

$$\omega_e = \omega - \frac{\omega^2 V}{g} \cos \beta \tag{38}$$

and it is necessary when representing the wave spectrum as a function of ω_e to present it in the form given by

$$A^2(\omega_e) = R^2\left[\omega(\omega_e)\right]J(\omega_e) \tag{39}$$

where $J(\omega_e)$ is the Jacobian given by

$$J(\omega_e) = \frac{1}{\sqrt{1 - \frac{4\omega_e V}{g} \cos \beta}} \tag{40}$$

(see [12] for details). The wave spectrum for a non-unidirectional sea, allowing for angular variation (a two-dimensional spectrum), will result in a modification to the basic frequency domain representation and that can be included in the program, depending on the form of the directional characteristic desired. The weighting function to account for the angular variation is given in terms of the angle β_w, which is measured from the direction toward which the wind is blowing (the predominant wave direction). For the case of a two-dimensional wave spectrum, the response spectrum (in terms of ω_e) for a particular ship heading measured relative to the wind direction is obtained by integrating with respect to the angle β_w (see [12] and [13] for details). The present computer program contains the elements that allow the complete spectral evaluation for the case of such short-crested seas, but the final steps have not been completely implemented. The effort required is very minimal, and in addition it requires knowledge of the particular angular spreading function representing the wave directional characteristics.

All of the statistical or probabilistic properties possessed by a particular linear Gaussian random response to the sea may be obtained, in principle, from the spectral density function $\Phi_i(\omega_e)$ for that response. For example, the total area E_i under the spectral density function curve, as defined above, given by

$$E_i = \int_0^\infty \Phi_i(\omega_e)\, d\omega_e \tag{41}$$

is equal to σ_i^2; i.e. the variance of the ordinates on the corresponding time-history curve.* Here the ordinate dispersion, or standard deviation, has been denoted by σ_i, which is the root-mean-square value of the deviations of the ordinates from the mean or average ordinate, which is assumed to be zero for consideration of all wave-induced effects in the theoretical computations outlined above. The mean amplitude of oscillation, the mean of the highest $\frac{1}{3}$ of such

* In the case of the Neumann wave spectrum input [12] the result is $E_i = 2\sigma_i^2$, which is a consequence of the definition of spectrum presented in [12].

amplitudes (known as the significant amplitude) and other statistical parameters of interest for a specified sea condition, ship speed, and relative heading are given in terms of E, e.g. the significant amplitude of the vertical wave-induced bending moment is expressed by

$$\overline{\left(BM_z\right)_{\frac{1}{3}}} = 2\sqrt{E_{BM_z}} \quad ** \tag{42}$$

where

$$E_{BM_z} = \int_0^{\infty} \Phi_{BM_z}(\omega_e)\,d\omega_e \tag{43}$$

The digital computer program developed for bending moment determination includes a subroutine for computing response power spectra for an arbitrary wave spectral density input. Appropriately spaced values of ω_e are selected and the necessary RAO's obtained, and these are weighted with the wave spectral density values at the same ω_e values in accordance with Equation (36) thereby producing the required response spectrum. An integration procedure is also included, together with operations to obtain $\sqrt{E_i}$, etc. in order to produce the necessary statistical data.

DIGITAL COMPUTER RESULTS FOR WAVE-INDUCED BENDING MOMENTS

As a means of checking the capability of the digital computer program developed in this study to predict the bending moments acting on ships in waves, a series of computer runs to determine the vertical wave-induced bending moments acting on a ship model were carried out. The first set of computations were made for a model of the SS WOLVERINE STATE, for which model tests in oblique waves were available [4]. The model was a 1/96 scale model corresponding to a 496 ft. LWL, a mean draft of 19.3 ft., maximum beam of 71.5 ft. and indicated displacement of 11,770 tons (29.8 lb. in model scale). The tests were carried out at a speed equivalent to 16 knots over a range of headings extending from 0° (following seas) to 180° (head seas), at 30° heading increments, and measurements made of the midship vertical and lateral bending moments.

**If the Neumann wave spectrum is used, the constant before the square root should be 1.41 rather than 2.

This same model data was used for comparison purposes in [1] with hand computations, at a speed of 17.5 knots in the computation, and relatively good agreement was found. On this basis it is anticipated that the computer program will also produce results that show good agreement, with a demonstration of the speed and versatility of the computer allowing investigation of various details, such as a larger number of wavelength conditions; the ability to include a different number of stations; an ease of making changes in the forward speed; different subroutines for local hydrodynamic force computation; the ability to obtain bending moments at any station, etc., etc. The weight curve of the ship model is shown in Figure 2, and data on ship lines was obtained from the test laboratory.

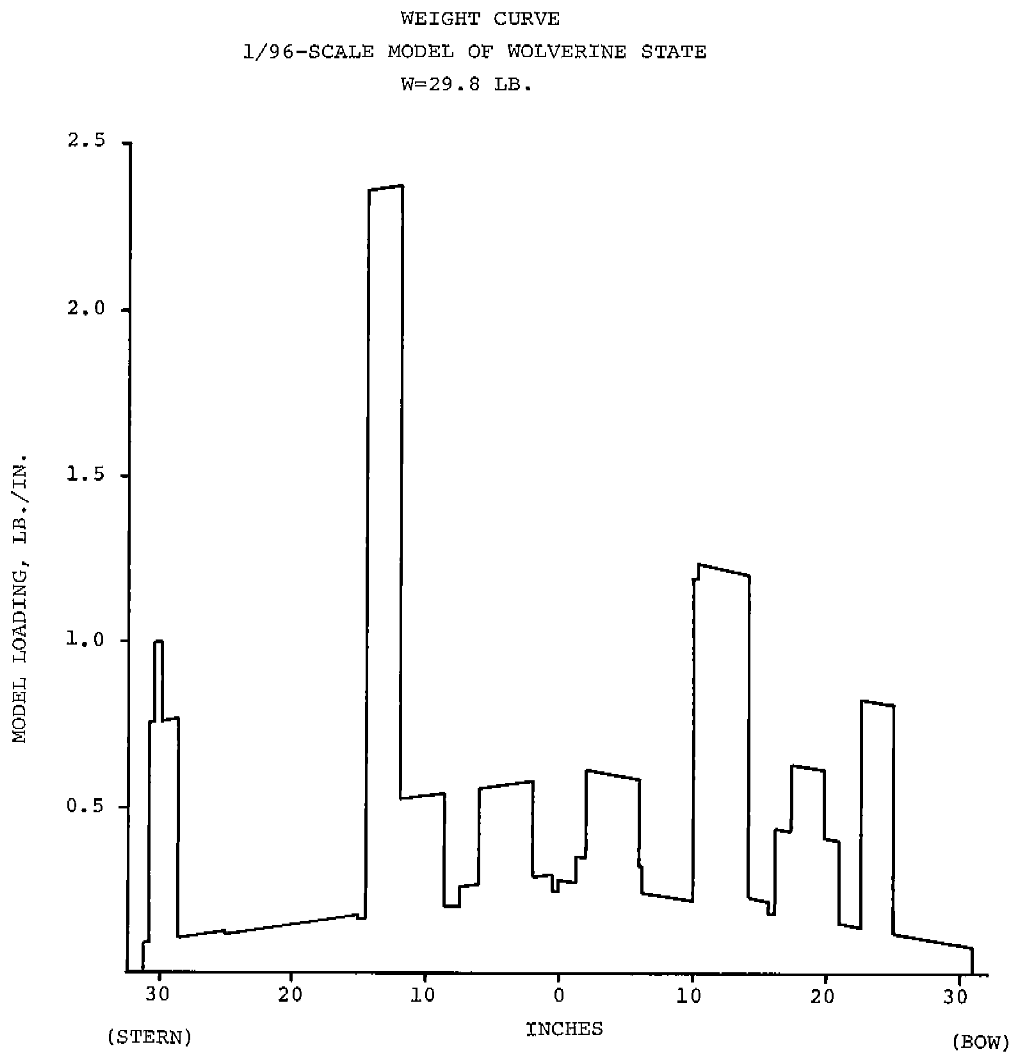

Fig. 2 Weight Curve of Wolverine State Model.

Illustrative results of the digital computer computations of midship vertical bending moment due to waves are shown in Figures 3-5, together with the experimental data at the same speed of 16 knots. The results are presented in the form of midship verti-

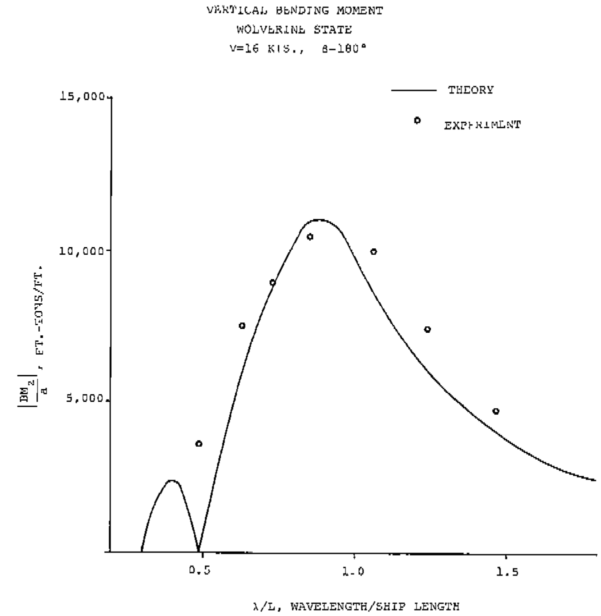

Fig. 3 Comparison of Theory and Experiment, Vertical Bending Moment, β=*180°.*

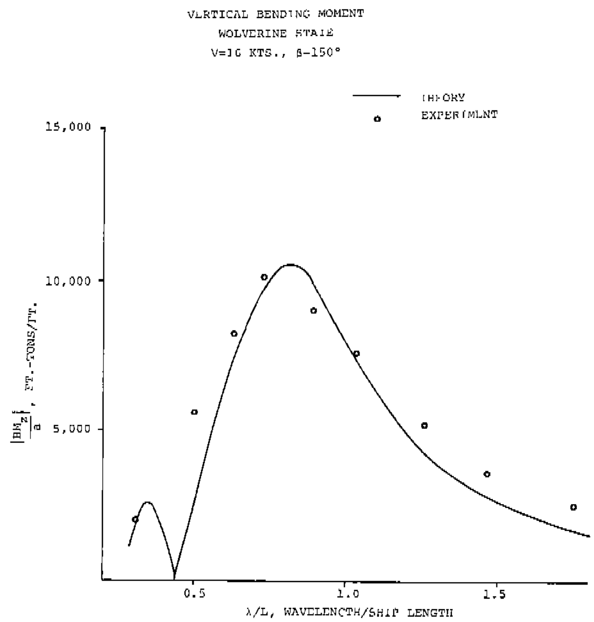

Fig. 4 Comparison of Theory and Experiment, Vertical Bending Moment, β=*150°.*

cal bending moment amplitude per unit wave amplitude, as a function of the ratio of wavelength to ship length, for three heading angles: 180° (head seas), 150°, and 120°. The agreement between theory and experiment in these cases is generally good, with a lack of agreement only at the short wave conditions ($\lambda/L \le .5$) for which the basic assumptions of strip theory are not expected to be valid. The present state of the art in ship motion prediction does not extend into that range, and since the magnitudes of various responses at those conditions is small this defect may not be significant. In addition, when considering spectral responses the frequencies of encounter corresponding to small wavelengths do not contain much wave energy and hence a small contribution to the resultant bending moment spectrum (and statistical response properties) is obtained from that region. Thus it appears that the present computer program will yield useful results in the spectral (i.e. frequency response domain for application to determining structural response

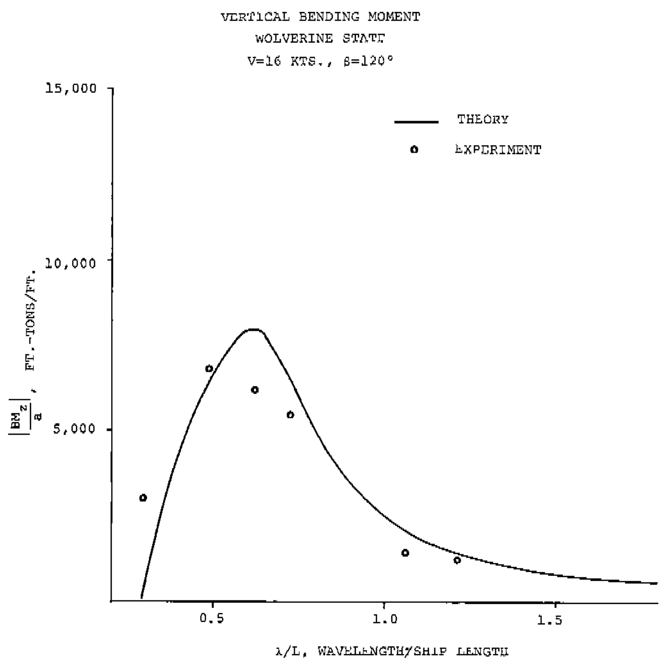

Fig. 5 Comparison of Theory and Experiment, Vertical Bending Moment, β*=120°.*

characteristics on the basis of the present comparison.

A specific requirement in this study is to obtain computer results for the vertical bending moment characteristics of the aircraft carrier USS ESSEX, for which extensive model tests were carried out in waves at NSRDC for a 6 ft. model (1/136 scale). Data on midship vertical bending moments directly induced by waves was obtained during these tests [3], and comparisons were made with digital computer results for the RAO characteristics and statistical properties at two forward speeds, 0 and 13.8 knots, in head seas. The model test data was obtained by spectral analysis of the ship responses and the waves, with the RAO characteristics extracted from these spectral characteristics. An average of the data from a series of 5 runs at each speed was obtained, and comparisons with the digital computer results are shown in Figures 6-9 for the pitch motion and the wave-induced vertical bending moment. In spite of the scatter of the data points, the computer results appear to have generally good agreement with the experimental data, with the least agreement for the zero speed case. However it is possible that the data obtained at zero speed, for a 6 ft. model in a 10 ft. wide towing tank, may be affected by wall reflections and similar interferences due to the experimental facility geometry that might invalidate the data. An examination of the pitch response, in terms of the ratio of pitch amplitude to wave slope $\left(\frac{2\pi a}{\lambda}\right)$, shows a proper asymptotic approach to the limit value 1.0 at low frequencies for the computed results and an erratic indication of the RAO data average for the zero speed condition, thereby indicating a possible extraneous influence on that data.

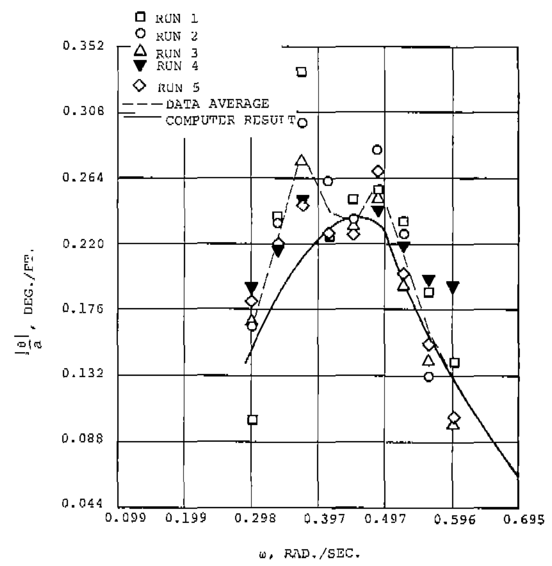

Fig. 6 Pitch Amplitude Response, Head Seas, V=0 Knots, USS Essex.

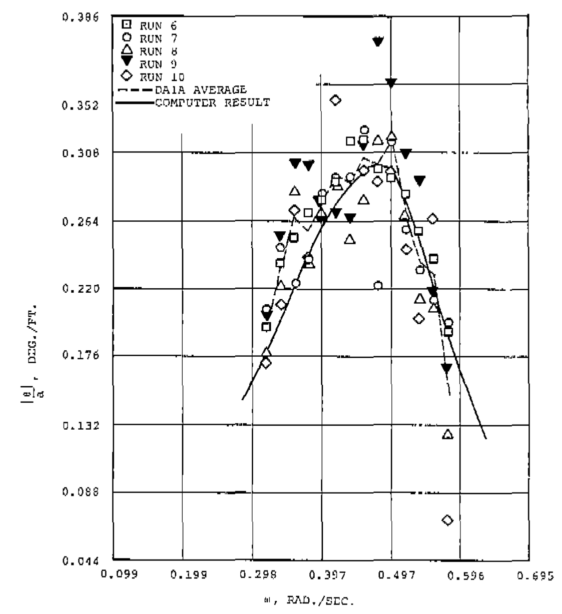

Fig. 7 Pitch Amplitude Response, Head Seas, V=13.8 Knots, USS Essex.

Further analysis of the properties of predicted response characteristics from computer results was obtained by determining the power spectra of responses to different sea state inputs, represented by the Neumann formulation. An example of the midship

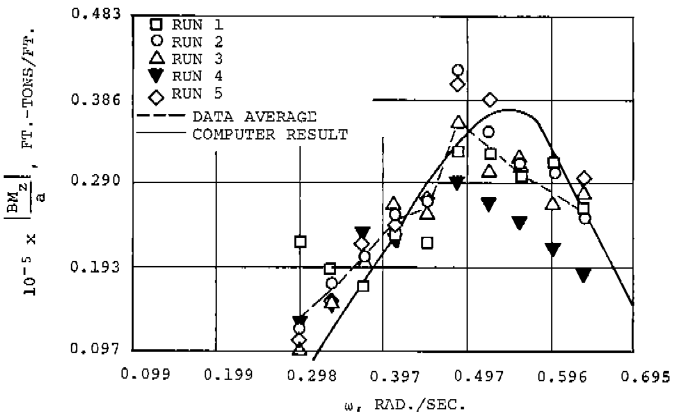

Fig. 8 Midship Bending Moment Response, Head Seas, V=0, USS Essex.

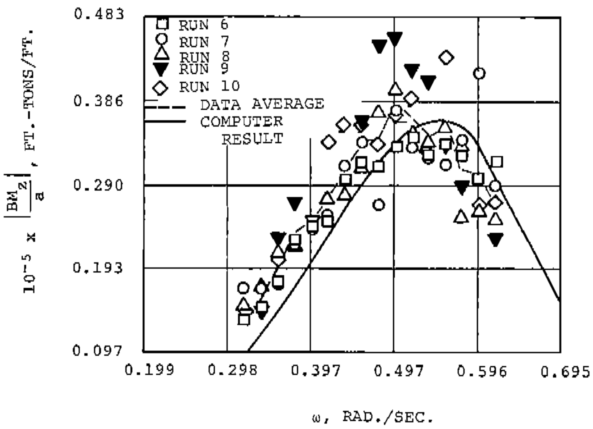

Fig. 9 Midship Bending Moment Response, Head Seas, V=13.8 Knots, USS Essex.

vertical bending moment power spectrum for the ESSEX in a Sea State 9, corresponding to a wind speed of 50 knots, for motion in head seas at a forward speed of 13.8 knots is shown in Figure 10. The square root of the spectral area for the vertical bending moment was obtained for that case and also for a Sea State 7 condition (wind speed = 30 knots) and those values are shown by the two points on Figure 11. These values obtained from the computer RAO are compared with the results obtained using model data and full scale RAO values in conjunction with Neumann wave spectra to represent $\sqrt{E_{BM_z}}$ values as a function of $\sqrt{E_\eta}$ (wave amplitude measure). Since the full scale data was obtained under 9 knot forward speed conditions, and the model test data at zero speed, the slightly increased value for the computer result at 13.8 knot forward speed is proper, and the agreement between theoretical predictions using computer results and those obtained from model tests and full scale measurements receives further verification.

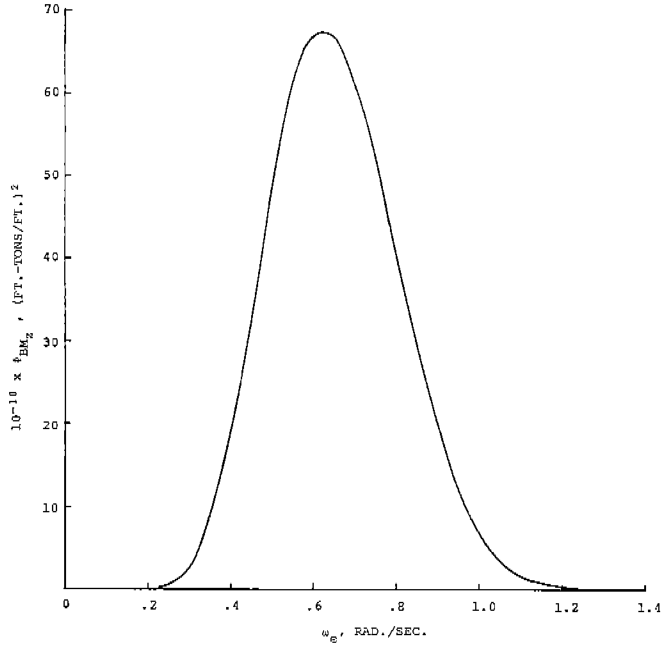

Fig. 10 Midship Vertical Bending Moment Spectrum, USS Essex, V=13.8 Knots, Head Seas, Sea State 9.

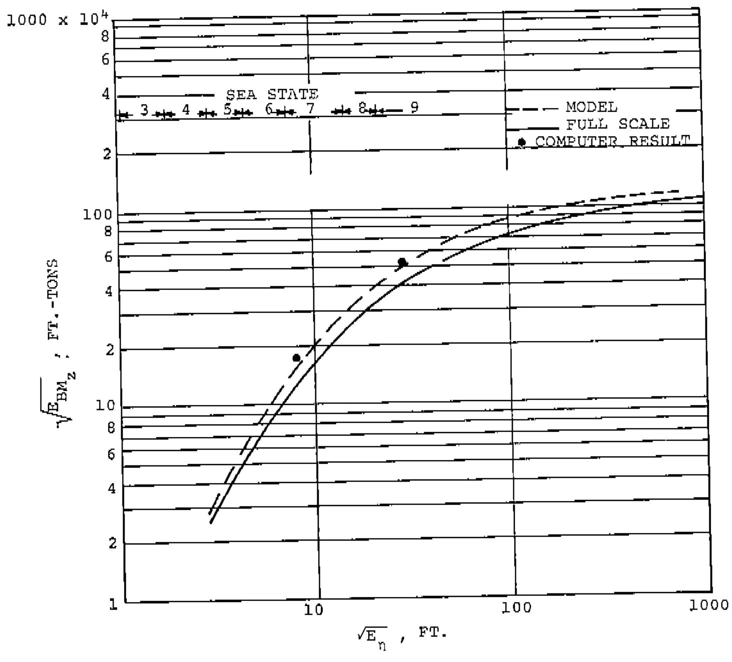

Fig. 11 Variation of RMS Bending Moment Amplitude With RMS Wave Amplitude for Model, Proto-Type, and Computer Results Using Linear Superposition.

COMPUTER SIMULATION OF SLAM-INDUCED BENDING MOMENTS

In the case of slam-induced bending moments only vertical plane motions and responses are considered, and the wave system orientation will be that of head seas. The basic equations are established on the basis of approximating the ship structure as an elastic beam with nonuniform mass and elastic properties distributed along its length.

The equations of motion governing the response characteristics of the ship are essentially the same as those presented in [14] and [15], with the main concern in the present study being the hydrodynamic forces acting as the excitation input. The basic equations are as follows:

$$\mu \frac{\partial^2 z_e}{\partial t^2} + c \frac{\partial z_e}{\partial t} + \frac{\partial V_s}{\partial x} = P(x,t) \quad , \tag{44}$$

where $\mu = \mu(x)$ is the sum of the ship mass and the added mass at a section; z_e represents the vertical elastic deflection; c is the damping coefficient; V_s is the shear force; and P(x,t) is the local input force due to ship-wave interaction.

$$\frac{\partial M}{\partial x} = V_s + I_r \frac{\partial^2 \gamma}{\partial t^2} \tag{45}$$

where M is the bending moment, I_r is the mass moment of inertia of a section; and γ is a deformation angle, with the last term on the right in Equation (45) representing the rotary inertia.

$$M = EI \frac{\partial \gamma}{\partial x} \tag{46}$$

is the fundamental elastic equation, with EI the bending flexural rigidity.

$$\frac{\partial z_e}{\partial x} = - \frac{V_s}{KAG} + \gamma \tag{47}$$

relates the bending and shear effects, where KAG is the vertical shear rigidity.

All of the above equations are partial differential equations, with the independent variables being x and t, so that the fundamental quantities of interest (i.e. bending moment, deflection, etc.) vary both temporally and spacially along the hull. The equations are appropriate to the same axes and coordinate system as was used in the analysis of vertical wave-induced bending moments, given by Equations (1)-(18), so that some of the expressions used there can be applied directly in the present case. The main emphasis in the consideration of the elastic response of the ship due to an impulsive force, arising from a slamming-like phenomenon associated with bow flare, is the determination of this force. The force input arises from interaction between the ship hull geometry and the wave, and the particular impulsive-type force must be distinguished from the ordinary wave-induced forces that cause the ship rigid body motions and the wave-induced bending moments. These latter forces are determined in accordance with linear theory, and they are found in terms of the ship geometry corresponding to an immersed portion defined by the still water equilibrium reference position, i.e. the mean value of the wave elevation. The fact that there is a difference in the actual immersed area, local form geometry, etc. due to the wave elevation and/or the resulting rigid body motions is not considered in the linear analysis that characterizes the work on wave-induced bending moments in [1], which is reproduced in the present report. Thus the input force $P(x,t)$ represented in Equation (44) will have all linear wave effects separated out, since they have already been accounted for in determining the vertical wave-induced bending moments.

The input force $P(x,t)$ is made up of two terms, one of which is of inertial nature while the other is due to buoyancy, and is represented by

$$P(x,t) = P_1(x,t) + P_2(x,t) \quad . \tag{48}$$

The force $P_1(x,t)$ is of inertial nature, and is represented by

$$P_1(x,t) = -\frac{D}{Dt}(\bar{m}_{n\ell} w_r) \tag{49}$$

where the operator

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - V\frac{\partial}{\partial x}$$

$\bar{m}_{n\ell}$ is the additional added mass at a section that is determined from the instantaneous immersion geometry of the ship section, after subtracting out the added mass determined from the still water (linear theory) reference geometry, and w_r is the relative velocity at the section, given by

$$w_r = \dot{z} + x\dot{\theta} - V\theta - w_o(x,t) \tag{50}$$

where the rigid body motions z and θ (and their derivatives) are determined from linear theory solutions (from Equations (9)-(11)) and $w_o(x,t)$ is the wave orbital velocity given by

$$w_o(x,t) = \frac{D\eta}{Dt} = -\frac{2\pi ac}{\lambda}\cos\frac{2\pi}{\lambda}\left[x + (V+c)t\right] \tag{51}$$

for the present head sea case (illustrated here for sinusoidal waves). The force $P_2(x,t)$, which is due to buoyancy, is represented by

$$P_2(x,t) = \rho g \bar{A}_{n\ell}(z_r;x) \tag{52}$$

where $\bar{A}_{n\ell}$ is the additional cross-sectional area at a section due to the difference between the area corresponding to the instantaneous submerged portion of the ship section and that corresponding to the still waterline, and after eliminating the linear buoyancy force terms. The quantity $\bar{A}_{n\ell}$ is determined, for a particular ship section, as a function of the relative immersion change

$$z_r = z + x\theta - \eta(x,t) \tag{53}$$

and it is expressed as

$$\bar{A}_{n\ell} = A - A_o + B^* z_r \tag{54}$$

where A is the instantaneous submerged area of a section, A_o is the area up to the still waterline, and $B^* z_r$ corresponds to the linear spring rate that is included in the determination of the direct wave-induced rigid body motions and the wave-induced vertical bending moment.

Since the bending moment due to whipping responses of the ship, resulting from slamming in the bow region due to nonlinear forces arising from bow flare, is a transient nonstationary process, it is best expressed as a time history output. The major simulation problem at first is the method of representing the nonlinear hydrodynamic forces defined in Equations (48)-(54) in time history form for a ship in an arbitrary seaway. The nonlinear buoyancy force defined in Equations (52)-(54) can be determined in tabular form at various stations from the ship lines drawing. Values of the added mass for various ship sections at different levels of immersion can also be calculated, with the added mass being that value appropriate to vibratory responses, i.e. a high frequency limit that will be independent of gravity effects, i.e. be frequency-independent. A mathematical procedure, which has been programmed for a digital computer, was developed for determining this added mass for arbitrary ship sections and is described below.

A multi-coefficient conformal mapping method was used to calculate the mapping coefficients for the various ship sections, where the method uses a series of points (x, y-coordinates) along the section contour as the input data and determines the various coefficients a_j that satisfy a conformal transformation similar to the Lewis [7] 2-parameter and the Landweber [17] 3-parameter mapping procedure. These methods are used in determining added masses of two-dimensional sections and the present program determines the coefficients so that the condition

$$\sum_{j=0}^{n} (x_j - x(\theta_j))^2 + (y_j - y(\theta_j))^2 = \text{minimum} \tag{55}$$

is satisfied, with the requirement

$$\frac{y_j}{x_j} = \frac{y(\theta_j)}{x(\theta_j)} \tag{56}$$

also satisfied, when x_j and y_j are the coordinates along the section and $x(\theta_j)$ and $y(\theta_j)$ are the mapped coordinates on the circle. The added mass, as defined by Landweber for a 2-parameter form, is given by

$$m_v = \frac{\rho\pi}{2}\left[(a_o+a_1)^2 + 3a_3^2\right] \quad , \tag{57}$$

and the general expression for coefficients up to $a_{2\ell-1}$ is

$$m_v = \frac{\rho\pi}{2}\left[(a_o+a_1)^2 + \sum_{\ell=2}^{n} (2\ell-1)a_{2\ell-1}^2\right] \tag{58}$$

An added mass coefficient is defined by

$$C_o = \frac{2m_v}{\rho\pi b^2} \tag{59}$$

where b is the half-beam of the section, and since b can be defined in terms of the mapping coefficients

$$C_o = \frac{(a_o+a_1)^2 + \sum_{\ell=2}^{n} (2\ell-1)a_{2\ell-1}^2}{\frac{1}{2}\rho\pi\left(1 + a_1 + \sum_{\ell=2}^{n} a_{2\ell-1}\right)^2} \tag{60}$$

A high degree of accuracy is obtained in fitting an arbitrary section, and this is limited by the number of points chosen along the section for a fit. For the present application a fit yielding 14 coefficients was used and the added mass value was judged by the degree of accuracy in fitting the section by the mapping technique, which was considered adequate for nearly all of the bow region sections (up to 20% of the ship length aft of the FP). The nonlinear added mass value can then be tabulated for different immersion levels at various ship sections by subtracting the added mass value corresponding to the still waterline (linear theory reference condition).

With the above data on nonlinear buoyancy and added mass known (in a tabular sense), it is then necessary to determine the instantaneous relative immersion (Equation (53)) and relative immersion velocity (Equation (50)) for the particular ship in an arbitrary seaway in time history form; evaluate the particular force elements in terms of the instantaneous motion time history; and combine the various effects along the ship hull to represent the distributed forces along the hull which are the input excitations to the dynamic equation (Equation (44)) of the partial differential equation system representing this interaction problem. A representation of the tabular data for the nonlinear added mass and buoyancy terms by means of an approximate fit on an analog computer function generation loop is laborious and also lacks generality. Similarly the generation of the instantaneous linear rigid body motions with proper frequency response characteristics, as well as accounting for phasing at various stations as the wave propagates along the advancing ship hull, is difficult for an analog computer in terms of component hardware requirements as well as ease of modification for a different ship, when considering extensive simulation work for many different ships. The linear ship motion characteristics in terms of frequency responses are determined from a digital computer solution in a simple manner, as described in the earlier sections of this report, and since this information is necessary for a complete structural response simulation it should be applied to other required phases of the overall project. Thus a digital computer approach, using its memory capability and function generation ability, would be most suitable for determining the nonlinear hydrodynamic forces, with the object to provide time histories of these quantities for use in the simulation.

A time history output of a linear ship response or any linear combination of such responses can be generated from knowledge of the frequency response of that quantity relative to the waves by use of a convolution integral operation in the time domain [18]. The wave motion time history at a point is the input data and a weighting function kernel operates on this input, as expressed mathematically by the following discussion, where the particular quantity considered is the ship pitch motion in this example. With knowledge of the pitch frequency response to a unit sinusoidal wave, as measured at a point x_1 ahead of the origin of coordinates (CG), given by

$$T_{\theta\eta}(\omega_e;x_1) = \left|\frac{\theta}{a}\right| e^{i[\phi_\theta - F(\omega_e)x_1]} \tag{61}$$

where $F(\omega_e) = \frac{2\pi}{\lambda}$ (with λ determined as a function of ω_e for a particular forward speed condition), the weighting kernel function is defined as the Fourier transform of the pitch motion frequency response operator, i.e.

$$K_\theta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} T_{\theta\eta}(\omega_e;x_1)\, e^{i\omega_e t}\, d\omega_e \tag{62}$$

in the frequency of encounter domain. The convolution integral operation is defined by

$$\theta(t) = \int_{-\infty}^{\infty} K_\theta(t-\tau)\,\eta_m(\tau)\,d\tau \tag{63}$$

which operates on the present and past history of the wave motion. The value of x_1 is chosen sufficiently far forward of the CG reference position such that the kernel functions will have no significant magnitude for negative values of their argument, thus avoiding any lags in the evaluation of instantaneous motions (i.e. the equivalent of a "realizable" filter). The location of the reference point x_1 is taken at 30 ft. ahead of the ship FP for the case of the ESSEX, but since this selection is only made to satisfy the requirements of the kernel function and is unrelated to real physical measurements it is not significant for the present problem, as long as all kernel functions are evaluated for that same reference point, thereby insuring proper phase relations for all quantities determined by use of these convolution integral operations. Hence the bending moment due to waves may also be represented in time history form by the relation

$$M_W(t) = \int_{-\infty}^{\infty} K_M(\tau)\,\eta_m(t-\tau)\,d\tau \tag{64}$$

where $K_M(t)$ is found as a Fourier transform of the bending moment frequency response, i.e. RAO and phase, with similar results for any other linear combination of responses that are linear with respect to wave height. The kernel function operates on $\eta_m(\tau)$, which is the wave time history as measured in the moving reference frame; i.e. at the reference point x_1 which is moving with the ship. Thus the frequency domain for carrying out the Fourier transforms is the frequency of encounter.

The wave record is generated by passing the output of a white noise generator through a filter whose amplitude characteristics are the same as the square root of the wave spectrum desired in the frequency of encounter domain. This insures a wave record that will be a representative sample of the family of possible time histories having that spectrum when a record of the wave measured at the moving point (at the desired forward speed) is analyzed.

A particular motion response such as the relative emmersion at station $18\frac{1}{2}$ (for the 20 station ESSEX), with station 20 located at the bow, going to 0 at the stern in accordance with the notation in [14]) is shown in the form of frequency response (amplitude and phase relative to the wave reference, in terms of frequency of encounter) in Figures 12 and 13 for a 13.8 knot forward speed. The

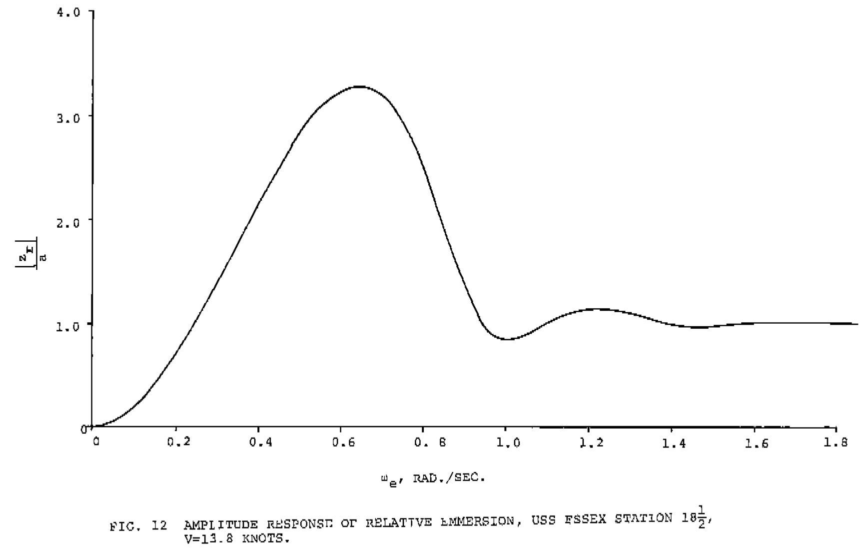

Fig. 12 Amplitude Response of Relative Immersion, USS Essex Station 18 1/2, V=13.8 Knots.

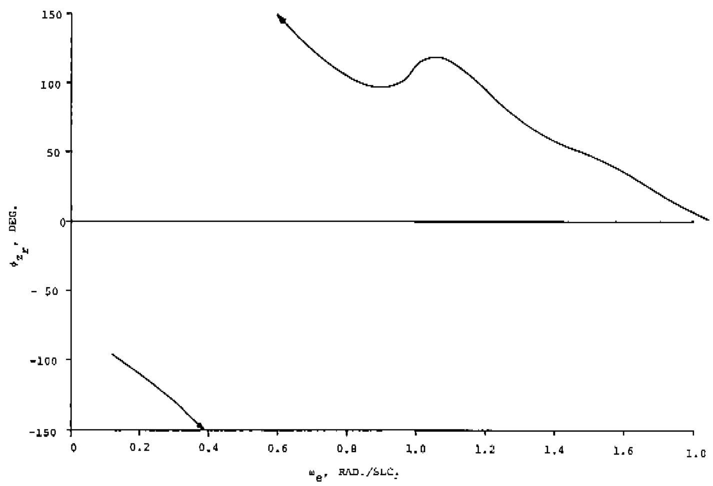

Fig. 13 Phase Angle of Relative Immersion, USS Essex Station 18 1/2, V=13.8 Knots.

amplitude is seen to asymptotically approach 1.0 as the frequency increases, as expected, and this must be truncated to zero at some finite frequency value in order to obtain a Fourier transform (see [19]). The frequency value chosen for the cutoff value (i.e. truncation point) was chosen as $\omega_e = 3.0$, which is high enough to have little effect on anticipated ship motion responses, as indicated by the power spectrum of that relative emmersion motion for a Sea State 7 (wind speed $V_w = 30$ knots), given in Figure 14. The kernel function for the relative emmersion at that station, based on the truncated response operator, is shown in Figure 15, where the

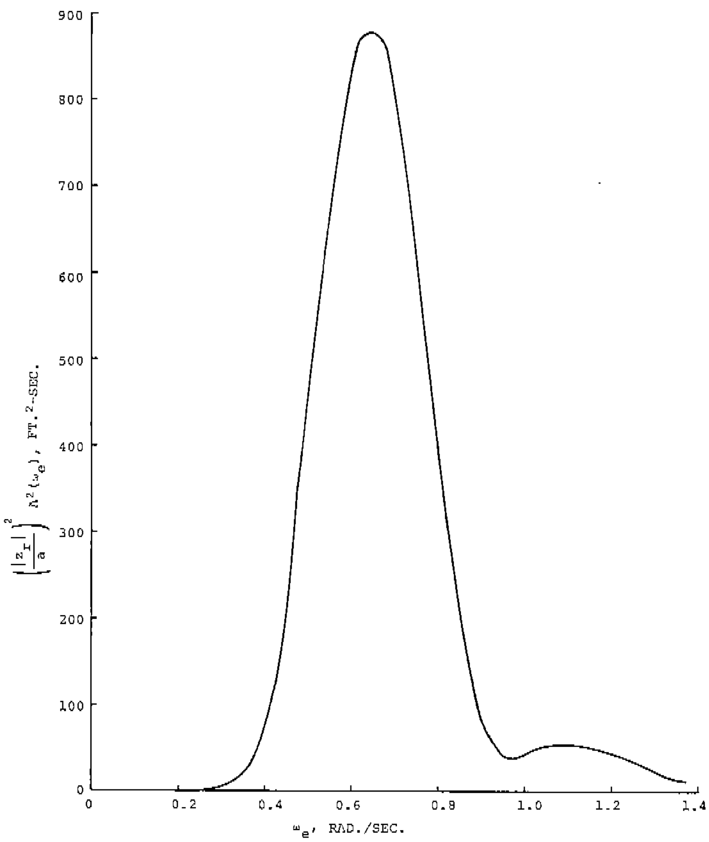

Fig. 14 Relative Immersion Spectrum, USS Essex Station 18 1/2, V=13.8 Knots, In Sea State 7.

values for negative arguments of time are to be neglected in the evaluation of the time histories. This is done in order to assure present time instantaneous values (see [18] for discussion) and the small values neglected for negative time have insignificant influence on the computed responses. Similarly a cutoff is also made for this particular kernel function at t = 20 (dotted line in Figure 15) since values of the kernel function at larger values of time give a negligible contribution. In each case when truncation was performed on the response amplitude, the resulting kernel was "inverted" to obtain the associated frequency response, and the accuracy of the kernels was found to be sufficiently good. To complete the presentation of the weighting kernel functions, a plot of the kernel function for the midship bending moment due to waves for the ESSEX at a 13.8 knot forward speed is presented in Figure 16, where the negative time portion is to be neglected in application to a wave record input.

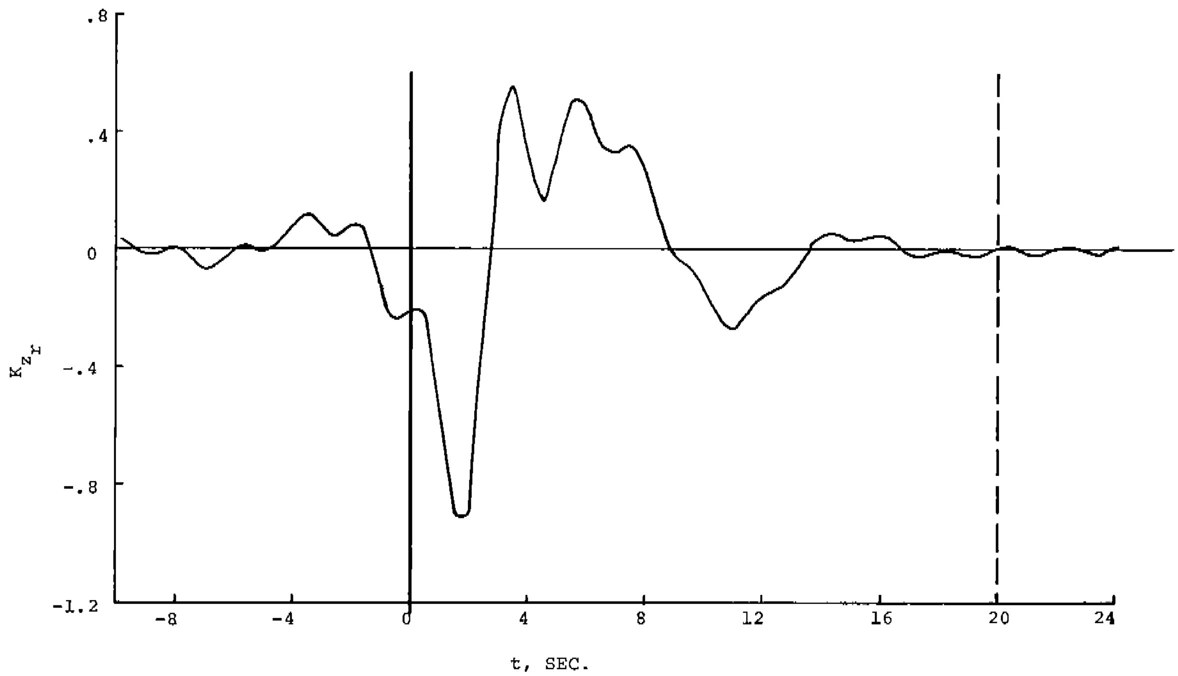

Fig. 15 Kernel Function for Relative Immersion, USS Essex Station 18 1/2, V=13.8 Knots.

The general technique initially proposed for determining time histories of bending moment due to bow-flare slamming was to use the digital computer technique described above to obtain the hydrodynamic force time history, i.e. use the digital computer as a function generator. The basic equations for the elastic response of the ship, Equations (44)-(47), were to be solved on an analog computer, with all the necessary interface equipment such as analog-to-digital (A-D) converters and D-A converters providing the linkage between the two computers, i.e. a hybrid computer simulation that makes effective use of the best capabilities of both types of computers. The basic procedures of this system are generally

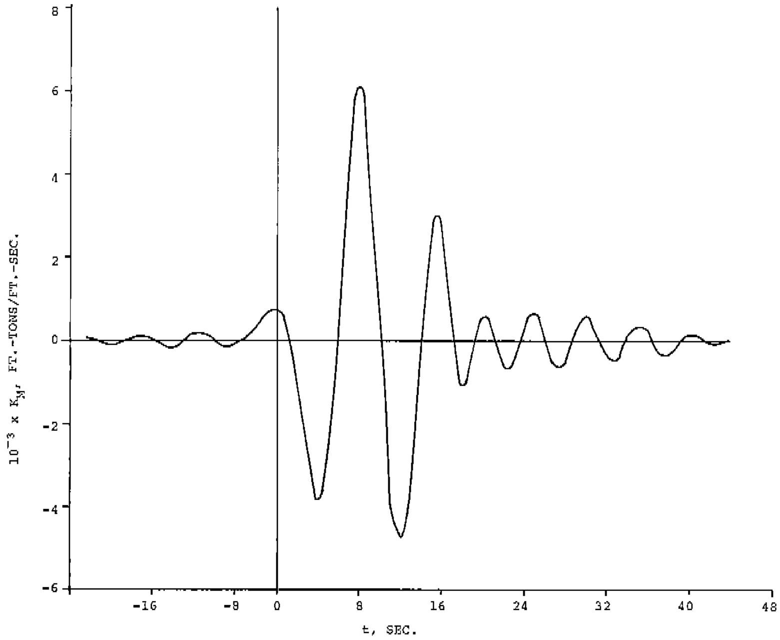

Fig. 16 Kernel Function for Wave-Induced Midship Bending Moment, USS Essex, V=13.8 Knots.

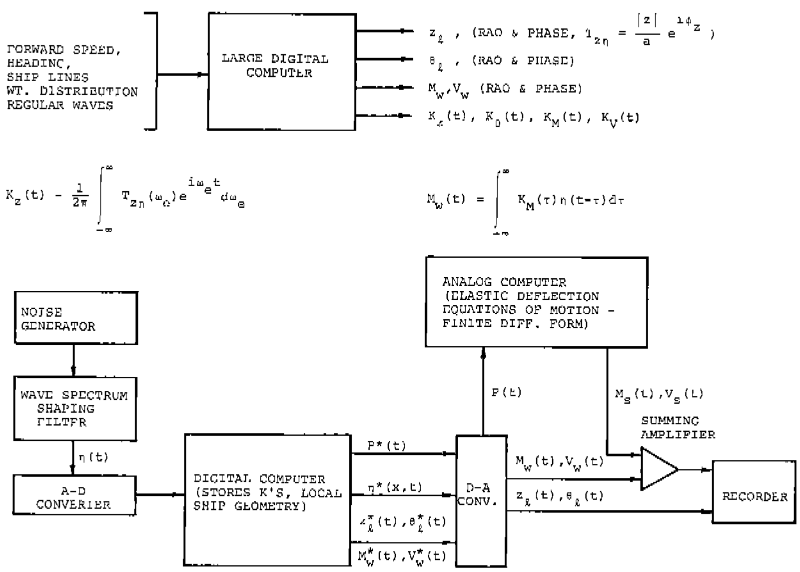

Fig. 17 Hybrid Computer Simulation Procedure.

described in Figure 17, where the origin and use of the various terms entering the simulation are shown. Since the digital function generation can be accomplished in real time, based on similar results in [18], it remains to determine the effectiveness of solving the partial differential elastic equations on the analog computer. The time scale of solution is important in that case, and the relation between the digital computer and analog computer time scales can then be examined.

The fundamental method of solution of the partial differential equations of elastic response is to convert those equations to ordinary differential-difference equations. The present case of a beam subject to dynamic loads can be viewed as a partial differential equation of the fourth order in the space variable and the second order in the time variable. The nodal approach breaks the beam into separate elements (20 segments are chosen in the present study), and a lumped-parameter system is assumed for each element, with a separate equation set for each element. The equations are functionally identical, with the only distinction being the different subscripts of the dependent variable in each equation. Thus each equation represents a different segment in time or space, depending on which variable has been retained as the continuous variable.

In the parallel method of computation, which is the classical technique applied to beam problems, a separate set of equations corresponds to each length segment of the beam, where the differencing is done with respect to the space variable. Thus a separate analog circuit is necessary for each differential equation, and this is reproduced for each segment, thereby requiring a relatively large amount of analog computer equipment. The entire set of equations is solved simultaneously (i.e. in parallel) with respect to time in this manner.

In the serial method of solution the equations are differenced with respect to time, and they are solved in an iterative fashion on a hybrid computer by time-sharing an analog circuit that represents a single differential equation. The iterative procedure updates the subscripts from run to run, stores appropriate values in the digital memory portion of the hybrid, and then obtains interpolated values of the dependent variable from the digital computer. This method requires great care in the finite difference scheme in the time variable; extensive logic control is required for the iteration procedures; and problems of instability occur in the space variable equations due to the requirements of satisfying a two point boundary value problem. In view of these difficulties the serial approach will not be considered for this present problem.

Carrying out the differencing in the space variable, the most suitable method of parallel solution on the analog computer is to form an equivalent set of four difference equations of the first order in the space variable. The finite difference equations are expressed as

$$\mu \ddot{z}_e\Big|_{n+\frac{1}{2}} + c\dot{z}_e\Big|_{n+\frac{1}{2}} + \frac{V_{n+1} - V_n}{\Delta x} = P_{n+\frac{1}{2}} \qquad (65)$$

$$\gamma_{n+1} = \frac{\left(z_e\right)_{n+\frac{3}{2}} - \left(z_e\right)_{n+\frac{1}{2}}}{\Delta x} \tag{66}$$

$$M_{n+\frac{1}{2}} = EI\Big|_{n+\frac{1}{2}} \frac{\gamma_{n+1} - \gamma_n}{\Delta x} \tag{67}$$

$$V_{n+1} = \frac{M_{n+\frac{3}{2}} - M_{n+\frac{1}{2}}}{\Delta x} \tag{68}$$

The beam (ship) under study was divided into twenty equal sections, the mechanization of which consists of twenty coupled oscillator loops. Each loop, in its uncoupled state, consists of 2 integrators and 3 amplifiers. It is recognizable as a sinusoidal generator, if we neglect the damping c, which is small. Figure 18 abstracts, as an example, Station $10\frac{1}{2}$ from the total mechanization.

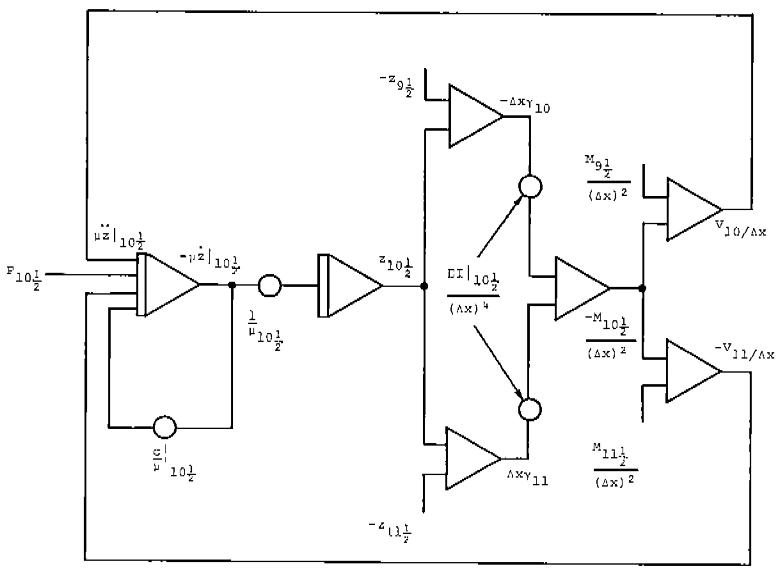

Fig. 18 Analog Computer Mechanization of Equations, USS Essex Station 10 1/2.

The loop gain defines the square of the angular frequency of the sinusoid and is given by the product of the potentiometer settings, namely

$$\omega_n^2 = \left.\frac{EI}{\mu}\right|_{10\frac{1}{2}} \cdot \frac{1}{\Delta x^4} \tag{69}$$

For the present example, using the data from [14], which is also tabulated in Table 1,

$$\Delta x = 41 \text{ ft.}$$

TABLE I PARAMETERS FOR USS ESSEX

Station	Mass μ ton-sec.^{2}/ft.^{2}	Bending Stiffness EI x 10^{-9} ton-ft.^{2}	Station	Shear Stiffness KAG x 10^{-5} tons
1/2	26.925	5.051	0	13.40
1 1/2	59.749	7.765	1	16.367
2 1/2	87.912	12.315	2	24.849
3 1/2	123.468	18.305	3	35.314
4 1/2	168.927	27.045	4	47.702
5 1/2	203.577	36.95	5	60.561
6 1/2	222.975	44.24	6	71.429
7 1/2	232.820	49.68	7	77.652
8 1/2	241.541	55.165	8	78.319
9 1/2	245.300	58.755	9	75.092
10 1/2	239.408	59.00	10	71.804
11 1/2	227.709	50.67	11	69.669
12 1/2	208.629	37.47	12	70.206
13 1/2	180.824	33.205	13	75.298
14 1/2	147.595	34.48	14	81.511
15 1/2	114.631	28.64	15	73.214
16 1/2	80.119	19.27	16	40.00
17 1/2	45.570	13.65	17	24.551
18 1/2	26.497	9.825	18	18.552
19 1/2	15.677	7.448	19	14.311
			20	12.694

$\frac{c}{\mu} = .055 \text{ sec.}^{-1}$

$$\mu_{10\frac{1}{2}} = 5.83922 \text{ ton-sec.}^2/\text{ft.}^2$$

$$EI\Big|_{10\frac{1}{2}} = 59(10^9) \text{ ton-ft.}^2$$

leading to

$$\omega_n^2 = 3576 \text{ (rad./sec.)}^2$$

The optimum time scale for the solution of the problem is generally one which gives loop gains in the neighborhood of unity. In this case, a "slowing down" of the problem by a factor of 60 gives a desirable loop gain of about 1.0. However a slowing down of the time scale is not the intention of the overall computer simulation, since an increase in the time scale is really desired, i.e. ship responses should be predicted at a faster rate than real time to make the computer approach attractive. The reasons for this difficulty must be determined and methods found to increase the time scale for this particular treatment to reach at least a real time capability.

One possible approach would be to decrease the number of segments to 10, resulting in an increase in the tolerable loop frequency by a factor of 4 since $\omega_n \sim \frac{1}{(\Delta x)^2}$. Other careful modifications can be made to increase the time scale so that real time solutions can be obtained. However the time scale is limited since the bandwidth of dc amplifiers is limited and instabilities in response occur due to imperfect frequency response, i.e. phase shifts that cause even a sine wave generator circuit to produce exponentially increasing outputs at higher frequencies. While the first mode of oscillation of the ship beam is of the order of 1 cps, higher modes are present in the 20 segment simulated structure. It is these higher modes that will be generated in the computer with attendant amplification errors, instabilities, etc. which will only produce "hash" in the resulting output. Thus bandwidth limitations of analog computer components are responsible for this difficulty, and even the newest analog computer hardware systems cannot be expected to produce outputs that will appreciably increase the time scaling capability of a 20 segment nodal model simulation of a ship. The sensitivity of the finite difference procedure for an elastic beam is also a problem, but the use of four separate first order equations overcomes that particular problem. Representing the equations with fourth order differences would lead to significant errors since the structural stiffness matrices are known to be ill-conditioned [20], and this is avoided by using first order differences. Thus the use of any computer simulation technique for transient elastic beam vibration via finite difference methods must be carried out very carefully in order to assure a successful simulation.

One method suggested in [15] makes use of a modal model to represent the beam, and this is easily accomplished if the effects of rotary inertia are deleted. Since rotary inertia has a very small influence on the first mode of motion, which is the predominant mode of ship elastic response [14], this method may be applicable here. The variables in the equations are represented in a product form as

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Returning to the forced motion, Equation (71), this can now be expressed as

$$z_e(x,t) = \sum_{i=1}^{\infty} q_i(t)X_i(x) \quad , \qquad M(x,t) = \sum_{i=1}^{\infty} q_i(t)M_i(x)$$

$$V(x,t) = \sum_{i=1}^{\infty} q_i(t)V_i(x) \tag{70}$$

where $X_i(x)$ is the normal mode shape of the ith mode. Substituting these expressions in Equations (44)-(47), eliminating and substituting, the equations can be shown to reduce to

$$\mu\ddot{q}_i X_i + c\dot{q}_i X_i + q_i\left[EI\ X_i'' + EI\left(\frac{V_i}{KAG}\right)'\right]'' = \frac{\mu Q_i X_i}{\bar{\mu}_i} \tag{71}$$

where

$$\bar{\mu}_i = \int_o^L \mu X_i^2(x)\,dx \tag{72}$$

and

$$Q_i(t) = \int_o^L P(x,t)X_i(x)\,dx \tag{73}$$

Considering the free motion of the beam, with no forcing function present, Equation (71) can be expressed as

different modes, and the reduced response for the higher modes is due to the increased values of ω_i^2. The major aspect of the structural modes in determining the dynamic slamming responses is then the eigenvalues rather than the eigenfunctions, and that may be used as guidance in selecting the number of modes in the computer simulation.

The equations for each mode can be solved in real time on an analog computer, for a particular transient input, and also in a faster than real time simulation, depending on the generation of the input function. Similarly the solution to Equation (77) can be expressed in closed form, for the initial conditions $q_i(0) = \dot{q}_i(0) = 0$, as

$$q_i(t) = \int_o^t \frac{Q_i(\tau)}{\lambda_i \bar{\mu}_i}\, e^{-\frac{c}{2\mu}(t-\tau)} \sin \lambda_i(t-\tau)\,d\tau \tag{80}$$

where

$$M_i = \int_o^x \int_o^x \mu(x)\omega_i^2 X_i(x)\,dx\,dx \tag{75}$$

is obtained. This double integral can also be expressed as a single integral

$$M_i = \omega_i^2 \int_o^x (x-s)\mu(s)X_i(s)\,ds \tag{76}$$

Returning to the forced motion, Equation (71), this can now be expressed as

$$\bar{\mu}_i\ddot{q}_i + C_i\dot{q}_i + K_i q_i = Q_i(t) \tag{77}$$

where $\bar{\mu}_i$ is defined by Equation (72), (since $\frac{c}{\mu}$ is assumed to be a constant),

$$C_i = \frac{c}{\mu}\,\bar{\mu}_i \tag{78}$$

and

$$K_i = \omega_i^2\bar{\mu}_i \tag{79}$$

Equation (77) can be easily solved for any input and solutions for the modal approach in the present case can be obtained for the first (i.e. predominant) mode and also as many of the higher modes that may be required, leading to a reduced computational effort. The number of modes that may be required for a complete representation is not known a priori, but many results of past analyses and experiments have shown that the first mode is the predominant effect. The important aspect that influences the response is the <u>frequency</u> associated with a particular mode, rather than the <u>mode shape</u> per se, since the slamming forces have a definite frequency content that has been found to be close to the first mode natural frequency value. The response of the higher modes is primarily that of a static displacement, and the amplitude is inversely proportional to the generalized spring constant K_i defined in Equation (79). Since the generalized mass $\bar{\mu}_i$, defined by Equation (72) depends on a weighted integral in terms of the square of the mode shape, the influence of whether the mode shape is symmetric or anti-symmetric is not significant. The values of $\bar{\mu}_i$ do not vary much for the different modes, and the reduced response for the higher modes is due to the increased values of ω_i^2. The major aspect of the structural modes in determining the dynamic slamming responses is then the eigenvalues rather than the eigenfunctions, and that may be used as guidance in selecting the number of modes in the computer simulation.

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$$q_i(t) = \int_0^t \frac{Q_i(\tau)}{\lambda_i \bar{\mu}_i} e^{-\frac{c}{2\mu}(t-\tau)} \sin \lambda_i (t-\tau) d\tau \qquad (80)$$

where

$$\lambda_i = \sqrt{\omega_i^2 - \frac{1}{4}\left(\frac{c}{\mu}\right)^2} \qquad (81)$$

While this solution can be obtained on a digital computer, for the present study and illustrative purposes the solution will be obtained from the analog. It is only necessary to have information on the normal mode shape for the first few normal modes, and that is provided for the ESSEX in [14]. For a general ship case such information can be obtained from available digital computer programs for solving eigenvalue and eigenfunction problems of structures, which are generally available at computer centers, or by applying the programs described in [21].

The use of a passive direct analog computer (network analyzer) discussed in [14] has the attributes of simple representation of the structural and mechanical properties of the hull, for a large nodal model. The representation of the hydrodynamic forces by use of analog elements required use of active elements, i.e. amplifiers, transfer function duplication, etc. which has the same limits as any analog system. Thus this particular type of computer network does not have the complete versatility as a computer technique that uses digital function generation for determining the hydrodynamic forcing functions. The advantage in computational speed of the passive analog portion, which allows solutions much faster than real time, is countered by this particular difficulty, as well as the greater labor of a computer operator required for problem preparation, check-out, etc., i.e. its versatility is again limited. Since the modal method described above can be applied in a time scale faster than real time (ideally), that particular advantage of the passive analog can be duplicated (to a degree) by simpler and more readily available computer systems.

HYBRID COMPUTER SOLUTIONS FOR SLAM-INDUCED BENDING MOMENTS

A number of runs were made for the USS ESSEX at a forward speed of 13.8 knots in waves with a hybrid computer setup composed of a PDP-8 digital computer, an EAI TR-48 analog computer, and DEC hybrid interface components.* These computer runs were made in two different sea states, corresponding to wind speeds of 30 knots and 50 knots (Sea States 7 and 9). The runs were of 20 minutes duration in real time, and separate outputs were obtained for the bending moments due to waves and due to slamming. The nonlinear hydrodynamic forces were applied to the first 4 of the 20 beam segments, and they were applied at the midpoints of each of the segments (stations $19\frac{1}{2}$, $18\frac{1}{2}$, $17\frac{1}{2}$ and $16\frac{1}{2}$) with the appropriate weighting according to Equation (73). A sample of the time history of the components of the local force at station $17\frac{1}{2}$, in terms of the wave record of Sea State 7 measured 30 ft. ahead of the bow, is shown in Figure 19. These results were obtained from the procedures outlined in Figure 17 using the pertinent equation definitions given in this report. The occurrence of large forces, associated with large waves, is seen from this figure, as well as the relative contribution of the two force components.

The output, in terms of the separate wave-induced and slam-induced midship bending moments, as well as their sum, is shown together with the wave record in Figure 20 for the case of Sea State 7. The occurrence of a slamming "event", in terms of bending moment response, is shown in this figure and the effect of the "whipping" action on the total midship bending moment is obvious. These occurrences can also be correlated with the nature of the associated forces by comparing with the data in Figure 19, which contains a related portion of the same wave record.

The result in Figure 20 is that due to the first mode response, and an investigation was made of the second mode response. This was done by applying the associated weighting function (second mode shape) to the local forces in accordance with Equation (73), and then as input to the dynamic second order equation (Equation (71)) with the appropriate values of the required constants. The results are shown in Figure 21, where it is seen that the second mode bending moment response is negligible compared to the first mode, which was anticipated due to previous investigations of this

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*The computer units mentioned were those used at Oceanics, Inc. in carrying out the work described herein. A discussion of the basic characteristics of computers that can be applied to this type of simulation problem is given in the next section of the report.

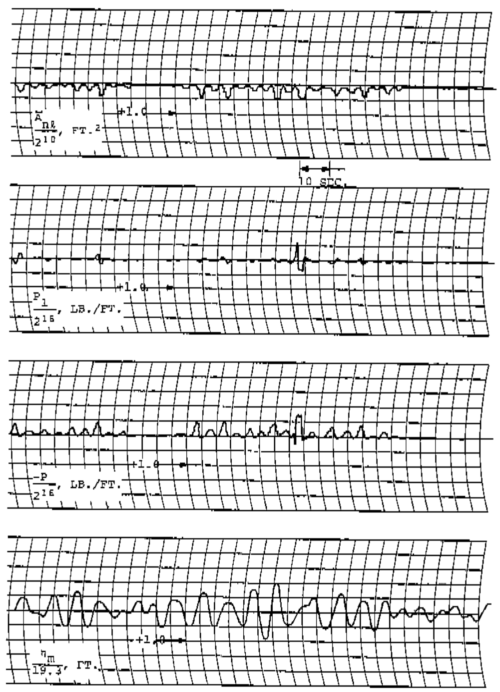

Fig. 19. Local Force Component Time Histories, USS Essex, Station 17 1/2, V=13.8 Knots, Sea State 7.

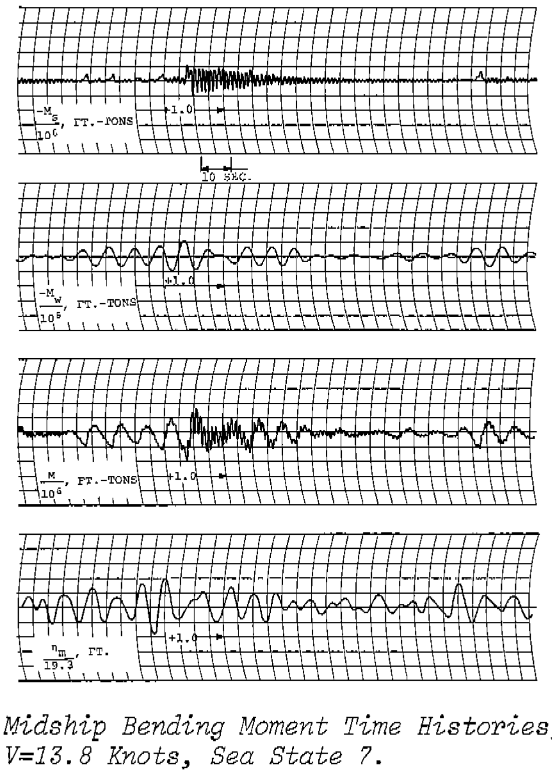

Fig. 20 Midship Bending Moment Time Histories, USS Essex V=13.8 Knots, Sea State 7.

problem (e.g. [14]). It has also been shown in [14] that the nonlinear added mass force has a "triplet" pulse shape that will excite the first mode of the ship, which has a frequency of .75 cps.

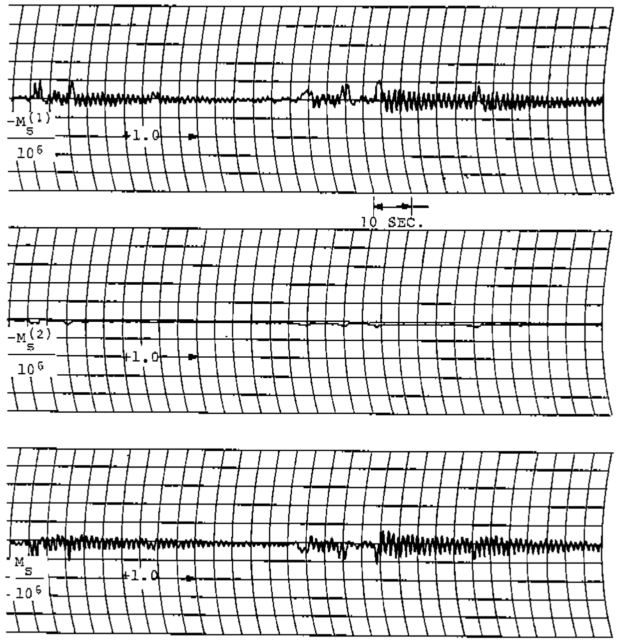

Fig. 21 Comparison of First and Second Mode Contributions to Total Slam-Induced Midship Bending Moment, USS Essex, V=13.8 Knots, Sea State 7.

A picture of the force output due to nonlinear effects is shown in Figure 22 for a 22 ft. height sinusoidal wave input at ω_e = .7 rad./sec. (the frequency of maximum relative emmersion), and the occurrence of this triplet pulse, at the appropriate frequency to excite the first mode of structural response, is evident. Thus the behavior indicated by other investigations of this problem, using other simulation techniques, is verified by these results in Figures 21 and 22.

A comparison of the bending moment output characteristics, as obtained from the hybrid computer simulation, with the experimental model data in [3] is shown in Figures 23 and 24, using the same method of presentation as in [3]. In Figure 23 the maximum whipping moments (peak to peak) due to slamming, as obtained from the computer run outputs, are divided by $2\sqrt{E}$ for the wave-induced bending moment and plotted as a function of $\sqrt{E_\eta}$, which is proportional to the rms wave height. The results are presented for the model tests at 0 and 13.8 knots forward speed, together with the data obtained from the computer output for a 13.8 knot forward speed in the two sea state conditions. Similarly the maximum total bending moment double amplitude (sum of wave-induced and slam-induced moments) at midship are presented as a function of $\sqrt{E_\eta}$ in Figure 24, where a similar comparison between the computer results and model test data is shown. Examination of these results shows quite good agreement between theory and experiment for the Sea State 7 condition and poor agreement for the Sea State 9 case. However this lack of agreement for the Sea State 9 case could be anticipated since the waves in that case are extremely large (significant height of 78 ft.), which indicates many occurrences where the forward portion of the hull would either be out of the water or have its flight deck covered with water. Such results were indicated in the time history

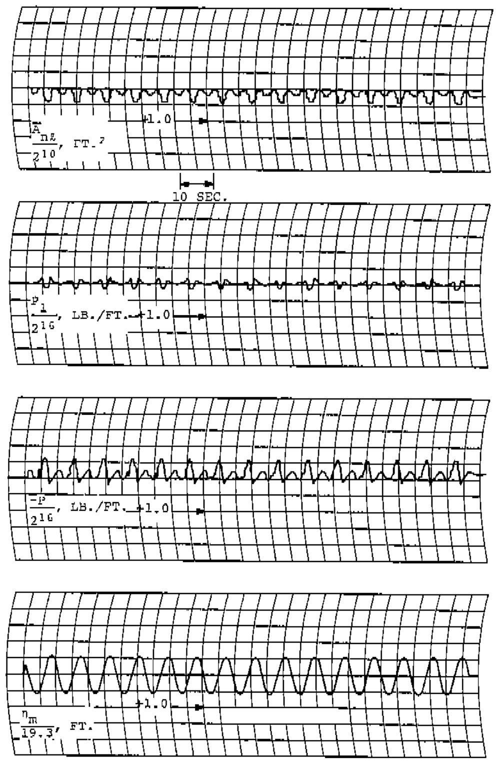

Fig. 22 Local Force Time Histories for Sinusoidal Input, USS Essex Station 17 1/2, V=13.8 Knots, ω_e*=.7 RAD./Sec.*

records for this case, where saturation levels in the hydrodynamic forces were shown since the data terminates when the hull is either out of the water or the entire ship section is submerged. In those cases bottom impact on the structure could be expected, and this type of slamming as an additional force input would result in larger whipping moments than those possible due to bow flare slamming above. Since no allowance was made to include any hydrodynamic mechanism to account for bottom impact forces, the lack of agreement is understandable. Furthermore the particular environmental condition is not

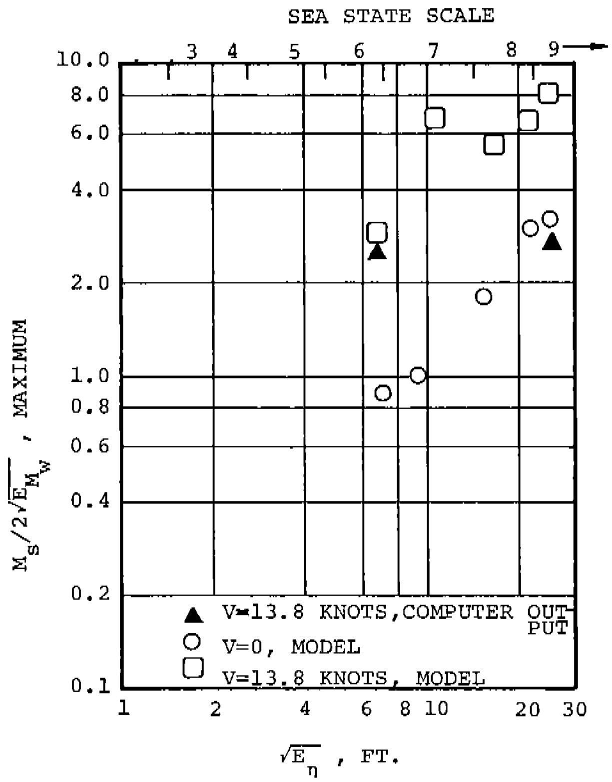

Fig. 23 Slam Induced Midship Bending Moment Characteristics, USS Essex.

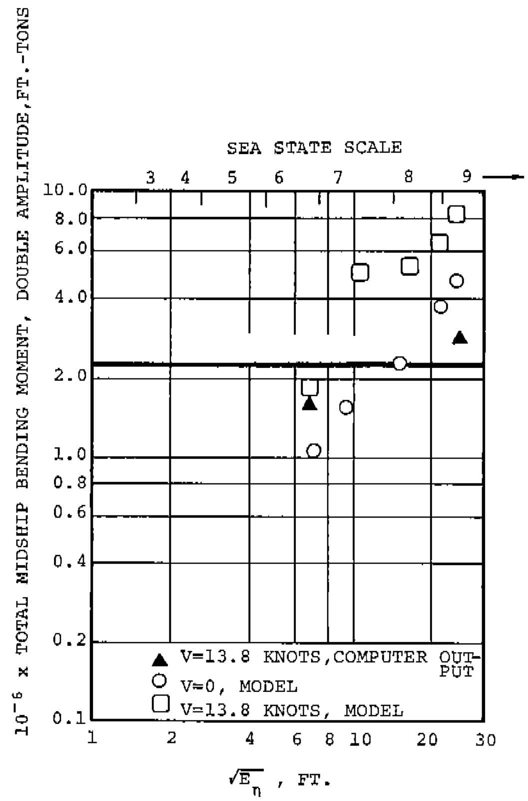

Fig. 24 Peak-To-Peak Total Midship Bending Moment Characteristics, USS Essex.

practical, since the ship would not necessarily proceed at such a forward speed in a head sea encounter with such severe waves. Thus it can be seen from the above results that the computer simulation outputs have good agreement with model test data for the conditions where the basic theory is applicable. Since the model test data has already been shown to be in agreement with full scale data [2], and further agreement between all results has been indicated in Figure 11, it can be seen that the present method of simulation provides a useful prediction of the structural responses of the USS ESSEX in a seaway.

DISCUSSION OF COMPUTER RESULTS AND CAPABILITIES

The results obtained in treating the two aspects of bending moments on ships in waves, i.e. the slowly-varying bending moments directly induced by the waves and those due to slamming, have shown good predictive ability with the computer techniques used for each case. While the particular machines and the manner of organization of the computational tasks may not have been the most advanced representatives of the state-of-the-art in either hardware or software, the basic methodology and types of computer systems used were certainly able to demonstrate the capability of simulating ship structural response characteristics. Means exist now, by use of the methods and type of computer equipment described previously to obtain spectra (and resulting rms values) for both vertical and lateral wave-induced bending moments in oblique seas for a ship moving at a prescribed forward speed and heading. This information can be readily extended to the case of short-crested seas if a particular representation of the short-crested sea spectrum is available. Similarly it is now possible to obtain time histories of both the wave-induced and slamming-induced bending moments with a hybrid computer system, so that analysis of computer output records will provide data on the effect of slamming (due to bow flare).

In order to judge the efforts required to apply the techniques developed herein on a large scale, some measure of equipment requirements and cost in time for providing such information must be established. Considering the linear bending moment induced by waves, the use of a digital computer such as the UNIVAC 1108 model used for the present computations would be representative of large digital computers of advanced capability that are presently available. The computational performance of that machine has been demonstrated in the present study to produce a complete output (i.e. amplitude and phase of heave, pitch, vertical shear and bending moment, at a particular station) within a time of 0.5 sec., for a ship at a particular forward speed and heading in a regular wave with a specific length. Assuming that 12 points (i.e. 12 wavelengths, or 12 different values of ω_e) are sufficient to represent the response operator curves such as those shown in Figures 3-9, the computational time is about 6 sec. If this were carried out for 6 different headings (15° increments, covering a 90° range), sufficient data would be available to allow evaluation of the bending moment spectrum in a short-crested sea with a specified spectrum, all of which could be accomplished within a time frame of about 1 minute (which will include integration of the resulting spectrum and evaluation of rms bending moment).

This type of result can also be obtained for the lateral bending moment at the cost of an additional 1 minute of computer time. Combined results of vertical and lateral wave-induced bending moments, such as the evaluation of deck edge stresses, could also be carried out with an insignificant amount of additional computer time. All of these results are found for a particular ship station, which the digital computer can be programmed to change, so that coverage of a number of stations with the time for complete evaluation at a particular station (since the rigid body motions are known from the first computation). Thus a versatile digital computer method can be applied to determine an extensive coverage of ship structural responses in a particular sea state within a matter of minutes, when considering the slowly-varying wave-induced bending moments.

For the vertical bending moment due to slamming, which arises from the nonlinear forces associated with bow flare, the computer method developed herein produces time history outputs by means of hybrid computer system. The various hydrodynamic forces are determined by operations on rigid body ship motions, which are expressed in terms of convolution integrals in the time domain. The weighting kernel functions are determined as by-products of the large digital computer solutions described above, and they are used in the digital computer portion of the hybrid system, which can be a much smaller machine for the hybrid applications. The computers used in the present study for this purpose were an Electronics Associates, Inc. TR-48 analog computer, a Digital Equipment Corp. PDP-8 digital computer, and a hybrid linkage (A-D and D-A converters) made by Digital Equipment Corp. The time requirements for carrying out the digital computation of the forcing functions will be considered initially, since that will have the major influence on the overall simulation time scale.

The time required to carry out the evaluation of two convolution integrals, representing the relative emmersion and relative velocity at a station, for a particular instant of time is about .025 sec. Since four sets of such results (for the four forward stations), together with a value of the time history of the wave-induced bending moment, are needed, and allowing for the time required for evaluating the nonlinear buoyancy and added mass from the tabulated values, the total time required for these operations is about .125 sec. The digital computation of the convolution integrals was carried out with a sampling time of .5 sec. (which is a proper value that would provide the correct frequency content in the hydrodynamic force terms), and hence this clock rate of the digital computer would allow a time speed-up by a factor of about 4:1 as compared to real time. Real time computations were carried out in the actual work due to capacity limitations of the particular computer equipment present (i.e. 4096 word memory, three D-A converters, etc.), but the complete capability could be achieved if increased components were available.

In addition to the 4:1 speed-up indicated above, the time scale could be further increased by a still larger factor. The evaluation of the convolution integral was primarily a series of multiplication operations which were carried out in double precision at this time to insure greater accuracy. However such accuracy may not be necessary for this problem, and the time scale could be increased by a factor of 10-12 times as fast by using single precision multiplication on the present machine or by use of a 16 bit machine rather than the present 12 bit computer. As a result it appears that the speed-up in the digital computer performance could be by a factor of 40-50 times as fast as real time. The analog computer can certainly be speeded up in its operation by that amount by a re-scaling of time, since that rate is well within the bandwidth characteristics of present solid state analog computers, as long as the modal method is used. Thus the digital computer time scaling for generation of the force inputs is the critical factor in this hybrid technique. It may even be possible, depending on capacity requirements, to obtain the complete solution by digital means according to the result in Equation (80), but that will require further investigation. Nevertheless the concept illustrated by the work described in this report, together with the analysis of time scaling, indicates an ability to provide time history outputs that will result in a rapid means of assessing the effect of slamming on bending moment response.

A hybrid computer system that is capable of carrying out the simulation of slamming responses in waves can be established using present state-of-the-art computer elements. The digital computer can be a small machine with a cycle time of the order of 1-2 microseconds, a memory (core) of 8,192 words, and a word length of 16 bits (or more). The linkage between the digital and analog computers is made up of A-D converters, a multiplexer, and D-A converters, and these units should be compatible in regard to speed, controllability, logic voltage level, etc. with the two basic computers. There are a number of small digital computers, together with the requisite hybrid interface units, that are commercially available now which can easily satisfy these requirements. As far as the analog computer portion of the hybrid system is concerned, almost any modern solid state machine made by commercial analog computer manufacturers for scientific applications will suffice for this work using the modal approach outlined previously.

Although a method was developed for presenting the bending moments due to slamming, the type of slamming was due to bow flare, which is not a very serious practical problem. Bottom impact slamming is a more significant problem, and the present techniques can be applied to that particular case as long as a method exists for representing the hydrodynamic force input. Some guidance as to the major mechanisms governing such forces is provided in [22] and [23], and similar techniques for force generation by a digital computer, as developed in this study, can be applied to that case. The equations for modal response of the dynamic and elastic structure can be applied, in conjunction with such an input, to produce information on slamming responses due to the bottom impact mechanism. Thus an extension of the method is possible for such nonstationary structural responses.

Similarly it also appears to be possible to extend the modal equations to predict the responses due to "springing" [24], which appear to arise from linear hydrodynamic forces associated with short waves. Small rigid body motions occur in that case, but the hydrodynamic wave force must be determined in detail along the hull for that case, which will require an extension of the slender-body theory (e.g. [25]) with similar terms as were illustrated in limited detail in [1]. In this case the resulting bending moments appear to be linear and stationary, and their effects can be included together with the lower frequency wave bending moments in a spectral form. This possible extension of computer simulation should be investigated further since it has important significance for particular ships, such as large Great Lakes vessels.

When considering extended capabilities of the present computer methods, it is also possible to include the effects of roll motion and data on vertical weight distribution so that complete lateral bending moments and also torsional moments induced by waves in oblique seas can be calculated. The basic mathematical models are described in [1], and only a small effort is necessary to extend the analysis for this purpose. More detailed programming of lateral hydrodynamic forces due to roll must be carried out, but the basic theory already exists ([9], [11]. An extension of this type to the digital computer method of predicting wave-induced bending moments will be an important addition, with applications to such vessels as container ships.

The preceding discussion has demonstrated the capabilities of the presently developed computer technique for predicting ship structural response, and it appears that a useful and efficient tool exists for determining the wave-induced bending moments.* While the slamming effects are not determined with the same ease, they are still amenable to solution and can be presented for any particular operating condition (with valid inclusion of all pertinent force mechanisms). The nature of the particular computer systems required for the final desired data is dependent on the extent of slamming information desired, and this should be explored for more cases rather than the present single aircraft carrier response. The extensions of the present methods to include the effects of bottom impact slamming and springing should be pursued, and similarly the prospect of computing torsional moments (and a more complete representation of lateral bending moments) in oblique waves should also be investigated.

CONCLUSIONS

The results obtained in the present study demonstrate the capability of providing information on ship structural response by means of computer simulation. Computed values have good agreement with available experimental data for the cases investigated, and the program has sufficient flexibility to allow variations that would encompass other ships. The particular computer system hardware requirements and the time requirements to obtain solutions for the different bending moment quantities have been delineated, and it thereby appears that present-day computers have the necessary capability for carrying out the required simulation work. Thus the present results provide a basis for further computer evaluations to produce additional verification and/or predictions for a larger class of ships.

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* A supplemental report [26] that lists the various digital computer programs used to determine wave-induced bending moments has also been prepared. This report has received limited distribution and is held on file by the Ship Structure Committee for internal use and information.

In addition to the ability to provide data on both vertical and lateral wave-induced bending moments in oblique seas, as well as slamming responses due to bow flare, the methods can be extended to other problems. These other applications are to bottom impact slamming, springing responses due to short waves, and torsional moments in oblique seas. These additional effects should be investigated by further analysis and computer simulation so that they can be treated in the same manner and with the same degree of success as the structural responses considered in detail in the present report.

Since the next phase of work in the Ship Computer Response project is aimed at obtaining verification of the presently developed computer techniques, these particular tasks should be carried out in that phase of work. The results obtained in that type of study will provide a further basis for establishing a computer method for determining ship structural responses, including important additional effects not considered previously in this study but which can be accomplished as a result of extensions of the present method.

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13. ABSTRACT

Methods of computer simulation of ship structural response in waves are described, with emphasis given to the slowly varying bending moments due to waves and to slamming responses. Analog, digital, and hybrid computer systems are analyzed, and results obtained by use of the most efficient computational procedures for each type of structural response. The vertical and lateral bending moments due to waves are determined by use of a digital computer, and sample computations illustrated for determining frequency domain outputs. Time history outputs of vertical bending moments due to nonlinear slamming are obtained using a modal model of the ship structural dynamic representations, together with time histories of the wave-induced vertical bending moment due to the same wave system. The capabilities of various computer systems to obtain the required responses, the form of the mathematical model appropriate for computational means, and the time requirements for carrying out the operations are also presented. The rapid assessment of spectral responses and their related statistical properties by means of digital computation, together with time history responses at rates faster than real time, provides a useful tool for determining many aspects of ship structural response characteristics by means of computer simulation.

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