SSC-215

A GUIDE FOR THE SYNTHESIS OF SHIP STRUCTURES
PART ONE
THE MIDSHIP HOLD OF A TRANSVERSELY-FRAMED DRY CARGO SHIP

SHIP STRUCTURE COMMITTEE

1970

SHIP STRUCTURE COMMITTEE

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ADDRESS CORRESPONDENCE TO:

SECRETARY
SHIP STRUCTURE COMMITTEE
U.S. COAST GUARD HEADQUARTERS
WASHINGTON, D.C. 20591

1970

Dear Sir:

The Ship Structure Committee has undertaken a series of research projects to develop analytical methods and computer programs which will apply modern high speed electronic computational techniques to ship hull structures.

Reported herein is the result of one of these projects, concerned with the midship section of a transversely framed ship.

Sincerely,

W. F. REA, III
Rear Admiral, U.S. Coast Guard
Chairman, Ship Structure Committee

SSC-215
Summary Report

on

Project SR-175, "Rational Ship Structural Design"

to the

Ship Structure Committee

A GUIDE FOR THE SYNTHESIS OF SHIP STRUCTURES

PART ONE

THE MIDSHIP HOLD OF A TRANSVERSELY-FRAMED DRY CARGO SHIP

by

Manley St. Denis
National Engineering Science Company

under
Department of the Navy
Naval Ship Engineering Center
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This document has been approved for public release and sale; its distribution is unlimited.

U. S. Coast Guard Headquarters
Washington, D.C.
1970

ABSTRACT

This report presents the design synthesis for a digital computer program that has been developed, based on the application of rational techniques, for the design of the optimum midship structure of a transversely-framed dry cargo ship. The merit of the design method used is that all empirical knowledge on the proportioning of hull structure to withstand the forces of the seaway finds expression in three factors; namely: separation of structure into primary, secondary and tertiary components.

The program is subject to the following qualifications: a) external loadings and wave induced bending moment must be entered as input data; b) design criteria are arbitrary and based solely on the overall strength of the hull; c) stress intensities under distributed loadings do not exceed the elastic limit of the material; d) the ship steams upright in head or following seas; c) impulsive loading from slamming is not taken into account explicitely, nor are stress concentrations, strength under localized loading, rigidity and corrosion allowances, *inter alia*.

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FOREWORD

This report is the first of what is intended to be a series of studies under a "Rational Ship Structural Design" program. The objective of this program is to develop a progressively more comprehensive and more rationally based computer code to design the entire structure of a dry cargo vessel. It is the present plan that the eventual result of this structural design program will mate with an ongoing "Ship Computer Response" program which will provide the proper hydrodynamic loading information. These will then give the maritime industry a single integrated program based wholly on rational techniques for structural synthesis after rational derivation of the loads to be expected. As a further step in this direction, a project is now underway to develop a similar code to analyze the structure of a longitudinally framed ship. The limitation of this report to transversely framed ships was particularly influenced by the availability within the Ship Structure Committee overall program of this type of ship stress data covering many years of service at sea. These data can thus be compared with the findings of the developed computer code.

The accompanying report, therefore, is only a first step and, as is pointed out in the report, the code does not incorporate all of the considerations, methods, procedures and constraints that will ultimately be required. The strength criteria, let alone the optimization criterion or criteria, while not completely arbitrary are obviously still far from fully determined. It was also necessary to make other compromises with empiricism to keep the size of the program within reason. Because of these recognized limitations, which will gradually be removed with further work, direct application of the code as presented in the report to the *Wolverine State* did not demonstrate a weight saving or appreciable change in scantlings over the actual ship. We do, however, believe that the approach proposed in this report is valid and merits this publication and circulation. Work toward the overall long-range objective is continuing as it is believed to hold much promise of appreciable eventual benefit to ship design practice in particular and to the industry in general.

Prof. R. A. Yagle,
Chairman, Ship Research Committee

July 1, 1970

1. INTRODUCTION

1.1 PURPOSE OF THE STUDY

The purpose of this study is to develop a method by which the structure of a transversely-framed dry cargo ship can be designed by the application of rational techniques so as to achieve optimization.

1.2 SCOPE

The scope of the study is restricted to the simple basic design given by the S. S. WOLVERINE STATE. Inputs to the study are:

a) The geometry of the hull and the internal disposition of decks and hatchways.

b) The time and space distribution history of the hydrodynamic loading.

c) The internal loading.

The desired result is the scantlings and disposition of structural material in the midship region of the ship so as to achieve an optimum structure. By optimum is meant, in this context, either:

a) A structure of minimum weight, or

b) A structure of minimum cost.

1.3 BACKGROUND

The structure of the merchantman of today is designed on the basis of a set of empirical rules which represent the judicious interpretation of the experience gained in the past with ships of similar construction and principal characteristics. To be sure, wisely interpreted experience is an excellent guide for the execution of projects, particularly in shipbuilding, where the loading to which a hull is subject and the hull structure itself are both so complex as to make the application of systematic rational procedures for the determination of scantlings an arduous and uncertain task. This in itself explains why in the past only the simplest rationalization has been introduced in procedures of ship structural design, a rationalization sufficient to provide an unsophisticated pattern against which to examine and interpret the experience slowly acquired over the years and at great expense.

The rules of design that resulted from the empirical evaluation of past hull syntheses, and which today govern the selection of hull scantlings, are no more than indices by which to compare one ship against the fleet of others that have seen satisfactory service. But design by comparison is design by hindsight and, although it would be folly to discard the lessons learned from the experience of yesteryears, it is a fair statement that the slowest way to advance is by looking backwards. Also, design by comparison rests on the assumption that subtle relationships between various scantlings remain preserved, though what these relationships are is never made explicit, so that as one departs from strict identity (both overall and specific), confidence suffers.

It was a century ago that Scott-Russell (1862) made the following philosophical observation: "Progress recently made in the art of building ships of war has illustrated curiously the tendency of the human mind to imitation merely of what has gone before, even when the conditions and materials of art have undergone a revolution." But few would disagree over the validity of the same observation transposed forward in time by five score and five years.

In fairness to the designers of the past, it should be observed that rationality, no matter how sophisticated, must have its roots in observation, hence, in experience. In the case of ships, the lapse of years from the inception of a conceptual design until the time the completed vessel has had a statistically significant number of voyages in heavy weather is great so that the accumulation of technical wisdom in the field of shipbuilding is a process whose dominant characteristic is an obdurate slowness. The corollary is that the evolution of ship design methods toward rationality is an unhurried adventure.

If it is recognized that the structure of a ship is the most complex of all engineering structures built to date and that the dynamic loading to which it is subject during its lifetime is highly unpredictable, it becomes easy to see why simple empirical rules must fail to suffice for its efficient design.

The alternative of pursuing wholly rational methods of design has not gained great momentum. To be sure, the literature bearing on the analysis of highly redundant ship structural elements is fairly exhaustive and libraries of computer programs are being compiled for the rapid execution of such analyses. But the analyses cannot be applied unless a design is in hand and the results they provide apply only to the geometric proportions of the basic design. Systematic procedures for designing, i. e., synthesizing hull structures are rare and a computer program for the rational synthesis of such structures does not exist at present.

Design synthesis implies to the execution of a procedure that will yield a solution fulfilling two conditions:

a) Insurance that no structural member is stressed or strained above its critical value.

b) Optimization of the structure with reference to a prescribed criterion.

It does not appear feasible to devise a direct or closed form solution which will insure the equitable proportioning of all scantlings in such manner that these conditions will be fulfilled. The alternative is to formulate an iterative solution which will converge to the desired design. Such a solution has not been pursued in the past because the amount of work required would have resulted in a manual task of prohibitive magnitude. However, the advent of computers is providing a golden means for the objective, hence necessarily sophisticated, analyses of ship structure and for the rapid synthesis of the structural elements of which the ship is compounded into an optimal entity.

1.4 PHILOSOPHY OF SOLUTION

The method of design to be developed has as point of initiation, the full knowledge of the hydrodynamic and inertial loadings that act on the hull structure. These loadings are obtained as outputs from the parallel study on ship response.

A synthesis implies that the hull scantlings derived must satisfy some optimum criterion. Because of the complexity and the high degree of redundancy of the hull structure, the problem becomes that of finding a multi-dimensional optimum.

The process for arriving at the optimum design of a hull structure consists in the following steps:

a) Definition of the hull geometry.

b) Mathematical representation of the structural assembly.

c) Formulation of design control criteria.

d) Formulation of design constraints.

e) Application of optimization technique.

f) Application of optimization criterion (or criteria).

The synthesis is for a specific hull geometry and changes are not considered in the geometric outline of the hull and in the internal arrangement of deck's innerbottoms, bulkheads and hatchways. These items are determined by other analyses and will be treated as invariant in the study. The only freedom allowed the ship structure synthesist is in the choice of spacing of frames and longitudinal girders. Thus, frame and longitudinal spacings are the basic design parameters.
To be introduced into the optimization process, the hull structure must be suitably modeled in mathematical form. It is in this step that the hull structure is represented by mathematical abstractions that bring out the essential relationships that exist mutually between the various components and between the structure itself and the loading. The complexity of the structure is such that, if one is not to be lost in the sterile pursuit of trivia, simplifications must be made. It is at this point that judgment must be exercised so what is of the essence is retained and what is inconsequential is discarded.

An important point is to be made in this connection: The essential item to be developed is the logical structure of the process. However, the mathematical models used to determine the scantlings of the various structural items, and which enter into the process in specific subroutines, can be changed without affecting the overall logic. Thus, as better models are found, they can be included in the process by simple substitution.

The design control criteria are set up to insure the structural sufficiency of the design.

The design constraints define the allowable choices in scantlings (plate widths and thicknesses, stiffener types and sizes, stanchion diameters, etc.) to suit mill standards. They also establish minimum frame and longitudinal spacings.

The optimization criterion provides the basis of selection of a design. As mentioned under scope, two optimization criteria are introduced: minimum weights and minimum cost. Since the synthesis relates only to the midship length of the ship, the optimization criteria become minimum weight and cost per unit length.

The amount of work required to obtain a solution depends critically on the optimization technique employed, hence, it is of paramount importance that the most efficient one be selected.

1.5 DESIGN PHILOSOPHY

The first and most important step is that of the design philosophy to be pursued for analyzing and synthesizing the hull structure. Almost without exception past methods of rational analysis of ship structure have been erected on the basic assumptions that:

a) The material is perfectly elastic and obeys Hooke's law of proportionality of stress and strain. The assumption of perfect elasticity has the effect of limiting the applicability of the analyses to such cases of loading for which the stress intensity at no point of the structure exceeds the yield point of the material. The assumption of the validity of Hooke's law implies that the principle of superposition applies. For shipbuilding steel this assumption is one of opportunity and convenience, not of reality.

b) The dynamic loading is slowly applied. Thus, the response of structural component to a dynamically applied load is assumed to be the same as that induced by a static load whose magnitude is equal to the peak value of the dynamic load. Otherwise stated, the dynamic load factor is unity.

Some remarks on these basic assumptions are pertinent.

So long as a structure is determinate, there is some validity to a method of design that inquires no further than the yield point of the material or the point of elastic buckling of critical members. Proportioning of a tensile member by relating the maximum expected load it is to carry to the yield strength of the material is justifiable because the margin of safety against its failure under load is simply determinable inasmuch as, for any grade of material, the ultimate strength bears a direct relation to the yield strength. Justification for proportioning a compressive member by substituting its elastic buckling strength for the yield strength employed above is similarly made, although the argument is now complicated by the geometry of the structure, and a simple relation between elastic buckling and plastic buckling strengths no longer obtains.

When the structure is redundant, such simple logic no longer holds. The margin of strength remaining above the yield point of the material or elastic buckling point of the structure depends in an essential manner on the degree of structural redundancy incorporated in the design. No simple rules can be formulated.

What appears to be the proper logic by which to design a hull structure is to proportion the scantlings so that the structural complex will have a collapse, or limit, load of a prescribed level above the maximum expected dynamic load. In other words, the criterion of strength adequacy is not based on yield or elastic buckling strength but on plastic collapse load. There is, unfortunately, a serious drawback to the use of such criterion: No adequate and manageable theory of limit design is presently in hand for use in the analysis of the highly redundant, complex assembly which is the hull structure of a ship.

At the present state of the art, one is left with the single choice of designing in accordance with the linear, or linearized, theory of elasticity and with the theory of elastic stability.

However, to safeguard oneself against disaster, one specifies that the steel to be used shall have a certain amount of ductility. (24 percent in 2 in for ordinary strength steel and 22 percent in 2 in for high strength steel). This ductility does not enter into the elastic analysis. It is introduced to account, in a subtle way, for those changes in the stress field introduced by the rolling, cutting and welding of the material; by the changes in temperature, loading and constraints during the period of fabrication and erection; and by the changes in the support reactions during building, launching and docking.

One is, evidently, justified in asking whether under this condition a design based on elastic theory can have any claim to being valid and useful. It is difficult to formulate a satisfactory reply. However, the following arguments can be adduced in support of a design by elastic theory, namely, that the real factor of safety of a redundant structure is higher than the apparent one calculated on the basis of elastic theory.

The loading acting on the hull structure is either static or dynamic. The static loading is introduced by the weight of the structure itself, machinery, equipment, cargo, consumables, etc.; and by the static buoyancy of the displaced water. The dynamic loading is either hydrodynamic or inertial. Inertial loading is induced by the motions of the hull in a seaway. Hydrodynamic loading is imposed by the interplay of hull and waves. Hydrodynamic loading can be classified as quasi-static or wave frequency and as impulsive. The first gives rise to the rigid body oscillations of the ship and to an elastic response of the hull structure varying with the frequency of the encountered waves, i. e., relatively slowly. The second is suddenly applied and of short duration - it generates a vibratory response of the hull structure and of its component parts.

The loading is not to be determined as part of the task: the program to be developed will be in terms of a nominal or general loading. But this categorization of loading bears emphasis because of the additional work of analysis that is associated with the impulsive loading. Whereas, the wave frequency loading can be treated as if it were statically applied (hence the qualification of quasi-static), the impulsive loading must first be reduced to an equivalent static load and to this end, the dynamic load factor must be determined.

Except for the static loading imposed by the weight of fixed items, the loading (both static and dynamic) can be discussed only in terms of certain statistical averages. The problem that arises is that of predicting the extreme values of these statistical averages. Again, in the development that follows, the statistical statements on the occurrence of extreme values are assumed to be given.

Acceptance of the linear theory of elasticity as a basis for design synthesis results in a powerful simplification of the process through introduction of the concept of the three types of structure: primary, secondary and tertiary corresponding to the hull girder as a whole, to cross-stiffened panels of plating and to the single plate itself.

This concept of structural classification appears to have been introduced by St. Denis (1954) and an exposition thereof is given in Appendix A.

The structure of the ship is essentially an assembly of plates and stiffeners, but the structural classification introduced makes it possible to analyze the components separately.

Design synthesis is made by application of the design control criteria.

In the midship region the plates are predominantly flat, except, of course, for the strakes in way of the turn of the bilge; they are of rectangular, or quasi-rectangular, shape and are loaded by forces acting in their plane as well as normally thereto.

A general theory for the analysis of flat plates under such complex loading does not appear to exist; however, if plate geometry and conditions of loading are such that the deflection of the center of the plate is less than about one fourth of its thickness, a moderate deflection solution is in hand due to Bengston (1939). These conditions are met for ship plates designed to withstand continuously or frequently applied normal loadings. This occurs with shell, decks and deep tanks. It does not occur with ordinary water-tight bulkheads. The theory of Bengston is outlined in Appendix B.

Some remarks relative to this theory are pertinent. The theory has been developed for idealized boundary conditions (all boundaries fixed or all boundaries simply supported). But in ship structures the boundary conditions are not ideal but intermediate and one is faced with the problem of the determination of the degree of edge fixety and with that of interpolating between solutions for ideal cases to obtain the solution for the specific case in hand.

Interpolation is easily accomplished on the basis of a linear relation connecting the solutions for the two extreme ideal cases provided the degree of fixity at the plate supports is known.

The degree of fixity of the plating at its supports is determined on the basis of the relative structural stiffnesses of plate and supporting stiffeners. The method is discussed in Appendix C.

An item of importance in the analysis of plating either acting as a stiffener or being, in turn, stiffened is the amount of material that can be considered effective for calculations of strength. A distinction is made between the case in which the stiffened plating is under normal load and that in which the plating is subjected to a compressive load in its own plane. The latter case is a problem of elastic stability alone, whereas the former is somewhat more complex and exists even in the absence of any plate instability. To distinguish between the two, one speaks of effective "width" when considering a plate loaded compressively in its plane and refers to as effective "breadth" when the case is that of a normally loaded stiffened plate. Discussion of these two aspects of analysis is contained in Appendix D.

The structure of the ship is essentially an assembly of panels of cross-stiffened plating. Three basic techniques have been developed for determining the elastic stress intensities in such panels:

a) Orthotropic plate (Huber-Schade)

b) Grillage: Finite element (Wah)

c) Grillage: Beam on elastic foundation (Schilling, Vedeler, Michelsen, Nielsen, Chang)

Since, a choice between these techniques needs be made, some general observations are introduced for guidance.

The orthotropic plate technique (or application of a theory) is the simplest of the three alternatives. The fundamental concept that a panel of cross-stiffened plating can be considered as an orthotropic plate is valid provided the stiffeners are fairly regular in each direction and closely spaced.

At the present, the orthotropic plate theory has been applied only to flat panels of idealized boundary conditions. The theory does not provide a way to estimate the degree of fixity at the panel supports. Also, the presence of stanchions and of irregularities in the scantlings and disposition of structure complicates the analysis severely.

The grillage methods are based on the philosophy that a system involving a discrete number of stiffeners should be analyzed by methods that take this discreteness into account fundamentally. In the finite element method, both frames and longitudinal girders are modeled as a net of structural elements of individual length spanning from one intersection to the next. In the beam-on-elastic foundation method, the closely spaced frames are assumed to distribute their action on the longitudinal girders over the full frame spacing. This simple artifice results in a strong simplification of the analysis.

These methods are fully applicable to three dimensional structure, regular or irregular, continuous or discontinuous. The several panels of cross-stiffened plating forming the structure of a ship can be connected together by slope-deflection techniques with the result that the boundary conditions at the panel supports are obtained as part of this solution.

To apply grillage techniques, influence coefficients must first be obtained. The limitations of grillage solutions depend on the ability to calculate influence coefficients. But it is observed that such ability transcends that of calculating stress intensities.

Another prerequisite to the application of grillage techniques is knowledge of the effective breadth to be used. This effective breadth is not determined as part of the grillage analysis and must be obtained independently. This is the weakness of the grillage technique.

Of the grillage techniques, the beam-on-elastic-foundation approach is more expeditious. The reason for this is that in a finite-element approach, all deflections at the intersections must be determined, whereas, in the beam-on-elastic-foundation approach, penalty is incurred only to the extent that frames depart from regularity. Since in actual ships, frames tend to be regular with but few exceptions, the advantage of the method is quite powerful. Note also that no degradation in accuracy results from the assumption of distributed frame reaction. Some recent work of Chang (1967), which gives comparisons of results obtained by the beam-on-elastic-foundation method with parallel results obtained by finite element and orthotropic plate theories, provides emphatic support to this statement.

Grillage analysis of the secondary structure is fairly complex since it involves the integration of a moderate number of separate calculations. This in turn, involves the development of a method for indexing the various elements. A simplified index system is presented in Appendix E. While Appendix F contains an overview of the grillage analysis, and Appendix G shows specific application to the S. S. WOLVERINE STATE.

2. OUTLINE OF THE PROCESS

The overall logic of the process is outlined in the sequence of steps that follows. The flow of logic is presented in Fig. 1.

Step A: Basic Inputs

These are the inputs that remain unaltered throughout the whole process. They relate to:

a) The geometry of the hull (hull envelope, arrangement of decks and inner bottom, bulkheads, hatchways, tankage, etc.)

b) The loading (both internal and external, both static and dynamic) to which the hull is subject.

c) The design constraints that must be observed in determining scantlings.

Basic inputs are discussed in Appendix H.

A structural item may be of one of three types:

a) Plating subject to normal loading, hence watertight or oil-tight.

b) Plating not subject to normal loading, hence non-tight.

c) Shape

Each structural item is designated accordingly.

Step B: Design Parameters

The design parameters are those design variables that characterize the design. They consist of:

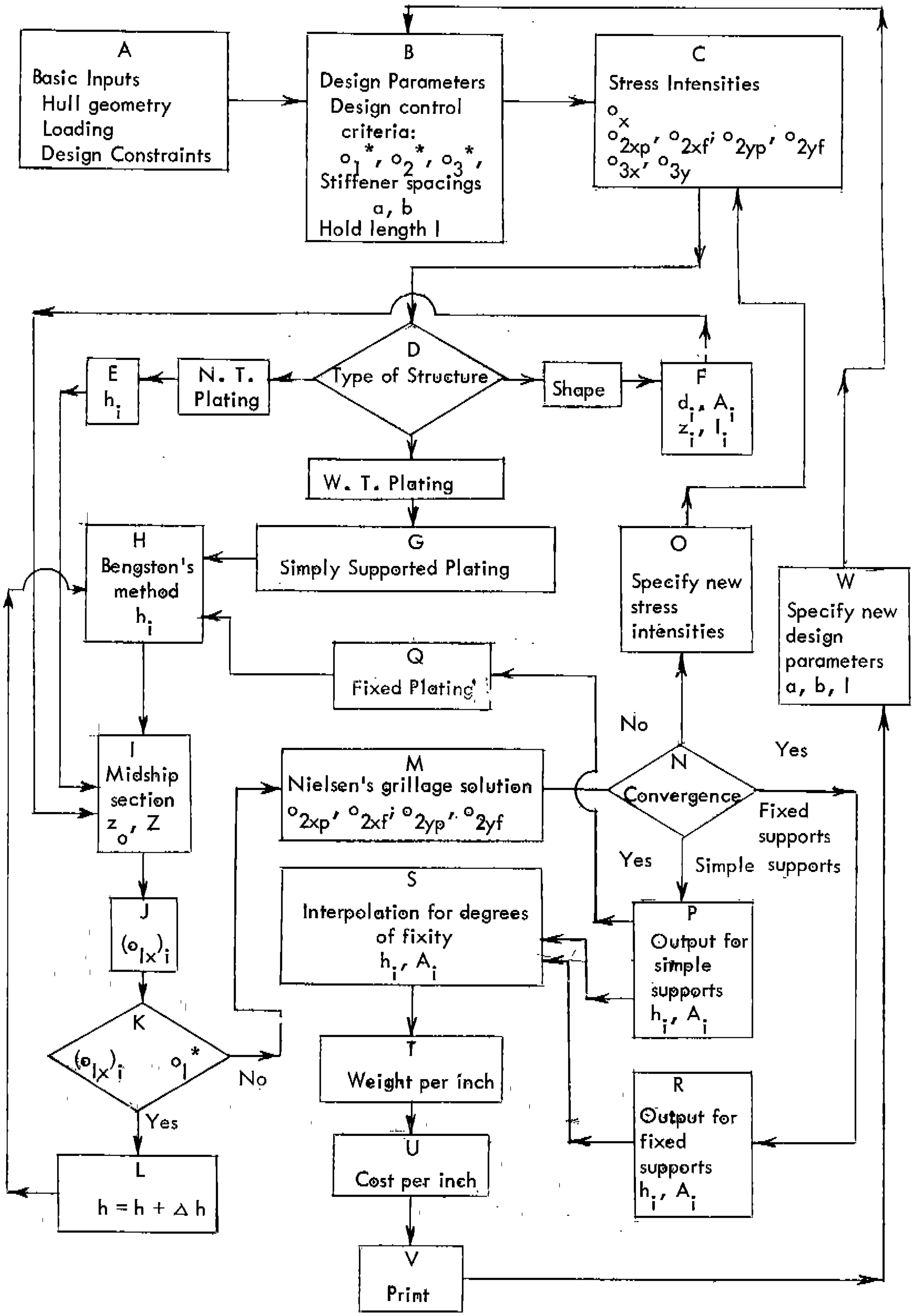

Fig. 1. Logic Flow for the Process

a) The criteria of design control, which state the maximum allowable primary, secondary and tertiary stress intensities (σ_1^*, σ_2^*, σ_3^*).

b) The spacing of frames (a) and of longitudinal girders (b). These are subject to systematic variation during the process. Criteria of design control are discussed in Appendix I.

Step C: Stress Intensities

This step specifies the primary, secondary and tertiary stress intensities (σ_{1x}, σ_{2x}, σ_{2y}, σ_{3x}, σ_{3y}) in each item. (In the case of freely standing shapes attached to plating, the secondary stress intensities in both plate and flange must be taken into account). In these symbols, x denotes the longitudinal, y the transverse coordinate.

To begin the calculations, the neutral axis is located from empirical data, see Appendix J. Also, the maximum allowable, or criterion primary stress intensity is assumed to obtain at either keel or deck in harmony with the location of the neutral axis. The maximum tertiary stress intensities (σ_{3x}, σ_{3y}) are determined for each item on the basis that the secondary stress intensities attain their maximum allowable, or criterion, values (σ_{2x}^*, σ_{2y}^*).

When secondary structure is composed of a combination of plate and freely-standing stiffener, the secondary stress intensity in the plating is minimal and is taken into account by an arbitrarily small allowance.

On successive iterations, when values of primary, secondary and tertiary stress intensities are available for each item from the previous solution, these are made to replace the initially assumed ones.

Step D: Classification of Structural Items

Since the method of calculation to be employed in determining the scantlings of the various items depends on their type, the items are so separated at this step.

If the item is a non-tight floor or double bottom longitudinal girder, the program goes to Step E; if a transverse or longitudinal shape, it goes to Step F; and if water (or oil) tight plating, then to Step G.

Step E: Thickness of Non-tight Plating

The non-tight longitudinal girders in the double bottom are proportioned to insure that their critical strength in compression exceeds by a specified margin the design level of primary stress intensity.

The non-tight floors are designed to carry the normal loading and have adequate margin of strength in shear, this strength being related both to the yield point of the material and to the critical buckling stress intensity (see Appendix L).

Step F: Scantlings of Shapes

The required section moduli of frames, deck beams and deck longitudinals are determined from the acting bending moments and the allowable stress intensities. The scantlings of the shapes are then obtained on the basis that they are T's cut from wide flange sections (empirical curves being used for convenience in relating section modulus to depth and cross sectional area of shape). These are discussed in Appendix J.

Steps G and Q: Assumption of Boundary Conditions for Watertight Plating

Determination of the thickness of plating is first made on the basis of idealized boundary conditions (simple-support and fixed). This step controls the boundary condition to be used. In the case of the double bottom, the solution is carried out for both ideal conditions, the interpolation between the two being made in Step S. For the rest of the structure, the plating is assumed to be simply supported except for "tween decks when laden with cargo (full load condition) and for the inner bottom when subjected to a normal pressure equal to the test head for which designed, in which cases, the plating is assumed to be fixed at its supports.

Step H: Watertight Plating

The plating thickness (for idealized boundary conditions) is obtained by application of the method of Bengston, which is outlined in Appendix B.

Step I: Properties of the Midship Section

The nontight longitudinal double bottom girders determined in Step E, the longitudinal hatch girder determined in Step F, and the watertight plating determined in Step H are integrated to form the midship section, and the neutral axis and midship section moduli are then calculated, as shown in Appendix K.

Step J: Distribution of Primary Stress Intensity

The midship section moduli to deck and keel being known, the distribution of primary stress intensity is obtained directly by application of the midship bending moment given in the statement of inputs.

Steps K and L: Criterion of Primary Stress Intensity

If the primary stress intensity in the bottom or main deck plating exceeds the corresponding criterion stress intensity ($\sigma_{1x} > \sigma^{*}_{1x}$), the plating is arbitrarily increased in thickness by a small amount in Step L. Following this, the process returns to Step H and the tertiary stress intensities are recalculated. This loop is repeated until the maximum value of primary stress intensity falls short or, at most, equals the criterion value.

If or when the maximum calculated value of primary stress intensity satisfies the criterion, the process continues to Step M.

Step M: Grillage Solution

Application of the grillage solution yields the secondary stress intensities in the structure (σ_{2xp}, σ_{2xf}, σ_{2yp}, σ_{2yf}; where the second subscripts p and f denote respectively plate and free flange). These stress intensities are for the scantlings derived in Steps E, F, and H.

Step N: Criterion of Convergence

If the maximum primary and secondary stress intensities are equal to those of Step C within a prescribed tolerance, the solution has been obtained and the program goes to Step P or R, otherwise, the program goes to Step O.

Step O: Specification of Maximum Stress Intensities

The primary stress intensities obtained in Step J and secondary stress intensities obtained in Step M and the resulting tertiary stress intensities of Step H are specified as the new set of stress intensities for the next iteration and the program returns to Step C.

Steps P and R: Output for Idealized Boundary Conditions

The scantlings resulting from the assumption of simply-supported plate boundaries are first obtained after which the program goes to Step O and the calculations are repeated for fixed plate boundaries, the outcome being reached in Step R. This calculation is initiated with the last set stress intensities obtained in Step C for the boundary conditions of simple support.

Step S: Interpolation for Degree of Fixity

The degree of fixity of the bottom shell plating at the supports is determined by the method of Appendix C. A linear interpolation is then made between the results obtained in Steps P and R.

Step T: Weight Intensity of Hull Structure

The structure having been determined corresponding to a choice of design parameters, its weight per inch of length results directly. This is discussed in Appendix M.

Step U: Cost Per Running Inch of Hull Structure

This cost is made up from the cost of material, that of fabrication and erection and that of welding. The first results directly from Step T; plausible empirical formulae are employed to assess the second; the last is simply derived provided the spacing of seams and butts is known or assumed. Weld material is considered in Appendix N, and cost of hull structure is taken up in Appendix O.

Step V: Documentation

The following results are listed for each combination of design parameters selected:

a) Thickness of plating and size of shapes.

b) Primary, secondary and tertiary stress intensities in each structural item.

c) Stress ratios corresponding to the stress factors f_1, f_2 and f_3.

d) Weight of material per inch of hull length.

e) Cost of material per inch of hull length.

Step W: Specification of Design Parameters

Having completed the calculations for one set of frame and longitudinal spacings on the basis of a selected set of design control criteria, the calculations are repeated for an additional set of spacings sufficient to cover a reasonably wide field of possibilities.

Step X: Optimization

For a given choice of design control criteria, the design parameters open to choice are the frame and longitudinal spacings. Thus, a simple optimization technique is sufficient to the purpose. This consists of plots of weight and cost per running inch of structure against the independent variables of frame and longitudinal spacing.

3. PROGRAM

3.1 GENERAL REMARKS

Computer runs were made with the code developed in this report for the midship hold of a ship having the basic geometry of the SS WOLVERINE STATE, the main and structural characteristics of which are presented in Tables 1 and 2 and in Figure 2. The principal variables are the geometric parameters of:

Frame spacing, a.
Spacing of longitudinal girders, b.
Length of hold, ℓ.
The criterion of primary stress intensity, σ_1^*, and
The parameter of elasticity, E.

The number of deck longitudinals was kept constant and the same for all decks throughout the series: each deck was supported by a single longitudinal girder located a hatch half-width away on each side of the vertical centerplane. The number of bottom longitudinals (n) was varied from a single center-keel (n = 1). to a center-keel and single side keelson on each side (n = 3), to a center-keel and two side keelsons on each side (n = 5). For convenience of reference, the number of bottom longitudinals (n) is preferable to their spacing (b).

Two values of the criterion of stress intensity were introduced:

$$\sigma_1^* = 19,000 \text{ lb. in}^{-2} \quad \text{and} \quad \sigma_1^* = 9,500 \text{ lb. in}^{-2},$$

the former being what is considered a normal value, the second being a drastic departure therefrom.

Three values of the parameter of elasticity were tried: 30, 20, and 10 million lb. in^{-2}. The first is that obtaining for steel, the last is somewhat lower than that of aluminum. Computer runs were made for the combination of parameters given in Table 3.

Table I
Wolverine State - Particulars

Type	C4-S-B5 (machinery aft dry cargo vessel)
Length, overall (ft)	520
Length, between perpendiculars (ft)	496
Beam, molded (ft)	71.5
Depth, molded (ft)	43.5

Condition	Design	Light Operating
Draft, molded (ft)	30.0	18.0
Displacement (tons)	20,000	11,130
Block Coefficient	0.654	0.610
Longitudinal Coefficient	0.664	0.628
Waterplane Coefficient	0.752	0.685

Machinery - Two Stage Turbine

Design power (hp)	9000
Normal propeller speed (rpm)	80 to 85
Normal operating speed (knots)	16 to 17

Builder: Sun Shipbuilding & Dry Dock Co., Chester, Pennsylvania
Owner: States Marine Lines

Table II
Scantlings of the *S.S. Wolverine State* as built

Midship frame spacing (in)	30
Number of double bottom longitudinals	5
Shell Plating	
Bottom shell (in)	0.78
Bilge strake (in)	0.78
Side shell (in)	0.72
Sheer strake (in)	0.91
Deck Plating	
Inner bottom (in)	0.56
Fourth deck (in)	0.31
Third deck (in)	0.31
Second deck (in)	0.44
Main deck (in)	1.06
Longitudinals	
Double bottom, oil-tight (in)	0.53
Double bottom, non-tight (in)	0.41
Fourth deck (in^2)	83.9
Third deck (in^2)	83.9
Second deck (in^2)	83.9
Main deck (in^2)	83.9
Transverse Stiffeners	
Floors, oil-tight (in)	0.53
Floors, non-tight (in)	0.41
Frames, above inner bottom (in^2)	6.73
Frames, above fourth deck (in^2)	6.73
Frames, above third deck (in^2)	5.74
Frames, above second deck (in^2)	5.74
Deck Beams (in^2)	8.75

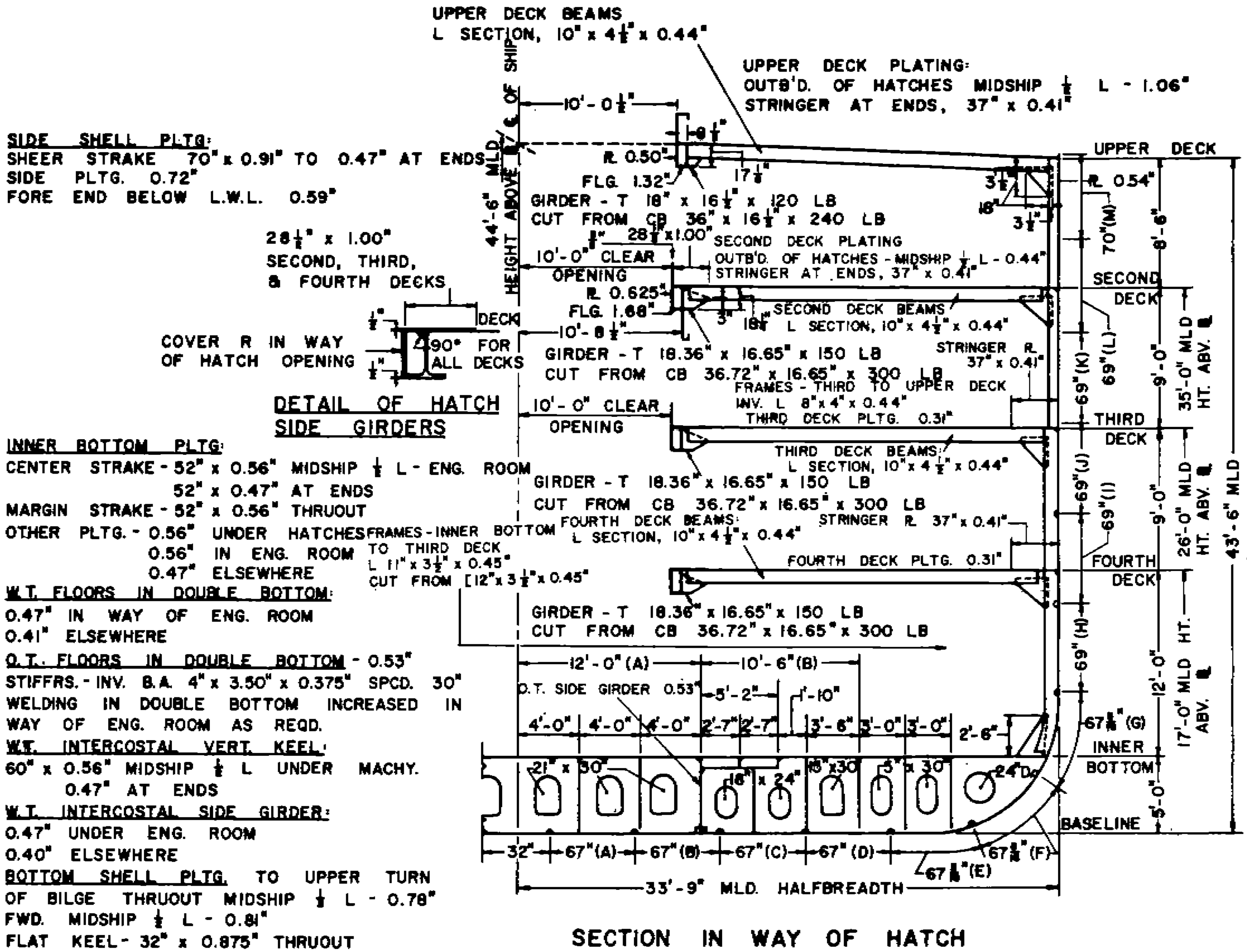

Fig. 2. *S.S. Wolverine State* - Scantlings as Built

Table III
Production Runs

Run	Frame Spacing	Number of Double Bottom Longitudinals	Length of Hold	Primary Stress Intensity Criterion	Modulus of Elasticity
	a (in)	n	l (ft)	σ_1^* (lb in^{-2})	E (10^6lb in^{-2})
1	30	3	60	19,000	30
2	20	3	60	19,000	30
3	40	3	60	19,000	30
4	30	5	60	19,000	30
5	30	3	60	9,500	30
6	30	3	40	19,000	30
7	30	3	80	19,000	30
8	30	1	60	19,000	30
9	20	3	60	9,500	30
10	40	3	60	9,500	30
11	30	3	60	19,000	20
12	30	3	60	19,000	10

3.2 ASSUMED INPUTS

The following inputs have been assumed to define the material, the loading, the design criteria and cost factors:

Materials

In addition to the modulus of elasticity which has been entered as a parameter, the following values have been used to describe the material properties:

Yield Strength, σ_{yp} = 35,000 (lb in^{-2})

Poisson's ratio, μ = 0.3

Loading

The external pressure head is given by

$$H \pm 0.4\,h - z \text{ (ft)}$$

where:

$H \equiv$ ship's draft (ft)

$h \equiv$ wave height (ft)

$z \equiv$ vertical coordinate measured positive upwards (ft) and where the plus sign is used for hog and the minus sign for sag. For the specific ship data, the external pressure head at the keel (H = 30 ft, z = 0 ft) amounts to 39.92 (ft) in hog and 20.08 (ft) in sag.

In hog, the above formula is complemented by the condition that the pressure head be at least 4.0 ft. In the hogging condition the external head on the bottom plating amounts to 17.76 lb in^{-2}. The internal loading on the decks has been taken at 2.1 lb in^{-2}, which corresponds to 7.5 ft head of cargo of a 40 lb ft^3 density. The hydrostatic head on the inner bottom has been taken equal to that caused by the external wave loading (15.54 lb in^{-2}). The interplay of external loading and internal masses results in maximum midship bending moments given by

$$M = k \rho g L^3 B (h/\lambda) \quad \text{(ft lb)}$$

where:

$\rho \equiv$ mass density of water (lb sec^2 ft^{-1})

$g \equiv$ gravitational acceleration (ft sec^2)

$L \equiv$ ship length (ft)

$B \equiv$ ship beam (ft)

$\lambda \equiv$ wave length (ft)

$k \equiv$ empirical constant which depends on the fullness of the ship and on the wave steepness

For the WOLVERINE STATE the value of

$$k = 0.0145$$

has been used for both hog and sag. This value is plausible but arbitrary. Application of the specific ship data yields a maximum hogging moment of 4.05×10^8 ft lb.

Cost

Cost factors have been assumed as follows:

Material cost	\$ 0.065 lb^{-1}
Weld cost	\$ 3.00 lb^{-1}
Labor rate	\$ 4.00 hr^{-1}

3.3 DESIGN CRITERIA

The stress criteria are given in terms of the yield strength of the material, namely:

$$\sigma_1^* = f_1 \sigma_{yp} \quad (\text{lb in}^{-2})$$
$$(\sigma_1 + \sigma_2)^* = f_2 \sigma_{yp} \quad (\text{lb in}^{-2})$$
$$(\sigma_1 + \sigma_2 + \sigma_3)^* = f_3 \sigma_{yp} \quad (\text{lb in}^{-2})$$
$$\tau_1^* = 0.6 f_1 \sigma_{yp} \quad (\text{lb in}^{-2})$$
$$(\tau_1 + \tau_2)^* = 0.6 f_2 \sigma_{yp} \quad (\text{lb in}^{-2})$$

where σ_1, σ_2, σ_3 denote respectively primary, secondary and tertiary normal stress intensities, while τ_1 and τ_2 denote primary and secondary shear stress intensities, where the asterisk denotes a criterion value and f_1, f_2, f_3 are stress intensity factors. These have been assigned the following arbitrary values:

$$f_1 = 0.56 \text{ and } 0.28$$
$$f_2 = 0.80$$
$$f_3 = 1.00$$

4. RESULTS

The results are presented in Tables 4 thru 10 and in Figures 3 thru 10. In making the comparison with the scantlings of the WOLVERINE STATE as built, two main points should be borne in mind:

a) The calculated scantlings are based on an arbitrary choice of external and internal loadings and wave-induced bending moments.

b) The calculated scantlings are based on an arbitrary choice of design criteria relating to the allowable primary, secondary and tertiary stress intensities.

c) The cost data are based on plausible but arbitrary cost formulations.

In addition, the following secondary points should be recognized:

a) The code calculates the thickness of side plating on the basis that the thickness changes at the level of decks and inner bottom.

b) Shapes are determined on the basis that they are T's cut from wide flange sections and bracketed at both ends.

c) Deck longitudinals, i. e., hatch girders, are sized for hull strength alone. In the WOLVERINE STATE as built, they appear to have been given scantlings determined upon other bases, such as rigidity.

Also note that:

a) No account has been taken of small items, such as brackets, flat bar stiffeners, access and drain openings in calculating the weight of the structure.

b) Cost data are valid only for comparing the results obtained with the production runs reported herein.

Table IV
Rationally-derived Scantlings
for Parametric Variations in
Frame Spacing

Run	1	2	3
Frame spacing (in)	30	20	40
Longitudinal girder spacing (in)	214.5	214.5	214.5
Number of longitudinals in double bottom	3	3	3
Length of hold (in)	720	720	720
Primary stress criterion (lb in^{-2})	19,000	19,000	19,000
Young's modulus (lb in^{-2})	$3x10^7$	$3x10^7$	$3x10^7$
Shell Plating			
Bottom shell (in)	0.92	0.74	1.12
Bilge strake (in)	0.92	0.74	1.12
Side shell above inner bottom (in)	0.51	0.35	0.66
Side shell above fourth deck (in)	0.40	0.29	0.48
Side shell above third deck (in)	0.50	0.36	0.60
Side shell above second deck (in)	0.58	0.43	0.70
Deck Plating			
Inner bottom (in)	0.81	0.62	1.00
Fourth deck (in)	0.52	0.43	0.60
Third deck (in)	0.60	0.45	0.73
Second deck (in)	0.76	0.61	0.71
Main deck (in)	0.85	0.60	1.02
Longitudinal Girders			
Double bottom, oil-tight (in)	0.67	0.51	0.78
Double bottom, non-tight (in)	0.48	0.40	0.52
Fourth deck (in^2)	19.4	22.8	16.4
Third deck (in^2)	21.2	25.1	17.4
Second deck (in^2)	23.9	28.8	18.7
Main deck (in^2)	27.8	34.2	20.4
Transverse Stiffeners			
Floors, oil-tight (in)	1.04	1.04	1.04
Floors, non-tight (in)	0.37	0.32	0.40
Frames, above inner bottom (in^2)	6.05	4.75	7.14
Frames, above fourth deck (in^2)	4.57	3.67	5.47
Frames, above third deck (in^2)	3.68	2.97	4.30
Frames, above second deck (in^2)	4.21	3.42	4.99
Fourth deck beams (in^2)	3.33	2.66	3.88
Third deck beams (in^2)	3.38	2.60	3.92
Second deck beams (in^2)	3.27	2.60	3.79
Main deck beams (in^2)	3.43	2.77	3.99

Table V
Rationally-derived Scantlings
For Parametric Variations in
Number of Double-bottom Longitudinal Girders

Run	1	4	8
Frame spacing (in)	30	30	30
Longitudinal girder spacing (in)	214.5	143	429
Number of longitudinals in double bottom	3	5	1
Length of hold (in)	720	720	720
Primary stress criterion (lb in^{-2})	19,000	19,000	19,000
Young's modulus (lb in^{-2})	$3x10^7$	$3x10^7$	$3x10^7$
Shell Plating			
Bottom shell (in)	0.92	0.90	1.27
Bilge strake (in)	0.92	0.90	1.27
Side shell above inner bottom (in)	0.51	0.50	0.50
Side shell above fourth deck (in)	0.40	0.41	0.47
Side shell above third deck (in)	0.50	0.48	0.55
Side shell above second deck (in)	0.58	0.55	0.67
Deck Plating			
Inner bottom (in)	0.81	0.80	1.03
Fourth deck (in)	0.52	0.50	0.45
Third deck (in)	0.60	0.59	0.58
Second deck (in)	0.76	0.70	0.76
Main deck (in)	0.85	0.78	0.78
Longitudinal Girders			
Double bottom, oil-tight (in)	0.67	0.67	0.76
Double bottom, non-tight (in)	0.48	0.45	0.49
Fourth deck (in^2)	19.4	17.7	13.8
Third deck (in^2)	21.1	18.9	15.0
Second deck (in^2)	23.9	19.4	16.9
Main deck (in^2)	27.8	24.2	19.8
Transverse Stiffeners			
Floors, oil-tight (in)	1.04	1.01	1.06
Floors, non-tight (in)	0.37	0.28	0.45
Frames, above inner bottom (in^2)	6.05	6.10	7.55
Frames, above fourth deck (in^2)	4.57	4.56	7.46
Frames, above third deck (in^2)	3.68	3.68	5.88
Frames, above second deck (in^2)	4.21	4.21	8.41
Fourth deck beams (in^2)	3.33	3.33	6.85
Third deck beams (in^2)	3.38	3.37	6.73
Second deck beams (in^2)	3.27	3.27	6.86
Main deck beams (in^2)	3.43	3.45	6.60

Table VI
Rationally-derived Scantlings
For Parametric Variations in
Length of Hold

Run	1	6	7
Frame spacing (in)	30	30	30
Longitudinal girder spacing (in)	214.5	214.5	214.5
Number of longitudinals in double bottom	3	3	3
Length of hold (in)	720	480	960
Primary stress criterion (lb in^{-2})	19,000	19,000	19,000
Young's modulus (lb in^{-2})	3×10^7	3×10^7	3×10^7
Shell Plating			
Bottom shell (in)	0.92	0.87	0.98
Bilge strake (in)	0.92	0.87	0.98
Side shell above inner bottom (in)	0.51	0.53	0.50
Side shell above fourth deck (in)	0.40	0.42	0.36
Side shell above third deck (in)	0.50	0.57	0.45
Side shell above second deck (in)	0.58	0.77	0.53
Deck Plating			
Inner bottom (in)	0.81	0.76	0.87
Fourth deck (in)	0.52	0.50	0.56
Third deck (in)	0.60	0.60	0.64
Second deck (in)	0.76	0.78	0.76
Main deck (in)	0.85	1.10	0.79
Longitudinal Girders			
Double bottom, oil-tight (in)	0.67	0.62	0.71
Double bottom, non-tight (in)	0.48	0.57	0.45
Fourth deck (in^2)	19.4	9.71	32.2
Third deck (in^2)	21.2	10.8	34.7
Second deck (in^2)	23.9	12.8	38.4
Main deck (in^2)	27.8	16.5	43.4
Transverse Stiffeners			
Floors, oil-tight (in)	1.04	1.04	1.04
Floors, non-tight (in)	0.37	0.37	0.37
Frames, above inner bottom (in^2)	6.05	6.05	6.06
Frames, above fourth deck (in^2)	4.57	4.58	4.55
Frames, above third deck (in^2)	3.68	3.66	3.69
Frames, above second deck (in^2)	4.21	4.17	4.19
Fourth deck beams (in^2)	3.33	3.34	3.32
Third deck beams (in^2)	3.38	3.38	3.37
Second deck beams (in^2)	3.27	3.27	3.27
Main deck beams (in^2)	3.43	3.40	3.45

Table VII
Rationally-derived Scantlings
for Parametric Variations in
Frame Spacing
at Reduced Primary Strength Criterion

Run	5	9	10
Frame spacing (in)	30	20	40
Longitudinal girder spacing (in)	214.5	214.5	214.5
Number of longitudinals in double bottom	3	3	3
Length of hold (in)	720	720	720
Primary stress criterion (lb in^{-2})	9,500	9,500	9,500
Young's modulus (lb in^{-2})	$3 x 10^7$	$3 x 10^7$	$3 x 10^7$
Shell Plating			
Bottom shell (in)	0.91	0.79	1.10
Bilge strake (in)	0.91	0.79	1.10
Side shell above inner bottom (in)	0.51	0.35	0.66
Side shell above fourth deck (in)	0.40	0.29	0.48
Side shell above third deck (in)	0.50	0.37	0.60
Side shell above second deck (in)	0.64	0.54	0.71
Deck Plating			
Inner bottom (in)	0.81	0.62	1.00
Fourth deck (in)	0.52	0.42	0.60
Third deck (in)	0.61	0.46	0.72
Second deck (in)	0.75	0.61	0.89
Main deck (in)	1.15	0.95	1.00
Longitudinal Girders			
Double bottom, oil-tight (in)	0.67	0.51	0.78
Double bottom, non-tight (in)	0.49	0.41	0.51
Fourth deck (in^2)	19.5	22.8	16.6
Third deck (in^2)	21.1	25.3	17.5
Second deck (in^2)	24.0	29.0	18.8
Main deck (in^2)	25.1	29.3	19.6
Transverse Stiffeners			
Floors, oil-tight (in)	1.04	1.04	1.04
Floors, non-tight (in)	0.37	0.32	0.40
Frames, above inner bottom (in^2)	6.05	4.75	7.14
Frames, above fourth deck (in^2)	4.55	3.67	5.47
Frames, above third deck (in^2)	3.68	2.96	4.30
Frames, above second deck (in^2)	4.18	3.42	4.99
Fourth deck beams (in^2)	3.33	2.66	3.88
Third deck beams (in^2)	3.37	2.69	3.92
Second deck beams (in^2)	3.28	2.60	3.79
Main deck beams (in^2)	3.43	2.76	3.99

Table VIII
Rationally-derived Scantlings
for Parametric Variations in
Young's Modulus

Run	1	11	12
Frame spacing (in)	30	30	30
Longitudinal girder spacing (in)	214.5	214.5	214.5
Number of longitudinals in double bottom	3	3	3
Length of hold (in)	720	720	720
Primary stress criterion (lb in^{-2})	19,000	19,000	19,000
Young's modulus (lb in^{-2})	$3x10^7$	$2x10^7$	$1x10^7$
Shell Plating			
Bottom shell (in)	0.92	0.96	1.03
Bilge strake (in)	0.92	0.96	1.03
Side shell above inner bottom (in)	0.51	0.51	0.52
Side shell above fourth deck (in)	0.40	0.37	0.37
Side shell above third deck (in)	0.50	0.52	0.67
Side shell above second deck (in)	0.58	0.67	0.87
Deck Plating			
Inner bottom (in)	0.81	0.85	0.94
Fourth deck (in)	0.52	0.62	0.80
Third deck (in)	0.60	0.68	0.87
Second deck (in)	0.76	0.86	1.02
Main deck (in)	0.85	1.10	1.35
Longitudinal Girders			
Double bottom, oil-tight (in)	0.67	0.71	0.81
Double bottom, non-tight (in)	0.48	0.59	0.81
Fourth deck (in^2)	19.4	19.6	19.7
Third deck (in^2)	21.2	21.0	20.6
Second deck (in^2)	23.9	23.4	22.6
Main deck (in^2)	27.8	27.0	25.5
Transverse Stiffeners			
Floors, oil-tight (in)	1.04	1.03	0.98
Floors, non-tight (in)	0.37	0.42	0.53
Frames, above inner bottom (in^2)	6.05	6.06	6.07
Frames, above fourth deck (in^2)	4.57	4.57	4.55
Frames, above third deck (in^2)	3.68	3.67	3.65
Frames, above second deck (in^2)	4.21	4.19	4.14
Fourth deck beams (in^2)	3.33	3.31	3.27
Third deck beams (in^2)	3.38	3.36	3.31
Second deck beams (in^2)	3.27	3.25	3.19
Main deck beams (in^2)	3.43	3.37	3.27

Table IX
Weight Per Inch of Hull Structure
(all weights in lb)

Run	Plating & Longitudinal Girders	Trans-verse Members	Total
1	1246	336	1582
2	980	422	1402
3	1480	285	1765
4	1344	297	1641
5	1288	337	1625
6	1267	348	1615
7	1306	332	1638
8	1335	406	1741
9	1038	422	1460
10	1464	357	1821
11	1388	357	1745
12	1445	401	1846
WOLVERINE STATE as built	1233	502	1735

Table X
Cost Per Inch of Hull Structure
(all costs in dollars)

Run	Material	Fabrica-tion & Erection	Welding	Total
1	158	1034	67	1259
2	140	1002	49	1191
3	177	1097	89	1363
4	174	1050	67	1291
5	163	1041	71	1275
6	162	1040	73	1275
7	164	1052	66	1282
8	183	1055	75	1313
9	146	1021	55	1222
10	182	1179	89	1450
11	175	1089	80	1344
12	185	1179	89	1453

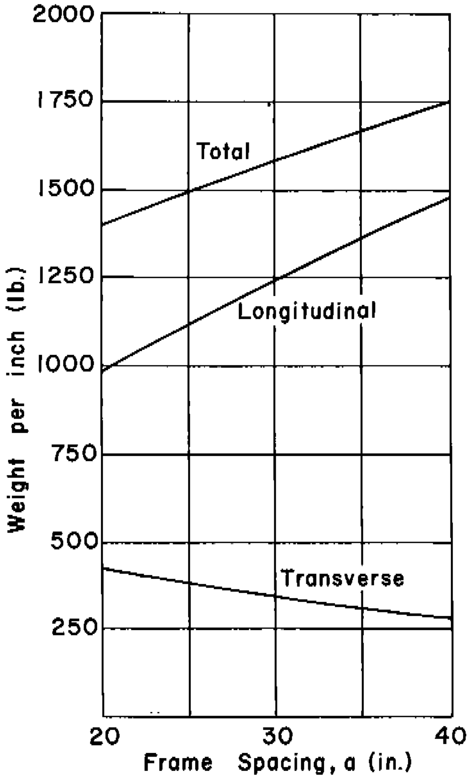

Fig. 3. Weight Per Inch of Hull Structure as a Function of Frame Spacing

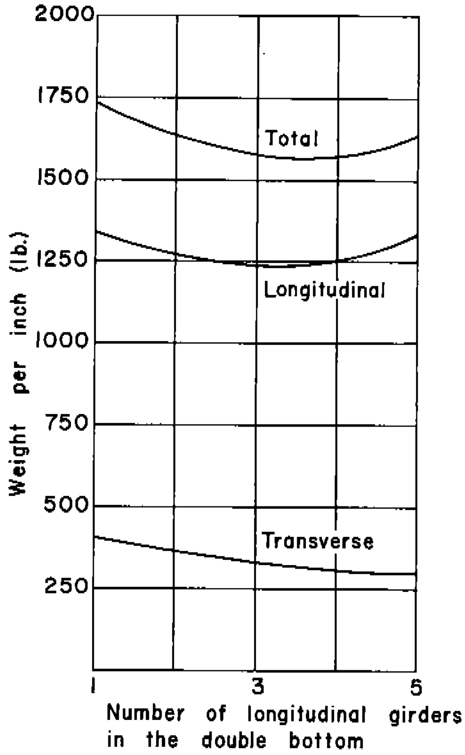

Fig. 4. Weight Per Inch of Hull Structure as a Function of Number of Longitudinal Girders in the Double Bottom

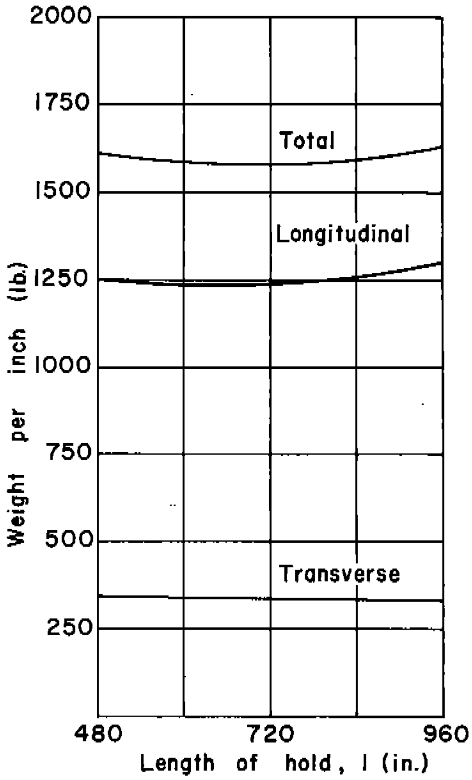

Fig. 5. Weight Per Inch of Hull Structure as a Function of Length of Hold

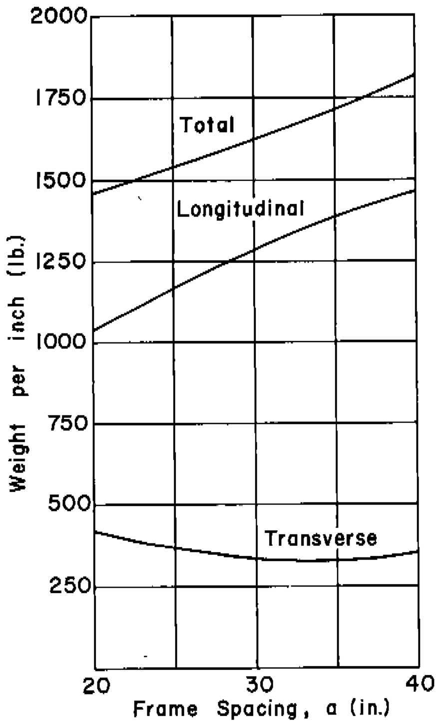

Fig. 6. Weight Per Inch of Hull Structure as a Function of Frame Spacing at the Reduced Primary Strength Criterion

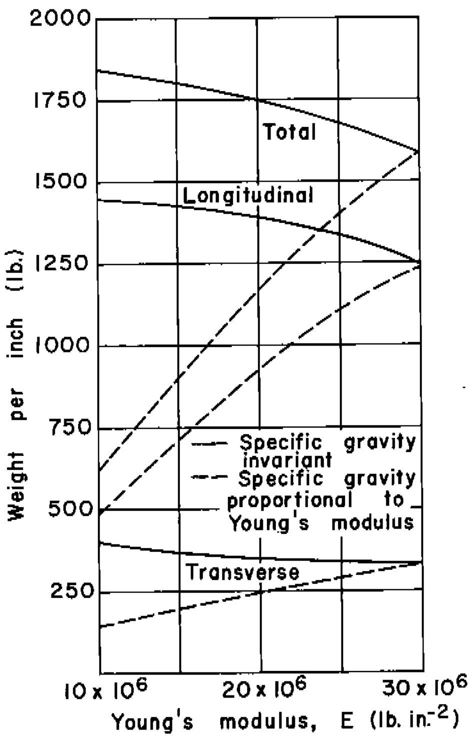

Fig. 7. Weight Per Inch of Hull Structure as a Function of Young's Modulus of the Material

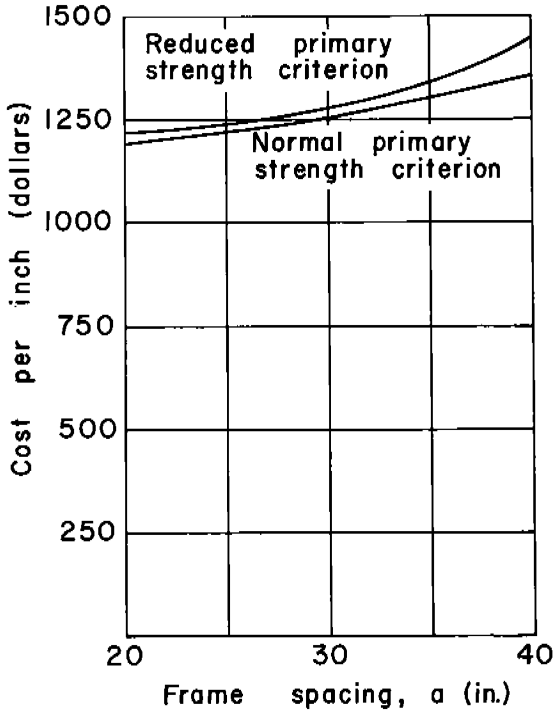

Fig. 8. Cost Per Inch of Hull Structure as a Function of Frame Spacing with Primary Stress Criterion as a Parameter

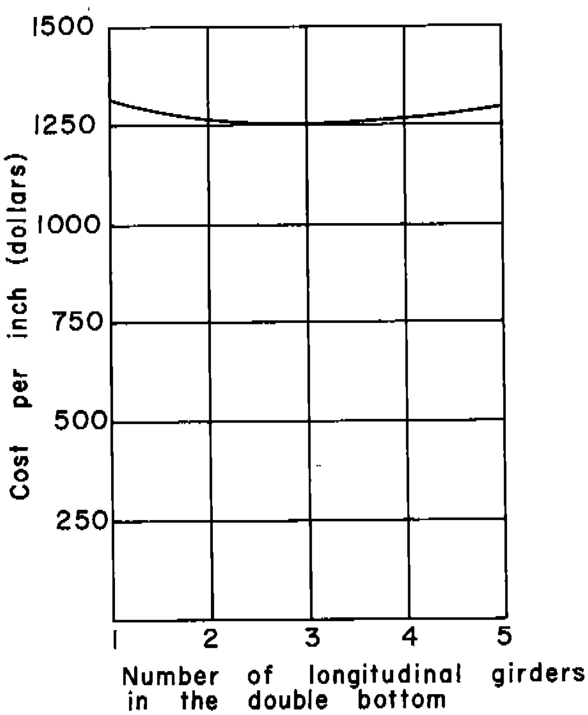

Fig. 9. Cost Per Inch of Hull Structure as a Function of Number of Longitudinal Girders in the Double Bottom

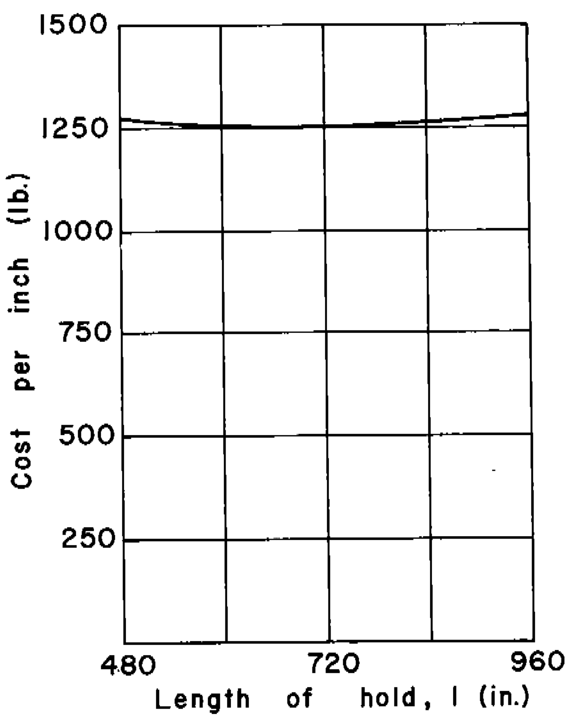

Fig. 10. Cost Per Inch of Hull Structure as a Function of Hold Length

Certain trends are observed:

a) The upward trend in weight per inch with frame spacing, Figures 3 and 6.

b) The dipping in weight per inch when the number of longitudinal girders in the double bottom is 3 (center keel and one keelson on each side), Figure 4.

c) The relative insensitivity of the weight per inch to variation in length of hold, Figure 5.

d) The downward trend in weight per inch with Young's modulus, provided the specific weight of the material is held constant, Figure 7. This figure also shows the trend in weight when the specific weight varies proportionally with Young's modulus.

5. DISCUSSION

The following comments are made with regard to these trends:

Frame Spacing

An increase in frame spacing decreases the transverse material inasmuch as the floors, frames and deck beams do not gain sufficiently in lightness to overcome the increase in weight density because of their closer spacing.

An increase in frame spacing brings about an increase in the scantlings of the double bottom longitudinal girders chiefly because of the greater shear they must carry as a consequence; however, the deck longitudinals drop in size because the stiffer deck beams now carry a greater proportion of the load.

Further, an increase in frame spacing results in a considerable increase in thickness of shell plating (proportional roughly to the square root of the frame spacing).

The final outcome is a fairly rapid increase in weight per inch as frames are spaced farther apart.

Number of Longitudinal Girders in the Double Bottom

When only a single longitudinal girder is fitted, the weight per inch is high because the girder is somewhat ineffective on account of the large shear lag that takes place in the secondary structure of the double bottom. The result is that bottom shell and inner bottom plating becomes relatively thick. When three longitudinal girders are fitted, the shear lag is considerably smaller and this is evidenced by the thinner double bottom flange plating. When five longitudinal girders are installed, the reduction in double bottom flange plating from the gain in beam efficiency due to a reduced shear lag is more than offset by the increase in weight from the more closely spaced webs.

The changes in bottom plating thickness with number of longitudinal girders arises almost entirely from the amount of bottom shell and inner bottom plating which is effective in compression. The results are listed for comparison in Table 11.

Table XI

Comparison of Results

Run	Number of Longitudi-nals in the Double Bottom	Width of Flange Plating Associated with the Web of Lon-gitudinals (in)	Effective Breadth Ratio	Effective Breadth of Bottom Shell (in)
1	3	214.5	0.79	676
4	5	143	0.93	795
8	1	429	0.42	359

In determining the primary stress intensities, only effective material was included in the compression flange of the hull girder. The low effective breadth ratio obtaining in Run 8 results in increasing the bottom plating over that for Run 1 by a considerable amount to keep the primary stress intensities within the criterion value. The effective breadth ratios of Runs 1 and 4 are much closer to each other.

The transverse structure decreases with number of longitudinal girders, as is to be expected, because of the additional support received.

Length of Hold

As the hold lengthens, the longitudinal members tend to become heavier. The consequence is that less material is required in the remaining members for primary girder strength. This is especially noticeable in the main deck and sheer strake. However, the overall impact on weight and cost is quite small.

The explanation for this behavior is that length of hold has a strong influence on the secondary stress intensities in the longitudinal girders and plating of the double bottom. As these are designed to conform with criteria of allowable stress intensities, the distribution of material, hence, the scantling over the whole section are affected. Because the top of the hull girder is considerably higher than the bottom, a change in bottom structure is reflected in a disproportionate change in top structure.

Young's Modulus

As is to be expected from a simple analysis of stability, the weight per inch decreases with increasing Young's modulus (roughly as the one-fifth power of the ratio of the moduli) so long as no change occurs in the specific gravity of the material. If the assumption is made that the specific gravity of the material varies proportionally with Young's modulus, the change in weight per inch appears to vary approximately with the four-fifth power of the ratio of the moduli.

Criterion of Primary Stress Intensity

If the maximum allowable primary stress intensity be lowered from

19,000 lb in^{-2} to half that value, there results a slight increase in weight per inch. The slightness of the amount is explained by the fact that in the former case the maximum actual primary stress intensity attains a value much below the criterion when the midship section is properly balanced out. This makes for but a slight drop in maximum actual stress intensity when the criterion stress intensity is lowered.

Comparison with the WOLVERINE STATE as Built

The code gives somewhat lighter side shell and main deck and considerably heavier internal decks and oil-tight longitudinal girders; also, somewhat lighter frames and deck beams. The code design for the same frame spacing and number of longitudinal girders as the ship as built gives a 5 percent lighter weight per inch than the actual ship when the length of hold is 720 in (1641 vs 1735 lb in).

Validity of a Comparison between Calculated and Actual Scantlings

Since there may be a temptation to compare the code-derived scantlings with the actual ones, some observations in the nature of warnings are pertinent:

a) The external and internal loadings and the wave-induced bending moment enter into the computer code as input data. Since such data were not available at the initiation of the production runs, some plausible values have been assumed. No further claim is made for the accuracy of these values. For checking out the computer code, the magnitudes of the loadings and bending moments are fairly irrelevant, and if the values selected are anywhere near reasonable, weights and cost trends corresponding to variations in the principal design parameters are meaningful.

The selected bending moment of 4.05×10^8 ft lb is somewhat less than the empirical value for the fictitious statical bending moment which is determined to be

$$\frac{2240}{35} \; W \; L = 6.40 \times 10^8 \text{ ft lb}$$

the symbol W denoting the ship's displacement in tons. But determination of the scantlings by the code requires that expected maximum loadings and bending moments be introduced, not fictitious ones.

b) The hull structure has been designed to an arbitrary set of criteria of allowable stress intensities. Whether the selected set of criteria is reasonable or extravagant and what, if any, changes should be made in its formulation can only be determined by interpreting in the light of experience a large number of computer runs made with parametric variations in the design criteria (and expected actual values of external loadings and wave-induced bending moments). Note that some of these runs can be made with a code devised for analysis rather than synthesis. Such a code is much simpler than the one used in this study and can be derived directly from it.

The set of design criteria relates solely to strength and elastic stability. Other design criteria, e. g., maximum allowable deflection, local strength, stress concentration or corrosion allowance, have not been taken into account in this initial study. The problem does not lie in the modification of the computer code to accomplish this, but rather in the formulation of such criteria.

The deviations between actual and calculated scantlings of main deck plating, longitudinal hatch girders and side shell plating appear to be attributable to the dominance of such complementary criteria.

c) The external loading and bending moment considered are those corresponding to a ship steaming upright into head or following seas. Consideration of the case of a ship steaming in oblique seas may well result in raising some scantlings because of the appearance of a lateral bending moment and of higher loadings on the sides induced by roll.

d) No explicit account has been taken of stress intensities induced by slamming. To some extent, these have been taken into account implicitely when formulating the pertinent criteria. But this is a gross and unreliable way to design structure. Eventually, an improved method must be found by which to design hull structure subject simultaneously to quasi-static and to impulsive dynamic loadings. Because of the implicit manner of accounting for slamming, it is not determinable whether the hull structure has been overdesigned or underdesigned with reference to this loading.

6. SUMMARY

The code for computing the optimum midship structure of a transversely framed, dry cargo ship is a workable one; however, it is subject to the following qualifications:

a) External loadings and wave-induced bending moment must be entered as input data.

b) The design criteria are arbitrary and based solely on overall strength of hull. Stress intensities under distributed loadings do not exceed the elastic limit of the material.

c) The ship steams upright in head or following seas.

d) Impulsive loading from slamming is not taken into account explicitely. Neither are stress concentrations, strength under localized loading, rigidity and corrosion allowances, inter alia.

The merit of the design method used (separation of structure into primary, secondary and tertiary components) is that all our empirical knowledge on the proportioning of hull structure to withstand the forces of the seaway finds expression in just these factors.

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8. NOTATION

Symbol	Definition	Dimensions
A_i	Cross sectional area	in^2
$\|A_{i,j}\|$	Grillage matrix	$in\ lb^{-1}$
a	Frame spacing	in
B	Beam of vessel	in
B_k	Component of bending moment corresponding to the external loading alone	in lb
b	Spacing of longitudinal girders	in
b_e	Effective width or effective breadth of plating	in
C	Constant $C = k/EI$	in^{-4}
	Cost	dollars in^{-1}
C_{fe}	Cost of fabrication and erection	" "
C_m	Cost of material	" "
C_w	Cost of welding	" "
D	Depth of vessel	in
	Flexural rigidity of plating $D = Eh^3/\{12\ [1-u^2]\}$	in lb
$\|Di\|$	Partition Submatrix $i = 1\ldots7$	
d	Depth of inner bottom	in
E	Young's modulus of the material	$lb\ in^{-2}$
$\|F_k\|$	Joint work matrix	in lb
f_i	Primary Stress factor	
f_2	Secondary stress factor	

Symbol	Definition	Dimensions
f_3	Tertiary stress factor	
$f(A_i)$	Time - size factor	
$f(h_i)$	vee weld function	
$f(w_i)$	Time - weight factor	lb^{-1}
H	Draft of vessel	in
	Hydrostatic head	in
h	Plating thickness	in
	Wave height	in
I	Second central moment of cross sectional area	in^4
K	Stiffness factor $K = I/\ell$	in^3
k	Foundation modulus	$lb\ in^{-2}$
L	Length of vessel	in
ℓ	Length of an element	in
M	Bending moment	in lb
M^F	Boundary bending moment corresponding to full fixity	in lb
m	Component weight of weld material	$lb\ in^{-1}$
$N_i(u)$	Nielsen function	
N_s	Number of seams	
P	Concentrated load	lb
p	Normal pressure	$lb\ in^{-2}$
Q	Shearing force	lb
$R_{i,j}$	Reaction at intersection i. j	lb
	Constant	
r	Coefficient of restraint	
	Labor rate	$dollars\ hr^{-1}$
s	Weight of structural component	$lb\ in^{-1}$
u	b - y	in

Symbol	Definition	Dimensions
W	Uniformly distributed load	lb in^{-1}
$w_{i,j}$	Deflection at the intersection i, j	in
$w^{o}_{i,j}$	Deflection at the intersection i, j of the released frame	in
$w^{*}_{i,j}$	Deflection at the intersection i, j caused by all the longitudinal girder reaction	in
x	Longitudinal coordinate	in
y	Transverse coordinate	in
y_e	effective breadth	in
y_h	Half-width of the hatchway	in
Z	Section modulus	in^3
$\alpha_{i,j}$	Influence coefficient	in lb^{-1}
β	Aspect ratio parameter	
γ	specific weight of steel	lb in^{-3}
Λ	Displacement of ship	tons
θ	Joint rotation	-
λ	Wave length	in
	Normalized effective breadth	-
μ	Poisson's ratio	-
σ	Axial (normal) stress intensity	lb in^{-2}
	σ_x in the longitudinal direction	
	σ_y in the transverse direction	
	σ_1, σ_{1x} primary stress intensity	
	σ_{2x}, σ_{2y} secondary stress intensity	
	σ_{3x}, σ_{3y} tertiary stress intensity	

Symbol	Definition		Dimensions
	σ_{2xf}, σ_{2xp} σ_{2yf}, σ_{2yp}	secondary stress intensity in the longitudinal and transverse directions in the flange and plating	
σ^*	Criterion stress intensity		lb in^{-2}
	σ_1^*	criterion primary stress intensity	
	σ_2^*	criterion secondary stress intensity	
	σ_3^*	criterion tertiary stress intensity	
τ	Shearing stress intensity		lb in^{-2}
ω	Function		-
ψ	Joint displacement		-

Subscript	Definition
b	bending
cr	critical
f	floor
h	in hogging
i	inner bottom
i, j	indices
m, n	indices
o	origin
	center of plating
r	restraint
rs	restraint at shell
ri	restraint at inner bottom
s	shell
	sagging
	fixed

Superscript	Definition
w	simply-supported
x	in the longitudinal direction
y	in the transverse direction
1	primary
2	secondary
3	tertiary
o	Unrestrained
*	Due to all longitudinal girders
	Criterion value
F	fixed.

APPENDIX A

THE CLASSIFICATION OF STRUCTURE

Following St. Denis (1954), the complex structural assembly which is the hull is subdivided into primary, secondary and tertiary structure. The first refers to the hull when considered in its totality, the second to the stiffened panels of plating bounded by side shell, transverse and longitudinal bulkheads and decks; the third is given by the unstiffened plating supported by transverse and longitudinal stiffness (frames, floors, longitudinal girders, etc).

This classification of structure under three basic types leads to a significant simplification of the work provided the interaction of one type of structure on the others is either negligible or can be determined. Fortunately, this is the case in hand.

By correspondence, stresses in primary structure are termed primary stresses; in secondary structure, secondary stresses; and in tertiary structure, tertiary stresses. Their intensities are represented herein by σ_1, σ_2, σ_3 respectively.

The absolute stress intensity at any point is obtained by the simple superposition of primary, secondary and tertiary stress intensities. This is a fundamental assumption.

APPENDIX B

DETERMINATION OF THE THICKNESS OF PLATING

B. 1 GENERAL

The plating thicknesses (h) are to be chosen so as to satisfy the criteria of structural adequacy. Economy in design requires that they satisfy all the criteria of structural adequacy by as slight an excess over the requirement as is practical.

In this appendix, equations are given for determining the stress intensities in plating under normal and planar loading for certain idealized boundary conditions. From a comparison of stress intensities so derived with those given by the set of criteria for structural adequacy, the required plating thickness is simply obtained. Unfortunately, neither actual boundary conditions nor planar loadings are known a priori. Thus, the process becomes an iterative one.

In shipbuilding, the spacing of the stiffeners and the thickness of the plating are usually of such dimensions that small deflection theories are not valid for assessing the strength of the plating. Thus, recourse must be had to large, or, at least, moderate deflection theories, an important characteristic of which is that they are nonlinear. However, the nonlinearity is of the progressive or non-essential type which implies that the linear solution provides a valid first approximation.

In the design of hull plating the loading is of one of the following types;

a) Normal loading alone

b) Planar loading alone, particularly compressive loading

c) Normal and planar loading combined.

The discussion that follows is limited to plates of rectangular geometry. Plating of form other than rectangular (or quasi-rectangular) is rare, particularly in the midship region.

So long as the central deflection under load is small (say less than one-quarter the thickness of the plating), specific solutions based on the classical (small-deflection) theory of Lagrange are available for all aspect ratios and boundary conditions of interest. (Schade, 1941). When the central deflection exceeds such value, only a few specific solutions, based on the large deflection (more properly, moderate deflection) theory of von-Karman are available. The general case has been treated only approximately, although exact solutions are available when the plate is square (a = b) or very wide (b >> a).

The most general moderate deflection theory presently available on the design of plating is due to Bengston (1939). The arbitrariness of some of the assumptions made in this theory has been criticized (e. g., Bleich, 1952). Nevertheless, there is some evidence which supports the theory (Levy, 1942).

Bengston's theory applies to the problem of the flat rectangular plate acted upon by in-plane and lateral loading, the former consisting of uniform compression applied along the edges, the latter, of uniformly distributed hydrostatic action over the surface. Since the parallel problem which involves tensile loading along the edges yields to simple solution by superposition, Bengston's theory provides the essential method for designing the plating.

To be sure, other methods are available for determining the stress intensity field when the in-plane loading is zero, or the critical buckling load when the lateral loading is absent, but the respective solutions can be obtained by setting the appropriate loading equal to zero in Bengston's solution. In establishing a computer code, it is usually preferable to provide a single, more general method which applies to all anticipated combinations of parameters, rather than several specific methods each capable of handling with greater efficiency a restricted combination of parameters. For this reason, only Bengston's method is considered herein.

Bengston presents solutions for the case of the rectangular plate simply supported along its boundaries and that of the rectangular plate fixed along its boundaries.

In applying this work, the question always arises as to what boundary conditions to assume. For the bottom plating in a ship, where the normal loading is large and always present, the condition of fixity along all the boundaries is approached because of the symmetry of the loading and because the plating is restrained from rotating at the supports by the floors and longitudinal girders. However, when the normal loading is small and the stiffeners shallow, as occurs in decks, the degree of boundary restraint can be quite small. The next appendix discusses the determination of the restraint at the plate boundaries. The magnitude of the normal loading at which the plating can no longer be considered to be fixed at its boundaries has not been determined as yet.

B. 2 THE SIMPLY SUPPORTED PLATE

The procedure for obtaining the plate stresses in the simply supported plate is outlined as follows:

a. Calculate the deflection w_o at the center of the plate by the formula:

$$\frac{w_o^3 C}{a^2} - \frac{w_o}{E}\left\{\frac{\pi^2}{2}[1-\mu^2]\left[\frac{b}{a}\bar{\sigma}_x + \frac{a}{b}\bar{\sigma}_y\right] - \frac{E R h^2}{a^2}\right\}$$

$$= \frac{8 p a b [1-\mu^2]}{\pi^2 E h}$$

in which

$$C \equiv 3.24 \frac{a^3}{b^3} + 3.24 \frac{b}{a} + 0.92 \frac{a}{b}$$

$$R \equiv \frac{\pi^4}{24} \cdot \frac{a}{b} \cdot \left[\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2 \right]$$

where p is the lateral pressure and where $\bar{\sigma}_x$ is the average compressive stress intensity in the x direction across side b and $\bar{\sigma}_y$ is the average compressive stress intensity in the y direction across side a.

Since at this point the thickness h is not yet known, the solution for w_o is obtained by introducing the value for h obtained for the plate under normal loading alone.

b. Determine the maximum bending stress intensities at the center of the plate by the relations:

In the x-direction

$$\sigma_{xb} = \frac{6 \pi^2 D w_o}{a^2 h^2} \cdot \left[1 + \mu \frac{a^2}{b^2} \right]$$

In the y-direction

$$\sigma_{yb} = \frac{6 \pi^2 D w_o}{b^2 h^2} \cdot \left[\frac{a^2}{b^2} + \mu \right]$$

In these expressions, D is the flexural rigidity

$$D \equiv \frac{E h^3}{12 [1 - \mu^2]}$$

c. Combine the axial and bending stress intensities by superposition. For the scantlings used in shipbuilding, the reduction in axial stress at the center of the sides is negligible.

d. The highest compressive stress intensity in the plating in the longitudinal direction occurs in way of the longitudinal supports ($y = \pm b/2$) at the center of panel length ($x = a/2$); it is given by

$$\sigma_x = \bar{\sigma}_x - \frac{E w_o^2}{a^2 [1 - \mu^2]} \cdot \frac{\pi^2}{8} \left[1 + \mu \frac{a^2}{b^2} \right]$$

(σ_x and $\bar{\sigma}_x$ are both negative). For transversely framed ships, the expression is simply,

$$\sigma_x = \bar{\sigma}_x - \frac{E w_o^2}{a^2 [1 - \mu^2]} \cdot \frac{\pi^2}{8}$$

e. The maximum transverse stress intensity is

$$\sigma_y = \bar{\sigma}_y - \frac{E w_o^2}{a^2 [1 - \mu^2]} \cdot \frac{\pi^2}{8} \cdot \left[\frac{a^2}{b^2} + \mu \right]$$

B. 3 THE FIXED PLATE

The procedure for obtaining the plate stresses in the fixed plate is as follows:

a. Calculate the deflection w_o at the center of the plate by the formula:

$$\frac{w_o^3 C}{a^2} - \frac{w_o}{E} \left\{ \frac{3 \pi^2}{8} [1 - \mu^2] \left[\frac{b}{a} \bar{\sigma}_x + \frac{a}{b} \bar{\sigma}_y \right] - \frac{E R h^2}{a^2} \right\} = \frac{p a b}{2 E h} [1 - \mu^2]$$

where:

$$C = 3.78 \frac{a^3}{b^3} + 3.78 \frac{b}{a} + 1.64 \frac{a}{b}$$

$$R = \frac{\pi^4}{24} \cdot \frac{a}{b} \cdot \left[3 \frac{a^2}{b^2} + 3 \frac{b^2}{a^2} + 2 \right]$$

b. Obtain the maximum bending stress intensities (at the middle of the sides) when the compressive stresses are of such magnitude as to cause buckling. These are given by:

In the x-direction by

$$\sigma'_{xb} = \frac{12 \pi^2 D w_o}{a^2 h^2}$$

In the y-direction by

$$\sigma'_{yb} \equiv \frac{12 \pi^2 D w_o}{b^2 h^2}$$

c. Obtain the maximum bending stress intensities when there are no compressive stresses in the plating. These are given by:

$$\sigma''_{xb} \equiv \frac{\varphi_x D w_o}{a^2 h^2} \qquad \sigma''_{yb} \equiv \frac{\varphi_y D w_o}{a^2 h^2}$$

where φ_x and φ_y are functions of the aspect ratio given by the following expressions:

For the stress intensity at the center of the longest side,

$$\varphi_x \equiv 192 + 51 \exp\left\{-2.5\left[\frac{b}{a} - 1\right]\right\}$$

For the stress intensity at the center of the shortest side

$$\varphi_y \equiv 243 \exp\left\{-0.59\left[\frac{b}{a} - 1\right]\right\}$$

d. Determine the critical buckling stress intensity from the equation

$$\left[\sigma_x + \frac{a^2}{b^2} \sigma_y\right]_{cr} = \frac{4\pi^2 D}{3hb^2}\left[3 \frac{a^2}{b^2} + 3 \frac{b^2}{a^2} + 2\right]$$

$$= 13.16 \left[\frac{D}{hb^2} + 3 \frac{a^2}{b^2} + 3 \frac{a^2}{b^2} + 2\right]$$

e. Obtain the expression based on the actual stress intensities

$$\bar{\sigma}_x + \frac{a^2}{b^2} \bar{\sigma}_y$$

f. Obtain the ratio

$$r_\sigma \equiv \frac{\sigma_x + \frac{a^2}{b^2} \sigma_y}{\left[\sigma_x + \frac{a^2}{b^2} \sigma_y\right]_{cr}}$$

g. Interpolate between Steps b and c in accordance with this ratio

$$\sigma_{xb} = \sigma''_{xb} + r_\sigma [\sigma'_{xb} - \sigma''_{xb}]$$

$$\sigma_{yb} = \sigma''_{yb} + r_\sigma [\sigma'_{yb} - \sigma''_{yb}]$$

h. Combine the axial and bending stress intensities by superposition. Again, the reduction in axial stress at the center of the sides is negligible.

i. The maximum longitudinal compressive stress intensity in the plating occurs in way of the longitudinal supports at the center of panel length and is

$$\sigma_x = \bar{\sigma}_x - \frac{E w_o^2}{a^2 [1 - \mu^2]} \cdot \frac{3 \pi^2}{32} \cdot \left[1 + \mu \frac{a^2}{b^2} \right]$$

(σ_x and $\bar{\sigma}_x$ are both negative).

j. The maximum transverse stress intensity is

$$\sigma_y = \bar{\sigma}_y - \frac{E w_o^2}{a^2 [1 - \mu^2]} \cdot \frac{3 \pi^2}{32} \cdot \left[\frac{a^2}{b^2} + \mu \right]$$

APPENDIX C

DETERMINATION OF THE DEGREE OF ELASTIC RESTRAINT OF PLATING AT ITS SUPPORTS

When plating and supporting stiffeners are connected, as occurs in all ship structures, a condition of simple support at the boundaries cannot obtain, for this would imply not only unrestrained rotation of the plating at the point of support but also of its stiffeners. Since rotation of the latter is to some degree restrained by their torsional rigidity, this restraint is in part experienced by the plating to which they are attached with the result that rotation of the plate boundaries is reduced.

So long as the stiffeners are small in size and their flanges are freely-standing, the elastic restraint imposed on the plating can be neglected in practical calculations. This case obtains when shell or deck plating is stiffened by ordinary frames or deck beams. But when the stiffeners are of large size and further when their flanges are wide and coupled, as occurs in the case of double bottoms, strong elastic restraints can be developed and since these can sensibly affect the solution for scantlings, they must be taken into account.

A method for computing the coefficient of restraint is proposed which is an extension of the method of Liou (1963). Consider the double bottom arrangement of structure of Fig. C-1 and focus attention on the bottom shell plating. If the normal pressure on the plating is sufficiently high, the plating will deflect as in (a) and this deflection will be influenced by the behavior of the floors, which may have been induced to rotate by the inner bottom, and if these deflect, elastic restraint will be exerted on the bottom plating. If the normal pressure is insufficient, the plating will deflect as in (b) and, again, this deflection will be influenced by the behavior of the floors.

The coefficient of elastic restraint (or degree of fixity) of the shell at the floor is

$$C_{rs} \equiv \frac{2}{\pi} \text{ arc cot } \left\{ 2 \left[\frac{h_s}{h_w} \right]^3 \cdot \frac{b}{a} \cdot r \right\}$$

while that of the inner bottom at the floor is

$$C_{ri} \equiv \frac{2}{\pi} \text{ arc cot } \left\{ 2 \left[\frac{h_i}{h_w} \right]^3 \cdot \frac{b}{a} \cdot r \right\}$$

where:

h_s ≡ shell thickness

h_w ≡ web thickness of floor

h_i ≡ inner bottom thickness

r ≡ coefficient defined as follows:

a) Far edges of restraining plate fixed

$$r \equiv r(1) \equiv \frac{1}{2\pi} \frac{\sinh^2(\pi\, d/b) - \pi^2[d/b]^2}{\sinh(\pi\, d/b)\cosh(\pi\, d/b) - [\pi\, d/b]}$$

b) Far edges of restraining plate simply supported

$$r \equiv r(0) \equiv \frac{1}{2\pi} \frac{\sinh(\pi\, d/b)\ \cosh(\pi\, d/b) - \pi\, d/b}{\sinh^2(\pi\, d/b)}$$

see Fig. C-2. In these expressions, d is the depth of the double bottom. In this notation, $C_{rs} \equiv C_{ri} \equiv 0$ for simply supported plating, $C_{rs} \equiv C_{ri} \equiv 1$ for fixed plating.

The procedure for determining the degree of elastic restraint is as follows:

a) Given the thicknesses of shell and inner bottom plating based on simply supported edges: $h_s(0)$, $h_i(0)$, and also based on fixed edges: $h_s(1)$, $h_i(1)$; and given the floor thickness based on both edges simply supported, $h_w(0,0)$, both edges fixed, $h_w(1,1)$ and one edge simply supported, the other fixed $h_w(0, 1)$ or $h_w(1,0)$, calculate the starting set of values:

$$C^o_{rs}[h_s(0),\ h_w(0,0),\ r_i(0)]$$

$$C^o_{rs}[h_s(0),\ h_w(0,1),\ r_i(1)]$$

$$C^o_{ri}[h_i(0)\ h_w(0,0),\ r_s(0)]$$

$$C^o_{ri}[h_i(0),\ h_w(0,1), r_s(1)]$$

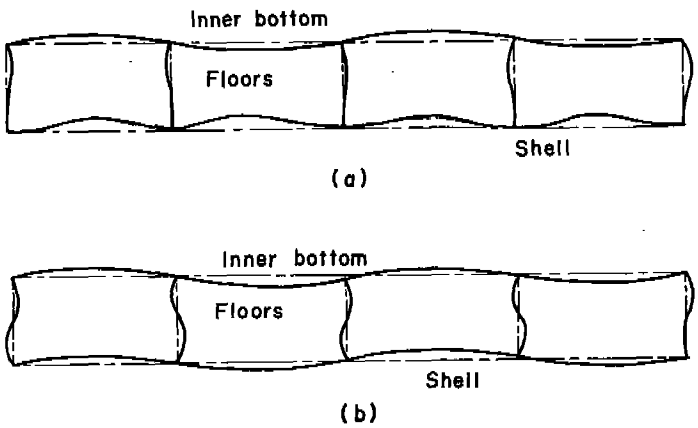

Fig. C.1. Modes of Bottom Shell Deflection

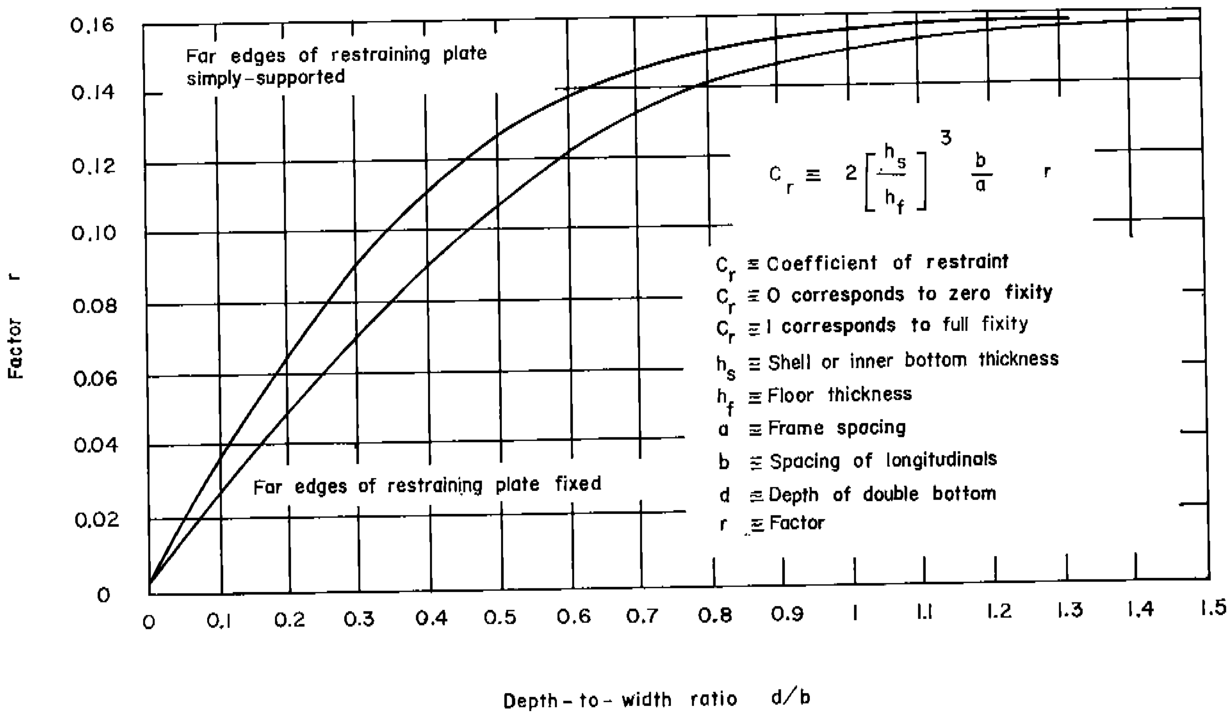

Fig. C.2. Coefficient of Restraint

b) The first approximations to the coefficients of restraint are:

$$C'_{rs} \equiv \frac{1}{2}\left\{ C^{o}_{rs}\ [\ h_s(0), h_w(0,0),\ r_i(0)] + C^{o}_{rs}\ [\ h_s(0),\ h_w(0,1),\ r_i(1)] \right\}$$

$$C'_{ri} \equiv \frac{1}{2}\left\{ C^{o}_{ri}\ [h_i(0), h_w(0,0),\ r_s(0)\ ] + C^{o}_{ri}\ \ h_i(0),\ h_w(0,1),\ r_i(0)\ ] \right\}$$

c) The first approximation to the plate thicknesses are:

$$h_s(C'_{rs}) \equiv C'_{rs}\ h_s(1) + [1-C'_{rs}]\ h_s(0)$$

$$h_i(C'_{ri}) \equiv C'_{ri}\ h_i(1) + [1-C'_{ri}]\ h_i(0)$$

$$h_w(C'_{rs}, C'_{ri}) \equiv \frac{1}{2}\ [C'_{rs} + C'_{ri}]\ h_w(1) + \left\{ 1 - \frac{1}{2}\ [C'_{rs} + C'_{ri}] \right\} h_w(0)$$

d) Calculate the coefficient r for the intermediate degree of fixity

$$r_s(C'_{rs}) \equiv C'_{rs}\ r_s(1) + [1-C'_{rs}]\ r_s(0)$$

$$r_i(C'_{ri}) \equiv C'_{ri}\ r_i(1) + [1-C'_{ri}]\ r_i(0)$$

e) The second approximations to the coefficients of restraint are obtained from

$$C''_{rs}[h_s(C'_{rs}),\ h_w(C'_{rs},\ C'_{ri}),\ r_i(C'_{ri})]$$

$$C''_{ri}[h_i(C'_{ri}),\ h_w(C'_{ri},\ C'_{rs}),\ r_s(C'_{rs})]$$

f) The second approximations to the plate thicknesses are:

$$h_s\ (C''_{rs}) \equiv C''_{rs}\ \ h_s(1) + [\ 1-C''_{rs}]\ \ h_s(0)$$

$$h_i\ (C''_{ri}) \equiv C''_{ri}\ h_i(1) + [\ 1-C''_{ri}]\ \ h_i(0)$$

$$h_w(C''_{rs},\ C''_{ri}) \equiv \frac{1}{2}\ [\ C''_{rs} + C''_{ri}]\ \ h_w(1)$$

$$+\left\{1 - \frac{1}{2}\ [\ C''_{rs} + C''_{ri}\ ]\right\} h_w(0)$$

This corresponds to step (c). One continues in this manner to convergence. The degree of fixity of the far edge of the floor (or web) does not greatly influence the degree of fixity of the plate. The exact restraint is found by successive approximations. To this end, consider the example for which:

$a = 30$ in	
$b = 120$ in,	$b/a = 4$
$d = 60$ in	$d/b = 0.5$
$h_f = 0.50$ in	
$h_s = 0.75$ in	$h_s/h_f = 1.25$
$h_i = 0.60$ in	$h_i/h_f = 0.80$

The coefficient of restraint of the shell plate when the inner bottom plating is simply supported is

$$C_r = \frac{2}{\pi}\ \cot^{-1}[\ 2(1.25)^3(4)(0.1275)] = 0.296$$

When the inner bottom plating is fixed at the floors, the coefficient of restraint of the shell is

$$C_r = \frac{2}{\pi}\ \cot^{-1}[\ 2(1.25)^3(4)(0.108)] = 0.341$$

Now, the coefficient of restraint of the inner bottom plating when the shell is simply supported is

$$C_r = \frac{2}{\pi} \cot^{-1}[2(0.80)^3(4)(0.1275)] = 0.694.$$

While when the shell is fixed it is

$$C_r = \frac{2}{\pi} \cot^{-1}[2(0.80)^3(4)(0.108)] = 0.735$$

The average coefficient of restraint for the inner bottom plating is

$$\frac{1}{2}[0.694 + 0.735] = 0.715$$

By linear interpolation, that for the shell plating is

$$0.296 + [0.341 - 0.296](0.715) = 0.328$$

This is the minimum degree of fixity of the shell plating at the floor supports and obtains when the normal pressure is insufficient to force a unilateral deflection pattern.

The foregoing exposition provides a method for determining the condition of fixity for double bottom structure. If the method is applied to side and deck structure, it is found that the fixity of the shell and deck plating at the frames and longitudinals is zero. This is an underestimate, for the method does not take into account the constraint to rotation of the plate edges provided by the torsional rigidity of the frames and longitudinals. However, since these are freely-standing and tend to be of small depth-to-length ratio, their torsional strength is quite negligible and, consequently, so is the constraint to rotation they are capable of imposing on the plating. Therefore, an assumption of simple-support appears to be close to correct and of a possibly slight conservative bias.

APPENDIX D

EFFECTIVE WIDTH & EFFECTIVE BREADTH OF PLATING

D. 1 EFFECTIVE WIDTH

The behavior of plating subjected to a uniform compressive load in the plane of the plating has received considerable attention, especially because of its applications in the aeronautical field. The presentation herein is based on Bengston's (1939) work. Again, consider the two cases of the rectangular plate simply supported at all boundaries and the rectangular plate with fixed boundaries.

D. 1. 1 The Simply Supported Plate

The effective width is given by

$$\frac{b_e}{b} = \frac{1}{1 + \dfrac{\pi^4\{1 + \mu\ [a^2/b^2]\}\ [b/a]}{16\ C_s}\left[1 + \dfrac{a^2}{b^2}\dfrac{\sigma_b}{\sigma_a} - \dfrac{\sigma_{cr}}{\sigma_a}\right]}$$

where

$$C_s \equiv 3.24\ \frac{a^3}{b^3} + 3.24\ \frac{b}{a} + 0.92\ \frac{a}{b}$$

$$\sigma_a \equiv \bar{\sigma}_x \equiv \frac{1}{b} \cdot \int_0^b \sigma_x(y)\ dy$$

and Poisson's ratio $\mu = 0.3$, also

$$\sigma_{cr} = \frac{2\ E\ R\ h^2\ [a/b]}{\pi^2\ [1 - \mu^2]\ a^2}$$

and

$$R \equiv \frac{\pi^4}{24}\ \frac{a}{b}\left[\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right]$$

Of course, $b_e/b \leqq 1$.

D. 1. 2 The Fixed Plate

The effective width is given by

$$\frac{b_e}{b} = \cfrac{1}{1 + \cfrac{9\,\pi^4 \{1 + \mu\ [a^2/b^2]\}\ [b/a]}{256\ C_f} \left[1 + \frac{a^2}{b^2}\frac{\sigma_y}{\sigma_x} - \frac{\sigma_{cr}}{\sigma_x} \right]}$$

where

$$C_s \equiv 3.78\ \frac{a^3}{b^3} + 3.78\ \frac{b}{a} + 1.64\ \frac{a}{b}$$

$$\sigma_{cr} \equiv \left[\sigma_x + \frac{a^2}{b^2}\ \sigma_y \right]_{cr} = \frac{4\ \pi^2\ D}{3\ h\ b^2} \left[3\ \frac{a^2}{b^2} + 3\ \frac{b^2}{a^2} + 2 \right]$$

Again, $b_e/b \leqq 1$.

Since the expression for the effective width of a fixed plate is developed from a single wave system, it is more limited in its application than is the parallel expression for the simply supported plate. In general, the expression holds for

a. Large variations in σ_x and σ_y when $a = b$

b. Large variations in a/b when $\sigma_y = 0$

The second case tends to be approached in transversely framed ships.

D. 2 EFFECTIVE BREADTH

The effective breadth of plating has been well presented by Vedeler (1945) and by Schade (1951) among others.

Vedeler gives the following expression for the effective flange width of a box-shaped beam of length ℓ and width s subjected to a sinusoidal bending moment

$$\lambda \equiv \frac{1 + \cfrac{\sinh(\pi\beta)}{\pi\ \beta}}{1 + \cosh(\pi\beta)}$$

where:

λ ≡ normalized effective breadth

$$\lambda \equiv \frac{s_e}{s}$$

s ≡ generic breadth

s_e ≡ generic effective breadth

β ≡ aspect ratio parameter

$$\beta \equiv \frac{s}{\ell}$$

The variation of λ with β is shown in Fig. D 1. The expression for λ is based on the assumption that the bending moment curve passes through zero at the ends of the beam. When this is not the case, the distance between points of zero bending moment is to be substituted for the length ℓ of the beam. Denoting this distance by $c\ell$, the aspect ratio parameter becomes $\beta \equiv s/c\ell$.

Schade gives the following expression for the normalized effective breadth of plating subject to uniform load

$$\lambda \equiv \frac{1.1}{1 + 2\beta^2} \qquad \text{but } \lambda \quad 1$$

Vedeler's expression for a loading that results in a sinusoidal bending moment and Schade's expression for a uniform loading give results that are close to each other.

Note that for these cases, the effective breadth is independent of the geometry of the section. This is a fortunate condition.

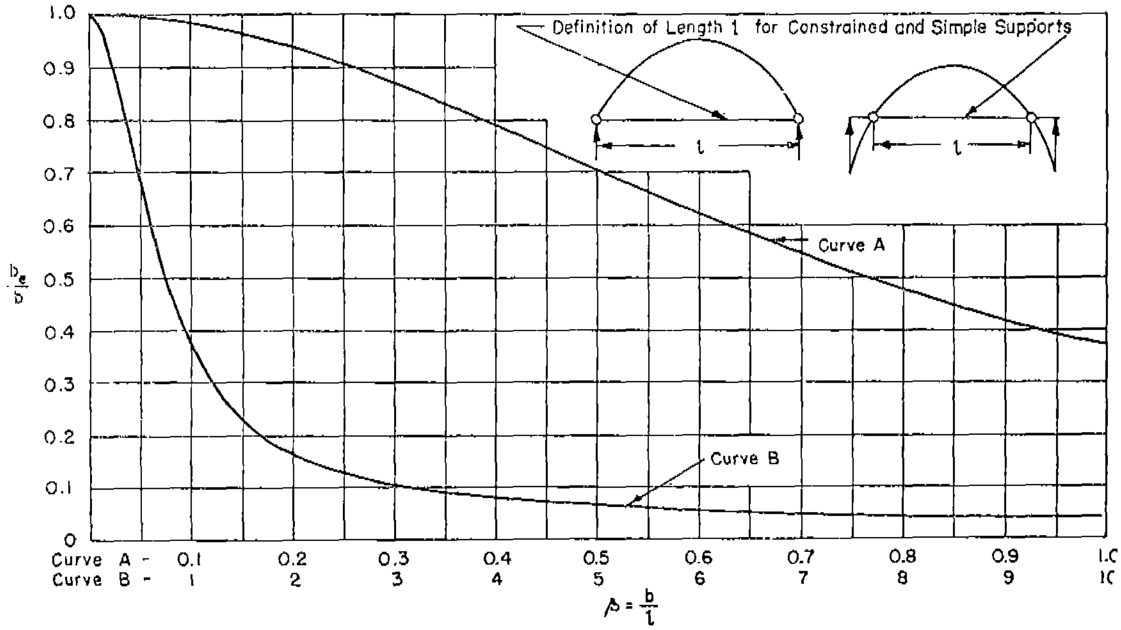

Fig. D.1. Effective Breadth According to Vedeler

When applying the concepts of effective width and effective breadth to a determination of ship scantlings, the following rules should be followed:

a) Determine by Bengston's formula the effective width $(b_e)_3$ of all tertiary plating when acted upon by primary and secondary stress intensities.

b) Determine by Schade's or Vedeler's formulae the normalized effective breadth λ_2 of all flange items of secondary structure associated with transverse and longitudinal stiffeners (decks, inner bottom, bottom shell). The length (ℓ) and breadth (s) to be used in determining the aspect ratio to be entered into the formula are furnished by the distance of separation of the points of zero bending moment in the pertinent orthogonal directions (ℓ is always in the direction of the stress intensity for which solution is being sought, i.e., when analyzing longitudinal stress intensities, ℓ is a fraction of hold length ℓ_h and s becomes the longitudinal girder spacing b; when analyzing transverse stress intensities, ℓ becomes a fraction of the beam and s becomes a). These points are furnished by the grillage solution.

c) The combined effective breadth-width of secondary structure is

$$(b_e)_3 \cdot \lambda_2$$

d) Determine the normalized effective breadth of the flange items of primary structure (decks, inner bottom and shell plating when considered as part of the ship girder). The aspect ratio in this case is given by B/L and the normalized effective breadth λ_1 is always close to unity.

e) The combined effective breadth width of primary structure is

$$(b_e)_3 \cdot \lambda_2 \cdot \lambda_1$$

APPENDIX E

INDEXING SYSTEMS

While it is recognized that a convenient and flexible system of indexing must eventually be set down that will account for the large number of structural members forming the hull structure, a somewhat simplified system can be pro tempore employed which is adequate for designing the structure in a midship hold. It was actually found convenient to employ more than one system depending on the aspect of the structure being designed. These systems are all ad-hoc, i.e., they apply only to the geometry of the WOLVERINE STATE, but can be readily extended. The reason for the choice of indexing systems introduced lies merely in the intent of reducing the demand on computer memory. The indexing systems are as follows, see Fig. E-1.

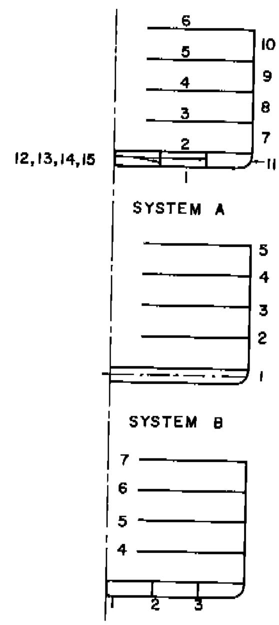

Fig. E.1. Indexing Systems Illustrated for Basic Design

a) System A

This system is used when determining the thickness of all plating and when computing the structural properties of the ship section which are directly related to the calculation of primary and tertiary stress intensities. The indexing is as follows:

Index	item
1	bottom shell
2	inner bottom
3	fourth deck plating
4	third deck plating
5	second deck plating
6	main deck plating
7	side shell inner bottom to fourth deck
8	side shell fourth to third decks
9	side shell third to second decks
10	side shell second to main decks

11	bilge plate
12	oil-tight floors
13	oil-tight longitudinal girders
14	non-tight floors
15	non-tight longitudinal girders

b) System B

This system is used for the frame solution in the grillage analysis, to which purpose, only the intersections of the side frames with the floors or deck beams (nodes) need be indexed. Bending moments are indexed in this system which reads:

Node Index	Node location at intersection of side frame with
1	double bottom
2	fourth deck
3	third deck
4	second deck
5	main deck

Note that the shell plating is assumed to change in thickness at the nodes.

c) System C

This system is also used in the grillage analysis, particularly for indexing the influence coefficients and the longitudinal girders. The system is:

Index	Item
1	vertical keel
2	first longitudinal double bottom girder
3	second longitudinal double bottom girder
4	fourth deck hatch longitudinal girder
5	third deck hatch longitudinal girder
6	second deck hatch longitudinal girder
7	main deck hatch longitudinal girder

d) System D

This system is used in the analysis of the sectional properties of transverse structure. The system is:

Index	Item
1	non-tight floor
2	oil-tight floor
3	fourth deck beam
4	third deck beam
5	second deck beam
6	main deck beam

APPENDIX F

REMARKS ON GRILLAGE ANALYSIS

F. 1 OVERVIEW

The purpose of the grillage analysis is to determine the secondary stress intensities in the deck, bottom and side structure.

The differential equations which describe the interactions in grillage networks have been developed by Schilling (1925) and have been applied by a large field of researchers among which Vedeler (1945), Suhara (1960), Nielsen (1965) and Chang (1967) to whom further reference is made in this exposition. Suhara calculated a two-deck cargo ship using the classical approach to the solution of the differential equations involved while Nielsen applied the more convenient Laplace transformation technique and Chang extended the work to account for the torsional rigidity of the stiffeners and for the stability of the grillage. In the present study, use is made of Nielsen's work.

Because of the relative complexity of the technique, a brief outline of its logic is provided as a frame of reference for the more detailed discussion to be made in the next appendix.

Under the action of external hydro-loading and of any internal loading, if present, the transverse stiffeners (frames, floor and deck beams) and the longitudinal girders jointly deform and support each other. Thus, all members are mutually coupled at their intersections. The solution to the field of stress intensity in the grillage system can be obtained by simple beam theory if the reactions are known. These are obtained from the condition that, at a generic intersection, the transverse stiffener and the longitudinal girder deflect the same amount under load. The equations governing the equilibrium under load are set up as follows: Assume that the closely-spaced transverse stiffeners carry the external load $p_i(y)$, where the symbol denotes the distribution along the transverse (y) axis of the load intensity (p) on stiffener (i), see Fig. F-1. Let the deflection at any point of the transverse stiffener under the load p(y) alone be $w_i^o(y)$: and, specifically, at its intersection with the longitudinal girder j denote it by $w_i^o(j)$. Furthermore, let the corresponding deflection at j of the transverse stiffener caused by all the longitudinal girder reactions be $w_i^*(j)$. The actual deflection is, then,

$$w_i(j) = w_i^o(j) - w_i^*(j)$$

Both $w_i^o(j)$ and $w_i^*(j)$ are calculated by simple beam theory, the former from the loading $p_i(y)$ and controlling boundary conditions, the latter by application of the frame influence coefficients; thus,

$$w_i^*(j) = \sum_n \alpha_{jn} R_i(n) = w_i^o(j) - w_i(j)$$

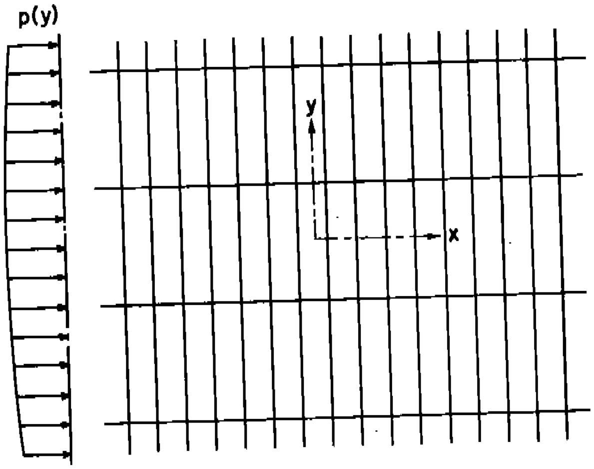

Fig. F.1. Cordinate and Index Systems

where:

α_{jn} ≡ frame influence coefficient (deflection at j caused by a unit load at n)

$R_i(n)$ ≡ loading on frame i imposed by longitudinal girder n

n ≡ index for the summation along the transverse (y) axis of the longitudinal girders

The equation governing the deflection of a longitudinal girder is

$$EI_j \frac{d^4 w_j}{dx^4} = \sum_m R_j(m) \cdot \delta(x-x_m)$$

where:

E ≡ Young's modulus of the material

I_j ≡ second central transverse moment of cross sectional area of longitudinal girder j

$R_j(m)$ ≡ loading on longitudinal girder j imposed by frame m

m ≡ index for the summation along the longitudinal (x) axis of the frame

$\delta(\,)$ ≡ Dirac delta function

The reaction is proportional to the displacement w* and is

$$R_j(m) = \frac{w_i^*(m)}{\alpha_{im}} = \frac{w_i^o - w_i(m)}{\alpha_{im}}$$

Upon multiplying the equation governing the equilibrium of the longitudinal girder by the proper influence coefficient and summing along the longitudinal axis, one has

$$\sum_m \alpha_{im} \cdot EI_m \frac{d^4 w_m}{dx^4} = \sum_m \alpha_{im} \cdot R_j(m) \cdot \delta(x-x_m)$$

$$= \sum_m [\, w_i^o(m) - w_i(m)\,] \;\delta(x - x_m)$$

Since the transverse stiffeners are closely spaced, no significant error is introduced by writing

$$\sum_m w_i(m) \cdot \delta(x-x_m) \cong \frac{w_i(x)}{a}$$

where a is the frame spacing (assumed to remain constant over the length of the longitudinal girder). This is equivalent to replacing a system of concentrated loads by a system of uniformly distributed ones. With this change, one has the Schilling equation

$$\sum_m \alpha_{im} \cdot EI_m \frac{d^4 w_m}{dx^4} + \frac{w_i(x)}{a} = \sum_m w_i^o(m) \;\delta(x - x_m)$$

A most convenient method for solving the set of differential equations is by application of the Laplace transform technique. The inverse transforms entering in the solution have been tabulated by Nielsen (1965) and by Michelsen and Nielsen (1965).

The foregoing exposition holds for the two-dimensional case. The extension to three dimensions is simply made by insuring that the index n is extended to cover all intersections at a generic frame i of longitudinal girders occurring at all deck levels.

From the foregoing it can be seen that the grillage system is resolved into two orthogonal systems of stiffeners linked by the conditions of equal deflections and of equal and opposite loading at their intersections. The solution rests fundamentally on the ability to derive the influence coefficients for the frame.

The sequential or chain linking of the longitudinal stiffeners at the boundary of bays is achieved by imposing the conditions of equal displacement and slope. This part is straightforward.

In discussing the technique, its application to a transversely framed ship will be kept in mind. This means that the frame spacing (a) is narrower than that of the longitudinal girders (b). Of course, the method is more general than this and its extension to longitudinally or mixed frame ships is readily made, but care must be taken to avoid possible confusion.

The steps involved in the process of calculating secondary stress intensities are:

a) Slope-deflection equations of the transverse stiffeners (frames).
b) Influence coefficients for the transverse stiffeners (frames).
c) Grillage deflection equations.
d) Grillage slope-deflection equations.
e) Stress intensities in the longitudinal girders.
f) Stress intensities in the transverse stiffeners (frames).

These steps are now discussed sequentially to provide a general outline of the method to be followed. Specific equations are provided in the next appendix.

F. 2 SLOPE-DEFLECTION EQUATIONS FOR THE FRAME

The deflections of the frame and the bending moment distribution to which it is subject under load can be developed by a variety of techniques of which a convenient one in the present context is that of slope-deflection. The treatment that follows is for a generic frame i and since in this step one is interested only in the transverse frames, floors and deck beams, and not in the longitudinal girders, a simplification can be made in the indexing system for temporary convenience.

The slope-deflection formulation for the frame has a three-fold purpose. It is used to calculate:

a) The influence coefficients of the frame when the latter is released from the action of the longitudinal stiffeners. The influence coefficients are required for the grillage calculation, item b .

b) The deflection of the released frame under the external water head and the deck loadings. The frame deflections at the intersections of frame and longitudinals are required for the grillage calculation, item c .

c) The unreleased frame moments once the longitudinal reactions are obtained from the grillage calculation.

The slope-deflection equations express the end moments and shear on a frame element in terms of the fixed end moments and the, as yet unknown, end slopes and deflections. The resulting equations are expressed in terms of N-functions which have been evaluated for the loadings of interest in the present study. See Table F. 1

The deflection along a generic frame of length ℓ, extending from y=-b to y=+b, is given by

$$w(u) = L(u) + w'''(0)\ N_1(u) + w''(0)\ N_2(u)$$

$$+ w'(0)\ N_3(u) + w(0)\ N_4(u)$$

Table F. 1.
N-Functions for
No Axial Load on the Beam
(After Michelsen and Nielsen, 1967)

$$N_0(\alpha, y) \equiv \frac{1}{C} [1 - N_4(\alpha, y)]$$

$$N_1(\alpha, y) \equiv \frac{\cosh(\alpha y) \sin(\alpha y) - \sinh(\alpha y) \cos(\alpha y)}{2 \alpha \sqrt{C}}$$

$$N_2(\alpha, y) \equiv \frac{\sinh(\alpha y) \sin(\alpha y)}{2 \alpha}$$

$$N_3(\alpha, y) \equiv \frac{\sinh(\alpha y) \cos(\alpha y) + \cosh(\alpha y) \sin(\alpha y)}{2 \alpha}$$

$$N_4(\alpha, y) \equiv \cosh(\alpha y) \cos(\alpha y)$$

$$N_5(\alpha y) \equiv - C N_1 (\alpha, y)$$

where:

$$C \equiv \frac{k}{E I} \qquad \alpha \equiv \frac{\sqrt[4]{C}}{\sqrt{2}}$$

$k \equiv$ foundation modulus
$E \equiv$ Young's modulus of the material
$I \equiv$ second central moment of area

where primes denote differentiation with respect to the axial coordinate. $L(u)$ is the load transform and $u \equiv b - y$. For a bending moment (M) applied at point y_1, i. e., u_1.

$$L(u) = \frac{M}{EI} N_2(u-u_1)$$

For a concentrated load (P) applied at point y_2, i. e., u_2

$$L(u) = \frac{P}{EI} N_1(u-u_2)$$

For a uniformly distributed load (W) applied over $y > y_3$, i. e., $u < u_3$

$$L(u) = \frac{W}{EI} N_0(u-u_3)$$

For a uniformly varying load (dW/dy) applied over $y > y_4$, i. e., $u < u_4$

$$L(u) = \frac{1}{EI} \frac{dW}{dy} N_{-1}(u-u_4)$$

The frame is treated as a simple beam and, for this case, the $N(\alpha, \beta, y)$ functions reduce to the well-known functions of y, namely:

$$N_{-1}(u-u_i) = (u-u_i)^5/120$$

$$N_0(u-u_i) = (u-u_i)^4/24$$

$$N_1(u-u_i) = (u-u_i)^3/6$$

$$N_2(u-u_i) = (u-u_i)^2/2$$

$$N_3(u-u_i) = (u-u_i)$$

$$N_4(u-u_i) = 1.$$

These expressions hold for $y_i > y$, i.e., $u_i < u$; otherwise the functions are zero.

The shear (Q) and bending moment (M) at the boundary y = b, i.e., u = 0 of the frame are:

$$Q(0) = Q^F(0) - \frac{E\,I}{D(u)}$$

$$\times \Big\{ w(0)\ N_3(u) - w'(0)\ N_2(u) - [N_3^2(u) - N_2(u)\ N_4(u)]\ w'(0) - [N_3(u)\ N_4(u) - N_2(u)\ N_5(u)]\ w(0) \Big\}$$

$$M(0) = M^F(0) - \frac{E\,I}{D(u)}$$

$$\times \Big\{ w'(0)\ N_1(u) - w(0)\ N_2(u) - [N_4(u)\ N_1(u) - N_2(u)\ N_3(u)]\ w'(0) - [N_1(u)\ N_5(u) - N_2(u)\ N_4(u)]\ w(0) \Big\}$$

where

$$D(u) \equiv N_1(u)\ N_3(u) - N_2^2(u)$$

The shear and bending moment at the boundary y = b, i.e., $u = \ell$, are obtained by symmetry.

The fixed end shear and moment are:

$$Q^F(0) = \frac{E\,I}{D(u)} [L(0)\ N_3(u) - L'(0)\ N_2(u)]$$

$$M^F(0) = \frac{E\,I}{D(u)} [L'(0)\ N_1(u) - L(0)\ N_2(u)]$$

where

$$L'(0) \equiv \left[\frac{dL(u)}{du}\right]_{u=0}$$

For application to grillages, it is more convenient to employ influence coefficients ($\alpha_{i,j}$) rather than foundation moduli, the former being the normalized inverse of the latter. Thus, in lieu of k, one introduces $(E I_i)/\alpha_{i,j}$ where

$$\alpha_{i,j} \equiv \frac{w^o_{i,j} - w_{i,j}}{R_j}$$

F. 3 FRAME INFLUENCE COEFFICIENT MATRIX

For a generic frame (i) the influence coefficient represents the deflection at the frame-girder intersection (i, j) caused by a unit concentrated load acting on the same frame at the point m. By Maxell's law of reciprocal deflections,

$$\alpha_{j,m} \equiv \alpha_{m,j}$$

The influence matrix $\| \alpha_{j,m} \|$ is obtained by application of the frame slope deflection equations in which all loads have been set equal to zero with the exception of the unit concentrated load at m. This operation yields the generic joint bending moment coefficient $w''_{j,m}$ and slope $w'_{j,m}$ for each concentrated load at m, where j is assigned by indexing system B.*

To obtain the frame influence coefficients, write the frame deflection. equations in the following manner, where $w'''_{j,m}(0)$ and $w_{j,m}(0)$ are determined from the condition of loading and from the boundary conditions at each joint.

$$w_{k,m}(u_k) \equiv \frac{P_m}{EI} N_1(u_k - u_m) + w'''_{j,m}(0)\, N_1(u_k) + w''_{j,m}(0)\, N_2(u_k)$$

$$+ w'_{j,m}(0)\, N_3(u_k) + w_{j,m}(0)\, N_4(u_k).$$

* The symbol ‖ ‖ is employed to denote a matrix, while [ ] is reserved for enclosing terms to be considered jointly.

where

$$u \equiv b - y$$

and

$$P_m \equiv 0 \text{ for } k \neq m$$

where k is the index of the longitudinal girder in accordance with index system C   For

$$P_m = 1, \quad w(u_k) = \alpha_{k,m}$$

F. 4 GRILLAGE DEFLECTION EOUATIONS

The system of differential equations of Schilling which describes the coupling of transverse and longitudinal stiffeners is

$$\| A_{i,j} \| \, \| w_j'''' (x) \| + \| w_j(x) \| = a \| w_j(x) \| \, \delta(x - x_j)$$

where the grillage matrix

$$\| A_{i,j} \| \equiv E\, a \| \alpha_{im} \| \, \| I_j \|$$

and

$$w_j'''' (x) \equiv \frac{d^4 w_j(x)}{dx^4}$$

A convenient way to solve this equation, which is linear and embodies constant coefficients is by application of the Laplace transform technique. This step has been carried out by Nielsen (1965) and by Nielsen and Michelsen (1965). When the grillage consists of N longitudinal stiffeners, the deflection of a generic longitudinal stiffener j is given by

$$w_j(x) = \| L_j(x) \| + \| B_1(x) \| \, \| w_j''' (0) \| + \| B_2(x) \| \, \| w_j''(0) \|$$

$$+ \| B_3(x) \| \, \| w_j'(0) \| + \| B_4(x) \| \, \| w_j(0) \|$$

$$\equiv \| L_j(x) \| + \sum_{p=1}^{4} \| B_p(x) \| \, \| w_j^{4-p}(0) \|$$

where the primes and the superscripts indicate differentiation with respect to the axial variable (x). The term $\| L_j(x) \|$ is a load matrix and the $\| B_p(x) \|$ represent matrices for the boundary value coefficients. The load matrix is

$$\| L_j(x) \| \equiv a \sum_{n=1}^{N} E(n) \sum_{m=1}^{M} w^o_{j,m} \; N_1(\lambda_n, \; x-x_m)$$

while the first matrix for the boundary value coefficients is

$$\| B_1(x) \| = \sum_{n=1}^{N} E(n) \; N_1(\lambda_n, \; x)$$

where

$$E(n) \equiv \frac{\| A^{**}_{m,j} (\lambda_n) \|}{\left| A_{m,j} \right| \prod_{\substack{m=1 \\ m=n}}^{N} (\lambda_m - \lambda_n)}$$

The remaining matrices are obtained from the relation

$$\| B_{p+1}(x) \| = \frac{d^p}{dx^p} \| B_1(x) \|$$

$$= \sum_{n=1}^{N} E(n) \frac{d^p}{dx^p} N_1(\lambda_n, \; x)$$

$$= \sum_{n=1}^{N} E(n) \; N_{1+p}(\lambda_n, \; x)$$

where N_{1+n} are given in Table F. 1. In these expressions:
$x_{m,j} \equiv$ point of application of the reaction of transverse stiffener m upon longitudinal stiffener j.

$$\| A^{**}_{m,j}(\lambda) \| \quad \text{is the adjoint of} \quad \| A^{*}_{m,j}(\lambda) \|$$

where

$$\| A^{*}_{m,j}(\lambda) \| \equiv \| A_{m,j}(\lambda) \| + \| I \|$$

is the characteristic matrix. Here the square matrix

$$\| A_{m,j}(\lambda) \| \equiv a \; EI_j \alpha_{m,j} \lambda$$

with $\alpha_{m,j}$ an influence coefficient. Also, $\| I \|$ is the identity matrix.

Denote by $\left| A_{m,j} \right|$ the determinant of the characteristic matrix, i. e., the characteristic function of the matrix $\| A_{m,j} \|$ The characteristic equation is obtained by setting this function equal to zero. This yields the roots λ_m and λ_n.

In the computational technique to be used, matrix inversion is carried out by the Cayley-Hamilton theorem.

The roots λ_m and λ_n are negative real and can be obtained by several computer techniques. Since the method of Newton's identities is used to determine the characteristic equation, the roots are conveniently established by Newton's root technique.

F. 5 GRILLAGE SLOPE - DEFLECTION EQUATIONS.

These equations are required to solve for the conditions obtaining at the boundary of two grillage systems. The equations are obtained by inverting the matrix for the slope and deflection of the longitudinal girders.

The deflection and slope at any point along a longitudinal girder is

$$\left\| \begin{matrix} \| w_j(x) \| \\ \| w_j'(x) \| \end{matrix} \right\| = \left\| \begin{matrix} \| L_1(x) \| \\ \| L_2(x) \| \end{matrix} \right\| + \left\| \begin{matrix} \| B_1(x) \| & \| B_2(x) \| \\ \| B_2(x) \| & \| B_3(x) \| \end{matrix} \right\| \left\| \begin{matrix} \| w_j'''(0) \| \\ \| w_j''(0) \| \end{matrix} \right\|$$

$$+ \left\| \begin{matrix} \| B_3(x) \| & \| B_4(x) \| \\ \| B_4(x) \| & \| B_5(x) \| \end{matrix} \right\| \left\| \begin{matrix} \| w_j'(0) \| \\ \| w_j(0) \| \end{matrix} \right\|$$

The shear and bending moment coefficients $w_j'''(0)$ and $w_j''(0)$ are expressed in terms of the slopes and deflections at both ends as follows:

$$\left\| \begin{matrix} \| w_j'''(0) \| \\ \| w_j''(0) \| \end{matrix} \right\| = \left\| \begin{matrix} \| B_1(\ell) \| & \| B_2(\ell) \| \\ \| B_2(\ell) \| & \| B_3(\ell) \| \end{matrix} \right\|^{-1}$$

$$\times \left\{ - \left\| \begin{matrix} \| B_3(\ell) \| & \| B_4(\ell) \| \\ \| B_4(\ell) \| & \| B_5(\ell) \| \end{matrix} \right\| \left\| \begin{matrix} \| w_j'(0) \| \\ \| w_j(0) \| \end{matrix} \right\| - \left\| \begin{matrix} \| L_1(\ell) \| \\ \| L_2(\ell) \| \end{matrix} \right\| + \left\| \begin{matrix} \| w(\ell) \| \\ \| w'(\ell) \| \end{matrix} \right\| \right\}$$

For fixed end conditions, the shear and bending moment coefficients become

$$\left\| \begin{matrix} \| w_j'''(0) \| \\ \| w_j''(0) \| \end{matrix} \right\|^{F} = - \left\| \begin{matrix} \| B_1(\ell) \| & \| B_2(\ell) \| \\ \| B_2(\ell) \| & \| B_3(\ell) \| \end{matrix} \right\|^{-1} \left\| \begin{matrix} \| L_1(\ell) \| \\ \| L_2(\ell) \| \end{matrix} \right\|$$

where the F superscript denotes fixity.

For notational and programming convenience, define partitioned matrices of equal order submatrices such that:

$$\begin{Vmatrix} \|D1\| & \|D3\| \\ \|D2\| & \|D4\| \end{Vmatrix} \equiv \begin{Vmatrix} \|B_1(\ell)\| & \|B_2(\ell)\| \\ \|B_2(\ell)\| & \|B_3(\ell)\| \end{Vmatrix}^{-1} \begin{Vmatrix} \|B_3(\ell)\| & \|B_4(\ell)\| \\ \|B_4(\ell)\| & \|B_5(\ell)\| \end{Vmatrix}$$

$$\begin{Vmatrix} \|D5\| & \|D6\| \\ \|D6\| & \|D7\| \end{Vmatrix} \equiv \begin{Vmatrix} \|B_1(\ell)\| & \|B_2(\ell)\| \\ \|B_2(\ell)\| & \|B_3(\ell)\| \end{Vmatrix}^{-1}$$

This substitution results in the grillage slope - deflection equations for the shear and bending moment coefficients given by

$$\begin{Vmatrix} \|w_j'''(0)\| \\ \|w_j''(0)\| \end{Vmatrix} = - \begin{Vmatrix} \|D1\| & \|D3\| \\ \|D2\| & \|D4\| \end{Vmatrix} \begin{Vmatrix} \|w_j'(0)\| \\ \|w_j(0)\| \end{Vmatrix}$$

$$+ \begin{Vmatrix} \|D5\| & \|D6\| \\ \|D6\| & \|D7\| \end{Vmatrix} \begin{Vmatrix} \|w_j(\ell)\| \\ \|w_j'(\ell)\| \end{Vmatrix} + \begin{Vmatrix} \|w_j'''(0)\| \\ \|w''(0)\| \end{Vmatrix}^F$$

The grillage deflection equations yield the deflection obtaining at any point along the span of the longitudinal girder in terms of the four boundary conditions at a support. The grillage slope - deflection equations express two of these boundary conditions (the shear and bending moment coefficients) in terms of the deflection and slope at the same boundary. If these are known a priori or can be established, the deflection is determined over the full span.

F. 6 STRESS INTENSITIES IN THE LONGITUDINAL GIRDERS

The analysis so far results in determining the deflection over the span of the slope and the shear and bending moment coefficients at the supports. Differentiation of the deflection yields the span - wise distribution of shear and bending moment. Given the section modulus of the longitudinal girder, the stress intensity field in the longitudinal girder is immediately derivable.

F. 7 STRESS INTENSITIES IN THE FRAMES

The reactions at the intersections of frames and girders are determined from the compatibility relation

$$\| R_{i,j} \| = \| \alpha_{i,j} \|^{-1} \| w^o_{i,j} - w_{i,j} \|$$

This relation plus the knowledge of the shear and bending moment at the boundaries of the grillage permits determination of the spanwise distribution of the grillage deflection $w_{i,j}$. The support bending moment and shear are then derived by the frame slope - deflection equations. Given the section modulus of the frame the stress intensity field is calculated directly for each element by ordinary beam equations

$$w''(u) = P''(u) + w'''(0)\ N_3(u) + w''(0)\ N_4(u)$$

APPENDIX G

GRILLAGE ANALYSIS

SPECIFIC APPLICATION TO THE S. S. WOLVERINE STATE.

G. 1 INTRODUCTION

Fig. G-1 illustrates the frame arrangement of the WOLVERINE STATE. It consists of 15 members on each side of the keel for the closed sections adjacent to the transverse bulkheads and of 11 members for the open sections in way of the hatch. For the optimization program, however, the number of longitudinal girders in the double bottom and supporting the decks is made flexible.

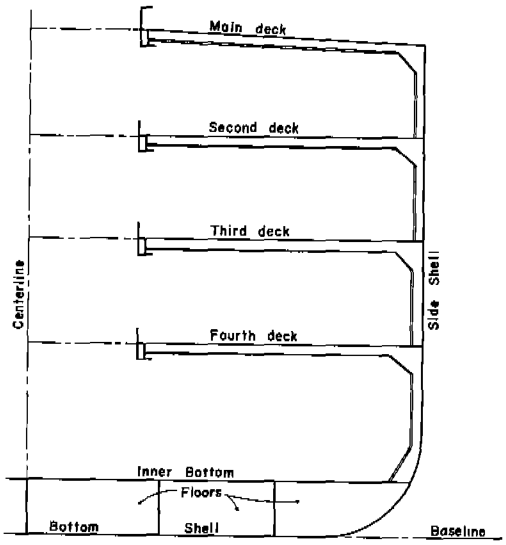

Fig. G.1. Framing Arrangement of *Wolverine State*

The general equations of the preceding appendix are now related to the arrangement in hand.

G - 2 SLOPE - DEFLECTION EQUATIONS FOR THE FRAME

A separate frame analysis is made depending upon whether the frame is located in a bulkhead section or in the open hatch section.

The only joints of interest occur at the intersection of frame and decks. Denote these in numerical sequence starting from the bottom.

Following the usual sign convention for slope-deflection equations, slopes are considered to be positive if clock-wise when measured from the zero deflection line. Likewise, moments are positive if they induce the same direction of rotation. Shear forces (Q) follow the usual beam convention, namely

$$Q = - E I \frac{d^2 w}{d x^2}$$

The slope-deflection equation gives the moment $M_{n/n+1}$ at a joint as

$$M_{n/n+1} = M^F_{n/n+1} + 2\,E\,K_{n/n+1}\left[2\theta_n + \theta_{n+1} - 3\psi_{n/n+1}\right]$$

where:

M^F ≡ fixed end moment
K ≡ stiffness factor

$$K_{n/n+1} \equiv \frac{I_{n/n+1}}{[y_{n+1} - y_n]} \quad \text{or} \quad \frac{I_{n/n+1}}{[z_{n+1} - z_n]}$$

$\left.\begin{matrix} y_{n+1} - y_n \\ z_{n+1} - z_n \end{matrix}\right\}$ ≡ length of stiffener (between joints)

θ ≡ joint rotation, positive clockwise
ψ ≡ joint displacement, positive upwards. When the section is loaded symmetrically, the joint displacement is zero.
n ≡ joint index

As to the fixed end moment, two cases are of interest: a uniformly distributed load of intensity Q gives

$$M^F_{n/n+1} = \frac{1}{12}\,Q\,[y_{n+1} - y_n]^2 \qquad \text{etc.}$$

while a concentrated load of magnitude P gives

$$M^F_{n/n+1} = P\,\frac{[y_p - y_n]\,[y_{n+1} - y_p]}{[y_{n+1} - y_n]^2}$$

where y_p is the location of the load, see Fig. G. 2. At an intersection, P will be the algebraic sum of the reactions R_{jk} and the concentrated load P_{jk}.

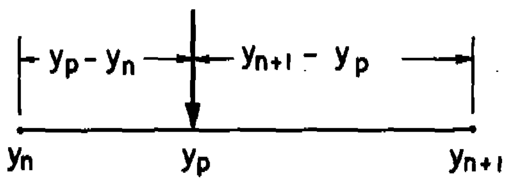

Fig. G.2. Definition Sketch

Certain observations are pertinent prior to applying the foregoing equations to the ship to be analyzed:

a) For this analysis, the loading over the whole section is symmetric about the centerplane, hence the relative joint displacement (ψ) is zero. Thus, to determine the influence coefficients it is sufficient to consider only half the section.

b) Since the ship structure is assumed to behave elastically, node rotations θ_k are identical for all elements intersecting at a node (k).

G. 2. 1 Section through a Hatchway (Open Section)

For the arrangement of the WOLVERINE STATE there are 13 bending moments to be determined. These moments are expressed in terms of the five joint rotations at the nodes, θ_k. The bending moments are written:

a) For the deck beams:

$$M_{k/k} = Q_k \cdot \frac{1}{2}\left[\frac{B}{2} - y_h\right]^2 + \sum_j \left[P_{j,k} - R_{j,k}\right] \left[\frac{B}{2} - y_{j,k}\right]$$

where:

$M_{k/k}$ ≡ bending moment on the deck beam at node k

Q_k ≡ uniformly distributed load on level k

y_h ≡ width of hatchway

$R_{j,k}$ ≡ reaction of joint j, k

$P_{j,k}$ ≡ concentrated load at joint j, k

For the design in hand, k = 2 through 5.

b) For the side frames:

$$M_{5/4} = 2\ E\ K_{5/4}\ \ 2\ \theta_5 + \theta_4 - M^F_{5/4}$$

$$M_{4/5} = 2\ E\ K_{4/5}\ \ 2\ \theta_4 + \theta_5 + M^F_{4/5}$$

$$M_{4/3} = 2\ E\ K_{4/3}\ \ 2\ \theta_4 + \theta_3 - M^F_{4/3}$$

$$M_{3/4} = 2\ E\ K_{3/4}\ \ 2\ \theta_3 + \theta_4 + M^F_{3/4}$$

$$M_{3/2} = 2\ E\ K_{3/2}\ \ 2\ \theta_3 + \theta_2 - M^F_{3/2}$$

$$M_{2/3} = 2\ E\ K_{2/3}\ \ 2\ \theta_2 + \theta_3 + M^F_{2/3}$$

$$M_{2/1} = 2\ E\ K_{2/1}\ \ 2\ \theta_2 + \theta_1 - M^F_{2/1}$$

$$M_{1/2} = 2\ E\ K_{1/2}\ \ 2\ \theta_1 + \theta_2 + M^F_{1/2}$$

where $M_{k/k\pm1}$ is the bending moment at node k on the side frame extending from k to $k\pm1$.

In these equations $M^F_{k/k\pm1}$ is the fixed end moment corresponding to the external head of water. If the external load in lb in^{-1} at the node k be defined by H_k , the fixed end moment has the following expressions depending upon whether the external head is lower or higher than the node k.

If the external head is lower than the node,

$$M^F_{k/k-1} = \frac{-H_{k-1}[H_k - z_{k-1}]^2}{(z_k)^2}\left[\frac{z_k}{12} - \frac{H_k - z_{k-1}}{20}\right]$$

$$M^F_{k-1/k} = H_{k-1}[H_k - z_{k-1}]^2$$

$$\left\{\frac{(z_k)^2}{6} + \frac{[H_k - z_{k-1}]^2}{20} - \frac{z_k[H_k - z_{k-1}]}{6}\right\}$$

The factor K is the structural stiffness and is given by

$$K_{k/k-1} \equiv \frac{I_{k/k-1}}{z_k - z_{k-1}}$$

where $I_{k/k-1}$ is the second central moment of cross sectional area of frame and effective shell plating. Of course,

$$K_{k/k-1} \equiv K_{k-1/k}$$

c) For the floors:

$$M_{1/1} = \frac{2 E I_1 \theta_1}{B} + \sum_j [P_{j,1} - R_{j,1}]$$

$$\left\{\frac{\frac{B}{2}^2 - \left[\frac{B}{2} - y_{j,1}\right]^2}{B}\right\} + \frac{1}{12} Q_1 B^2$$

If the external head is higher than the node,

$$M^F_{k/k-1} = -[H_{k-1} - H_k]\frac{z_k}{30} - H_k \frac{(z_k)^2}{3}$$

$$M^F_{k-1/k} = [H_{k-1} - H_k] \frac{(z_k)^2}{20} + H_k \frac{(z_k)^2}{3}$$

For equilibrium, the sum of the moments at each node must equal zero, hence,

$$M_{5/5} + M_{5/4} = 0$$

$$M_{4/4} + M_{4/5} + M_{4/3} = 0$$

$$M_{3/3} + M_{3/4} + M_{3/2} = 0$$

$$M_{2/2} + M_{2/3} + M_{2/1} = 0$$

$$M_{1/1} + M_{1/2} = 0$$

G. 2. 2 Section adjacent to a Bulkhead (Closed Section)

The equations for the closed section are similar to those for the open section except that account must now be taken of the condition that the deck beams are no longer cantilevered. This results in the following changed expressions for the bending moments acting on the deck beams:

$$M_{k/k} = \frac{1}{12} Q_k B^2$$

$$+ \sum_j [P_{j,k} - R_{j,k}] \frac{\left[\frac{B}{2}^2 - \frac{B}{2} - y_{j,k}\right]^2}{B}$$

where $k = 2$ through 5.

For equilibrium,

$$M_{5/5} + M_{5/4} = 0$$

$$M_{4/4} + M_{4/5} + M_{4/3} = 0$$

$$M_{3/3} + M_{3/4} + M_{3/2} = 0$$

$$M_{2/2} + M_{2/3} + M_{2/1} = 0$$

$$M_{1/1} + M_{1/2} = 0$$

Solution of these systems of linear equations yields the five rotations θ_k.

Knowledge of the slopes at each node leads directly to the nodal moments M_k. These, along with the loading, yield the shear force Q_k

at each node. Application of beam theory gives the deflection, slope, bending moment and shear force distribution along each element. Derivation of the specific expressions employed in the analysis is as follows:

G. 2. 3 Horizontal or Deck Elements

Consider a generic deck beam or floor at level k and extending transversely from point j-1 to point j. Its length is given by

$$(y_j - y_{j-1})_k$$

Other definitions are given in Fig. G. 3 The external loadings acting on the element are

$$P_{j-1/j,k} \qquad Q_{j-1/j,k} \qquad \text{and } R_{j-1/j,k}$$

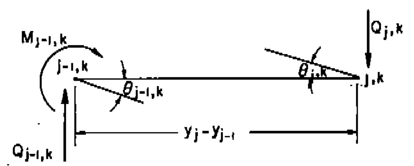

Fig. G.3. Definition Sketch

The sum of moments about the joint j is

$$M_{j-1,k} + Q_{j-1,k}\,[y_{j,k} - y_{j-1,k}]$$

$$- [P_{j-1,k} - R_{j-1,k}] \cdot [y_{j,k} - y_{j-1,k}]$$

$$- Q_k \frac{[y_{j,k} - y_{j-1,k}]^2}{2} = 0$$

Thus,

$$Q_{j-1,k} = - \frac{M_{j-1,k}}{y_{j,k} - y_{j-1,k}} + P_{j-1,k} - R_{j-1,k}$$

$$+ \frac{1}{2} Q_k [y_{j,k} - y_{j-1,k}]$$

The deflection equation is, then

$$w_{j,k} = \frac{1}{E\, I_{k/k}} \left\{ \left[- Q_{j-1,k} + \sum_j [P_{j-1,k} - R_{j-1,k}] \right] \frac{[y_{j,k} - y_{j-1,k}]^3}{6} \right.$$

$$+ Q_{j-1,k} \frac{[y_{j,k} - y_{j-1,k}]^4}{24} - M_{j-1,k} \frac{[y_{j,k} - y_{j-1,k}]^2}{2}$$

$$+ \quad \theta_{j-1.k} \quad [y_{j,k} - y_{j-1,k}] \Bigg\}$$

where the summation is from j=1 to the joint of interest. While the bending moment equation is

$$E\, I_{k/k} \quad w_{j,k} =$$

$$\left[Q_{j-1,k} - \sum_{j} [P_{j-1,k} - R_{j-1,k}] \right] [y_{j,k} - y_{j-1,k}]$$

$$- \; Q_k \quad \frac{[y_{j,k} - y_{j-1,k}]^2}{2} \quad + \quad M_{j-1,k}$$

G. 2. 4 Vertical or Side Elements

Considering a generic vertical frame extending from level k to k+1. Its height is

$$z_{k+1} - z_k$$

Other definitions are given in Fig. G. 4. The lateral loading on the side frame is due to the external head of water and its distribution at

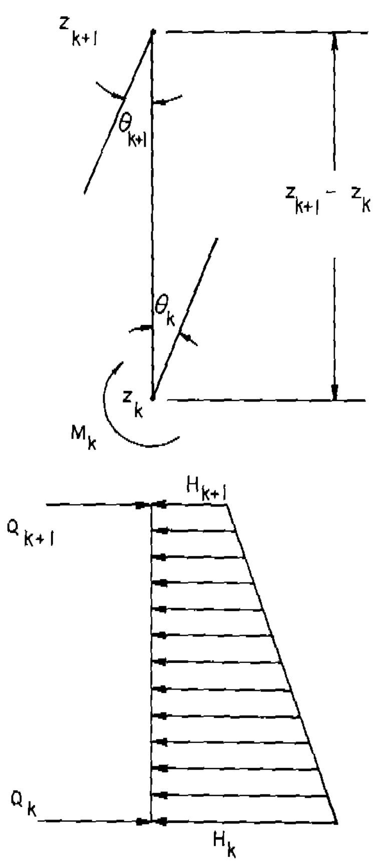

Fig. G. 4. Definition Sketch

node k is written H_k etc.

The shear force results from simple statics as

$$Q_k = \pm \frac{1}{2} H_{k+1} [z_{k+1} - z_k] + \frac{1}{3} [H_k \mp H_{k+1}] [z_{k+1} - z_k]$$

While the bending moment is

$$M_{k/k+1} = M_k + Q_k [z_{k+1} - z_k] \pm \frac{1}{2} H_{k+1} [z_{k+1} - z_k]^2$$

$$+ \frac{1}{6} [H_k \mp H_{k+1}] [z_{k+1} - z_k]^3$$

G. 3 FRAME INFLUENCE COEFFICIENTS

By way of exposition, consider the case of a deck longitudinal in a hatch bay. A unit load placed at the edge of the hatchway on the main deck gives the following nodal moments when all other loads are equal to zero (note that the B system of indexing is employed):

$$M_{5/-5} = \frac{B}{2} - y_{4/2,5}$$

$$M_{4/-4} = 0$$

$$M_{3/-3} = 0$$

$$M_{2/-2} = 0$$

$$M_{5/4} = 2 E K_{5/4} \| 2\theta_5 + \theta_4 \|$$

$$M_{4/5} = 2 E K_{4/5} \| 2\theta_4 + \theta_5 \|$$

$$M_{4/3} = 2 E K_{4/3} \| 2\theta_4 + \theta_3 \|$$

$$M_{3/4} = 2 E K_{3/4} \| 2\theta_3 + \theta_4 \|$$

$$M_{3/2} = 2 E K_{3/2} \| 2\theta_3 + \theta_2 \|$$

$$M_{2/3} = 2 E K_{2/3} \| 2\theta_2 + \theta_3 \|$$

$$M_{2/1} = 2 E K_{2/1} \| 2\theta_2 + \theta_1 \|$$

$$M_{1/2} = 2 E K_{1/2} \| 2\theta_1 + \theta_2 \|$$

$$M_{1/1} = -2 E I_1 \theta_1 / B$$

Inversion of the moment matrix leads to the determination of the nodal slopes θ_k, and these, in turn, yield the nodal moments. The equilibrium conditions for the hatch bay give the following set of equations:

$$E \begin{Vmatrix} 4K_{4/5} & 2K_{4/5} & 0 & 0 & 0 \\ 2K_{4/5} & 4[K_{4/5}+K_{3/4}] & 2K_{3/4} & 0 & 0 \\ 0 & 2K_{3/4} & 4[K_{3/4}+K_{2/3}] & 2K_{2/3} & 0 \\ 0 & 0 & 2K_{2/3} & 4[K_{2/3}+K_{1/2}] & 2K_{1/2} \\ 0 & 0 & 0 & 2K_{1/2} & \frac{2I_1}{B}+4K_{1/2} \end{Vmatrix}$$

$$\begin{Vmatrix} \theta_5 \\ \theta_4 \\ \theta_3 \\ \theta_2 \\ \theta_1 \end{Vmatrix} = \begin{Vmatrix} F_k \end{Vmatrix}$$

which can be denoted by

$$\begin{Vmatrix} \text{Moment matrix} \end{Vmatrix} \begin{Vmatrix} \theta_k \end{Vmatrix} = \begin{Vmatrix} F_k \end{Vmatrix}$$

where $\begin{Vmatrix} F_k \end{Vmatrix}$ is the joint work matrix. This matrix has the following expressions:

a) For the α_{77} influence coefficients

$$\begin{Vmatrix} F_k \end{Vmatrix} = \begin{Vmatrix} b' \\ 0 \\ 0 \\ 0 \\ 0 \end{Vmatrix}$$

b) For the α_{76} influence coefficients

$$\left\| F_k \right\| = \left\| \begin{matrix} 0 \\ b' \\ 0 \\ 0 \\ 0 \end{matrix} \right\|$$

c) For the α_{75} influence coefficients

$$\left\| F_k \right\| = \left\| \begin{matrix} 0 \\ 0 \\ b' \\ 0 \\ 0 \end{matrix} \right\|$$

d) For the α_{74} influence coefficients

$$\left\| F_k \right\| = \left\| \begin{matrix} 0 \\ 0 \\ 0 \\ b' \\ 0 \end{matrix} \right\|$$

e) For α_{71} influence coefficients

$$\left\| F_k \right\| = \left\| \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{\frac{B}{2} - (b')^2}{B} \end{matrix} \right\|$$

In these expressions

$$b' \equiv \frac{B}{2} - y_h$$

where y_h is the halfwidth of the hatchway.

The influence coefficients read as follows:

$$\alpha_{77} = \left\{ \frac{1}{2} M_{5/-5} (b')^2 + V_5 (b')^3 \right\} \frac{1}{E I_7} + \theta_5 b'$$

$$\alpha_{67} = \theta_4 b'$$

$$\alpha_{57} = \theta_3 b'$$

$$\alpha_{47} = \theta_2 b'$$

$$\alpha_{17} = -\frac{1}{EI_1} M_{1/1} \left[\frac{B}{2} \right]^2 + \theta_1 \frac{B}{2}$$

where

$$M_{1/1} = \frac{B}{2} E I_1 \theta_1$$

Note that in the foregoing expressions two systems of indexing have been employed: B and C. The influence coefficients on the left hand side of the equations follow the latter system, while the moments, shear and rotations on the right side of the equation are in the former system of indexing. Such heterogenous indexing poses no problem to the computer.

Similar influence coefficient arrays are derived by placing a unit load at 6, 5, 4 and 1 in succession. These five arrays make up the square matrix of influence coefficients.

G. 3. 1 Released Frame Deflections

The deflections of the frame released from the action of the longitudinal girders are obtained by application of the slope-deflection equations for the frame supported only by primary structure and acted upon by the live load on the deck and the external head of water on the sides and bottom. Thus, the loadings imposed by the longitudinal girders are set to equal to zero. The derived joint deflections $w_i''(j)$ are subsequently used in the grillage calculation. The procedure is the same as for the influence coefficients, but the result is a somewhat different expression for the work matrix which reflects the condition that the loads are no longer of unit magnitude but have, instead, a specific distribution. The work matrix is now expressed as

$$\left\| F_k \right\| = \left\| \begin{matrix} B_5 - M^F_{5/4} \\ B_4 + M^F_{4/5} - M^F_{4/3} \\ B_3 + M^F_{3/4} - M^F_{3/2} \\ B_2 + M^F_{2/3} - M^F_{2/1} \\ B_1 + M^F_{1/2} + \frac{1}{12} Q_1 \; B^2 + f\,[P_1] \end{matrix} \right\|$$

where f (P_1) is the bending moment resulting from the action of all concentrated loads on the bottom frame and where B_k is the component of bending moment related to the external loading alone. The first symbol is expressed as

$$f(P_1) \equiv \sum_j P_{1,j} \frac{\left[\frac{B}{2}\right]^2 - \left[\frac{B}{2} - y_{1,j}\right]^2}{B}$$

where $P_{1,j}$ and $y_{1,j}$ refer respectively to the concentrated load and its transverse location. The second symbol has two expressions depending on the value of k. For k = 1,

$$B_k \equiv \frac{1}{2} Q_k \; b' + \sum_j P_{i,k} \left[\frac{B}{2} - y_{k,j}\right]$$

For k = 1

$$B_1 \equiv \frac{1}{2} Q_1 \frac{B}{2} + \sum_j P_{1,j} \left[\frac{B}{2} - y_{1,j}\right]$$

Given this matrix, the slopes θ_k are obtained by inversion of the moment matrix. From these the joint moments, joint shear forces and the intersection deflections at each longitudinal are immediately derived.

G. 3. 2 Frame Bending Moments

The grillage calculation yields the reactions at the intersections. The frame slope-deflection equations yield in turn the moments of the supported frame caused by the external head by the live loads on the decks and by the reactions imposed by the longitudinal girders. The work matrix now becomes

$$\left\| F_k \right\| = \left\| \begin{matrix} B_5 - M^F_{5/4} \\ B_4 + M^F_{4/5} - M^F_{4/3} \\ B_3 + M^F_{3/4} - M^F_{3/2} \\ B_2 + M^F_{2/3} - M^F_{2/1} \\ B_1 + \frac{1}{12} Q_1 B^2 + f(P_1, R_1) \end{matrix} \right\|$$

where, for k = 1

$$B_k \equiv \frac{1}{2} Q_k b' + \sum_j [P_{j,k} - R_{j,k}] \left[\frac{B}{2} - y_{j,k} \right]$$

G. 4 SOLUTION OF THE SLOPE-DEFLECTION EQUATIONS.

The following solution is obtained on the assumption that the longitudinals are fixed at the bulkheads and that the structure is longitudinally symmetric both in geometry and in loadings, see Fig. G-5 Thus, the following set of boundary conditions obtains at the bulkheads:

$$\left\| w_j(0) \right\| = \left\| w_j(\ell) \right\| = 0$$

$$\left\| w_j'(0) \right\| = \left\| w_j'(\ell) \right\| = 0$$

In addition, the sums of forces and moments are both zero at x_1 and x_2.

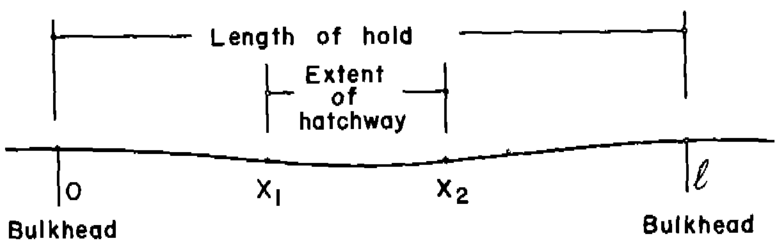

Fig. G.5. Definition Sketch

G. 4. 1 Solution for the Hatch Bay.

The origin of the hatch bay is taken at x_1. The following conditions apply at the boundaries thereof:

$$\left\| w_j(x_1) \right\| = \left\| w_j(x_2) \right\|$$

$$\left\| w_j'(x_1) \right\| = - \left\| w_j'(x_2) \right\|$$

$$+ \left\| \begin{matrix} \| w_j'''(x_1) \| \\ \| w_j''(x_1) \| \end{matrix} \right\|_h^F$$

where the subscript h denotes pertaining to the hatch bay and the superscript F denotes fixity. The corresponding solution for $w_j''(x_2)$ and $w_j'''(x_2)$ is obtained from the relations:

$$\| w_j''(x_2) \| = \| - w_j''(x_1) \|$$

$$\| w_j'''(x_2) \| = \| - w_j'''(x_1) \|$$

The shear forces and bending moments at x_1 and x_2 are:

$$\left\| \begin{matrix} \| Q_j(x_1) \| \\ \| M_j(x_1) \| \end{matrix} \right\|_h = - E \| I_j \|_h \left\| \begin{matrix} \| w_j'''(x_1) \| \\ \| w_j''(x_1) \| \end{matrix} \right\|_h$$

$$\left\| \begin{matrix} \| Q_j(x_2) \| \\ \| M_j(x_2) \| \end{matrix} \right\|_h = - E \| I_j \|_h \left\| \begin{matrix} \| w_j'''(x_2) \| \\ \| w_j''(x_2) \| \end{matrix} \right\|_h$$

G. 4. 1 Solution for the Bulkhead Bay

To preserve positive directions, the origin of the bulkhead bay is taken at x_2 and the equations are set up for the region (x_2, ℓ). The bay from $x = 0$ to x_1 is solved by symmetry.

The shear force and bending moment coefficients are:

$$\left\| \begin{matrix} \| w_j'''(x_1) \| \\ \| w_j''(x_1) \| \end{matrix} \right\|_h = \left\| \begin{matrix} \| - \| D3 \| + \| D5 \| \|, & \| - \| D1 \| - \| D6 \| \| \\ \| - \| D4 \| + \| D6 \| \|, & \| - \| D2 \| - \| D7 \| \| \end{matrix} \right\|_h \left\| \begin{matrix} \| w_j(x_1) \| \\ \| w_j(x_1) \| \end{matrix} \right\|_h$$

The boundary conditions at the ends of a symmetrically disposed hatchway are:

$$\left\| w_j(x_1) \right\| = \left\| w_j(x_2) \right\|$$

$$\left\| w_j'(x_1) \right\| = -\left\| w_j'(x_2) \right\|$$

while at the ends of the hold

$$\left\| w_j(0) \right\| = \left\| w_j(\ell) \right\|$$

Further, it is assumed that

$$\left\| w_j'(0) \right\| = -\left\| w_j'(\ell) \right\|$$

The coefficients of shear force and bending moment at x_2 are:

$$\left\| \begin{matrix} \| w_j'''(x_2) \| \\ \| w_j''(x_2) \| \end{matrix} \right\|_b = - \left\| \begin{matrix} \|D3\| & \|D1\| \\ \|D4\| & \|D2\| \end{matrix} \right\|_b \left\| \begin{matrix} \| w_j(x_2) \| \\ \| w_j'(x_2) \| \end{matrix} \right\|_b + \left\| \begin{matrix} \| w_j'''(x_2) \| \\ \| w_j''(x_2) \| \end{matrix} \right\|_b^F$$

where the subscript b indicates pertaining to the bulkhead. The shear force and bending moment at x_2 are, consequently:

$$\left\| \begin{matrix} \| Q_j(x_2) \| \\ \| M_j(x_2) \| \end{matrix} \right\|_b = - E \left\| I_j \right\|_b \left\| \begin{matrix} \| w_j'''(x_2) \| \\ \| w_j''(x_2) \| \end{matrix} \right\|_b$$

The coefficients of shear force and bending moment at $x = \ell$ are:

$$\left\| \begin{matrix} \| w_j'''(\ell) \| \\ \| w_j''(\ell) \| \end{matrix} \right\|_b = \left\| \begin{matrix} \|D5\| & \|D6\| \\ \|D6\| & \|D7\| \end{matrix} \right\|_b \left\| \begin{matrix} \| w_j(x_2) \| \\ \| w_j'(x_2) \| \end{matrix} \right\|_b + \left\| \begin{matrix} \| w_j'''(\ell) \| \\ \| w_j''(\ell) \| \end{matrix} \right\|_b^F$$

The shear force and bending moment at $x = \ell$ are, therefore:

$$\left\| \begin{matrix} \| Q_j(\ell) \| \\ \| M_j(\ell) \| \end{matrix} \right\|_b = - E \left\| I_j \right\|_b \left\| \begin{matrix} \| w_j'''(\ell) \| \\ \| w_j''(\ell) \| \end{matrix} \right\|_b$$

G. 4. 3 Joint Solution

The unknowns $\left\| w_j(x_2) \right\|$ and $\left\| w_j'(x_2) \right\|$ are obtained from the conditions of compatibility

$$\left\| w_j(x_2) \right\|_h = \left\| w_j(x_2) \right\|_b$$

$$\left\| w_j'(x_2) \right\|_h = \left\| w_j'(x_2) \right\|_b$$

and of equilibrium

$$\left\| w_j''(x_2) \right\|_h = \left\| w_j''(x_2) \right\|_b$$

$$\left\| w_j'''(x_2) \right\|_h = \left\| w_j'''(x_2) \right\|_b$$

These yield:

$$\left\| \begin{matrix} \left\| -\left\| D3 \right\|_h + \left\| D5 \right\|_h - \left\| D3 \right\|_b \right\|, & \left\| -\left\| D1 \right\|_h + \left\| D6 \right\|_h - \left\| D1 \right\|_b \right\| \\ \left\| -\left\| D4 \right\|_h + \left\| D6 \right\|_h - \left\| D4 \right\|_b \right\|, & \left\| -\left\| D2 \right\|_h + \left\| D7 \right\|_h - \left\| D2 \right\|_b \right\| \end{matrix} \right\|$$

$$\left\| \begin{matrix} \left\| w_j(x_2) \right\| \\ \left\| w_j'(x_2) \right\| \end{matrix} \right\| - \left\| \begin{matrix} \left\| w_j'''(x_2) \right\|_h + \left\| w_j''(x_2) \right\|_b \\ \left\| w_j''(x_2) \right\|_h + \left\| w_j''(x_2) \right\|_b \end{matrix} \right\|^F = 0$$

Finally, $\left\| w_j(x_2) \right\|$ and $\left\| w_j'(x_2) \right\|$ are obtained by inverting the left-hand matrix. The joint forces are obtained by introducing these values into the slope-deflection equations for the hatch and bulkhead bays.

G. 5 GRILLAGE ELEMENT SOLUTION

The deflection equation for the grillage in the bulkhead bay ($0 \le x \le x_1$ and $x_2 < x < \ell$) results as

$$\left\| w_j(x) \right\|_b = \left\| L_1(x) \right\| - \left\| B_1(x) \right\| \left\| w_j'''(\ell) \right\| + \left\| B_2(x) \right\| \left\| w_j''(\ell) \right\|$$

$$- \left\| B_3(x) \right\| \left\| w_j'(x_2) \right\| + \left\| B_4(x) \right\| \left\| w_j(x_2) \right\|$$

while the bending moment equation is

$$- E \left\| I_j \right\|_b \left\| w_j''(x) \right\|_b = - E \left\| I_j \right\|_b \Big\{ \left\| L_3(x) \right\| - \left\| B_3(x) \right\| \left\| w_j''(\ell) \right\|$$

$$+ \left\| B_4(x) \right\| \left\| w_j''(\ell) \right\| - \left\| B_5(x) \right\| \left\| w_j'(x_2) \right\|$$

$$+ \left\| B_6(x) \right\| \left\| w_j(x_2) \right\| \Big\}$$

Similarly, the deflection equation for the grillage in the hatch bay ($x_1 \le x \le x_2$) is

$$\left\| w_j(x) \right\|_h = \left\| L_1(x) \right\| + \left\| B_1(x) \right\| \left\| w_j'''(x_1) \right\| \left\| B_2(x) \right\| \left\| w_j''(x_1) \right\|$$

$$+ \left\| B_3(x) \right\| \left\| w_j'(x_1) \right\| \left\| B_4(x) \right\| \left\| w(x_1) \right\|$$

while the bending moment equation becomes.

$$E \left\| I_j \right\|_h \left\| w_j''(x) \right\|_h = - E \left\| I_j \right\|_h \Big\{ \left\| L_3(x) \right\| + \left\| B_3(x) \right\| \left\| w_j'''(x_1) \right\|$$
$$+ \left\| B_4(x) \right\| \left\| w_j''(x_1) \right\| + \left\| B_5(x) \right\| \left\| w_j'(x_2) \right\|$$
$$+ \left\| B_6(x) \right\| \left\| w_j(x_2) \right\| \Big\}$$

G. 6 BENDING MOMENTS AND STRESS INTENSITIES IN THE FRAMES

It is now possible to determine the frame reactions $R_{i,j}$. These are calculated from

$$\left\| R_{i,j} \right\| = \left\| \alpha_{i,j} \right\| \left\| w^o_{i,j} - w_{i,j} \right\|$$

where:

$\alpha_{i,j}$ ≡ influence coefficient array of the released frame

$w^o_{i,j}$ ≡ deflection array of the release frame

$w_{i,j}$ ≡ grillage deflection at the intersection i, j

With the reactions in hand, the frame can be analyzed as a two-dimensional beam as described in Section F. 2.

APPENDIX H

BASIC INPUTS

1. HULL GEOMETRY

The basic geometric features of the hull for which a structure is to be synthesized are listed in Table H-1. The basic geometry of the midship section is shown in Fig. G. 1

The hull structural arrangement is described by the following equations. (All dimensions in inches).

1.1 Midship Section

a) Shell

$z = 0 \qquad 0 \leq y \leq 309$

$z = \sqrt{1200 - [y - 309]^2} \qquad 309 \leq y \leq 434, \quad 0 \leq z \leq 120$

$y = 434 \qquad 120 \leq z \leq 522$

b) Inner Bottom

$z = 60 \qquad 0 \leq y \leq 413$

c) Fourth Deck

$z = 204 \qquad 0 \leq y \leq 434$

d) Third Deck

$z = 312 \qquad 0 \leq y \leq 432$

e) Second Deck

$z = 420 \qquad 0 \leq y \leq 434$

f) Main Deck

$z = 534 \qquad 0 \leq y \leq 120$

$z = 534 - 0.0388\,[y - 120] \qquad 120 \leq y \leq 434$

Table H-1

Basic Geometric Features of
S.S. Wolverine State

Type

C4-S-B5 Machinery aft, dry cargo vessel

Principal Dimensions

Length, overall (ft)	520 (6240 in)
Length, between perpendiculars (ft)	496 (5952 in)
Beam, molded (ft)	71.5 (868 in)
Depth, molded (ft)	43.5 (522 in)

Design Condition

Draft, molded (ft)	30.0 (360 in)
Displacement (tons)	20,000
Block coefficient	0.654
Longitudinal coefficient	0.664
Waterplane	0.752

Midship Section Particulars

Half-girth to upper deck, molded (ft)	75.8 (910 in)

The longitudinal extent of the hatchway is approximately one-third of hold length.

The frames and longitudinal girders are assumed to be uniformly spaced.

2. DESIGN CONSTRAINTS

The basis independent variables being the frame and longitudinal girder spacing and the length of hold, the most important design constraints relate to their permissible values.

There is a minimum frame spacing below which one cannot go without risk of impairing workmanship, particularly in way of the double bottom. What this minimum is depends in part on the height of the double bottom, on the spacing of the longitudinal girders and on whether the floors and longitudinals are non-tight, and therefore, lightened, or whether they are water or oil-tight. No simple empirical formulation appears to be available. The ABS Rules call for a minimum frame spacing of 21 in (when the length of the ship is 100 ft) and it does not appear to be worthwhile to consider frame spacings much less than this value.

The same arguments hold for the longitudinal girders.

The spacing of frames is maintained uniform within a hold. The spacing of the longitudinal girders in the double bottom is also maintained uniform. The spacing of deck longitudinals, if any are fitted, is made uniform but not necessarily the same as that obtaining in the double bottom.

Another constraint is that, wherever feasible, the stiffeners are to be derived from standard rolled sections.

An additional constraint is that the stiffeners are of uniform section throughout their span. Thus longitudinal girders, on the one hand, and frames, floors and deck beams on the other do not change in scantlings between points of support.

The maximum hold length is determined by the requirements of racking strength and of internal subdivision. The latter is an external factor not readily translatable into a specific figure. The former is an aspect not considered in this study inasmuch as racking does not occur when the hydrodynamic loading is transversely symmetric as is the case herein studied. The ABS Rules limit the hold length to a maximum of 100 ft. This constraint is accepted as a pro-tempore limitation.

3. LOADING

The externally applied load at a generic point of the hull envelope is made up of a hydrostatic and of a hydrodynamic component, the latter being in turn divisible into a wave-induced and a motion-induced component. By extension, integration of the externally applied load over the hull gives rise to the hydrostatic component of the still water bending moment distribution and to the wave and motion-induced bending moment distribution experienced in a seaway. When to such external load and moment distributions are added the parallel distributions of static weight and dynamic inertia loading, the still water and the sea-induced load and moment distributions result. The schema of Fig. H-2 illustrates this subdivision into components.

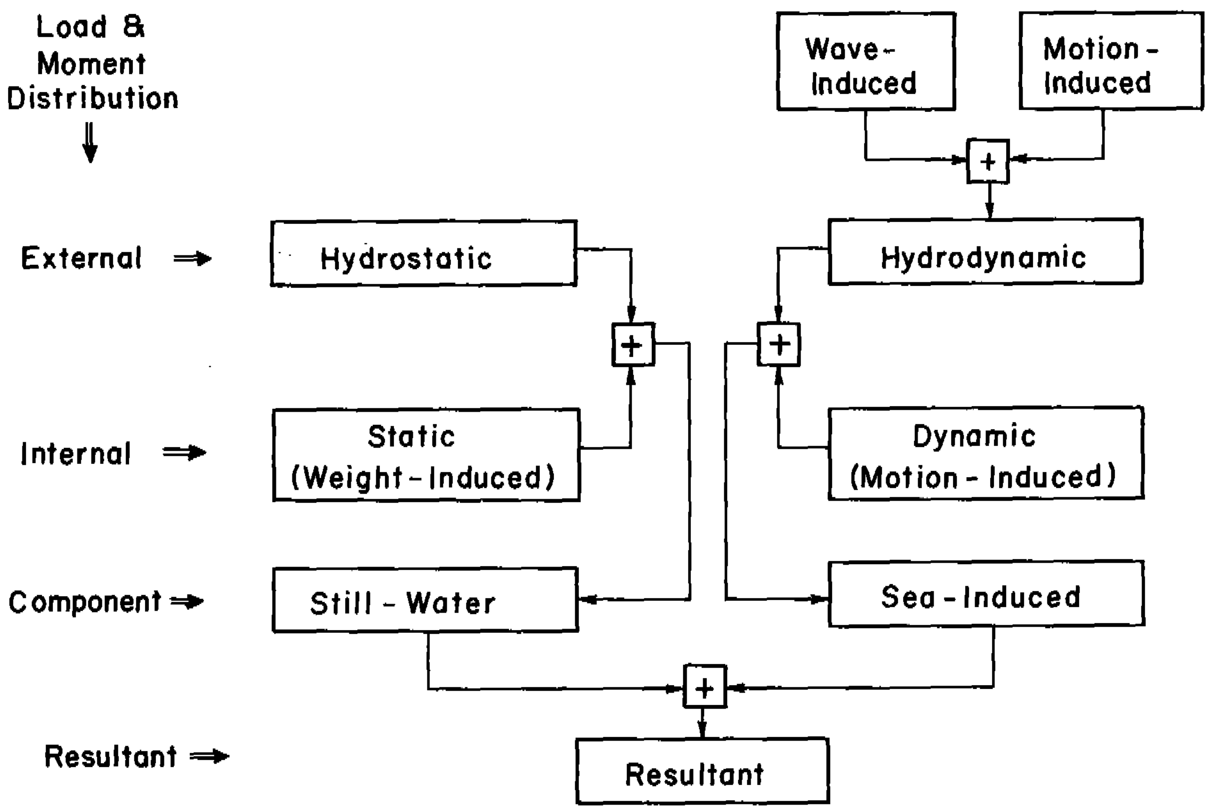

Fig. H.2. Force and Moment Components Acting on the Hull

The resultant pressure, shear force and moment distributions are obtained as outputs of a study on ship response and are assumed to be known for the present study. They are denoted respectively by

$$p\ (x,\ s,\ t)$$
$$Q\ (x,\ s,\ t)$$
$$M\ (x,\ s,\ t)$$

where s is the ordinate of a point on the surface of the hull and t represents time. When such inputs are not available, it becomes necessary to estimate their values empirically. Empirical formulae which are useful to this end are discussed in Appendix J.

Application of these formulae yields:

External hydrostatic pressure on bottom = 17.76 lb in^{-2}

External hydrostatic pressure on sides = 17.76 - 0.445 z lb in^{-2}

Minimum external hydrostatic pressure = 1.78 lb in^{-2}

Internal hydrostatic pressure on inner bottom = 15.54 lb in^{-2}

Internal loading in decks = 2.1 lb in^{-2}

Shear force amidships = negligible

Wave bending moment amidships = 4.05×10^8 ft lb

Here z is the height above keel.

4. MATERIAL PROPERTIES

The material properties which enter explicitly into design of the ship's structure by elastic theory are: Young's modulus (E), Poisson's ratio (μ), Yield strength (σ_{yp}). The following values of the material parameters were used for the initial set of computer runs:

$$E = 3 \times 10^7,\quad 2 \times 10^7,\quad 10^7 \text{ lb in}^{-2}$$

$$\mu = 0.33$$

$$\sigma_{yp} = 35,000 \text{ lb in}^{-2}$$

APPENDIX I

DESIGN CONTROL CRITERIA

The design control criteria are specified as maximum allowable values of primary, secondary and tertiary stress intensities. They are stated as follows:

a) The maximum allowable primary stress intensity at any point is

$$\sigma_1^* = f_1 \sigma_{yp}$$

where f_1 is a primary stress factor

b) The maximum allowable secondary stress intensity is such that at any point

$$(\sigma_1 + \sigma_2^*) \leq f_2 \sigma_{yp}$$

i. e.

$$\sigma_2^* \leq f_2 \sigma_{yp} - \sigma_1$$

where f_2 is a secondary stress factor and σ_1 is the actual maximum primary stress intensity at the point.

In addition, the critical buckling strength of any panel of plating of any plate and stiffener combination shall be

$$\sigma_{cr} \geq \sigma_{yp}$$

$$\geq \frac{\sigma_1^* + \sigma_2^*}{f_2}$$

whichever is less.

c) The maximum allowable tertiary stress intensity at any point is such that

$$(\sigma_1 + \sigma_2 + \sigma_3^*) \leq f_3 \sigma_{yp}$$

i. e.,

$$\sigma_3^* \leq f_3 \sigma_{yp} - \sigma_1 - \sigma_2$$

where f_3 is a tertiary stress factor and σ_1, σ_2 are maximum actual stress stress intensities at the point.

The stress factors f_1, f_2 and f_3 control, in effect, the distribution of material over the section as a whole and between plating and stiffeners. They are of the nature of (the reciprocal of) factors of safety so, because it does not follow that if the stress intensities exceed the critical values σ_1^*, σ_2^*, σ_3^* collapse will automatically follow. The inability to determine the loads, hence the stress intensities, at failure, makes the stress factors useful for design only if they can be interpreted properly. They provide the advantage that the whole background of experience with ship structures can be reduced to a definition of permissible ranges of values of stress factors. It is in these factors that empiricism is chiefly introduced, a necessary step because of the basic assumption of elasticity as a basis for design.

In the process to be developed, the stress factors are considered as parameters since it does not appear possible at this time to justify specific values which might be assigned to them. A choice of stress factors can be made only after a large number of analyses have been correlated with experience.

The calculations have been based on the following initial values of the stress intensity factors:

$$f_1 = 0.56$$
$$f_2 = 0.80$$
$$f_3 = 1.00$$

Of course, except for the last, these are not the factors that necessarily result when the structure is designed.

APPENDIX J

EMPIRICAL FORMULAE

J.1 GENERAL

This appendix contains a compilation of empirical formulae useful for supplying approximate input data, for initiating the rational design process and for selecting commercial scantlings.

J.2 INPUT DATA

The program requires the following inputs:

a) The external head of water at any point of the hull surface

b) The internal loading

c) The Shear force distribution

d) The maximum bending moment on the hull girder

J.2.1 External Pressure Head

The hydrodynamic pressure distribution with depth is assumed to be replaceable by an equivalent hydrostatic pressure distribution. The maximum external pressure head amidships obtains in the hogging condition. When the sea is from ahead or astern, the hydro-loading is transversely symmetric and, according to St. Denis (1954), the pressure head at any point of the hull surface is given by

$$H + 0.4\, h_w - z$$

where h_w is the wave height. The minimum pressure head is arbitrarily taken to be 4 ft. The pressure distribution is further assumed to be invariant along the length of the hold and to be symmetric with respect to the longitudinal centerplane. Refer to Fig. J.1.

This assumption reflects standard practice for the determination of the numeral of stress intensity which is associated with the statical method of calculating the wave-induced bending moment. It has not been proved that such practice is either relevant or reliable.

J.2.2 The Internal Loading

Also assumed known are the internal loads on the decks and tank boundaries.

The maximum normal deadload pressure acting on a deck can be taken equal to the 'tween deck height multiplied by a cargo density factor. The cargo density factor depends on the intended service of the ship and is purely empirical. For a full scantling ship the value of 75 lb ft^{-3} is normally used, for a shelter deck ship the value of 40 lb ft^{-3}.

The maximum hydrostatic load acting on tank boundaries corresponds to the test head.

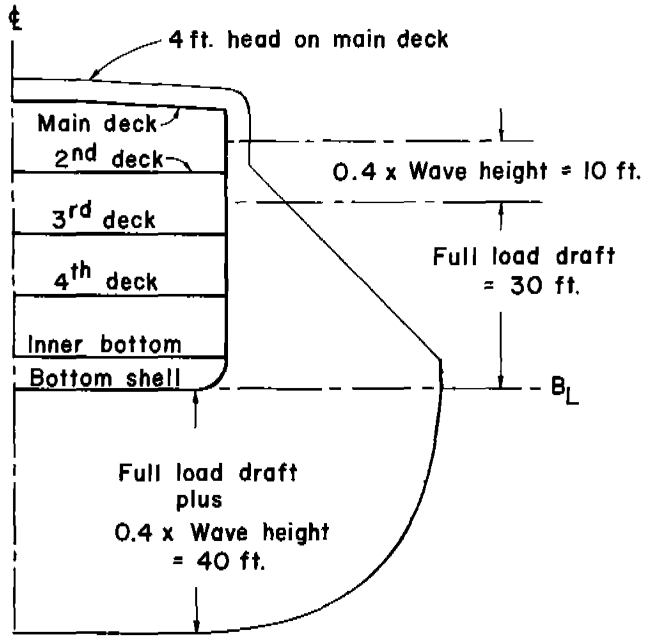

Fig. J.1. External Pressure Head

The ABS Rules spell out this head to be for inner bottom tanks up to:

a) The freeboard deck.
b) The bulkhead deck.
c) The highest point to which the tank contents may rise in service.

whichever is the greatest and for deep tanks up to:

a) The overflow.
b) The load line.
c) Two-thirds the distance from tank top to the bulkhead or freeboard deck.

whichever is the greatest.

J. 2. 3 Shear Force Distribution

In the midship region, the vertical shear force is close to zero and does not influence the scantlings. The horizontal shear force is zero by symmetry of loading. Consequently, both distributions can be neglected.

J. 2. 4 Maximum Bending Moment Amidships

A summary of maximum midship wave bending moment experienced by models in extreme regular waves is due to Dalzell (1963). The models tested were those of a cargo vessel of the MARINER class, a tanker and a destroyer, hence the findings are useful to the present study.

The wave bending moments in hog (h) and sag (s) are written

$$M_h = k_h \rho g L^3 B$$

$$M_s = k_s \rho g L^3 B$$

where:

ρ ≡ density of water (lb sec^2 ft^{-4})

g ≡ acceleration of gravity (ft sec^{-2})
L ≡ ship length (ft)
B ≡ ship beam (ft)
k_h, k_s ≡ hog and sag coefficients

The hog and sag moment coefficients depend on the ship's fullness of form, on her freeboard, on the distribution of internal load and on the encountered wave steepness.

An average value for the sag coefficient is:

$$k_s = 0.0145 \left[\frac{h}{\lambda}\right]$$

where:

h ≡ wave height (ft)

λ ≡ wave length (ft)

This value is increased some 10 percent by unfavorable distribution of cargo (concentration amidships or near the ends). However, when a tanker is in the fully-laden condition, the cargo distribution is not unfavorable. An average value for the hog coefficient is

$$k_h = 0.0080 \left[\frac{h}{\lambda}\right]$$

but unfavorable loading (midship or end concentration) can increase it by some 50 percent. Also, fullness tends to have a powerful influence. To account for this, the average value for the hog coefficient determined above is to be multiplied by

$$\left[\frac{C_b}{0.61}\right]^3$$

where C_b is the block coefficient.

Application of the foregoing formulae to the WOLVERINE STATE (C_b = 0.66) yields

$$k_s = 0.0145 \left[\frac{h}{\lambda}\right]$$

$$k_h = 0.0101 \left[\frac{h}{\lambda}\right]$$

for favorable loading. Thus, the bending moment in the sagging condition dominates and its value is used to determine the primary stress intensities to be measured against the design control criteria.

J. 3 INITIATION DATA

To initiate the computation process, an approximation is required for the height above keel of the neutral axis.

J. 3. 1 Neutral Axis

Evans (1958) interprets the ABS rules as being based on a height of neutral axis equal to

$$z_o \doteq 0.40 \ D$$

where D is the depth of the vessel. The specific data of the WOLVERINE STATE give $z_o = 209$ in.

J. 4. SELECTION OF COMMERCIAL SIZES OF PLATES AND STIFFENERS

Two items requiring determination are the average width of plating to be used, inasmuch as this width determines the number of seams, and the choice of stiffeners.

J. 4. 1 Plate Widths.

The location of seams must be made in accordance with certain rules of a practical nature which reflect shipyard practice. Although, in a final computer program these rules should be entered as constraints, it is preferable to disregard them initially and to locate the seams arbitrarily. The computational work is materially simplified at, what appears to be, but a trivial loss in flexibility. Accordingly, the following rule on plate widths is observed:

Bottom plating - from centerline to turn of bilge
Bilge strake - from baseline to inner bottom
Side shell - from inner bottom to fourth deck
from fourth deck to third deck
from third deck to second deck
from second deck to main deck
Second deck - from hatch coaming to side
Third deck - from hatch coaming to side
Fourth deck - from hatch coaming to side
Inner bottom - from centerline to side

However, when calculating weld material and cost, such simplification is not permissible and or, at least, more reasonable plate widths must be introduced. For such computations, the plate width has been established arbitrarily as just under 6 ft. This gives 13 strakes of bottom and side plating on each side if the keel be excluded.

J. 4. 2 Stiffeners

Selection of stiffener depends on its geometric properties when coupled to the plating it supports. The geometric characteristic pertinent to a structural analysis of stiffener and plating combinations are:

a) Stiffener depth (d_s), cross-sectional area (A_s), centroid to toe distance (χ_s), and second central moment (I_s).

b) Thickness of shell or deck plating (h) and stiffener spacing a or b

c) Effective breadth (ℓ_e) and area (A_{pe}) and effective second central moment (I_{pe}) of plating associated with each stiffener.

The effective breadth is discussed in Appendix D. The effective area of plating is, consequently,

$$A_{pe} = \ell_e h$$

The second central moments of effective plating about the faying surface is

$$I_{pe} = \frac{1}{3} A_{pe} h^2$$

If the stiffeners are to be of standard rolled sections, the following possibilities are available:

a) American standard I-sections (I-ST)
b) Wide flange I-sections (I-WF)
c) H-sections (H)
d) Cut American standard I-sections (i.e., faying flange reduced to approximately half width) (cut I-ST)
e) Cut wide flange I-sections (cut I-WF)
f) Cut H-sections (cut H)
g) T-sections cut from American standard I-sections (T-ST)
h) T-sections cut from wide flange W-sections (T-WF)

Note that the depth of T sections is half of the depth of the I section from which cut.

Of the listed geometries, the most efficient from the viewpoint of strength are the T-sections cut from wide flange I-sections (T-WF); they are however, fairly deep in comparison with uncut or partially cut I and H sections of equivalent strength. This may pose a problem in that such frames reduce the net 'tween deck height somewhat more than do shallower (but heavier) sections of equal strength. Resolution of this problem is not sought within the scope of this analysis. since it involves external inputs either not available or not readily formulable. However, this step must eventually be made if the rational design process is to yield a practical solution.

Figure J.2 presents a plot sectional area (A_s) against depth of section (d_s) for T-sections cut from wide flange shapes (T-WF). For the lightest and, therefore, structurally more efficient sections, the empirical fit

$$A_s = 0.20\ d_s^{1.585}$$

obtains.

Fig. J.3 is a plot of the distance from centroid to toe of section for which the following empirical fit obtains

$$\chi_s = k_s \, d_s$$

where

$$k_s = 0.70 + \frac{1}{2 \, d_s}$$

Fig. J.4 is a plot of second central moment of the frame section about the toe for which the fit is

$$I_s = 0.12 \, d_s^{3.70}$$

for the lightest sections.

The centroid of the combined plate and frame is at a distance from the center of the plate given by

$$\chi_{ps} = \frac{A_s \, k_s \, d_s - h \, A_{pe}/2}{A_{pe} + A_s}$$

This makes the second moment of transference equal to

$$I_t = [A_{pe} + A_s] \, \chi_{ps}^2$$

The effective second moment of the plate frame combination is

$$I_{pf} = I_{pe} + I_s - I_t$$

The web thickness of the frame sections is given by the simple relation

$$h_w = 0.036 \, d$$

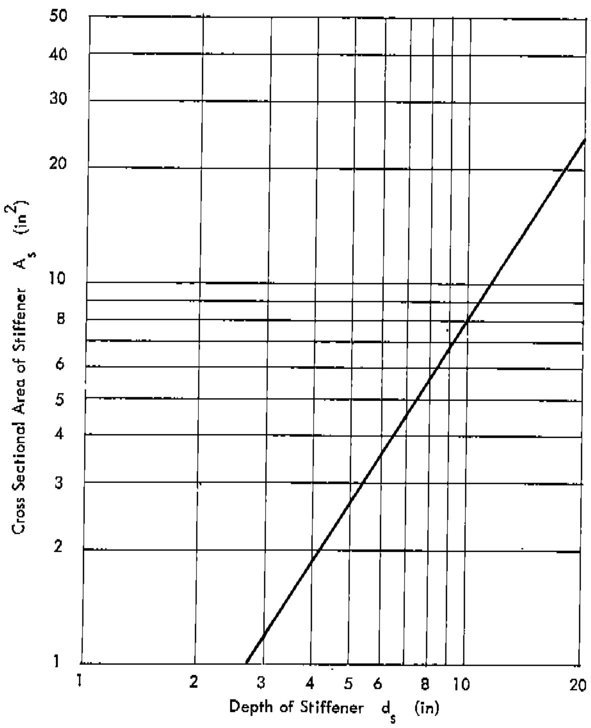

Fig. J.2. Cross Sectional Area of T-Sections Cut From Wide Flange Shapes

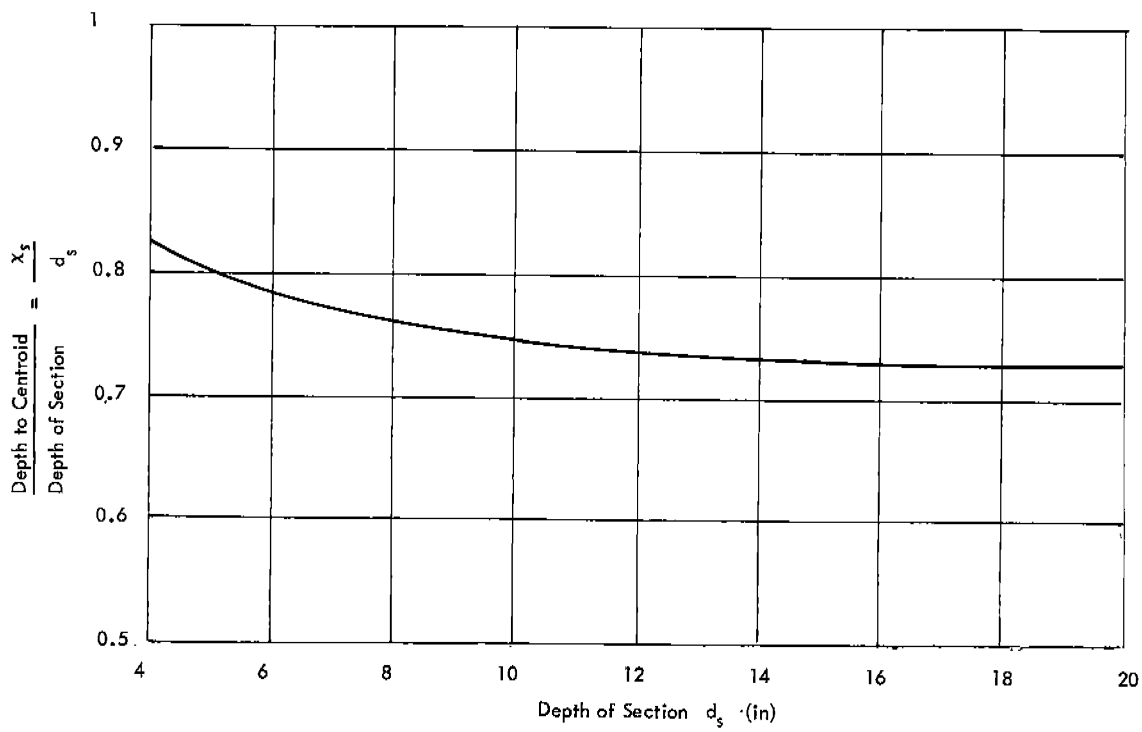

Fig. J.3. Plot of Distance From Centroid To Toe of Section vs Depth of Section

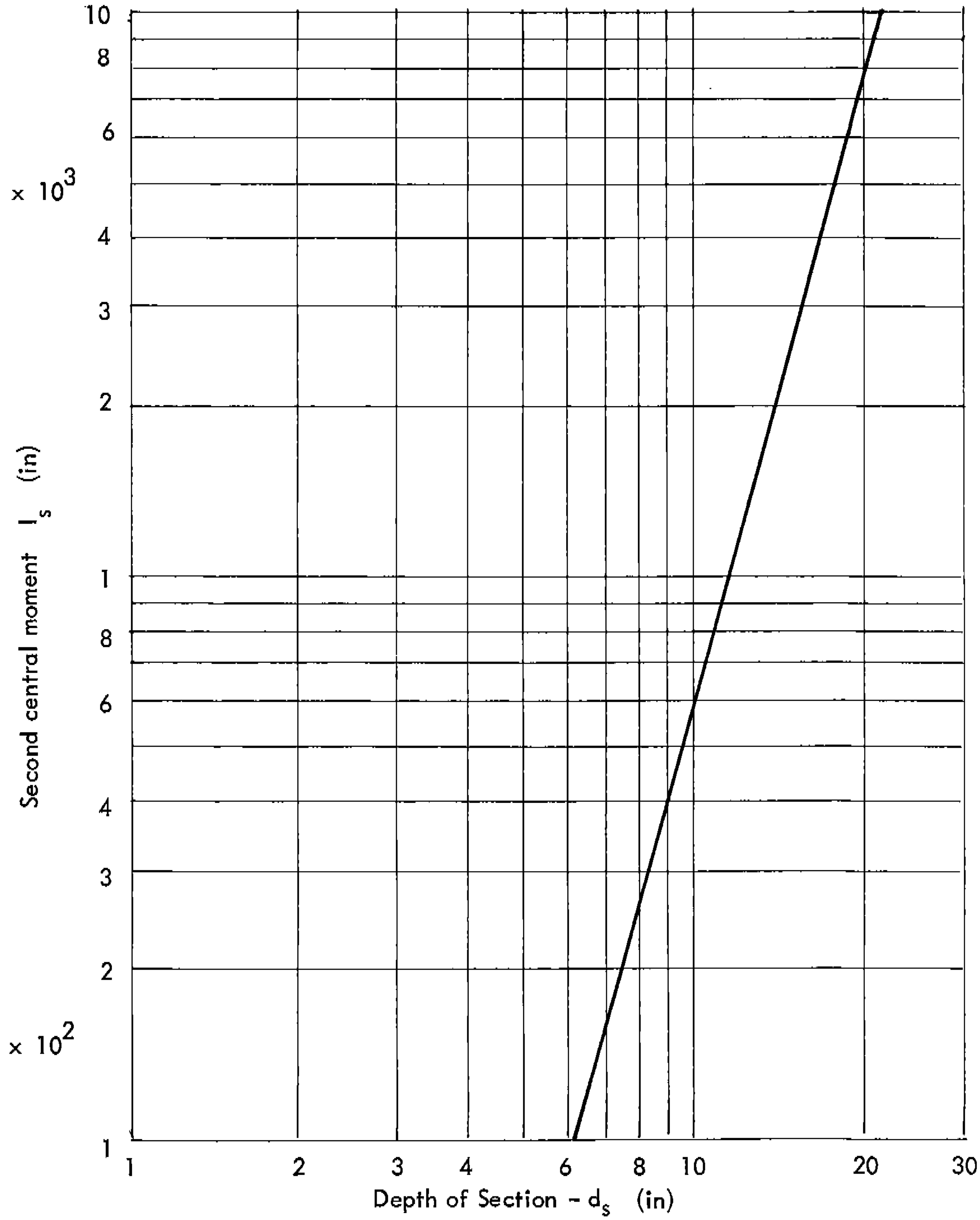

Fig. J.4. Second Moment of Cross Section Area About the toe of a T-Section vs Section Depth

APPENDIX K

PROPERTIES OF THE MIDSHIP SECTION

To determine the primary stress intensities in the midship section, the section modulus must first be obtained. This calculation follows the standard procedure with the exception that only effective material is taken into account. The effective breadth of plating is discussed in Appendix D. For the WOLVERINE STATE, the calculation yields $b_e/b = 0.85$ which means that only this fraction of the flange material (i.e., all longitudinally continuous material except the side shell) can be considered as fully effective. The calculations are as follows:

- Effective cross-sectional area

$$A \equiv \sum_i (b_e)_i \; h_i + \sum_i (A_s)_i$$

where:

h ≡ plate thickness
b_e ≡ effective breadth of plate

$b_e = b$ for side shell plates

$b_e = 0.85b$ for all other plates

A_s ≡ cross sectional area of stiffener

The first summation is over all plating and the second summation is over all longitudinal shapes.

- Moment about baseline

$$M \equiv \sum_i z_i \; (b_e)_i \; h_i + \sum_i (A_s)_i \; z_i$$

where z is the vertical coordinate to the centroid of a plate or shape.

- Neutral axis

$$\bar{z} \equiv \frac{M}{A}$$

Second central moment of area about neutral axis

$$I \equiv \sum_i (b_e)_i \; h_i \; z_i^2 + \sum_i (A_s)_i \; z_i^2 - A \; z_i^2$$

Section moduli to deck and keel respectively

$$Z_d \equiv \frac{I}{z_d - z} \qquad\qquad Z_k \equiv \frac{I}{z}$$

APPENDIX L

ELASTIC STABILITY

L.1 GENERAL

Plating and shapes subject to in-plane loading in compression and/or shear must be of adequate size to withstand buckling. The design of plating subject to such in-plane loading and to normal loading is discussed in Appendix B. In this appendix formulae are presented for determining the elastic stability of plating not subject to normal loading and of shapes.

The critical stress intensity depends in an essential manner on geometry and on degree of fixity at the supports. The disposition of stiffening is such that only rectangular plates need be considered.

L.2 PLATING IN COMPRESSION

The only case of interest obtains when the compression is uni-axial.

The critical stress intensity of a rectangular plate is given by

$$\sigma_{cr} = k \frac{\pi^2 D}{b^2 h}$$

where:

D ≡ flexural rigidity

$$D \equiv \frac{E h^3}{12 [1 - \mu^2]}$$

E ≡ Young's modulus
h ≡ plate thickness
μ ≡ Poisson's ratio
b ≡ plate width
k ≡ factor dependent on aspect ratio and conditions of fixity

For simply supported plating

$$k \equiv \left[m \frac{b}{a} + \frac{1}{m} \frac{a}{b} \right]^2$$

where:

a ≡ plate length (in direction of compression)
m ≡ integer chosen so as to minimize the value of the expression.

For $a/b < 1$, $m = 1$ and

$$k \equiv \left[\frac{b}{a} + \frac{a}{b} \right]^2$$

For $a/b > 1$, $k \cong 4$

This is the only case that needs be considered.

L. 3 PLATING IN SHEAR

The critical shearing stress intensity of a rectangular plate is

$$\tau_{cr} = k \frac{\pi^2 D}{b^2 h}$$

where for a simply supported plate

$$k = 5.35 + 4 \left[\frac{b}{a}\right]^2$$

while for fixed edges

$$k = 8.98 + 3.85 \left[\frac{b}{a}\right] + 2.56 \left[\frac{b}{a}\right]^2$$

L. 4 STIFFENERS IN COMPRESSION

The critical compressive stress intensity will exceed the yield strength of the material when lateral supports are spaced a pitch distance given by kw where w is the width of the flange and k is a factor dependent on the ratio of flange width to section depth (w/d). For medium steel

$$k = 19 + 12 \left[\frac{w}{d}\right]$$

while for high tensile steel

$$k = 17 + 10 \left[\frac{w}{d}\right]$$

In this study, such a criterion applies to the deck longitudinals. Since support is provided by the frames, one needs only insure that

$$a < kw$$

APPENDIX M

WEIGHT OF HULL STRUCTURE

To obtain the weight of hull structure per unit of ship length (the inch being used as the dimension), requires but a simple accounting procedure once the scantlings have been established. This procedure is outlined herein.

The structural items are grouped in 13 homogeneous sets. The total weight per inch is obtained by simple summation of the component sets and an allowance for miscellanea.

a) Shell plating

$$s_1 \equiv 2\gamma \sum_i w_i h_i$$

where:

$w_i \equiv$ width of plate i
$h_i \equiv$ thickness of plate i
$\gamma \equiv$ specific weight of steel = 0.283 lb ft^{-3}

The summation is over the plates covering a half girth of the section.

b) Inner bottom

$$s_2 \equiv \gamma B h$$

where h is the inner bottom thickness

c) Deck plating in a bulkhead bay

$$s_3 \equiv \gamma B c \sum_i h_i$$

where:

B $\equiv$ beam
c $\equiv$ factor

$$c \equiv \frac{\ell_{hold} - \ell_{hatch}}{\ell_{hold}}$$

$\ell_{hold} \equiv$ length of hold
$\ell_{hatch} \equiv$ length of hatch
$h_i \equiv$ thickness of deck i in way of bulkhead bay

The summation is over all decks.

d) Deck plating in the hatch bay

$$s_4 \equiv \gamma \; [B - 2y_h] \; [1 - c] \sum_i h_i$$

where:

$2y_h \equiv$ width of hatchway

$h_i \equiv$ thickness of deck i in way of hatch bay

The summation is over all decks.

e) Double bottom longitudinal girders

$$s_5 \equiv \gamma \; d \sum_i h_i$$

where:

$d \equiv$ depth of inner bottom

$h_i \equiv$ web thickness of longitudinal girder i

The summation is over all longitudinal girders.

Non-tight girders have cut-outs for access and drain openings. These are not taken into account on the consideration that the refinement is not worth the effort.

f) Deck longitudinals

$$s_6 \equiv \gamma \sum_i N_i \, A_i$$

where:

$N_i \equiv$ number of longitudinals supporting deck i

$A_i \equiv$ cross sectional area of longitudinals supporting deck i

The summation is over all decks. Note that the longitudinal hatch girders are not included.

g) Longitudinal hatch girders

There being two such, their weight per inch of ship length is

$$s_7 \equiv 2 \gamma \sum_i A_i$$

where A_i is the cross sectional area of the longitudinal girder supporting deck i and the summation is over all decks.

h) Floors

$$s_8 \equiv \gamma \frac{B \; d}{\ell_{hold}} [N_t \, h_t + N_n \, h_n]$$

where:

N_t ≡ number of (oil or water) tight floors in a hold

h_t ≡ thickness of tight floors

N_n ≡ number of non-tight floors in a hold

h_n ≡ thickness of non-tight floors

i) Frames

$$s_9 \equiv \frac{\gamma}{a} \sum_i A_i \ell_i$$

where

A_i ≡ cross sectional area of frame supporting deck i

ℓ_i ≡ length (i. e., 'tween deck height) of frame supporting deck i

The summation is over all 'tween deck heights.

j) Deck beams in a bulkhead bay

$$s_{10} \equiv \frac{\gamma B c}{a} \sum_i A_i$$

where A_i is the cross sectional area of the beam supporting a bulkhead bay of deck i and the summation is over all decks.

k) Deck beams in the hatch bay

$$s_{11} \equiv \frac{\gamma}{a} [1 - c] [B - 2y_h] \sum_i A_i$$

where A_i is the cross sectional area of the beam supporting a hatch bay of deck i and the summation is over all decks.

l) Transverse hatch girders

There are two such at each deck level, hence,

$$s_{12} \equiv \frac{2\gamma}{\ell_{hold}} B \sum_i [A_i^* - A_i]$$

where A_i^* is the cross sectional area of the transverse hatch girder at deck i and A_i is that of the deck beam at the same deck. The summation is over all decks.

m) Stanchions

There are four stanchions at each deck level, hence,

$$s_{13} = \frac{1.132}{\ell_{hold}} \sum_{i} A_i \ell_i$$

where A_i is the cross sectional area and ℓ_i is the height of stanchion supporting deck i. The summation is over all decks.

n) Miscellaneous

There are a large number of small items, such as brackets, reinforcement of openings, etc., which remain to be taken into account. Allowance for the weight of these items can best be made by proportioning them to the total. In the calculations, the factor is taken as 3 percent. Note that this manner of accounting does not affect the relative standings with regard to weight of the designs analyzed.

The total weight of hull structure per inch of length is

$$S = 1.03 \sum_{j=1}^{13} s_j$$

APPENDIX N

WELD MATERIAL

In calculating the weld material, distinction is made between transverse and longitudinal welds. The former include the butt connections in the shell plating and the connections to shell, inner bottom and decks of floors, frames, deck beams and transverse hatch girders. The latter consist of the seam connections in the shell plating and the connections of double bottom and deck longitudinals and longitudinal hatch girders to shell, inner bottom and or decks. The material is calculated in accordance with the rules of the American Bureau of Shipping except that step changes in weld size and spacing as a function of plate thickness have been replaced by a continuous smooth change.

Some considerations preface the calculation of weld material: the length and width of plating to be used in the construction of shell, inner bottom and decks; and the types of welds to be employed.

The length of plating depends essentially on the practice at the specific shipyard where the ship is built. Present practice is to make the plate length about 30 feet and this figure is used herein, inasmuch as even substantial variations from this value do not affect critically the overall welding cost that results. Thus, all plating is assumed to have butt connections every 30 feet.

The width of plating also depends on shipyard practice; however, it is not possible in this case to state that the overall cost of welding is fairly insensitive to width of plating. The calculations have been carried out on the assumption that the average width of plating is near 6 feet.

The welding connections of interest are:

a) Continuous reinforced 60 deg. vee weld.
b) Continuous double fillet weld.
c) Intermittent double fillet weld.

The continuous reinforced (60 deg.) vee weld has a weight per inch of run equal to

$$0.20\,h^2 + 0.04\,h \equiv f_1(h)$$

where h is the plating thickness. When plates of dissimilar thickness are joined, the average thickness of the two plates is used.

The continuous double fillet weld has a weight per inch of

$$0.283\,h^2 \equiv f_2(h)$$

where h is the thickness of the discontinuous plate. The weight per inch of the intermittent double fillet weld is

$$0.07\, h^2 \equiv f_3\,(h)$$

based on a weld spacing of 4 times the run.

The weight of welds per inch of ship length are calculated as follows:

a) Butt welds in the shell

$$m_1 \equiv \frac{1}{180} \sum_i w_i \cdot f_1\,(h_i)$$

where:

w_i ≡ width of plate i

h_i ≡ thickness of shell plate i

and where the summation extends over the shell plates in one half of the cross section.

b) Butt welds in the inner bottom and in the decks in a bulkhead bay

$$m_2 \equiv \frac{B}{360} \sum_i f_1\,(h_i)$$

where:

B ≡ beam of ship

h_i ≡ thickness of deck (or inner bottom) i and where the summation extends over all decks and inner bottom.

c) Butt welds in the decks in a hatch bay

$$m_3 \equiv \frac{B - 2y_h}{360} \sum_i f_1\,(h_i)$$

where $2y_h$ is the width of the hatchway. and where the summation extends over all the decks (but not inner bottom).

d) Connections to the shell, inner bottom and longitudinal girders of a non-tight floor

$$m_4 \equiv \left\{ \frac{2}{a} \left[B + N_g\, d\right] f_3(h) \right\} \frac{N_n}{\ell_{hold}}$$

where:

a ≡ frame spacing

N_g ≡ number of longitudinal girders in double bottom

d ≡ depth of double bottom

h ≡ floor thickness

e) Connections to the shell, inner bottom and longitudinal girders of an oil-tight or water-tight floor

$$m_5 \equiv \frac{2}{a} [B + N_g \, d] \; f_2(h)$$

where h is the floor thickness.

f) Frame connections to the shell

$$m_6 \equiv \frac{2}{a} \sum_i \ell_i \cdot f_3 (h_i)$$

where:

ℓ_j ≡ length ('tween deck height) of frame i
h_i ≡ web thickness of frame i and where the summation extends over the frames in one-half of the section.

g) Connection of deck beams to deck plating in a bulkhead bay

$$m_7 \equiv \frac{B}{a} c \sum_i f_3(h_i)$$

where:

h_i ≡ web thickness of the beam supporting deck i
c ≡ factor

$$c \equiv \frac{\ell_{hold} - \ell_{hatch}}{\ell_{hold}}$$

ℓ_{hold} ≡ length of hold
ℓ_{hatch} ≡ length of hatch

The summation is over all decks.

h) Connection of deck beams to deck plating in the hatch bay

$$m_8 \equiv \frac{B - 2y_h}{a} [1 - c] \sum_i f_3 (h_i)$$

where h_i is the web thickness of the beam supporting deck i and where the summation extends over all decks.

i) Transverse hatch girder connections to deck plating.

There being two such girders per hold, the excess of weld over that corresponding to an ordinary frame is distributed uniformly over the hold length

$$m_9 \equiv \frac{B}{x_h} \sum_i [f_2(h_i) - f_3(h_i)]$$

where:

x_h ≡ length of hold
h_i ≡ web thickness of transverse hatch girder supporting deck i
h_i ≡ web thickness of ordinary frame supporting deck i and where the summation extends over all decks

j) Plate seams in the shell

$$m_{10} \equiv 2 \sum_i f_1 \left[\frac{h_i + h_{i+1}}{2}\right]$$

where h_i is the thickness of plate i and where the summation extends over all the shell plating in the half girth (from center-keel at base to gunwale).

k) Plate seams in the inner bottom

$$m_{11} \equiv N_s \cdot f_1(h)$$

where:

N_s ≡ number of seams in inner bottom
h ≡ inner bottom thickness

$$N_s = \frac{B}{\bar{w}} + 1$$

where $\bar{w}$ is the average width of the inner bottom plating ($B/\bar{w}$ is an integer)

l) Seams in the deck plating in a bulkhead bay

$$m_{12} \equiv c\left[\frac{B}{\bar{w}} + 1\right] \sum_i f_1(h_i)$$

where:

h_i ≡ thickness of plating in way of bulkhead bay of deck i
$\bar{w}$ ≡ average width of deck plating and the summation extends over all decks. Note that B/w must be an integer, the value of $\bar{w}$ being adjusted to this end.

m) Seams in the deck plating in the hatch bay

$$m_{13} \equiv [1 - c]\left[\frac{B - 2y_h}{\bar{w}}\right] \sum_i f_1(h_i)$$

where:

$h_i \equiv$ thickness of plating in way of hatch bay of deck i

The summation extends over all decks. Note that $[B-2y_h]/\bar{w}$ must be an integer, the value of $\bar{w}$ being so adjusted.

n) Double bottom oil or watertight longitudinal girders

$$m_{14} \equiv 2N_{gt} \cdot f_2 (h)$$

where h is the web thickness of the longitudinal girder and N_{gt} is the number of oil or waterlight longitudinal girders in the double bottom.

o) Double bottom non-tight longitudinal girders

$$m_{15} \equiv 2N_{gn} \cdot f_3 (h)$$

where h is the web thickness of the longitudinal girder and N_{gn} the number of non-tight longitudinal girders in the double bottom.

p) Deck longitudinals in a bulkhead bay

$$m_{16} \equiv c N_\ell \sum_i f_3 (h_i)$$

where:

$N_\ell \equiv$ number of longitudinals in a bulkhead bay
$h_i \equiv$ web thickness of longitudinal in bulkhead bay supporting deck i

The summation extends over all decks.

q) Deck longitudinals in the hatch bay

$$m_{17} \equiv [1 - c] N_\ell \cdot \sum_i f_3 (h_i)$$

where:

$N_\ell \equiv$ number of longitudinals in a hatch bay
$h_i \equiv$ web thickness of longitudinal in hatch bay supporting deck i

The summation extends over all decks.

r) Longitudinal hatch girder connections to deck plating

There are two such girders, therefore,

$$m_{18} \equiv 2 \sum_i f_2 (h_i)$$

where h is the web thickness of the hatch girder and the summation extends over all decks.

s) Miscellaneous

There are a large number of small items, such as brackets, stanchions, reinforcement of openings, etc. which remain to be taken into account. The weld material for these items is best estimated as a fraction of the total. In the calculations this factor is taken as 3 percent.

The total weight of weld material is simply

$$M = 1.03 \sum_{j=1}^{18} \dot{m}_j$$

APPENDIX O

COST OF HULL STRUCTURE

In computing the cost of hull structure, only the material and labor charges are taken into account, the cost of engineering and the burden on these charges are ignored altogether.

The material cost of plates and shapes depends on the quality of steel used. This shows itself in a correlation (though not necessarily a simple one) between cost and yield point of material. Medium steel is used in the calculations reported herein. Its present price is about 0.065 dollars per pound. Although there is a small difference between the price of steel of plates and shapes, this is not taken into account. Thus, material cost is obtained directly by multiplying the weights of the structural components by the cost of material. Thus,

$$C_m \equiv 0.065\ S$$

where S is the weight per inch of hull structure, see Appendix L

The material cost of electrodes is conveniently included in the labor cost of welding.

Labor cost is divided into two components: cost of fabrication and erection and cost of welding. The cost of fabrication and erection does not appear to be directly proportional to weight but follows, instead, a more complex relation.

The literature does not abound with description of quantitative methods for estimating the cost of fabricating and erecting ship structures. The few methods presented are at variance with each other and range from making this cost proportional to plate surface to making it proportional to plate weight. Personal communications have led to a choice intermediate between these extremes. The empirical formulae presented below are plausible and heuristic: no argument is made as to their range of validity. They also are flexible and can be readily substituted by others in the subroutine for calculating cost. The formulae for the cost per inch of ship of fabrication and erection (excluding welding) are as follows:

a) Longitudinal plating

$$C_{pl} \equiv r \sum w_i\ f(h_i)$$

where:

r ≡ hourly rate of labor
w_i ≡ width of plate i
$f(h_i)$ ≡ time-thickness factor

The summation is over all longitudinal plates including inner bottom longitudinal girders. The time-thickness factor has been taken equal to

$$f_5(h_i) \equiv 0.04\ h_i^{2/3}$$

Also, the hourly rate has been taken at

$$r = 4.00 \text{ dollars per hour.}$$

b) <u>Longitudinal shapes</u>

$$C_{sl} \equiv r \sum_i f(A_i)$$

where A_i is the cross sectional area of the shape and $f(A_i)$ is a time size factor. The empirical expression assumed for this factor is

$$f(A_i) \equiv 0.04\, A^{2/3}$$

c) <u>Transverse plating (floors)</u>

$$C_{pt} \equiv \frac{N}{\ell_{hold}} \sum_i d_i \cdot f(h_i)$$

where d_i is the depth of the inner bottom and B is the beam of the ship

d) <u>Transverse shapes (frames, deck beams)</u>

$$C_{st} \equiv \frac{r}{a} \sum_i f(A_i)\, \ell_i$$

where ℓ is the 'tween deck height.

e) <u>Stanchions</u>

There are four to each level in a hold, hence

$$C_s \equiv \frac{4\, r}{\ell_{hold}} \sum_i w_i \, f(w_i)$$

where:

w_i ≡ weight of stanchion supporting deck i

$f(w_i)$ ≡ time-weight factor

The summation is over all decks. The time-weight factor is taken as

$$f(w_i) \equiv 0.02 \text{ hours per lb.}$$

The total labor cost of erection and fabrication (exclusive of welding) is by summation

$$C_{fe} \equiv C_{p\ell} + C_{s\ell} + C_{pt} + C_{st} + C_s$$

The cost of welding is directly proportional to the weight of welds, the constant of proportionality depending on the quality of the steel, hence on the electrodes, to be used. For welds in medium steel, the present labor rate, including cost of electrodes, is about 3 dollars per pound of deposited weld and the calculations are based on this figure. Thus, the cost of weld is

$$C_w = 3.00\ M$$

where M is the total weight of weld material; see Appendix M.

The total cost of hull structure is, consequently,

$$C = C_m + C_{fe} + C_w$$

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13. ABSTRACT

This report presents the design synthesis for a digital computer program that has been developed, based on the application of rational techniques, for the design of the optimum midship structure of a transversely-framed dry cargo ship. The merit of the design method used is that all empirical knowledge on the proportioning of hull structure to withstand the forces of the seaway finds expression in three factors; namely: separation of structure into primary, secondary, and tertiary components.

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