SSC-301

PROBABILISTIC STRUCTURAL ANALYSIS OF SHIP HULL LONGITUDINAL STRENGTH

SHIP STRUCTURE COMMITTEE

1981

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Ship
Structure
Committee

An Interagency Advisory Committee
Dedicated to Improving the Structure of Ships

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Uncertainties are unavoidable in any engineering design. Limitation on the control of material properties, mill tolerances in plate and extruded shape thickness, time-dependent effects such as deterioration due to corrosion, cracking, wear and tear are only some of the factors that contribute to the uncertainties associated with the actual strength of a ship's hull. Ship designers and naval architects usually treat these items in a qualitative sense as very few attempts have been made to quantify them.

Based on previous experience, the qualitative assessment of the uncertainties does not lend itself to systematic improvement of design procedures. Therefore, the Ship Structure Committee initiated this project to develop a computer program to analyze the uncertainties associated with ship hull strength. The development of the program and its contents are presented.

Henry H. Bell
Rear Admiral, U.S. Coast Guard
Chairman, Ship Structure Committee

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16. Abstract

Existing probabilistic structural design methods are reviewed, their applicability to ship hull structural design considered and the most promising probabilistic analysis techniques are identified.

The current state of knowledge concerning structural modes of failure and load distribution is considered with respect to its impact on probabilistic structural analyses. The emphasis is on longitudinal strength considerations.

Factors influencing strength, in terms of uncertainties in ship strength distribution, are reviewed. Different methods are proposed to obtain coefficients of variation for various types of data on the uncertainties.

Sample calculations are performed for a number of ships using an approximate probabilistic method and yielding safety margins for each. This method requires that only the coefficients of variation of the strength and load be known.

A computer program is developed to perform this calculation for any ship subjected to any load or mode of failure.

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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

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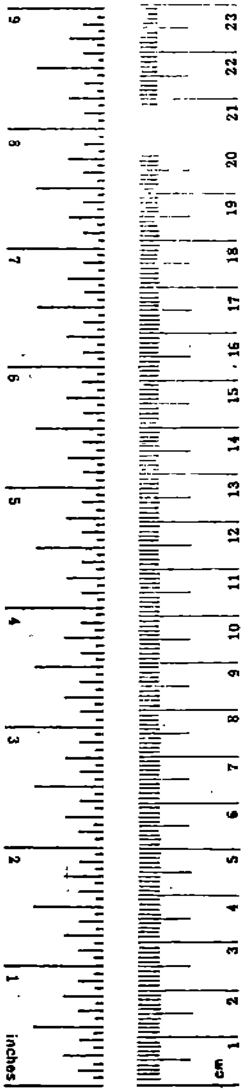
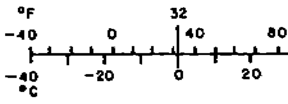
*1 in ≡ 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions

Symbol	When You Know	Multi
		LEN
mm	millimeters	0.0
cm	centimeters	0.4
m	meters	3.3
m	meters	1.1
km	kilometers	0.0
		AR
cm^2	square centimeters	0.
m^2	square meters	1.
km^2	square kilometers	0.
ha	hectares (10,000 m^2)	2.
		MASS (
g	grams	0.0
kg	kilograms	2.2
t	tonnes (1000 kg)	1.1
		VOL
ml	milliliters	0.0
l	liters	2.1
l	liters	1.0
l	liters	0.2
m^3	cubic meters	35
m^3	cubic meters	1.3
		TEMPERAT
°C	Celsius temperature	9/5 add



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NOMENCLATURE

A_F	Area of Flanges
A_w	Area of Webs
B	Beam of Ship
D	Depth of Ship
E	Mean Square Value
$f_Q(q)$	Density Function of Load
$F_Q(q)$	Distribution Function of Load
$f_S(s)$	Density Function of Strength
$F_S(s)$	Distribution Function of Strength
f_L	Density Function of Load
F_L	Distribution Function of Load
h_e	Height of Static Wave that Yields Average Irregular Wave Longitudinal Bending Moment
k	Strength Factor or Parameter
l	Parameter
L	Length of Ship
m_M	Mean of Margin of Safety
M	Margin of Safety,(S-Z)
m_Z	Mean of Load
m_S	Mean of Strength
m	Mean of Still Water Bending Moment; (m_o - Deterministic SWBM)
N	Deck or Bottom Section Modulus;or Number of Data Points
P_f	Probability of Failure
Pr	Probability
p	Probability Density Function

Q	S/Z		
R	Reliability		
R_c	Rate of Corrosion		
S	Failure Governing Strength		
$\acute{S}$	Nominal Strength Under Idealized and Standard Test Conditions		
SM	Required Section Modulus of Ship Hull		
s_y	Tensile Yield Stress of Material		
t	Thickness		
t_c	(For Corrosion Allowance)	t_o	(Original)
t_f	(Of Flange)	t_w	(Of Web)
t_n	(For Limiting Stress)	$\bar{t}_n$, $\bar{t}_c$	(Means of t_n and t_c.)
tol	Tolerance		
V_S	Coefficient of Variation (COV) of Strength, $({}^{\sigma}S/_{m_S})$		
V_Z	Coefficient of Variation (COV) of Load, $({}^{\sigma}Z/_{m_Z})$		
V_x	Coefficient of Variation of x		
V_{xo}	Coefficient of Variation of Objective Uncertainties of x		
V_{xs}	Coefficient of Variation of Subjective Uncertainties of x		
$\bar{x}$	Mean of Variable x		
Z	Failure Governing Load		
δ	Coefficient of Variation		
ϕ_{Y_n}	Density Function of the Extreme Wave Bending Moment		
Φ_{Y_n}	Distribution Function of the Extreme Wave Bending Moment		
ϕ_{Z_n}	Density Function of Extreme Load Composed of Wave Bending and Still Water Bending		
Φ_{Z_n}	Distribution Function of the Extreme Load of Wave Bending and Still Water Bending		

ε	Random Variable Representing Constituent Parts of Strength
θ	Central Safety Factor, (m_S/m_Z)
$\phi(z)$	Density Function of Load
$\Phi(z)$	Distribution Function of Load
τ	Time
δ_S	Coefficient of Variation of Strength
σ_M^2	Variance of Margin of Safety
σ_N	Average Failure Stress
σ_Z^2	Variance of Load
σ_S^2	Variance of Strength
σ^2	Variance of Still Water Bending Moment or General Variance
σ_N	Average Failure Stress of Hull Material
γ	Safety Index, (m_M/σ_M)
ρ	Correlation Coefficient
ψ_S	Standard Tabulated Normal Function
μ	Mean
ξ	Initial Lateral Deflection of Plating

SECTION 1.0

INTRODUCTION

The conventional methods of performing longitudinal structure designs of ships make use of accumulated experience from previously built ships of similar size and function. The accumulated experience is mostly expressed in the form of semi-empirical formulas contained in classification society rules and design specifications. The designs resulting from this approach are uncertain as to the degree of structural adequacy they afford even though the ship designs based on these approaches have given acceptable service. The uncertainty stems from the assumptions made regarding parameters affecting the environment and the strength of the ship. Many years of design experience have shown that by using appropriate empirical margins for strength over expected load, the unknowns can be accounted for and ships with acceptable risk or probability of failure levels designed.

With the advent of new ship types, and the resultant lack of "accumulated experience" on vessels of similar size and function, it has become a professional responsibility to look into a more scientific, or rational, approach to longitudinal strength design of ship hulls. In this context, various investigators in the ship research community have adopted probabilistic structural analysis procedures from mechanical and civil engineering. In the "probabilistic approach", since the quantitative values of many of the factors affecting the strength of the structure and the magnitude of the load are statistically determined, the resulting measure of the adequacy of the design is also statistical in nature.

In the study presented in this report, various facets of probabilistic structural design were investigated with emphasis on applicability to ships.

Section 2.0 gives a statement concerning the detailed objectives of the study. In Section 3.0, probabilistic structural analysis is reviewed from a general standpoint and its applicability to ships is noted. Section 4.0 discusses the possible structural modes of failure of a ship that pertain to longitudinal strength. The present situation with information on ship loads as they relate to structural design is discussed in Section 5.0, and the probabilistic structural analysis procedures that show promise for ship applications are presented in Section 6.0. In Section 7.0, the investigations, analyses and collected information performed and obtained as part of this study in the area of the uncertainties of hull strength with respect to the statistical description of the strength are presented. Section 8.0 gives sample calculations for different ships using a probabilistic structural analysis procedure embodied in a computer program included in the Appendix. Sections 9.0 and 10 0 give the conclusions and recommendations respectively arrived at as a result of these studies.

The references cited in the report are listed in Section 11.0

SECTION 2.0

STATEMENT OF OBJECTIVES

The objectives of this study were modified by the Ship Structure Committee during the course of the project to be commensurate with what was found to be available and possible within the rather small funding allocated.

The final objectives can be stated as follows:

- Survey the existing literature on reliability analysis and probabilistic design methods in structures. Comment on the applicability to ships.
- Develop a method, or use an existing method, for the formulation of strength in terms of the means and variances of its uncertainties. Although a mathematical distribution of strength is not required, observations are to be made with respect to the impact of using only means and variances.
- Relate the existing bending moment distributions calculated from existing data to the developed strength distributions using an existing method for structural reliability analysis. Use available statistical strength parameter means and variances and make assumptions for any strength or load parameters for which no statistical data are available.
- Develop a FORTRAN IV computer program to perform the above procedure with the objective of determining the safety level of a given ship subjected to a given load.
- Apply the developed computerized procedure to actual ships.
- On the basis of obtained results, suggest further research to develop suitable longitudinal strength criteria for future designs.

SECTION 3.0

PROBABILISTIC APPROACH TO STRUCTURAL DESIGN

3.1 General

The objectives of this study include the analysis of uncertainties associated with ship hull strength and the development of expressions for structural reliability. Such analyses require the adoption of a probabilistic structural design approach since a purely deterministic approach cannot yield the desired information.

In the deterministic design of structures, the strength of the structure is always increased above that which would just survive the greatest expected load by an empirical margin. The ratio of the latter to the former strength is usually termed the factor of safety. It accounts for all the unknowns in the load and strength and yields a structure that should have an acceptable performance based on past experiences.

The fundamental aims of a probabilistic approach are to more clearly and rationally define the necessary margin, or factor of safety, and obtain a quantitative measure of performance through a rational rather than empirical analysis. The measure of performance is usually called the probability of failure or reliability. With such aims, it is not necessary that a probabilistic analysis be exhaustive in that rationalization of even only one of the unknowns in the factor of safety will put it on a sounder footing. In this vein, the ultimate result of improved probabilistic analysis procedures, as far as designers are concerned, will probably be rational factors of safety based on desired quantitative levels of performance. The probabilistic analysis itself need not be executed by the designers, although this could be possible.

A complete probabilistic structural analysis would proceed in the following manner [9]*:

- Conduct an analysis of failure modes, effects, and criticality.
    - Identify all significant failure modes of the structure.
    - List the cause of these failure modes.
    - Identify all parameters contributing to these causes.
    - Determine the criticality of all siginficant failure modes to the success of structures.
    - List the most critical failure modes in order of priority.
- Formulate the relationship between the critical parameters and the failure-governing criteria involved.
- Determine the failure-governing load function.
- Determine the failure-governing load distribution.
- Determine the failure-governing strength function.
- Determine the failure-governing strength distribution.
- Calculate the probability of failure or reliability associated with

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* Numbers in brackets indicate similarly numbered references in Section 11.0.

the failure-governing load and strength distribution for each critical failure mode.
- An upper bound of the total probability of failure or a lower bound of the reliability will be the sum of the individual probabilities of each of the critical failure modes under the assumption that these modes are mutually exclusive events.

Because of the difficulty associated with the determination of the failure-governing load and strength functions and distributions, a number of probabilistic approaches or methods have evolved. They differ fundamentally in the two primary aims of any probabilistic analysis as mentioned above:

- Quantitative measure of performance
- Rational quantification of load and strength

Actually, not all the approaches are necessarily probabilistic in the mathematical sense in that for some, probability densities and distributions are not needed, and the output is not a probability.

These methods may be grouped as follows:

- Classical probabilistic approach
- Safety index approach
- Strength reduction and load magnification factors approach

The presentation in this section is divided into three groups. The first group discusses the general approach used in obtaining the quantitative measure of performance of a structure given the load and strength statistics. The next groups each deal with details of the strength and load formulations respectively, in a general sense. More specific mention of these considerations, as applicable to ships, is given in Sections 5.0 thru 7.0, respectively for loadings, longitudinal strength, and for uncertainties in the strength of the ship's hull.

The literature contains abundant sources of probabilistic structural analyses. Most of the work has been done in the areas of civil and mechanical engineering but has more recently spread to naval architecture.

Probabilistic design concepts for structures were first proposed in the U.S. in 1947 [1]. Since then, several investigators have presented further considerations for applications in civil engineering, References [2] thru [6], mechanical engineering, references [7] thru [9], and more recently in naval architecture, reference [10].

Within the framework of the present study, a brief review of the numerous methods as cited was performed to identify the ones which would seem appropriate for future consideration in probabilistic structural analyses of ships from the standpoint of design.

3.2 Probabilistic Methods

3.2.1 Quantitative Measure of Performance

As previously mentioned, the existing probabilistic structural analysis methods differ in the output measure of performance of the structure being considered.

Those methods that are more probabilistic in the mathematical sense, generally, are of the classical type. Their measure of performance is in terms of a probability defining failure or reliability.

The other methods have evolved primarily due to the difficulties associated with executing a fully probabilistic procedure. Their measure of performance is not a probability at all, instead, it is a number indicating either a margin of safety or reduction and magnification factors for strength and load, respectively. These numbers do not have a physical significance like probability of failure or reliability, but they can be compared to each other for previous successful and unsuccessful designs to obtain limiting values.

3.2.2 Classical Approach

The one common point in all probabilistic structural analysis procedures is the definition of the probability of failure and reliability. If the failure-governing load is Ƶ and the failure-governing strength S, then the probability of failure, P_f, is given by all probabilities that the failure-governing load exceeds the failure-governing strength:

$$P_f = P\ (Z > S) \qquad (1)$$

The probability of failure is also called the unreliability, while the reliability, R, becomes:

$$R = 1 - P_f = P\ (S > Z) \qquad (2)$$

Equation (1) is presented in much of the literature, for example in [10], as directly applicable to ships in the following manner:

$$\begin{aligned} P_f &= P\ [S < Z] = P\ [\tfrac{S}{Z} < 1] = P\ [Q < 1] \qquad (3) \\ &= P\ [(S - Z) < 0] = P\ [M < 0] \end{aligned}$$

The terms "Q" and "M" of Equation (3) are functions of two random variables: the strength, S, and the load, Ƶ, and themselves random variables whose probability must be determined by joint probability density and distribution functions. However, there seems to be a universal agreement to consider the load and strength statistically independent so that the statistics of M and Q can be directly determined from those of S and Ƶ. This assumption appears to be reasonable for most strength considerations as long as the effects on the structure of being in an aqueous environment with waves for a long period of time are accounted for in the strength. If $\phi(z)$ and $\Phi(z)$ are the probability density and distribution functions of the load, respectively, and $f_S(s)$ and $F_S(s)$ those of strength, then it can be shown that the density and distribution functions of Q are, [10]:

$$f_Q\ (q) = \int_0^\infty \phi(z)\ f_S\ (qz)\ z\ dz \qquad (4)$$

$$F_Q\ (q) = \int_0^\infty \Phi(z)\ F_S\ (qz)\ dz \qquad (5)$$

and the probability of failure becomes:

$$P_f = \int_0^\infty \phi(z)\ F_S\ (z)\ dz \qquad (6)$$

$$= 1 - \int_0^\infty \Phi(z)\ f_S\ (z)\ dz \qquad (7)$$

Equations (6) and (7) are rather simple and could easily be evaluated provided the density and distribution functions of load and strength are known. This is where the crux of the matter lies and will be discussed later in Sections 3.3 and 3.4. The methods that make use of Equations (6) and (7) vary significantly in complexity and effort required for execution.

Equation (7) can be evaluated for each mode of failure and, as noted previously, the sum of all probabilities of failure for all modes will give an upper bound. To do better would require the joint probability density function of strength in the various failure modes which would be at best very difficult to obtain. A lower bound on the probability of failure can be determined by assuming that the modes of failure are perfectly correlated.

3.2.3 Safety Index Approach

The difficulty in obtaining load and strength density and distribution functions has led investigators to develop approaches which minimize the effort required. For instance, in the area of ships, [13] contains an approximate semi-probabilistic design method which was motivated, among other things, by the lack of data on loads and strength and by the controversial status of forms of load and strength distributions. The method requires that only the means and variances of the load and strength be known.

This "approximate" approach considers the margin of safety M of Equation (3) as a random variable with mean m_M and variance σ_M^2.

$$P_f = P\ [M<0] = P\ [\frac{M-m_M}{\sigma_M} < \frac{-m_M}{\sigma_M}] = P\ [G \leq -\gamma] = F_G\ (-\gamma) \qquad (8)$$

By using the error distribution of M, [16], discussed in more detail in Section 3.3, the mean and the variance of M can be written:

$$m_M = m_S - m_Z \qquad (9)$$

$$\sigma_M^2 = \sigma_S^2 + \sigma_Z^2 \qquad (10)$$

where:

m_S, σ_S = mean and variance, respectively of strength.

m_Z, σ_Z = mean and variance of total load.

The following results are obtained by algebraic processes:

$$\gamma = \frac{m_M}{\sigma_M} = \frac{m_S - m_Z}{\sqrt{\sigma_S^2 + \sigma_Z^2}} = \frac{\theta - 1}{\sqrt{\theta^2 V_S^2 + V_Z^2}} \qquad (11)$$

$$\theta = \frac{1+\gamma(V_S{}^2+V_Z{}^2-\gamma^2 V_S{}^2 V_Z^2)^{1/2}}{1-\gamma^2 V_S{}^2} \tag{12}$$

$$m_S = \theta\ m_Z \tag{13}$$

$$SM = \frac{m_S}{\sigma_N} \tag{14}$$

Where: γ = safety index = m_M/σ_M

θ = central safety factor = m_S/m_Z

V_S = coefficient of variation (COV) of strength = $\frac{\sigma_S}{m_S}$

SM = required section modulus of the ship hull

V_Z = COV of load = $\frac{\sigma_Z}{m_Z}$

SM = required section modulus of ship hull

σ_N = average of failure stress of hull material

m_M = mean of the margin of safety

σ_M^2 = variance of the margin of safety

From Equation (8), it can be seen that each value of the safety index γ is associated with some probability of failure. However, Equation (8) cannot be evaluated since the distribution function F_G is not known. If enough information were available to determine F_G, then Equations (6) and (7) of the classical approach could be used directly. From Equations (11) through (14), it can be seen that the inputs needed to obtain a hull design strength are the strength and load COV's, mean of the bending moment, and the safety index γ. The amount of computation is insignificant.

The safety index γ is a single number that must be obtained on the basis of many technical factors. It has previously been proposed [13] to determine this value from existing designs to take into account the vast accumulated experience. In addition, if the probability of failure associated with past designs is socially acceptable, then this aspect is also considered.

3.2.4 Strength Reduction and Load Magnification Factors

This method, discussed in [5,62,63], is similar to the approximate method described above in that only means and variances of the load and strength are used to obtain relative and semi-probabilistic measures of the structure's performance. In this case, the measures of performance are the strength reduction and load magnification factors.

The strength reduction factor, f_S, and load magnification factor, f_Z, can be defined as follows:

$$f_S = \frac{\text{minimum strength}}{\text{average strength}} = \frac{m_S - K_S \sigma_S}{m_S} = 1 - K_S V_S ; \; f_S < 1 \qquad (15)$$

$$f_Z = \frac{\text{maximum load}}{\text{average load}} = \frac{m_Z + K_Z \sigma_Z}{m_Z} = 1 + K_Z V_Z ; \; f_Z > 1 \qquad (16)$$

where: K_S, K_Z = Factors giving the number of standard deviations between the average and the minimum strengths and the maximum loads, respectively.

For a safe design, the minimum strength must exceed or equal the maximum load:

$$m_S f_S \geq m_Z f_Z \qquad (17)$$

The values of acceptable strength reductions factors and load magnification factors could be obtained from past designs in a similar fashion to the safety index of the previous section.

In [5], this approach has been extended to fatigue for both the constant range and the random loads.

Similarly to the safety index approach, the analyses required to execute this method are quite limited in extent and complexity.

3.3 Strength Statistics

3.3.1 General

It must be first stated that the strength of the hull girder may or may not vary with time depending on the failure mode being considered. Time invariant strengths will include yielding and buckling. Time variant strengths will include fracture, fatigue, and reduced strengths due to corrosion. For ships, time variant strengths will also normally include random loadings of low or high cycles, and possibly thermal loadings. This scenario should cover the most significant modes of hull girder failure which need to be addressed.

3.3.2 Strength Equation

The strength of a structure is principally described in two different ways in the numerous probabilistic structural design methods to be found in the literature.

$$S = f\,(\varepsilon_1, \varepsilon_2, \text{----}. \varepsilon_n) \qquad (18)$$

or:

$$S = k_1 k_2 k_3 \text{----} k_n S' \qquad (19)$$

where: $\varepsilon_1 ---- \varepsilon_n$ = Constituent parts of the strength which are assumed to be random variables

S' = Nominal strength determined under idealized and standard test conditions

$K_1 ---- K_2$ = Strength factors to convert the nominal strength to actual strength. (These factors are assumed to be random variables).

The K factors account for physical variables such as size, forming and manufacturing processes, surface finish, load, heat treatment, direct surface environment, temperature, time, corrosion, etc.

The approach given by Equation (18) has been used in ships, but the actual examples developed have been such that only the explicit functional strength constituents, ε, have been considered as random variables or uncertainties in the strength. As the probabilistic analyses become more comprehensive and more uncertainties become identified, some of these may not appear as constituents in the strength equation, and the approach depicted in Equation (19) may have to be adopted in addition to that in Equation (18).

3.3.3 Strength Distributions

Equations (18) and (19) give general expressions for the strength, but since the strength is statistical in nature, the probability density and distribution function must be specified to completely characterize it and allow the probability of failure to be evaluated by Equations (6) and (7).

The probabilistic structural analysis approaches found in the literature assume that the strength distribution can be determined in one of the following ways:

- Actual component strength distribution determined by actual testing under the exact geometry, application, and operational environment in which the component shall function.
- Component strength distribution synthesized from the known distributions of the constituent parts and strength factors as given in Equations (18) and (19).
- An assumption made as to what type of distribution the strength will follow, i.e. normal, lognormal, Weibull, etc.
- An assumption made that all that can be determined of the strength is its COV.

The first of the above approaches is used extensively in machine design and some of the test equipment required is described in [7]. This approach would hardly seem realistic for ships because of the large size of the structure, the implication of using the whole ship as a discardable test component, and the large data sample required for conclusive results. Whether or not components of the ship structure could be tested and results extrapolated to the whole ship appears questionable. In the case of welded ship grillages under compressive load [64]:

> "Further experimental evaluation of grillage strength also has a key part to play but cannot be expected to provide direct statistical descriptions of grillage strength; large-scale tests of the type described in the present paper are too expensive to carry out in sufficient numbers and small-scale tests are statistically unrepresentative for the reasons mentioned above. It is suggested that the main role of further grillage tests should, therefore, be to guide the development of improved analysis methods and to check the accuracy of such methods and design data with provision of empirical corrections where necessary."

The second approach requires that the distributions of the constituent parts and strength factors be known. It may, for example, be necessary that the distribution of the dimensions of depth, beam, and the area of flanges be known. Such quantities are much more amenable to scrutiny in ships than the overall testing of the hull girder. As discussed in Section 7.0, however, not much data presently exist for many of the variables, and consequently the distributions themselves cannot be identified. This would seem to be a promising area in the future, if an effort is made to collect such data.

If the distribution of the constituent parts and functions are known, there are various methods for synthesizing their distribution to obtain the overall strength distribution. Reference [7] gives eight methods:

- The algebra of normal function method
- The change of variable method
- The moment generating function method
- The Fourier transform, convolution, and inversion method
- The Mellin transform, convolution and inversion method
- The characteristic function method
- The cumulative distribution function method
- The Monte Carlo method

The Monte Carlo method will always give results even for complex functions of non-identically distributed random variables although the length and complexity of the computations will reportedly be quite extensive and possibly unrealistic.

The third approach requires that assumptions be made concerning the distribution of the strength. Of course the same could be done with the constituent parts and factors, and the second method used to construct the strength distribution. This approach seems to be universal in the literature for civil engineering and naval architecture. It is natural that these two disciplines would make greater use of this last approach because of the size and complexity of the structure analyzed.

This approach requires the adoption of a distribution (such as the normal, lognormal, Weibull, etc.) and the specification of necessary parameters of the distribution to obtain numerical values from tabulated density and distribution functions. The necessary parameters are at least the first and second moments of the distribution, the mean and variance.

Most of the assumed distributions in the literature on structural analysis are the normal and the lognormal distributions. It would seem natural for investigators to make such assumptions since experimental measurements in science and engineering seem to approximate, rather well, the normal law. However, the integrations of Equations (6) and (7) for the probability of failure involve important constituent parts at the tail end of the distributions which can vary greatly depending on the assumed distributions. In reference [4], it is stated that for the probability of failure $P_f \leq 10^{-5}$, the calculated probability is sensitive to the assumed distribution and the results can only be used relatively. On the other hand for probabilities of failure $P_f > 10^{-3}$, such problems would not be too serious.

As reported in [15], the record of world ship catastrophes indicate a current probability of failure for ships in the order of 10^{-3} so that these approximations may not be a problem in the case of ships if the historical safety levels are considered adequate.

If the strength is assumed to be normally distributed, the probability density and distribution functions are:

$$f_S(s) = \frac{1}{\sigma_S\sqrt{2\pi}} \exp -1/2\left(\frac{s-m_S}{\sigma_S}\right) \qquad (20)$$

$$F_S(s) = \int_{-\infty}^{s} f_S(s)\,ds = \Psi_S\left(\frac{s-m_S}{\sigma_S}\right) \qquad (21)$$

where:

m_S = mean of strength S

σ_S = standard deviation of strength S

Ψ_S = standard tabulated normal function.

Consequently, under such an assumption, the only quantities that need to be determined are the means and variances of the strength. Then, the probability of failure given by Equations (6) and (7) can be evaluated (provided the load distribution is known). The latter statement is not trivial since, in fact, the means and variances of ships strength are not easily determinable.

The approach, in general, has been to expand the strength function in terms of its constituents in a Taylor Series about the means of the constituents:

$$S = f(\varepsilon_1, \varepsilon_2, ----, \varepsilon_n)$$

$$= f(\bar{\epsilon}_1, \bar{\epsilon}_2, ----, \bar{\epsilon}_n) + \sum_{i=1}^{n} (\epsilon_i - \bar{\epsilon}_i) \left(\frac{\partial f}{\partial \epsilon_i}\right)_{\bar{\epsilon}_i} \qquad (22)$$

$$+ 1/2 \sum_{i=1}^{n} (\epsilon_i - \bar{\epsilon}_i)^2 \left(\frac{\partial^2 f}{\partial \epsilon_i^2}\right)_{\bar{\epsilon}_i} + \ldots . + (\text{Remainder})$$

in which the derivatives are evaluated at the constituent means, $\bar{\epsilon}_1, \bar{\epsilon}_2$,----$\epsilon_n$ and the remainder consists of the higher derivatives.

If it is assumed that the higher derivatives are small or zero and that the coefficients of variation of the constituents are small, in the order of 15 per cent or less [16], then Equation (22) can be linearized and the following obtained:

$$m_S \cong f(\bar{\epsilon}_1, \bar{\epsilon}_2, ----\bar{\epsilon}_n) \qquad (23)$$

$$\sigma_S^2 \cong \sum_{i=1}^{n} \sigma_{\epsilon_i}^2 \left(\frac{\partial f}{\partial \epsilon_i}\right)^2 + \sum_{i \neq j}\sum c_{ij}\sigma_{\epsilon i}\sigma_{\epsilon j} \left(\frac{\partial f}{\partial \epsilon_i}\right) \left(\frac{\partial f}{\partial \epsilon_j}\right) \qquad (24)$$

Where ρ_{ij} is the correlation coefficient between ϵ_i and ϵ_j. These assumptions may not turn out to be correct for all ships for all modes of failure. It is indicated in [14] that the inclusion of nonlinearities in the strength distribution causes various changes only in the predictions of long-term probability of failure.

Further, making the assumption that the constituent parts are statistically independent, the correlation becomes zero and Equation (24) reduces to:

$$\sigma_S^2 = \sum_{i=1}^{n} \sigma_{\epsilon_i}^2 \left(\frac{\partial f}{\partial \epsilon_i}\right)^2 \qquad (25)$$

Equations (23) and (25) have been used in ship studies to date. The assumption of zero correlation inherent in Equation (25) may be reasonable for many of the constituent parts. For example, in the case of the strength defined by Equation (27), the beam (B) should have no effect on the depth (D) and similarly both D and B should have no effect on plate thicknesses t_f and t_w On the other hand, as an example, the strengths in different failure modes of the same panel may be highly correlated [66].

If Equation (25) is written in terms of a coefficient of variation (COV):

$$\delta_S^2 = (\sigma_S/m_S)^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial s_i} \cdot \frac{\bar{\epsilon}_i}{m_S}\right)^2 \cdot \delta_{\epsilon_i}^2 \qquad (26)$$

where: $\delta_S = \frac{\sigma_S}{m_S} =$ strength COV

$\delta_{\epsilon_i} = \frac{\sigma_{\epsilon_i}}{\bar{\epsilon}_i} =$ COV's of constituent parts

Equations (23), (25), and (26) then give the strength parameter's mean, variance, and COV respectively in terms of the means and variances of the constituent parts, (ϵ). These must be determined from data or by estimation as discussed in detail in Section 7.0. The definition of the strength is then complete and the probability of failure can then be evaluated. The greatest amount of effort is needed in determining the strength COV, and is only a fraction of that required by the first two approaches. One would, of course, have a lesser degree of confidence in the results.

The fourth approach requires only that the COV or the mean and variance of the strength be known. The procedure to obtain these was just given above. These data can only be used in the semi-probabilistic methods outlined in Sections 3.2.3 and 3.2.4. This approach requires the least computational effort to obtain its results.

3.3.4 Time Dependent Strengths

In general, whenever a critical failure mode involves a time variant strength such as it does in the cases of fracture, fatigue, thermal effect, and corrosion, the variations with time must be accounted for. If the strength can be treated as a function of time, the general probabilistic procedures presented previously can be utilized.

Mechanical reliability for components exposed to fatigue is discussed in [8] and [9]. The approach therein is to use the form of strength given by Equation (19) which would take care of some time-dependent effects through the K coefficients; this is implied but not stated.

From the standpoint of fatigue, the following problems are directly addressed in these references:

- Fatigue under a fixed alternating load level, given the "cycles to failure" distribution of the component.
- Fatigue for a specified life given the broad band strength and load distributions for that life.
- Cumulative fatigue under sequential groups of stresses, each group having a specific number of cycles and the same maximum and mean alternating stress levels.

The approaches to solving these problems are identical to those previously discussed herein in that all analyses are performed at a given time in the life of the component and at a constant load level.

Reference [17] reports on studies conducted to investigate time-varying structural probabilistic strengths in the jet engine field. The basis of the general procedure proposed is a computational sequence to determine probability of failure vs time consisting of two phases: the first is a failure probability phase and the second a degradation of strength phase. Thus, a probability of failure calculation is made, followed by a strength degradation calculation reflecting some operation time. The sequence can be repeated indefinitely. The crux of the procedure revolves around identifying a time-varying strength degradation scenario. Several types are proposed but the analyses reported in that paper were of a "preliminary" nature. It is noted that additional work was in progress at that time.

During the course of the study presented herein, a potential scenario for corrosion of ship hulls was envisioned. If the mode of failure under consideration is that of yielding during bending of the hull girder as a "free-free" beam, it can easily be shown that the strength equation is:

$$\begin{aligned} S &= f(D,\ t_f,\ B,\ t_w,\ s_y) \\ &= N s_y \equiv D(t_f B + 1/3\, t_w D)\, s_y \end{aligned} \tag{27}$$

where:

N = deck or bottom section modulus

s_y = tensile strength

D = section depth

B = section beam

A_f = area of flanges

A_w = area of webs

t_f = $A_f/2B$ = equivalent thickness of one flange

t_w = $A_w/2D$= equivalent thickness of one web

If corrosion is introduced, then A_f, A_w, t_f and t_w become functions of time as the plating corrodes.

The plate thicknesses may then be considered a function of time as follows:

$$t(\tau) = t_o - \tau R_c(\tau) \tag{28}$$

where:

$t(\tau)$ = Plate thickness in time -

t_o = Original thickness at τ=o

R_c = Rate of corrosion, also a random variable

The strength would then become a function of time and the probability of failure could be estimated at various times during the ship's life using the probabilistic theory previously presented. Alternatively, the original strength at time t=o could be multiplied by a factor k_c, reflecting equation (28), also a random variable, to account for a specific reduction in strength at a certain time in the vessel life.

Another approach to consider the effect of corrosion which does not result in a time dependent strength is to take the total plate thickness as the sum of the thickness required for limiting stresses,

t_n, plus a thickness for corrosion allowance, t_c [62]:

$$t = t_n + t_c \qquad (29)$$

Which by Equation (26) yields:

$$\delta_t^2 = \left(\frac{\bar{t}_n}{\bar{t}}\right)^2 \cdot \delta_{t_n}^2 + \left(\frac{\bar{t}_c}{\bar{t}}\right)^2 \delta_c^2 \qquad (30)$$

where:

δ_{t_n} = COV of the plate thickness due to production tolerances

δ_c = COV due to corrosion

As pointed out in [62], the corrosion rate will vary from one group of strength members to another and this has been addressed by others using a Monte Carlo simulation technique [65].

In [19], a method is presented for probabilistic analysis of fatigue-crack initiation at a butt-welded joint. The procedure is used for analyzing both the longitudinal and transverse structural members of a tanker subjected to random still water and wave loads. This reference represents the only source found during the course of this study which gives a probabilistic evaluation of ship structure fatigue. The strength function given therein is based on Miners' law and on the coefficients of a logarithmic linear approximation of the S-N curve, which are regarded as random variables. A sensitivity analysis on these random variables is also presented. The degradation of strength in time by factors other then fatigue is not considered and it is noted that:

"because of lack of sufficient amount of statistic data or quantitative information on unexpected defects in hull structure, this study is limited to within a range of treating only a standard ship which is built through sound workmanship of well quality-controlled fabrication and is put into service with satisfactory maintenance under normal operating conditions. It should, therefore, be clearly born in mind that the results obtained by this analysis will provide information on the reliability of ship structures merely on the basis of design-oriented point of view." [19].

The approach used in [5], as previously discussed in Section 3.2.4., has been extended therein to constant stress range and random fatigue.

3.4 Load Statistics

3.4.1 General

As discussed in Section 2.0, the objectives of this study do not include details concerning the load distribution. However, since the load is one of the two major considerations of any probabilistic structural design, it will be discussed here from the standpoint of characteristics and mechanics that must be considered for application in probabilistic structural design. The literature on loads does not address this point extensively. A qualitative appraisal of the situation with respect to loads applied to ships is included in Section 5.0 of this report.

The types of loads applied to the hull girder consist of the following [59]:

- Calm water due to weight and buoyancy.
- Ship's own wave train.
- Thermal effects.
- Quasi-static wave induced (low frequency).
- Dynamic (high frequency): including slamming, whipping, springing, and propeller induced vibration.

3.4.2 Equations and Distributions

Equations (1) through (7) deal with expressions for the probability of failure, reliability, and margin of safety. In these expressions strength and load carry the same weight and require the same type of expressions for their mathematical description. Hence, all that has been stated for the strength equations and distributions would apply in most cases to the load distributions as well.

With respect to ships, the procedures of synthesizing distributions of the constituent parts into that of the whole should be emphasized. The procedure for combining still water and wave bending moments, springing, slamming, and thermal effects should be similar to that presented in 3.3.3 for strength distributions.

The analyses to be found in the literature on probabilistic structural design of ships have only considered still water and wave bending moments directly. This is primarily due to lack of information applicable to other types of loads, as discussed further in Section 5.0. It should be pointed out here, however, that in any complete probabilistic analysis, the total load must be considered.

In the case of longitudinal strength, this total load will include the effects of local loadings, such as that due to water head, since this will add a random load toward increasing the overall load and hence, the stress.

With respect to specific distributions proposed in the literature, those found in [10] have been used in probabilistic structural analyses of ships presented therein; the wave bending moments and still-water bending moments have been considered. The amplitudes of the wave bending moments are assumed to follow a Rayleigh distribution in the short term, and an exponential probability law in the long term. Using the Weibull distribution, both the short-term and the long-term wave distribution and density functions, respectively, are given as follows:

$$f_L(x) = (\ell/k)\cdot(x/k)^{\ell-1}\, e^{-(x/k)} \quad x \geq 0 \tag{31}$$

$$F_L(x) = \int_0^x f_L(x)\, dx = 1 - e^{-(x/k)} \quad x \geq 0 \tag{32}$$

where:

$\ell = 2$ for short term
$\;\;= 1$ for long term
$k = \sqrt{E}$ for short term
$k = \lambda$ for long term
E = mean square value of L taken over a short period of time
λ = expected value of L taken over a long period of time

It should be pointed out that in [60] it is shown that the exponential law underestimates the data measured onboard an Ore/Bulk/Oil carrier. Therein, it is concluded that mathematical models based on the normal or general Weibull distributions give excellent agreement with statistical data for the ship analyzed. Reference [61] shows that for two other ships, the Weibull distribution does not exactly fit the data.

In Reference [12], "order statistics" are used to obtain the extreme wave bending moment density and distribution functions using equations (31) and (32). These extreme functions become:

$$\phi_{Y_n}(y) = \frac{n\ell}{k}\,(y/k)^{\ell-1}\cdot e^{-(y/k)^{\ell}}[1-e^{-(y/k)^{\ell}}]^{n-1} \quad y \geqslant 0 \tag{33}$$

$$\phi_{Y_n}(y) = P[Y_n \leqslant y] = [1-e^{-(y/k)^{\ell}}]^{n} \quad y \geqslant 0 \tag{34}$$

where n is the number of wave records considered.

The still-water bending moment is incorporated first as deterministic and then as a normally distributed random variable. The combined still-water and wave bending moment probability density and distribution functions in the deterministic case are:

$$\phi_{Z_n}(z) = \frac{n\ell}{k}\left(\frac{z-m_o}{k}\right)^{\ell-1}\cdot e^{-\left(\frac{z-m_o}{k}\right)^{\ell}}\cdot[1-e^{-\left(\frac{z-m_o}{k}\right)^{\ell}}]^{n-1} \tag{35}$$

$$= 0,\ \text{otherwise},\ z \not\geqslant m_o$$

$$\phi_{Z_n}(z) = [1-e^{-\left(\frac{z-m_o}{k}\right)^{\ell}}]^{n} \tag{36}$$

$$= 0\ \text{otherwise},\ z \not\geqslant m_o$$

where m_o is the deterministic bending moment.

The probability density and distribution functions in the normally distributed case are:

$$\phi_{Z_n}(z) = \frac{nl}{k}\cdot\frac{1}{\sigma_s\sqrt{2\pi}}\int_0^{\infty}(y/k)^{l-1}\cdot e^{-(y/k)^{l}-\frac{1}{2}\left(\frac{z-y-m}{\sigma_s}\right)^2}\cdot[1-e^{-(y/k)^{l}}]^{n-1}dy \tag{37}$$

and

$$\Phi_{Z_n}(z) = \frac{nl}{k}\cdot\frac{1}{\sigma_s\sqrt{2\pi}}\int_0^{\infty}(y/k)^{l-1}\cdot e^{-(y/k)^{l}}\cdot[1-e^{-(y/k)^{l}}]^{n-1}\cdot\int_{-\infty}^{z}e^{-\frac{1}{2}\left(\frac{t-y-m}{\sigma_s}\right)^2}dt\,dy \tag{38}$$

where m and σ are the mean and standard deviation of the still water bending moment respectively.

SECTION 4.0

MODES OF HULL FAILURE

4.1 General

It is well known that the design of a ship's hull girder from the standpoint of longitudinal strength is usually performed by considering yield failure of the hull girder as a free-free beam in bending. The load is normally determined by balancing the ship on an "extreme wave" for both hogging and sagging conditions and the resulting stress must remain below an allowable level. Factors of safety based on experience are contained in the loads and the allowable stresses. Experience has shown that such an approach leads to probabilities of commercial ship failures in the order of 10^{-3}, [15, 59], although the modes of the failures are not all known.

In turning to probabilistic structural design, as pointed out in Section 3.0, all conventional factors of safety must be stripped away and accurate distributions of load and strength must be determined. Further, all potential modes of failure must be analyzed in separate calculations.

This last aspect may appear subtle to some; but one must remember that the yield failure of the hull girder as a beam is not the only potential mode of failure of a ship hull girder. With the historical conventional factor of safety approach on this yield failure mode, other modes of failure may also be automatically taken care of but with smaller margin and, therefore, with less of an effective factor of safety. This, of course, is the major shortcoming of the conventional factor of safety approach and is rooted in its empiricism.

Consequently, in the probabilistic structural analysis, all the potential modes of failure of the hull girder must be identified and analyzed. The output may again be a factor of safety, but its determination would be on a more rational basis.

4.2 Modes of Failure of the Hull Girder

Modes of failure of the hull girder from a longitudinal strength standpoint can be grouped into the following:

- Yield failure due to bending of the ship considered as a free-free beam
- Compression instability buckling
- Brittle fracture
- Fatigue fracture
- Ultimate plastic collapse

As previously stated, longitudinal strength in hull girder design is usually based on the deterministic evaluation of beam bending with factors of safety to prevent a yield failure. However, it is interesting to note that various investigators have indicated this not to be the most significant mode of failure, [21] and [23].

In [21], it is shown that compressive and tensile strengths of even poorly built ships are adequate to withstand the most severe wave bending moments. With respect to brittle fracture, it is noted that fractures cannot initiate because the quality of workmanship today is high and the nominal stresses are usually low. However, if higher allowable stresses in hull materials are used, then means of arresting cracks will have to be considered. The feeling is that the brittle-fracture problem can be eliminated by proper use of crack-arresting steels under any circumstances. In the future, the problem may be restricted to fatigue cracks and how large they may be allowed to get without leading to unstable fracture. Fracture-mechanics investigations are proposed for this analysis. A statement made in (21] is of interest:

> "So much for the brittle-fracture problem. It is quite possible that within 10 or 20 years it has disappeared from shipbuilding. Then the level of permissible stresses will be to a large extent determined by fatigue considerations. In fact it does so already nowadays together with brittle fracture, buckling of bulkheads and webs of deep frames and bottom damage due to slamming. It seems that not everyone is aware of this fact. There are even investigators, dedicating their time to wave bending moments, who are not much interested in fatigue."

In [22], a method is presented for the determination of the ultimate plastic moment of the hull girder. It is stated that elastic stresses from the conventional approach:

> "may be influenced by residual reaction or thermal effects to such an uncertain extent that the stresses thus calculated are sometimes regarded as having only comparative rather than absolute value. The ultimate strength of a ship is likely to be influenced by these uncertainties to a much smaller extent, so that the calculated hull bending moment should give a reliable indication of the true bending strength of the hull. It must be emphasized again, however, that the possibility of premature failure by major hull fracture must be guarded against by proper design and construction details and control of material quality. If this is true, then overall hull girder failure can only occur through yielding and buckling, in the way assumed in this analysis."

However, discussions of the cited reference indicate that buckling has been eliminated to a very great extent and brittle fracture is the principal hazard, [23], and that low-cycle fatigue leading to local failure and hastening the complete "breaking its back" before the ideal ultimate failure load is the primary problem, [24].

It is proposed in [11] that the fracture modes of failure can be avoided providing care is taken in material selection and inspections are made periodically. In conclusion, it is in effect stated that only adequate safeguard against the occurence of plastic collapse need be provided. This is tantamount to the considerations of compression instability which are summarized as strut-panel and tripping of stiffeners locally as well as overall grillage buckling.

In [18], the importance of analyzing various modes of failures and damage to ship structures is pointed out. Results of a 20-year-lifetime probability analysis are given with respect to yielding, local buckling, total collapse, and fatigue type failures. Effects of local water pressure are also included. The results, quoting from the afore-mentioned study, were that "The probability of fatigue crack initiation is comparatively high, whereas for ductile failures, probability of local collapse of bottom longitudinals is fairly significant, followed by the yield failure of deck or bottom plating, and very low probability of total plastic collapse of the hull girder."

As a further complication to the problem, one must also remember that many of the proposed modes of failure have been investigated from a "stress at a point" view and due to primary hull stresses only. However, the hull girder has the capability of redistributing stresses once it yields at a point. The total principal stress must, therefore, be determined by the superposition of primary, secondary, and tertiary stresses. Again these considerations are not important in the usual empirical approach to longitudinal strength but are of great concern in any precise structural analysis.

4.3 Conclusion

It is obvious from the foregoing that the mode of failure for a ship hull girder is not specifically known. In fact, it seems perfectly plausible that the mode of failure may vary depending on the design as is generally experienced in structural design. Furthermore, the overall probability of failure requires that all probabilities of failure of individual modes of failure be known and combined, and that the total stresses including any local stresses must be considered.

As opposed to this situation, in the examples of probabilistic structural design for machine parts, such as the one in [8], the mode of failure and various stress components acting on the parts are exactly known; and it is emphasized that this must be the case. Section 3.0 discusses this subject in more detail.

SECTION 5.0

LOADINGS

The present study is not concerned with any investigation of loadings on the hull girder other than to obtain, from a review of the literature, data on loadings needed to perform an example calculation. Yet, this point must be addressed in principle, since it shares an equal portion with the strength of structures in the probabilistic structural design theory. In other words, in order to perform probabilistic structural analyses, all must be known about both the load and the strength.

It was stated in Section 3.0 that the load considered must be the total load acting on the structure to cause the particular failure in question. In relation to the longitudinal strength of a ship hull girder, such loads would include still-water bending moments, wave-induced bending moments, springing induced bending moments, slamming induced bending moments of all types, transient deck loads due to weather, thermal effects, and bending moments due to the ships own wave train. Except for the wave loads, there is very little in the literature concerning the statistical data for these various loadings.

From the standpoint of analytically determining lifetime wave loads, Reference [12] presents a procedure for determining the extreme values of the wave bending moment using "order statistics" and assuming that the distribution of the maximums is of the Weibull type.

Several investigators have presented statistical full-scale data measurements of wave bending moments for actual vessels, [25], [26], [27]. In the measurements presented, the effects of springing and whipping were filtered out. The results are curves of cumulative long-term distribution of the average bending moments which show the probabilities, per cycle of load [27], of exceeding different levels of these bending moments during a ship's lifetime. Figure 1 is an example reproduced from [26]. A method for converting these loads per cycle to a cumulative probability curve for the ship's lifetime is indicated in [28]. Following this procedure, a form of long term distribution must be assumed. As discussed in Section 3.4, the different assumed long-term-distribution shapes fit the measured data differently [10, 60, 61].

There does not appear to be enough data nor any analytical methods in the literature for determining the statistical distribution of the other loads mentioned above. The ship structural reliability studies presented in [10] assumed both deterministic and normally distributed still-water bending moments. Reference [29] discusses the computations of wave slamming and springing bending moments in the context of a probabilistic structural analysis, but it is pointed out that much verification must be made with respect to slamming and springing before the procedures can be used. It is also noted that with regard to the structural probabilistic analysis, springing and slamming were not incorporated although they might easily be.

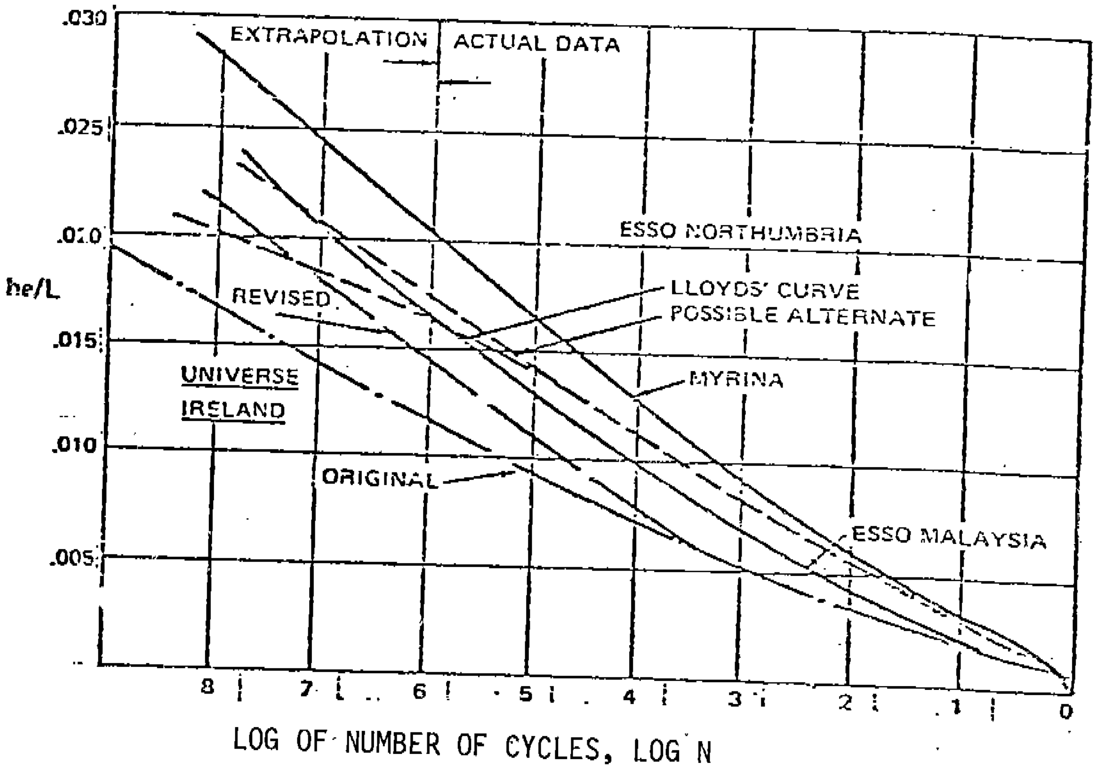


FIG. 1 - CUMULATIVE LONG TERM DISTRIBUTION OF AVERAGE BENDING MOMENTS

In conclusion, it is to be noted that the total load scenario for a ship is not clearly established, particularly in the probabilistic sense. An absolute or completely rational probabilistic analysis, from the standpoint above, does not seem possible at this time.

SECTION 6.0

PROBABILISTIC STRUCTURAL ANALYSIS OF SHIP HULL LONGITUDINAL STRENGTH

6.1 General

All of the most essential considerations for probabilistic structural design, discussed in preceding sections, would apply to transverse and torsional hull strength as well as longitudinal. However, transverse and torsional hull strength are beyond the scope of this study (Section 2.0).

It is clear that there are significant problems concerning the data, theory, and techniques that stand in the way of a completely rational probabilistic hull girder longitudinal strength analysis. This is to say that the probabilities of failure from such an analysis could only be used in a relative sense; and even then the comparison of modes of failure might be questionable due to possible better input to one mode of failure analysis than the other.

Other investigators have discussed this point. It is stated in [18] that the relative assessment of probabilities of failure may be one of the useful methods of evaluation of ship structures. In Reference [13], one of the motivations behind the approximate approach presented therein was that "probabilistic analysis of structural safety for ships is difficult at the present time because the available data are too limited to provide the exact forms of the probability distributions of the bending moment and the ship strength." Reportedly, the sample size required is of the order of multimillion pieces of records or data [30]. Two more recent papers, [62] and [63], also discuss this point.

One other aspect of probabilistic design which has received mention but not much analysis is the problem of determining the acceptable limiting value of the probability of failure. It was mentioned previously that the current level, based on actual occurrences, was determined in a study [15, 59].

The two emerging problems, i.e. the lack of available data and techniques to perform an accurate probability of failure analysis and the absence of an acceptable limit to the probability of failure, point to a need for the following three overall efforts:

- Continue to develop techniques and obtain data for both load and strength for probabilistic analysis.
- Perform absolute probability of failure analyses for different ships, compare and update the results as better data and techniques are developed.
- From the data presently available on ship failures of all types for all types of ships, perform semi-probabilistic analyses to identify safety factors of current and past ships.

The first of the above is needed for the advancement of probabilistic structural analysis methodology.

The second is necessary to obtain results using the latest collected data and the theoretical and testing methods in a developing analysis procedure, and to compare these results with what is known and with each other. Ultimately, the results of such a procedure could be used, in conjunction with non-structural aspects, to obtain an acceptable limit for the probability of failure. This would lead to the determination of the resulting factors of safety to be used by designers.

The third and last effort is needed to reap immediate fruits from the probabilistic design approach. As mentioned earlier, current longitudinal strength procedures only consider ductile yielding of the hull girder due to an equivalent wave imposing a vertical bending moment. An extensive analysis of ships, particularly those that have failed in longitudinal strength, considering all the modes of failure and using known loads, can produce a better understanding of which modes of failure are most significant, what the factors of safety are for these modes, and possibly indicate trends with respect to ship type, size, area of operation, etc.

It should be noted that the factor of safety discussed in conjunction with the second effort above is different from that discussed in the third. Factors of safety that come from an exact probabilistic analysis are to be based on the exact knowledge of the load and strength. This would be the case with the second item but in the case of the third, uncertainty concerning the load and strength would be tied into that safety factor but it should involve less uncertainty than in current procedures. As more becomes known about the strength and load, the results of the third item can of course be updated.

The proposed approaches for the above three areas of effort are presented below in greater detail.

6.2 Development of a Probabilistic Structural Analysis Methodology

In Sections 3, 4 and 5, the areas were identified where more theoretical studies need to be performed and data collected from full-scale experimental measurements. These areas can further be divided into strength and load distributions, strength equations, and time-dependent analyses.

6.2.1 Strength and Load Distributions

As discussed in Section 3.0, if the exact form, and the magnitudes of the strength and load distributions are known, it is simple to determine the probability of failure. The problem is that for both strength and load, these distributions do not exist. The problems for load are discussed in Section 5.0, and will not be elaborated upon here since this is not the specific area addressed by this project and is being addressed elsewhere [32].

In the case of the strength, the procedure has been to synthesize the distribution either by estimating only the coefficients of variation of strength variables, i.e. constituent parts whose distributions are not known, or by making an assumption as to what type of distribution the strength follows. Higher level syntheses discussed in Section 3.3.3 consist of the determination of strength statistics from the assumed distribution of the constituent parts, from the known distribution of constituent parts, or the determination of strength statistics from actual testing. The latter would give the greatest accuracy but, as discussed previously, is very nearly impossible due to its extent.

6.2.2 Strength Equations

A simple strength equation for ductile yielding due to bending of the hull girder is given in Section 3.0, Equation (27). The whole structural description rests with equations like these and the variables they include, since it is the statistics of these variables which are used to synthesize the overall strength distribution. Such an equation can be lacking in the simplifying assumptions associated with its derivation or in the number of variables it contains. Equations can have the same simplifying assumptions, which contribute to subjective uncertainties as discussed in Section 7, but a different number of variables. For instance consider the following:

$$S = f(N, s_y) = N s_y \tag{39}$$

$$N = g(D, t_f, B, t_w) \tag{40}$$

$$S = f[g(D, t_f, B, t_w), s_y] \cong D(t_f B + 1/3 t_w D) s_y \tag{41}$$

where:
N = deck or bottom section modulus
s_y = tensile strength
D = section depth
B = section beam
A_f = area of flanges
A_w = area of webs
$t_f = A_f/2B$ = equivalent thickness of one flange
$t_w = A_w/2D$ = equivalent thickness of one web

Equation (39) only considers the section modulus and tensile strength and is based on the simplifying assumptions of engineering beam theory. To synthesize a distribution from these, one would need the statistics of the section modulus and of the tensile strength. The former would be nearly impossible to obtain accurately since ships or structural models would have to be tested. By breaking the section modulus into variables as in Equation (40), the strength also becomes composed of more variables amenable to more direct scrutiny as far as their statistics are concerned, Equation (41). This can naturally be extended to much more subtle variables.

The objective of such an approach is to define all the variables which can more easily be measured and for which statistics can be determined. The methods discussed in Section 3.3.3 should then enable one to synthesize an accurate strength distribution.

The example of Equation (41) is usually intended for elastic bending of the hull girder as a beam. In the cases of fracture, fatigue, and ultimate strength, more work will probably have to be done to obtain an accurate expression for hull strength for these modes of failure.

6.2.3 Time-Dependent Strength Analyses

It was discussed in Section 3.3.4 that there is not much available in the literature regarding time-dependent strengths of ships. It is, therefore, suggested that research be performed to develop an accurate probability of failure procedure for the analysis of time-variant ship strengths, i.e. in the case of fatigue, corrosion, etc.

6.3 Application of Probabilistic Structural Analysis Methodology

6.3.1 General

In order to obtain an immediate and clear idea of ship longitudinal strength from the practical standpoints of modes of failure, safety margins, and probability of failure, the probabilistic structural design approach can be used most fruitfully in a semi-probabilistic type of analysis.

The semi-probabilistic analysis approach would be the easiest to apply and be consistent with the assumptions that would be necessary. Applying a more rigorous and time-consuming technique with an equal amount of additional assumptions may not add insight or accuracy, and it may even detract from the efforts.

A potential basis for such an approach has been presented in [13]. It is implied there that the procedure might actually be used in structural design. From the standpoint of a designer, this seems highly unlikely in the near future since most ship longitudinal strength determinations are presently based on classification society rules, or specifications which have been developed through years of experience, and in all likelihood they would not be changed until a different approach that offers advantage could be established and well proven.

However, in the interest of obtaining improved analysis methods and insight, it is important for researchers and designers to have a better appreciation for the basic structural phenomena associated with hull girder longitudinal strength. As pointed out, the present procedure of designing for ductile yielding of the hull girder in vertical bending may be mythical in some cases; and only coincidentally, an adequate strength for other modes of failure may have been accounted for.

6.3.2 Method of Approach

The "Approximate Probabilistic Method" of Reference [13] does not require assumptions concerning the types of distribution for load and strength. This method is a candidate for the semi-probabilistic probability studies discussed for the short term. The details of this procedure are presented in Section 3.2.3.

The procedure was presented specifically for vertical bending of the hull girder as a beam. But it seems plausible that the approach might be extended, with some modification, to other modes of failure. For the specific purposes of obtaining better knowledge on more rational factors of safety and on critical modes of failure, this procedure yields the safety index as given by Equation (11) of Section 3.0. If this index can be evaluated for existing ships, including those that have failed, for various modes of failure, then the safety factors of ship longitudinal strength will be better understood. The results of limited analyses of this type are presented in [13] and [31] for oil tankers and [63] for naval designs. It is of interest to note that Figure 2, reproduced from [13], gives an indication of the probability of failure for a given safety index when different distributions are assumed.

The procedure for arriving at the safety index would then be as follows:

- Determine the mean strength for the mode of failure in question

$$m_S = \sigma_N S \tag{42}$$

where:

σ_N = average failure stress

S = hull strength in question

m_S= mean of strength

- Determine central safety factor.

$$\theta = \frac{m_S}{m_Z} \tag{43}$$

where:

m_Z = mean of total load

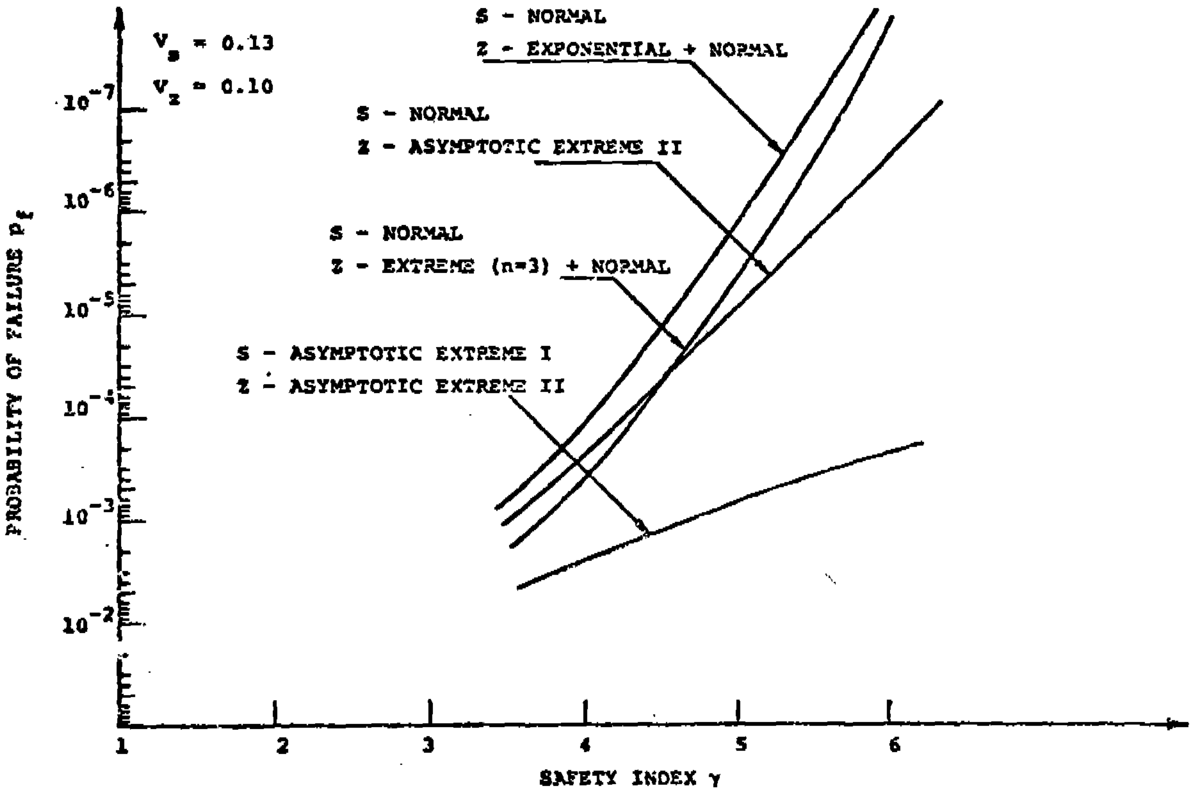


FIG. 2 - PROBABILITY OF FAILURE VERSUS THE SAFETY INDEX

° Determine safety index.

$$\gamma = \frac{\theta-1}{(\theta^2 V_S^2 + V_Z^2)^{1/2}} \qquad (44)$$

where: V_S = COV of strength

V_Z = COV of load.

The quantities σ_N and S should be obtainable from the design calculations of the ship. m_Z can be determined by measured data, empirical procedures, or by an analytical approach such as the one in [10]. A correction for loads not considered by these approaches and the COV's of load and strength can be determined from existing data. An example of this procedure along with a computer program are presented in Section 8.0.

The safety index γ will be directly indicative of the safety factor, and since it will be derived from statistics of both the load and the strength, for various modes of failure, it will give much more information than the singular approach of current design based on vertical bending only.

SECTION 7.0

UNCERTAINTIES IN HULL STRENGTH

7.1 General

As mentioned repeatedly in the preceding sections, a suitable approach to ship probabilistic structural analysis requires the determination of strength variable means and variances. The ratio of standard deviation to mean is termed the coefficient of variation or, COV.

The statistical nature of the variables has led to their being termed "uncertainties."

Reference [3] classifies the uncertainties into two types:

- Objective - "Measurable or quantifiable, such as observed statistical variabilities and deductive probabilistic information."
- Subjective - "No factual information is available or the uncertainty is not amenable to quantitative description and must be described and handled subjectively on the basis of judgement and intuition."

Once the means and variances of the objective and subjective uncertainties have been established, they can be combined as follows:

$$V_x = \sqrt{V_{xo}^2 + V_{xs}^2} \qquad (45)$$

where:
V_x = COV of x
V_{xo} = COV of objective uncertainties of x
V_{xs} = COV of subjective uncertainties of x

7.2 Objective Uncertainties

7.2.1 General

The objective uncertainties of the hull longitudinal strength are divided into three groups: mill practice, shipyard practice, and operational occurrences.

- Mill Practice
  - Variation in physical properties of materials including ductile, fatigue and fracture characteristics.
  - Variation in material thickness and shape dimensions.
- Shipyard Practice
  - Variations in material scantlings.
  - Variations in fabrication tolerances.
  - Variation in weldments.
  - Residual stresses.
  - Initial Deflections

- Operational Occurrences
  - Corrosion
  - Wear and Tear

With the goal of collecting data for these various uncertainties, requests were made to steel mills and the literature was surveyed. From the steel mills, statistical data in the form of either distributions, means and variances, raw data points, or tolerances for at least the following quantities were requested:

- Tensile, Yield, and Ultimate Strength
- Young's Modulus, Tangent Modulus, and Shear Modulus
- Ductility
- Corrosion Resistance
- Dimensions of Manufactured Items (Plate Thicknesses, Shape Dimensions, etc.)
- Poisson's Ratio

Eleven major steel producers in the U.S. were contacted. The responses received [34] did not include any new data, but instead made reference to [35] through [39]. Generally speaking, the steel producers indicated that they do not collect the type of data requested. Manufacturing of steels is controlled within the limitations of References [37] and [39].

The literature survey disclosed a number of pertinent references that are of value. These references contain information on strengths, fatigue, corrosion, dimensional accuracy, and welding stresses.

Since most ship-related probabilistic structural analyses make use of only a few uncertainties and usually assume values for their COV's, some of the references cited were used to obtain more accurate estimates of several COV's. Although there is not an abundance of data, it may be possible to uncover more numerical values and do more comprehensive an analysis than what is reported here. This work should, therefore, be continued.

7.2.2. Forms of Existing Data

The data for the uncertainties considered in this study fell into three categories:

- Means and Variances
- Data points
- Tolerances

For each of these a different approach was used to compute the respective COV.

In the first case the COV can simply be computed by dividing the square root of the variance by the mean.

In the second case, the data points can be used to directly compute the mean and variance of the uncertainty, which of course would directly yield the COV:

$$\mu = \frac{1}{N} \sum_{n=1}^{N} x_n \qquad (46)$$

$$\sigma^2 = \sum_{n=1}^{N} \frac{(x_n - \mu)^2}{N-1} \qquad (47)$$

$$\mathrm{COV} = \frac{\sigma}{\mu} \qquad (48)$$

where: N = number of data points
x_n = n^{th} data point
μ = mean of variable x
σ_x^2 = variance of variable x

In the third case, if it is assumed that the uncertainties arise because of many individual variables, it can be shown by the "Central Limit Theorem" that the uncertainties will be normally distributed. It has been determined for some ship-related uncertainties that the tolerance limits generally encompass 99.7% of the events [53]. For a normally distributed random variable, this corresponds to the following:

$$\Pr = \int_{\mu-\mathrm{tol}}^{\mu+\mathrm{tol}} p(x)\,dx = \int_{\mu-\mathrm{tol}}^{\mu+\mathrm{tol}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] dx = 0.997 \qquad (49)$$

where: p (x) = probability density function and tol = tolerance limit.

By the change of variable $t = \frac{x-\mu}{\sigma}$, equation (49) reduces to:

$$\Pr = \int_{-\frac{\mathrm{tol}}{\sigma}}^{+\frac{\mathrm{tol}}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp(-t^2/2)\,dt = 0.997 \qquad (50)$$

Equation (50) represents a zero mean, normally distributed process with a standard deviation of 1.0. It is known that the .997 probability level is contained within ± 3 standard deviation. Therefore:

$$\pm \frac{\mathrm{tol}}{\sigma} = \pm 3 \qquad (51)$$

$$\sigma = \frac{\mathrm{tol}}{3} \qquad (52)$$

It is mentioned in [54] that tolerance limits are usually taken to be $\mu \pm 3\sigma$. Consequently, on the basis of the foregoing, the mean, μ, can be chosen from the context and the standard deviation computed by dividing the tolerance by 3.

The uncertainties which were analyzed are listed below in groups corresponding to the type of data that were available:

- Means and Standard Deviations
  - Depth of stiffener web [53]
  - Breadth of stiffener flange [53]
  - Breadth and length of plate [53]
  - Yield strength [35]
  - Tensile strength [35]
  - Initial deflections in plates [67]
- Data Points
  - Depth of ship [40]
  - Beam of ship [40]
  - Flange breadth [40]
- Tolerances
  - Depth of ship [40]
  - Beam of ship [40]
  - Thickness of plate (receipt inspection) [40]
  - Thickness of plate (undercut) [40]

7.2.3 Determination of COVs

The means and standard deviations from [35] are presented in Tables 1 through 3 along with the computed COVs.

In the case of the means and variances available for flange breadth, web depth, and length of plate, it is assumed that the variance is not a function of the absolute size of the member since it is not presented in this manner. Consequently, the mean and the COV of a dimension "L" are given by:

$$\mu = \overline{x}+L \qquad (53)$$

$$\delta = \mathrm{COV} = \frac{\sigma}{\mu} \qquad (54)$$

Figures 3 through 5 give the data base and expressions for the uncertainties.

The determinations of COVs from data points for flange breadth, ship depth, and ship beam are shown in Tables 4 through 6. Note that the results of Table 4 and Figure 4 agree well in that they both show COV around 1% for flange breadth. It should be noted that the COVs for ship beam and depth are extremely small. Reference [10] assumed a value of zero which appears reasonable.

The determination of COVs from tolerances for ship depth, ship beam, thickness of plate (receipt inspection), and the thickness of plate (undercut) are given in Tables 7 through 10.

7.2.4 COVs From Literature Survey

Reference [11] presents some results for objective uncertainties for materials, scantlings, and manufacturing imperfections. These are shown in Table 11.

TABLE I - CARBON STEEL PLATES

TENSILE STRENGTH

	Official Tensile Strength, ksi Under 60	60-70 excl.	70-80 excl.	80 and over
Number of Tests	329	1,119	703	152
Official Test Average, psi	57,187	65,321	74,519	84,488
Average Difference, psi	-115	+108	+15	-367
Standard Deviation, psi	1,577	2,166	3,245	3,058
COV	.028	.033	.044	.036

YIELD POINT

	Official Yield Point; ksi Under 30	30-40 excl.	40-50 excl.	50 and over
Number of Tests	50	1,265	813	168
Official Test Average, psi	28,557	35,653	43,412	55,646
Average Difference, psi	+732	-142	-1,427	-1,915
Standard Deviation, psi	2,711	2,673	3,253	4,017
COV	.095	.075	.075	.072

TABLE II - AS-ROLLED PLATE

TENSILE STRENGTH

	Tensile Strength At No. 2 Corner Position, ksi Under 60	60-70	70-80	80 & Over
Number of Tests	487	1,174	368	120
Official Test Average, psi	55,744	64,579	74,352	83,925
Average Difference, psi	+399	+100	-23	-38
Standard Deviation, psi	1,546	1,803	1,829	2,451
COV	.028	.028	.025	.029

YIELD POINT

	Yield Point At No. 2 Corner Position, ksi Under 40	40-50	50 & Over
Number of Tests	1,170	831	150
Official Test Average, psi	36,015	44,226	55,836
Average Difference, psi	+107	-196	-360
Standard Deviation, psi	2,020	2,183	2,171
COV	.056	.049	.039

TABLE III - CARBON STEEL WIDE FLANGE SHAPES

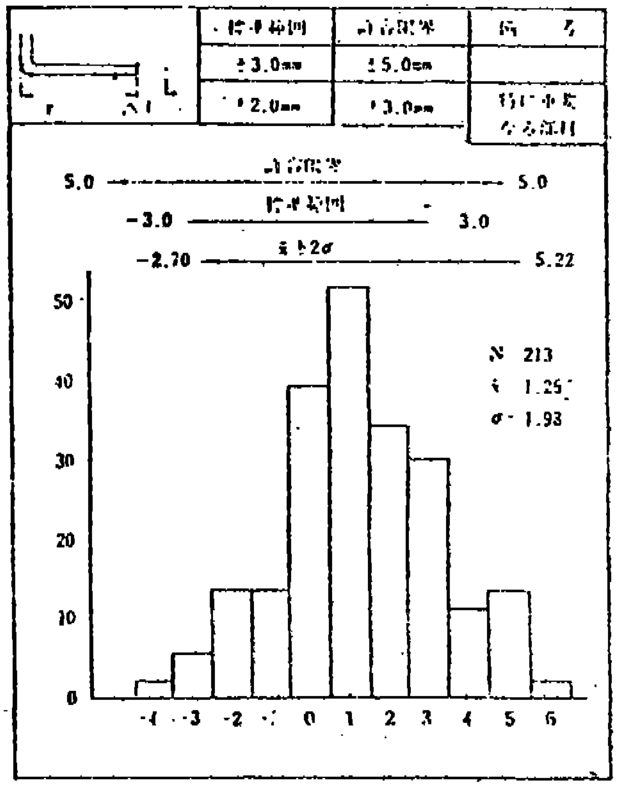
	TENSILE STRENGTH Under 65	65-70	70 & Over	YIELD POINT Under 40	40-45	45-50	50 & Over
FLANGE TESTS							
Number of Tests	159	356	164	118	333	138	89
Official Test Average, psi	62,748	67,170	72,726	37,688	42,193	47,031	52,497
Average Difference, psi	+794	-1,142	-3,357	+162	-2,247	-4,249	-5,845
Standard Deviation, psi	2,512	2,690	3,632	2,649	3,600	3,182	3,863
COV	.04	.04	.05	.07	.085	.068	.074
WEB TESTS							
Number of Tests	160	362	167	117	338	145	89
Official Test Average, psi	62,671	67,182	72,769	37,697	42,193	47,020	52,566
Average Difference, psi	+1,966	+161	-2,133	+1,344	+516	-686	-2,015
Standard Deviation, psi	2,627	3,113	4,257	2,334	2,853	3,603	4,069
COV	.042	.046	.059	.062	.068	.077	.077

FIGURE 3

DETERMINATION OF COEFFICIENT OF VARIATION FOR THE UNCERTAINTY

DEPTH OF STIFFENER WEB

(Data from reference 53)



From FIG. 1.1

$$\bar{x}_{\Delta\ell} = 1.26 \text{ mm}$$

$$\sigma_{\Delta\ell} = 1.98 \text{ mm}$$

$$\therefore \quad \delta_\ell = \frac{\sigma_\ell}{\mu_\ell} = \frac{\sigma_{\Delta\ell}}{\bar{x}_{\Delta\ell} + \ell_{web}}$$

$$\delta_\ell = \frac{1.98}{1.26 + \ell_{web}} \qquad (1)$$

where: ℓ_{web} = stiffener depth

FIGURE 4

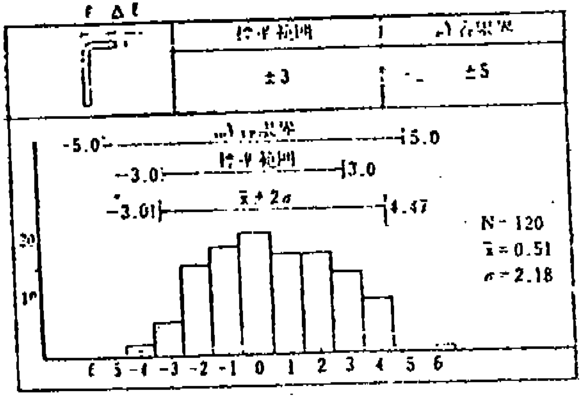
DETERMINATION OF COEFFICIENT OF VARIATION

FOR THE UNCERTAINTY

BREADTH OF STIFFENER FLANGE

(Data from reference 53)

From FIG. 1.2



$$\bar{x}_{\Delta \ell} = 0.51 \text{ mm}$$

$$\sigma_{\Delta \ell} = 2.18 \text{ mm}$$

$$\therefore \quad \delta_{\ell} = \frac{\sigma_{\ell}}{\mu_{\ell}} = \frac{\sigma_{\Delta \ell}}{\bar{x}_{\Delta \ell} + \ell_{\text{flange}}}$$

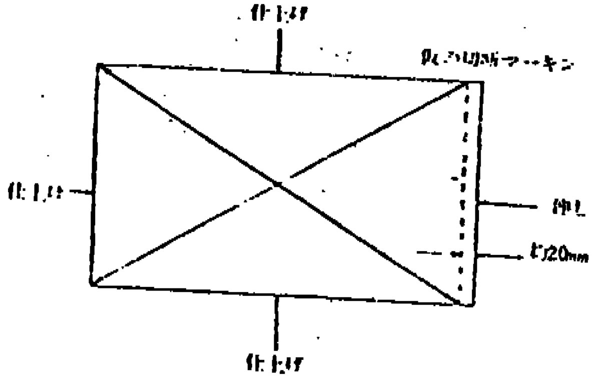
$$\delta_{\ell} = \frac{2.18}{0.51 + \ell_{\text{flange}}} \qquad (2)$$

where: ℓ_{flange} = breadth of stiffener flange

FIGURE 5

DETERMINATION OF COEFFICIENT OF VARIATION FOR THE UNCERTAINTIES BREADTH AND LENGTH OF PLATE

(Data from reference 53)



$\bar{x}_{\Delta\ell} = -0.33$ mm (breadth of plate)

$\sigma_{\Delta\ell} = 2.36$ mm

$$\delta_{B_{PL}} = \frac{\sigma_\ell}{\mu_\ell} = \frac{\sigma_{\Delta\ell}}{\bar{x}_{\Delta\ell} + \ell_{B_{PL}}} = \frac{2.36}{-0.33 + \ell_{B_{PL}}}$$

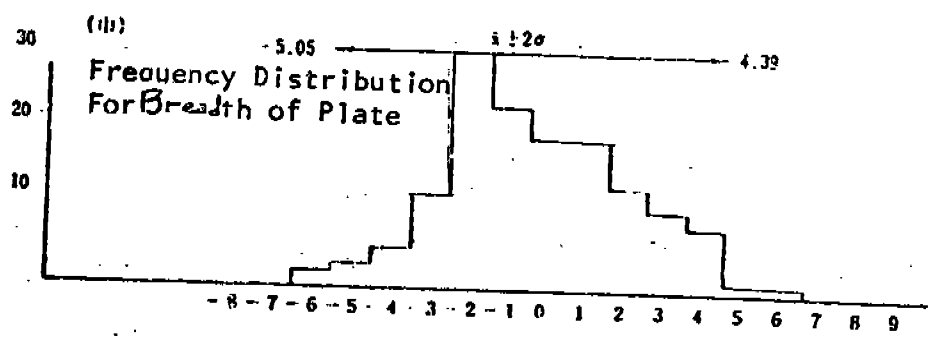
$\bar{x}_{\Delta\ell} = -0.95$ mm (length of plate)

$\sigma_{\Delta\ell} = 2.69$ mm

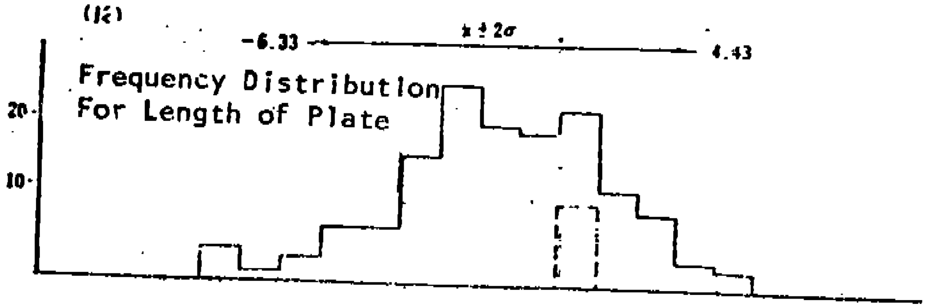
$$\delta_{L_{PL}} = \frac{\sigma_\ell}{\mu_\ell} = \frac{\sigma_{\Delta\ell}}{\bar{x}_{\Delta\ell} + \ell_{L_{FL}}} = \frac{2.69}{-0.95 + \ell_{L_{PL}}}$$

where: $\ell_{L_{PL}}$ = length of plate

$\ell_{B_{PL}}$ = breadth of plate



N = 146
$\bar{x}$ = -0.33
σ = 2.36



N = 150
$\bar{x}$ = -0.95
σ = 2.69

TABLE 4

FLANGE BREADTH UNCERTAINTY

Data Point	x (inch)	$(x - \bar{x})^2$
1	1/8	0.000961
2	1/8	0.000961
3	1/4	0.008836
4	1/8	0.000961
Σ	5/8	0.01172

w_f = flange breadth

$$\bar{x} = \frac{5/8}{4} = 0.156''$$

$$\sigma = \left[\frac{\Sigma (x - \bar{x})^2}{N-1}\right]^{1/2} = \left[\frac{0.01172}{3}\right]^{1/2}$$

if N<25

$$\sigma = 0.0625''$$

$$\mu_{\substack{\text{flange}\\\text{breadth}}} = \bar{x} + w_f = 0.156 + w_f$$

$$\delta = \frac{\sigma}{\mu} = \frac{0.0625}{0.156 + w_f}$$

For $w_f = 6''$:

$$\delta = \frac{0.0625}{0.156 + 6} = 0.0102 \text{ or } 1.02\,\%$$

Comparison with Figure 4:

Compare $\ell = 1.02\%$ with that using formulation from Japanese standard data

$$w_f = 6'' = 152.4 \text{ mm} \qquad (1 \text{ inch} = 25.40 \text{ mm})$$

$$\delta_\ell = \frac{2.18}{0.51 + \ell_{\text{flange}}} = \frac{2.18}{0.51 + 152.4} = 0.0143 \text{ or } 1.43\%$$

TABLE V - DETERMINATION OF $^{\delta}D$ (COV OF DEPTH) FROM THE MEAN AND STD. DEV. OF MEASURED DATA

DEPTH UNCERTAINTY

Data Point	Depth	Depth Meas. Dev.
1	36'	1.5"
2	46'	1% = 0.552"
3	16'	5/8"
4	83'	-

NON-DIMENSIONALIZE ABOVE DATA

Pt. 1

$$\frac{36}{36} + \frac{1.5}{36 \times 12} = 1 + 0.003472$$

Pt. 2

$$\frac{46}{46} + \frac{0.552}{46 \times 12} = 1 + 0.001$$

Pt. 3

$$\frac{16}{16} + \frac{0.625}{16 \times 12} = 1 + 0.003255$$

N	Y_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
1	1.003472	0.000896	8.02816×10^{-7}
2	1.001	-0.001576	24.8378×10^{-7}
3	1.003255	0.000679	4.61041×10^{-7}
	$\Sigma = 3.007727$		$\Sigma = 37.47637 \times 10^{-7}$

$$\bar{x} = \frac{\sum^{N} Y_i}{N} = \frac{3.007727}{3} = 1.002576$$

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N-1}} = \sqrt{\frac{37.47637 \times 10^{-7}}{2}} = 0.00136887$$

$$\delta = \frac{\sigma}{\bar{x}} = \frac{0.00136887}{1.002576} = 0.001365 \text{ or } 0.1365\ \%$$

TABLE VI - $^{\delta}$B (COV OF BEAM) FROM THE MEAN AND STD DEV. OF MEASURED DATA

Data Point	Beam	Beam Measured Dev.	
1	55'	±2"	(2 ships - measured data)
2	95'	0.1% or 1.14"	
3	75'	1/2"	
4	145'	-4"	

NON-DIMENSIONALIZE ABOVE DATA

Data Point #1

$$\frac{55'}{55'} \pm \frac{2''}{55 \times 12} \doteq 1 \pm 0.003$$

Data Point #2

$$\frac{95}{95} + \frac{1.14}{95 \times 12} = 1 + 0.001$$

Data Point #3

$$\frac{75'}{75'} + \frac{0.5}{75 \times 12} = 1 + .000555$$

Data Point #4

$$\frac{145'}{145'} - \frac{4}{145 \times 12} = 1 - 0.002299$$

N	x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
1	1.001	0.001248	1.5575×10^{-6}
2	1.000555	0.000803	0.6448×10^{-6}
3	0.997701	-0.002051	4.2066×10^{-6}
	$\Sigma = 2.999256$		$\Sigma = 6.4089 \times 10^{-6}$

$$\bar{x} = \frac{\Sigma x_i^n}{N} = \frac{2.999256}{3} = 0.999752$$

$$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{N-1}} = \sqrt{\frac{6.4089 \times 10^{-6}}{2}} = 0.001790$$

$$\delta = \frac{\sigma}{\bar{x}} = \frac{0.001790}{0.999752} = 0.001791 \text{ or } 0.1791\%$$

TABLE 7

UNCERTAINTY - DEPTH OF SHIP

Data Point	Tol. (Inch)	σ (Feet)	μ (Mean) (Feet)	Coef. of Variation δ	δ (Percent)
1	1/4"	0.00694	20.0	0.000347	0.0347
2	1/2"	0.0139	36.0	0.000386	0.0386
3	0.1%	0.012	36.0	0.000333	0.0333
4	1/2%	0.0139	26.0	0.000535	0.0535
5	3/8	0.0104	91.0	0.000114	0.0114
6	1/2	0.0139	50.0	0.000278	0.0278
				δ avg. =	0.0332%

Table 8

UNCERTAINTY - BEAM OF SHIP

Data Point	Tol. (Inch)	δ (Feet)	μ (Mean) (Feet)	Coef. of Variation δ	δ (Percent)
1	.1%	0.024	72"	0.000333	0.0333
2	1/2	0.0139	200	0.0000695	0.00695
3	1/2	0.0133	75	0.000177	0.0177
4	1/2	0.0139	96	0.000145	0.0145
				δ avg =	.0181

Table 9

UNCERTAINTY - THICKNESS

(RECEIPT INSPECTION)

Data Point	Tol. (Inch)	δ (Inch)	μ(Mean) (Inch)	δ	δ(%)
1	1/8	0.0417	t	0.0417/t	4.17/t
2	.1t	0.0333t	t	0.0333	3.33
3	1/32	0.0104	t	0.0104/t	1.04/t
4	1/64	0.0052	t	0.0052/t	0.52/t
5	1/8	0.0417	t	0.0417/t	4.17/t

TABLE 10

UNCERTAINTY - THICKNESS

(UNDERCUT)

Data Point	Tol. (Inch)	δ (Inch)	μ (Mean) (Inch)	δ	δ (%)
1	1/32	0.0104	t	0.104/t	1.04/t
2	1/16	0.0625	t	0.0625/t	6.25/t
3	1/32	0.0104	t	0.0104/t	1.04/t
4	1/32	0.104	t	0.0104/t	1.04.t
5	1/32	0.0104	t	0.0104/t	1.04/t
6	1/32	0.0104	t	0.0104/t	1.04/t
7	1/16	0.0625	t	0.0625/t	6.25/t
8	1/32	0.0104	t	0.0104/t	1.04/t
9	1/16	0.0625	t	0.0625/t	6.25/t
				δ avg =	2.78/t

TABLE 11: Objective Uncertainties

Type	Variable	Source	COV
Material	Modulus of Elasticity, E	Unknown	2.5%
	Yield Strengths (Royal Navy B Steel)	histograms (two)	6-8%
Scantlings	Plate thickness (0.25" plate)	[55]	3.6%
	Plate thickness (2" plate)	[55]	0.7%
	Plate thickness (All)	[56]	4%
Manufacture Imperfections	Residual Welding Stress	Unknown	10-15%

Reference [10] gives identical information on objective uncertainties as shown in Table 12.

TABLE 12: Objective Uncertainties

Type	Variable	Source	COV
Material	Yield Strength (23 ksi steel)	[57]	6-8%
	Yield Strength (32.6 ksi mean yield)	[41]	6.7%
	Yield Strength	[58]	7.9%

Reference [67] gives information on initial lateral deflections in plating in the bottom of a universal bulk carrier and a tanker as shown in Table 12-A. It was shown by analysis of the histograms by the χ^2 - criteria that the initial delfections obey a Gaussian law.

TABLE 12-A

Ship type	Dead weight	$\frac{a}{b}$	Type of Structure	t (mm)	b/t	B	$\bar{\xi}$	σ_{ξ_0}	$\frac{\sigma_{\xi}}{\bar{\xi}}$
UBC	25,000	2.0	bottom	18	46.0	1.740	0.194	0.0287	0.15
Tanker	100,000	4.4	deck	22	40.9	1.547	0.145	0.0624	0.43
			bottom	24	37.5	1.418	0.136	0.0618	0.45

TABLE 13

SUBJECTIVE UNCERTAINTIES [11]

Mode of Failure	Ship Bending*	Gross Panel**
Tension yield........................	3	0
Plate Buckling		
Strut Panel Beam Column	4	6
Grillage Buckling....................	4	7.5

* c.o.v. of subjective uncertainties in strength arising from ship bending actions. (%)

** c.o.v. of subjective uncertainties in strength arising from gross panel actions. (%)

7.3 Subjective Uncertainties

The subjective uncertainties, quoting from [3], "include the nonmeasurable inaccuracies of engineering analyses, any non-measured variances of construction and fabrication, and the unavoidable and non-determinable errors associated with the prediction of future conditions."

In Reference [10], which contains ship longitudinal strength analyses, for tensile yield ductile bending and inelastic compression buckling, the COV of the subjective uncertainties was taken as 3% and 5% respectively. In the case of bending, the uncertainty was attributed to:

"1. The use of the simple beam theory which is based on Navier hypothesis. This hypothesis excludes any shear lag or shear deformation effects.
2. The presence of small cutouts and openings in the deck.
3. The residual stresses due to welding.
4. The cracks, voids, and other flaws in the material."

For buckling, in addition to the above, the uncertainty in initial deflections of the plate was added. No mention of how the uncertainties were specifically determined is made.

In Reference [11], subjective uncertainties are given for various strengths, including ultimate tension, bending, and compression failure due to plate and grillage buckling. The discussion therein cites a number of references from which data was obtained and the COV's estimated. The results are presented in Table 13.

7.4 Conclusions

Future efforts, in the long and short terms, should be directed towards identifying and quantifying more uncertainties. Most subjective uncertainties are really "as-yet-unquantified" objective uncertainties. They possess the property of being measurable although data may not exist or may be sparse and difficult to locate.

The data presented herein verifies the assumption in the literature that the variability in principal ship dimensions is small. The additional results for material yield strength also agree well with what has been presented before.

SECTION 8.0

SAMPLE CALCULATIONS

8.1 General

It was stated in Section 6.3.2 that the "Approximate Probabilistic Structural Analysis Method" of Reference [13] can be used to obtain more insight into ship hull longitudinal strength through determination of the safety index, γ.

In this section, the method is applied to the ten different ships in vertical hull bending mode of failure which were analyzed using the method found in Reference [31]. Table 14, reproduced from this reference, gives the input data and the safety index results.

This section presents a computer algorithm which embodies the subject method. An equation for the strength COV for ductile yield vertical bending is developed in terms of the COV's of the uncertainties. Results of the computations with the computer algorithm are presented for nine ships using the data from Table 14. In the case of Ship #10, the formula developed herein for the strength COV is utilized along with the remaining data of Table 14 to obtain the safety index.

8.2 Computer Algorithm

The computer algorithm represents the computational sequence of Equations (42) thru (44) of Section 6.3.2. Within this framework, all quantities in the equations are input. However, in the case of the strength COV, the option exists of evaluating it from the strength COV equation and the input COV's of the strength uncertainties.

The algorithm is structured as shown in Figure 6. The main program consists of Equations (42) thru (44) noted above. The subroutines for the strength COV's are based on individual derivation of the strength COV determined by the technique given by Equation (26) of Section 3.3.3 and the strength equation. Only the ductile yielding of the hull girder due to vertical bending is considered here. Equation (20) of Section 3.3.3 is the strength equation for this mode of failure of the hull girder. If the areas of the flange and the web, A_f and A_w, are respectively expressed as functions of additional variables, the following equations would result:

$$A_f = 2[Bt_d + M_f t_s(\ell_{w1} + \ell_{f1})] \qquad (55)$$

$$A_w = 4[Dt_w + M_w(\ell_{w2} + \ell_{f2})] \qquad (56)$$

$$S = [DBt_d + DM_f t_s(\ell_{w1} + \ell_{f1}) + (2/3)D^2 t_w + (2/3)DM_w t_s(\ell_{w2} + \ell_{f2})]\,s_y \qquad (57)$$

SHIP	LBP (ft)	m * (ft-ton)	λ * (ft-ton)	V_z	V_s	γ
1	594.00	206,000	34,500	0.1595	0.11	5.619
2	656.20	328,500	42,500	0.1400	0.11	5.840
3	700.65	396,500	51,500	0.1402	0.11	5.796
4	719.10	357,500	48,850	0.1444	0.11	5.769
5	754.70	519,500	60,500	0.1325	0.11	5.993
6	775.00	610,000	61,000	0.1229	0.11	5.856
7	800.00	613,500	65,500	0.1268	0.11	6.103
8	1,000.00	1,474,500	113,250	0.1105	0.11	6.152
9	1,069.25	1,718,300	118,400	0.1068	0.11	6.279
10	1,076.00	1,906,500	131,400	0.1068	0.11	6.314

TABLE 14: "APPROXIMATE PROBABILISTIC METHOD"
(Input and Results from Reference [31])

NOTE
*

m = Mean of Still Water Bending Moment
λ = Average Value of Long Term Wave Bending Moment
Assumed: $m_Z = m+\lambda$

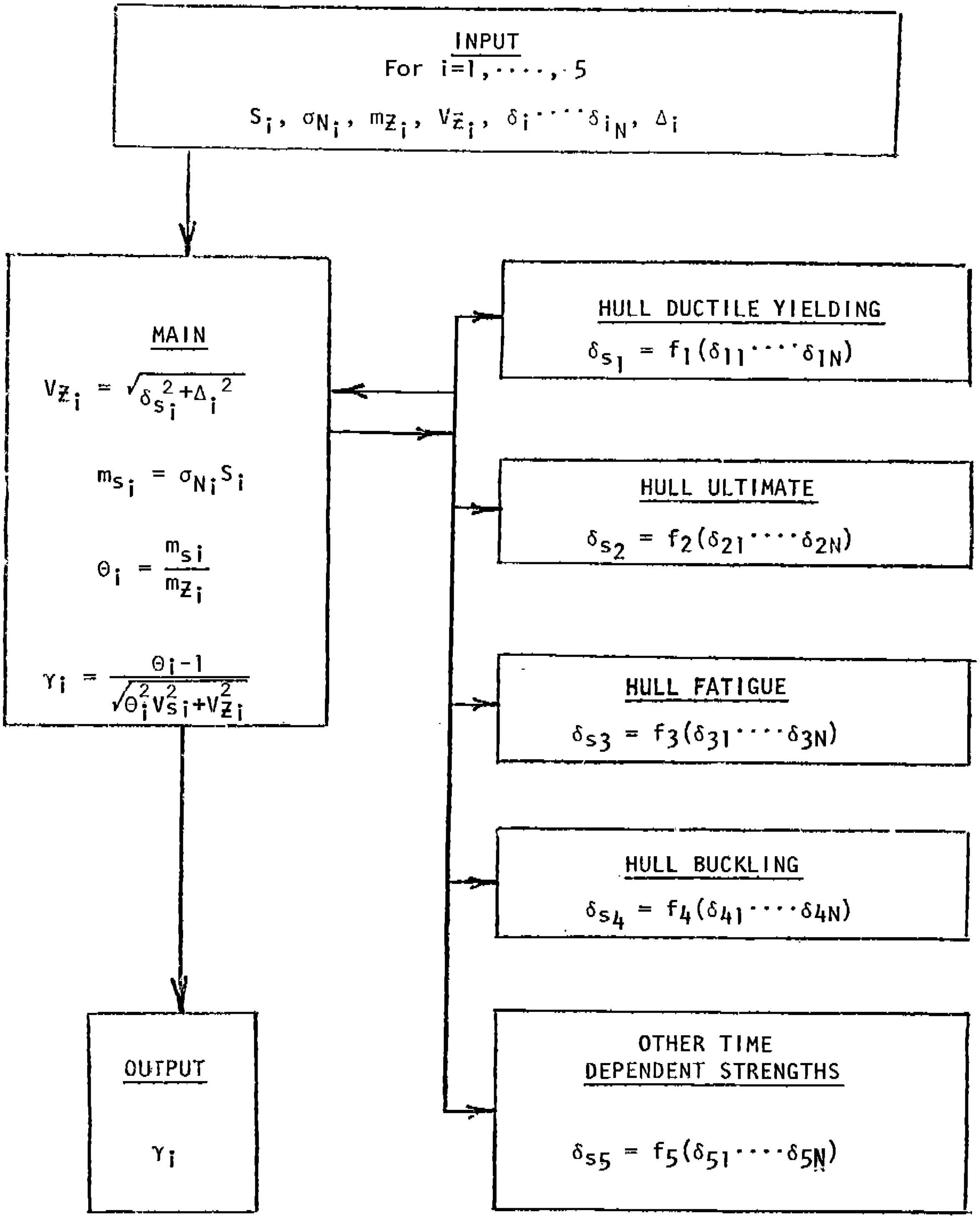


Figure 6: "Approximate Probabilistic Method" Algorithm

where all terms are as previously noted and in addition:

B = Beam of Ship

D = Depth of Ship

t_d = thickness of deck plating

t_s = thickness of stiffeners

t_w = thickness of webs (side plating on longitudinal bulkheads)

ℓ_{w1} = length of deck or bottom stiffener web

ℓ_{w2} = length of side or longitudinal bulkhead stiffener web

ℓ_{f_1} = length of deck and bottom stiffener flange

ℓ_{f_2} = length of side or longitudinal bulkhead stiffener flange

M_f, M_w = number of stiffeners along deck and side plating, respectively

s_y = tensile yield strength

The coefficient of variation of the bending strength as given by Equation (57), is shown in Appendix A to be:

$$\begin{aligned}\delta_s{}^2 = \frac{9}{(3A+3B+2C+2D)^2}[1/9(3A+3B+4C+2D)^2\delta_D{}^2 \\ +A^2(\delta_B{}^2+\delta_{td}{}^2)+E^2\delta^2_{\ell w1} \\ +F^2\delta^2_{\ell f1}+1/9(3E+2G)^2\delta^2_{ts}] \\ +\delta^2_{sy}+\frac{4(C^2\delta^2_{tw}+G^2\delta^2_{\ell w2}+H^2\delta^2_{\ell f_2})}{(3A+3B+2C+2D)^2}\end{aligned} \tag{58}$$

where:

$A = Bt_d$

$B = M_f t_s(\ell_{w1} + \ell_{f1})$

$C = Dt_w$

$D = M_w t_s(\ell_{w2} + \ell_{f2})$

$E = M_f t_s \ell_{w1}$

$F = M_f t_s \ell_{f1}$

$G = M_w t_s \ell_{w2}$

$H = M_w t_s \ell_{f2}$

No attempt has been made to include strength equations for hull ultimate, compression, fatigue and other time-dependent strengths due to the limited nature of this project. As mentioned in Section 6.0, it is considered plausible that strength equation can be adopted for each of these strengths which would conform to an analysis by the "Approximate Probabilistic Method".

As the additional strength equations and the expressions for strength COV are developed for different modes of failure, they can be added to the algorithm presented herein as subroutines. It is noted, however, that the algorithm is simple and could be coded quickly for any type of computing machine.

A listing of the algorithm written in FORTRAN-IV for an IBM computer is given in Appendix B along with the documentation.

8.3 Analysis and Results

The computer algorithm was verified by re-computing the results of Table 14 with the input data given therein, including V_S, the strength COV. In the case of Ship #10, scantling plans were available and V_S was also computed by the subroutine representing Equation (58).

Ship #10, the "UNIVERSE IRELAND" is a large oil tanker which has been used as a subject in many research studies. Table 15 gives the principal characteristics of the "UNIVERSE IRELAND".

Figure 7 represents the approximate midship section used to simplify the application of equation (58). Since the vessel is full at midships, the error introduced by this assumption should not be significant.

Table 16 gives structural variables determined from the drawings.

The next few steps of the calculation were actually performed by the computer. The process, however, consists simply of inserting the above values into Equation (58) to yield the following:

$$\delta_s{}^2 = 1.23\ \delta_D^2 + 0.798\ \delta_B^2 + 0.798\,\delta_{t_d}^2 + 0.189\,\delta_{\ell_{w_1}}^2$$

$$+ 0.03\,\delta_{I_{f_1}}^2 + 0.33\ \delta t_s{}^2 + \delta s_y{}^2 + 0.012\,\delta^2\,t_w + 0.005\ \delta^2{}_{I_{w_2}}$$

$$+0.0004\,\delta^2 I_{f_2} \qquad (59)$$

Table 17 gives the uncertainty COV's determined from the data presented in section 7.0.

Equation (59) yields $\delta_s = .0849$. Assuming subjective uncertainties to be 3%, the total strength COV becomes

$$\text{COV} = \sqrt{\delta_s{}^2 + \Delta_s{}^2} = \sqrt{(.0849)^2 + (.03)^2} = 0.09 \qquad (60)$$

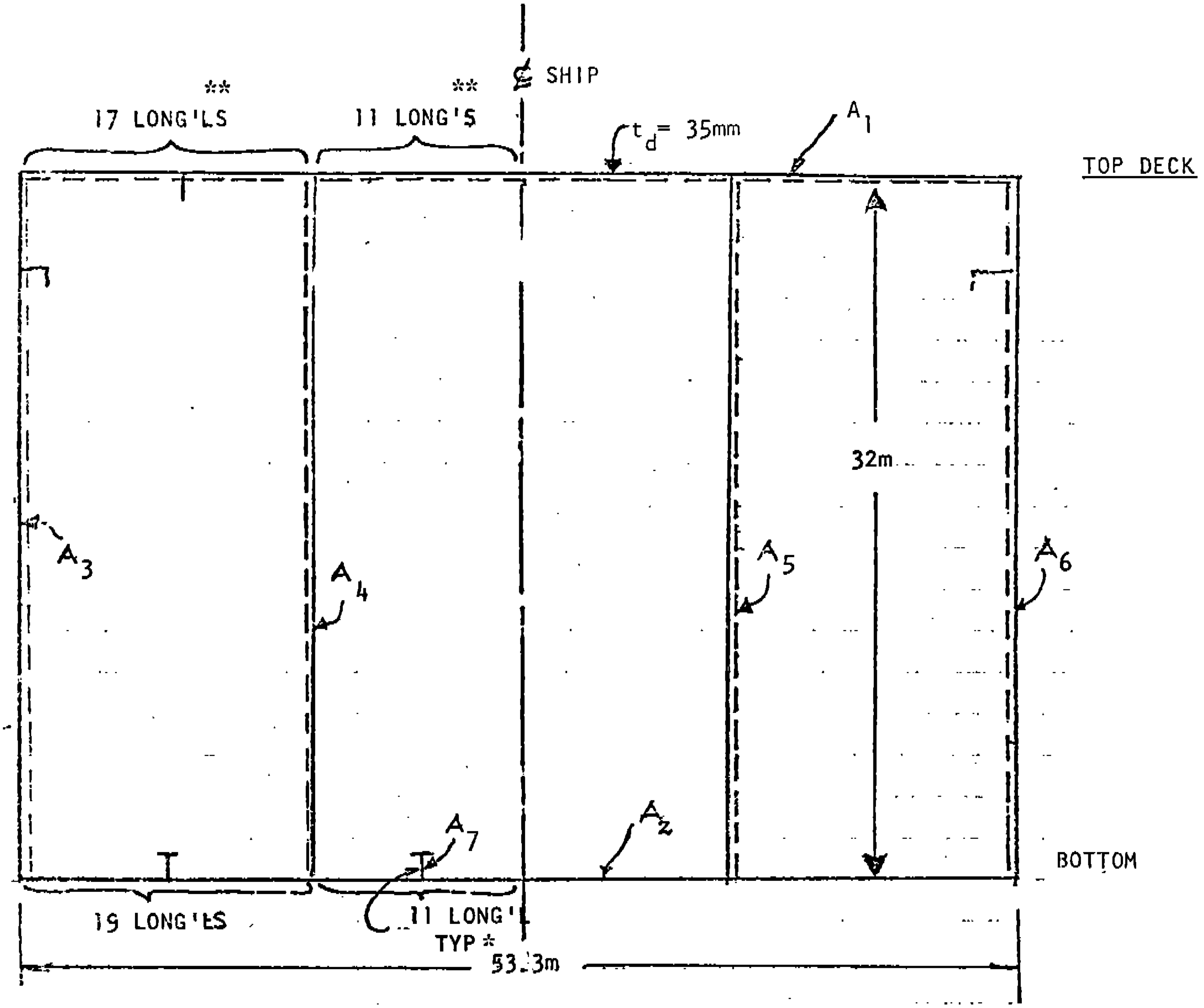
It is interesting to note the relative importance of uncertainties as given by equations (59) and (60). First, the assumed 3% COV for subjective uncertainties does not significantly affect the uncertainty that would be obtained from objective considerations only. Table 18 gives the values of the individual terms of equation (59).

As can be seen in the case of the "UNIVERSE IRELAND", the yield strength of the material far outweighs the consideration of other variables. Additional variables of some importance are thickness of deck plating and stiffeners and the length of flanges on side and longitudinal bulkhead stiffeners.

The results of the safety index computation employing the computer program are plotted in Figure 8.

TABLE 15: UNIVERSE IRELAND CHARACTERISTICS

TYPE	TANKER
Approximate dwt, tons	326,585
Overall length, ft.	1,135.17
L_{BP}, ft.	1,076
Breadth, ft.	174.87
Dept. ft.	105
Design draft (keel), ft.-in.	81'-5"
Builder	Ishikawajima Harima
Block coefficient (L_{WL})	0.86
Section modulus, top, in^2-ft.	566,794
Maximum stillwater bending moment in full load condition (long voyage) (sagging)	= 1,940,000 LT-FT
Maximum stillwater bending moment in full load condition (short voyage) (sagging)	= 2,355,000 LT-FT
Maximum stillwater bending moment in normal ballast (hogging)	= 1,858,000 LT-FT



* Typical longitudinal: Web 764 x 17.5mm, flange 320 x 40mm

** These longitudinals have no flanges

FIG. 7 - APPROXIMATE MIDSHIP SECTION FOR "UNIVERSE IRELAND"

Table 16: "UNIVERSE IRELAND" Structural Variables

$B = 53.3\text{m}$

$D = 32.0\text{m}$

$t_s = .026\text{m}, \quad t_d = .035\text{ m}.$

$t_w = .02\text{m}$

$\ell_{w_1} = .4\text{m}$, main deck, .764m, bottom

$\ell_{f_1} = 0.0$ main deck, .23m bottom

$\ell_{w_2} = .55\text{m}$

$\ell_{f_2} = .138\text{m}$

$M_f = 60$

$M_w = 30$

Table 18: Strength COV Equation Terms

$1.23\delta_D^2 = 1.23\ (.001)^2 = 1.23 \times 10^{-6}$

$.798\ \delta_B^2 = .798\ (.0015)^2 = 1.8 \times 10^{-6}$

$.798\ \delta^2_{t_d} = .798\ (.0302)^2 = 912.04 \times 10^{-6}$

$.189\delta^2_{l_{w_1}} = .189\ (.003)^2 = 1.7 \times 10^{-6}$

$.03\delta^2_{l_{f_1}} = .03\ (.093)^2 = 259.5 \times 10^{-6}$

$.33\ \delta^2_{ts} = .33\ (.0409)^2 = 552.03 \times 10^{-6}$

$\delta^2_{s_y} = (.0750)^2 = 5625.0 \times 10^{-6}$

$.012\ \delta^2_{t_w} = .012\ (.053)^2 = 33.7 \times 10^{-6}$

$.005\delta^2_{l_{w_2}} = .005\ (.0036)^2 = .06 \times 10^{-6}$

$.0004\delta^2_{l_{f2}} = .0004\ (.0157)^2 = .1 \times 10^{-6}$

Table 17: "UNIVERSE IRELAND" Uncertainty COVs

$\delta_D = 0.001$

$\delta_B = 0.0015$

$\delta_{t_d} = .0302$

$\delta_{\ell_{w_1}} = .003$

$\delta_{\ell_{s_1}} = .093$

$\delta_{s_y} = .0750$

$\delta_{t_w} = .053$

$\delta_{\ell_{w_2}} = .0036$

$\delta_{\ell_{f_2}} = .0157$

$\delta_{t_s} = .0409$

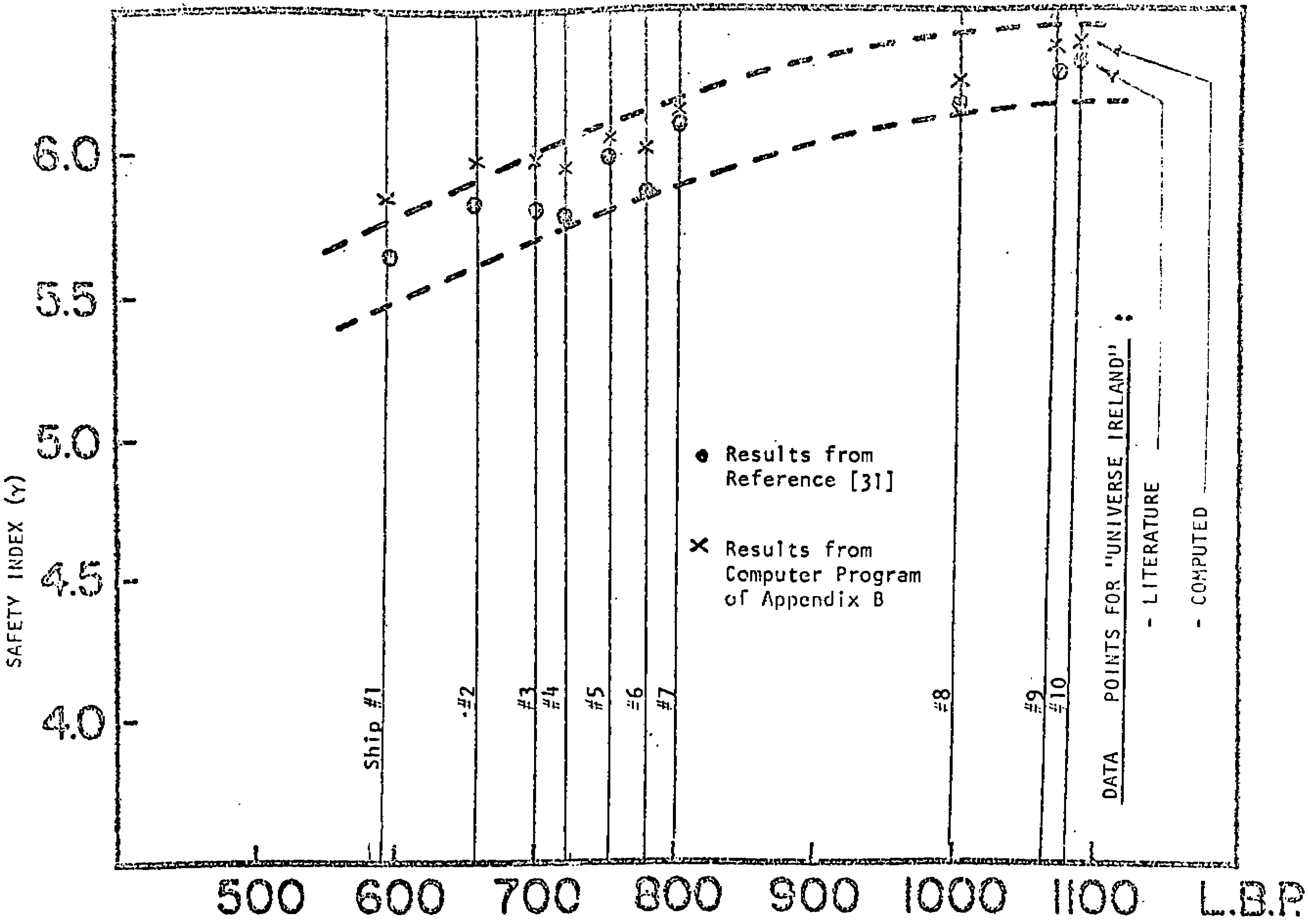


FIG. 8 - RELIABILITY ANALYSIS - DISTRIBUTION-FREE METHOD
(COMPARISON OF RESULTS FROM REF. (31) AND FROM COMPUTER PROGRAM OF APPENDIX B)

SECTION 9.0

CONCLUSIONS

The conclusions reached in the course of this study are summarized below under the major subheadings from which they were derived.

General Probabilistic Structural Analysis:

- Mechanical, Çivil, and Ship probabilistic analyses of structure have all utilized similar methods which differ in the assumptions concerning load and strength distributions.
- Strengths can be described from known distributions, synthesized from known distributions of constituent parts, synthesized from assumed distribution of constituent parts, or synthesized in terms of means and variances from those of the constituent parts. The accuracy decreases from the former to the latter approaches.
- Since probability of failure, margin of safety, and reliability involve integration under the tails of distribution curves, the shapes of the distribution may be important.
- Not much is available in the literature concerning time dependent strength analyses in general.

Modes of Failure:

- The most significant mode of failure of ship hulls from a longitudinal strength standpoint is not known. The probable modes include ductile yield bending of the hull as a free-free beam, ultimate plastic collapse buckling, fatigue and fracture.
- Modes of failure may vary according to ship type, size, etc.
- Local stresses must be superimposed on primary stresses when considering modes of failure i.e. principal stresses must be known.
- The lack of knowledge of ship modes of failure for longitudinal strength point to the need to obtain more insight into these types of failures to be better able to establish guidelines for new and structurally different ships of the future.

Loadings

- The total load scenario for a ship at sea is not clearly known in terms of statistical distributions.

Structure Statistics:

- Strength and uncertainty distributions for ships are not available except for limited cases of the latter. Assumptions are made in analyzing individual structures.

- Strength statistics in terms of means and variations have been generated for ships using an approximate method to obtain the mean and variance from those of the uncertainties in the strength.

- Little has been done with strengths other than static bending strength and plate buckling strength.

Strength Uncertainties:

- It is difficult to obtain data on strength uncertainties.

- Dimensional uncertainties in principal characteristics of ships appear negligible.

- Strength variations in yield and tensile properties of steels appear significant.

- It is difficult to estimate subjective uncertainties; not much data are available.

- Uncertainty statistics can be obtained from tolerances in production.

Ship Analyses:

- Most ship probabilistic structural analyses have been concerned with vertical bending and yielding.

- Due to the lack of statistics concerning ship stregths and loads, the development of an analysis approach which does not require a distribution shape appears warranted. This would, therefore, be a semi-probabilistic approach.

- Analyses using an approach of the latter type could be used to compare past and present designs, modes of failure, etc., to obtain more insight into ship longitudinal strength.

SECTION 10.0

RECOMMENDATIONS

An overall recommendation is that investigations should continue in the area of ship longitudinal strength so that any structurally different ship of the future can be properly designed with confidence. Although the probabilistic structural analysis of ships will probably have to remain at a more or less simplistic or semi-probabilistic level for the near future due to lack of methods and data, the efforts to improve the approach should nevertheless be undertaken if technology is to forge ahead.

Some specific recommendations are presented below for both the short- and long-term goals:

Long Term

- Continue to develop techniques and obtain data for both load and strength for probabilistic analysis.
- Perform classical probability of failure analyses for different ships; compare and update the results as better data and techniques are developed

Strength and Load Distributions:

- Determine conclusively whether it would be practical or not to determine strength distributions from small scale structural models.
- Obtain accurate coefficient of variation estimates for strength variables.
- Synthesize strength distributions using coefficients of variation of strength variables and assumed distributions of these variables.
- Determine the exact distributions of strength variables and synthesize strength distributions by the methods, discussed earlier, to obtain a more accurate distribution than otherwise available with the assumed distribution approach.
- Determine whether or not specific tabulated distributions are accurate for ship strengths.

Strength Equations:

- Develop accurate strength formulas for hull failure in ductile beam bending, compression buckling, ultimate strength failure, fracture and fatigue.

- Incorporate into these formulas as many uncertainty variables as possible.

Time-Dependent Analyses:

- Develop an accurate probability of failure procedure for the analysis of time-variant ship strengths for the cases of fatigue, corrosion, etc.

Short Term

- Using a probabilistic structural analysis method of complexity consistent with required assumptions, analyze past and current ships, including those that have failed in longitudinal strength, for various modes of failure. This method should embody a semi-probabilistic approach. Determine the corresponding safety factors.
- Compare results of the analyses to gain more insight into ship longitudinal strength.

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SECTION 11.0

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APPENDIX A

DERIVATION OF EXPRESSION FOR STRENGTH COV FOR DUCTILE YIELD OF THE HULL DU VERTICAL BENDING

Yield Strength= $\frac{D}{2}(A_f + 1/3\ Aw)\ s_y$ (1)

D= section depth (between neutral axes of combined plate a
beams of deck and bottom)

B= section beam

A_f= total area of flanges (deck and bottom)

Aw= total area of webs (sides and longitudal bulkheads)

S_y = yield stress

Take into account the following uncertainties in the above strength for

- o D& B: as described above
- o t_d: thickness of deck plating
- o t_s: thickness of stiffêner
- o t_ω: thickness of webs (side plating or longitudal bulkhea
- o l_ω: length of stiffener web
- o l_f: length of stiffener flange

M_f, M_w= number of stiffeners along deck and number of stiffen
along side plating respectively.

$$A_f = [Bt_d + M_f\ t_s\ (lw_1 + lf_1)] \cdot 2 \qquad (2)$$

$$Aw = [Dt_w + M_w\ (lw_2 + lf_2)] \cdot 4 \qquad (3)$$

INSERTING INTO (1):

$$S = \frac{D}{2}\{[(Bt_d + M_f t_s (lw_1 + l_{f1})] \cdot 2 + 1/3 [Dt_w + M_w t_s (lw_2 + lf_2)] \cdot 4\}\ s_y$$

$$S = [DBt_d + DM_f t_s (l_{w_1} + l_{f_1}) + 2/3\ D^2 t_w + 2/3\ DM_w t_s (lw_2 + lf_2)]\ s_y \qquad (4)$$

FROM SECTION 3.3.3:

$$\delta_s^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial \varepsilon_i} \cdot \frac{\bar{\varepsilon}_i}{m_S}\right)^2 \cdot \delta_{\varepsilon_i}^2 \qquad (5)$$

Consequently, the following partial derivatives must be determined:

$$\frac{\partial s}{\partial D}, \frac{\partial s}{\partial B}, \frac{\partial s}{\partial t_d}, \frac{\partial s}{\partial t_w}, \frac{\partial s}{\partial t_s}, \frac{\partial s}{\partial l_{w_1}}, \frac{\partial s}{\partial l_{f_1}}, \frac{\partial s}{\partial s_y}, \frac{\partial s}{\partial l_{w_2}}, \frac{\partial s}{\partial l_{f_2}}.$$

$$\frac{\partial s}{\partial D} = [Bt_d + M_f t_s\ (lw_1 + l_{f_1}) + 4/3\ Dt_w + 2/3\ M_\omega t_s\ (lw_2 + l_{f_2})]\ s_y \qquad (6)$$

$$\frac{\partial s}{\partial B} = Dt_d\ s_y \qquad (7)$$

$$\frac{\partial s}{\partial t_d} = DB\ s_y \qquad (8)$$

$$\frac{\partial s}{\partial t_w} = 2/3\ D^2\ s_y \qquad (9)$$

$$\frac{\partial s}{\partial t_s} = [DM_f\ (l_{\omega_1} + l_{f_1}) + 2/3\ DM_w\ (l_{\omega_2} + l_{f_2})\ s_y] \qquad (10)$$

$$\frac{\partial s}{\partial l_{w_1}} = DM_f t_s\ s_y \qquad (11)$$

$$\frac{\partial s}{\partial l_{f_1}} = DM_f t_s\ s_y \qquad (12)$$

$$\frac{\partial s}{\partial s_y} = DBt_d + DM_f t_s (lw_1 + lf_1) + 2/3\ D^2 t_\omega + 2/3\ DM_\omega t_s (lw_2 + l_{f_2}) \qquad (13)$$

$$\frac{\partial s}{\partial lw_2} = 2/3\ DM_\omega t_s\ s_y \qquad (14)$$

$\frac{\partial s}{\partial l_{f_2}} = 2/3\ DM_\omega t_s\ s_y$ (15); Equation (5) then becomes:

$$\delta s^2 = \left(\frac{\partial s}{\partial D} \cdot \frac{\bar{D}}{S}\right)^2 \delta_D^2 + \left(\frac{\partial s}{\partial B} \cdot \frac{\bar{B}}{S}\right) \delta_B^2 + \left(\frac{\partial s}{\partial t_d} \cdot \frac{\bar{td}}{S}\right)^2 \delta^2 td + \left(\frac{\partial s}{\partial t_\omega} \cdot \frac{\bar{tw}}{\bar{s}}\right)^2 \delta^2 t$$
$$+ \left(\frac{\partial s}{\partial t_s} - \frac{\overline{ts}}{\bar{S}}\right)^2 \delta^2 t_s + \left(\frac{\partial s}{\partial lw_1} \cdot \frac{\overline{lw}_1}{\bar{S}}\right)^2 \delta^2_{l_{w_1}} + \left(\frac{\partial s}{\partial l_{f_1}} \cdot \frac{l_{f_1}}{\bar{s}}\right)^2 \delta^2 l_{f_1}$$
$$+ \left(\frac{\partial s}{\partial lw_2} \cdot \frac{\overline{lw}_2}{\bar{S}}\right)^2 \delta^2 l_{w_2} + \left(\frac{\partial s}{\partial l_{f_2}} \cdot \frac{\overline{l_{f_2}}}{\bar{S}}\right)^2 \partial^2 l_{f_2} + \left(\frac{\partial s}{\partial s_y} \cdot \frac{\overline{s_y}}{\bar{S}}\right)^2 \delta^2 s_y \qquad (16)$$

let: $A = B t_d$ $\qquad E = M_f t_s l_{w_1}$

$B = M_f t_s (l_{w_1} + l_{f_1})$ $\qquad F = M_f t_s l_{f_1}$

$C = D t_w$ $\qquad G = M_w t_s l_{w_2}$

$D = M_w t_s (l_{w_2} + l_{f_2})$ $\qquad H = M_w t_s l_{f_2}$

$$\begin{aligned}
\delta^2_S = {} & \left(\frac{A + B + 4/3C + 2/3\,D}{A + B + 2/3C + 2/3\,D}\right)^2 \cdot \delta^2_D + \left(\frac{A}{A + B + 2/3C + 2/3\,D}\right)^2 \cdot \delta^2_B \\
& + \left(\frac{A}{A + B + 2/3\,C + 2/3D}\right)^2 \cdot \delta^2_{t_d} + \left(\frac{2C}{3A + 3B + 2C + 2D}\right)^2 \cdot \delta^2_{t_w} \\
& + \left(\frac{E}{A + B + 2/3C + 2/3D}\right)^2 \cdot \delta^2_{l_{w_1}} + \left(\frac{F}{A + B + 2/3C + 2/3D}\right)^2 \cdot \delta^2_{l_{f_1}} \\
& + \left(\frac{2G}{3A + 3B + 2C + 2D}\right)^2 \cdot \delta^2_{l_{w_2}} + \left(\frac{2H}{3A + 3B + 2C + 2D}\right)^2 \cdot \delta^2_{l_{f_2}} \\
& + \left(\frac{\left[\frac{E}{t_s} + 2/3\,\frac{G}{t_s}\right] t_s}{A + B + 2/3C + 2/3D}\right)^2 \delta^2_{t_s} + \delta^2_{s_y} \qquad (17)
\end{aligned}$$

$$\begin{aligned}
\delta^2_S = {} & \frac{9}{(3A + 3B + 2C + 2D)^2} \cdot \Big[1/9\,(3A + 3B + 4C + 2D)^2 \delta^2_D + A^2 (\delta^2_B + \delta^2_t) + E^2 \delta^2_{l_{w_1}} \\
& + D^2 \; l_{s_1}^2 + F^2 \delta^2_{l_{f_1}} + 1/9\,(3E + 2G)^2 \delta^2_{t_s} + \delta^2_{s_y} \\
& + \frac{4(C^2 \delta^2_{t_w} + G^2 \delta^2_{l_{w_2}} + H^2 \delta^2_{l_{f_2}})}{(3A + 3B + 2C + 2D)^2} \qquad (18)
\end{aligned}$$

Equation (18) is the final expression for the strength COV as a function of the COV's of the uncertainties.

APPENDIX B

LISTING AND DOCUMENTATION OF COMPUTER PROGRAM

INTRODUCTION

The computer algorithm was developed as described in Section 8.0. The program has several features that should be noted. Any number of ships may be analyzed for any number of modes of failure. Each ship and mode of failure requires its own data set, however. Further, with respect to modes of failure, there is an option to input the strength COV or have it computed by inputting the uncertainty COV's and computing the strength COV by subroutines for that purpose. Although the only such subroutine presently included in the program is that for ductile yielding of the hull girder in vertical bending, the program has been structured to allow for additional subroutines to be added for other modes of failure.

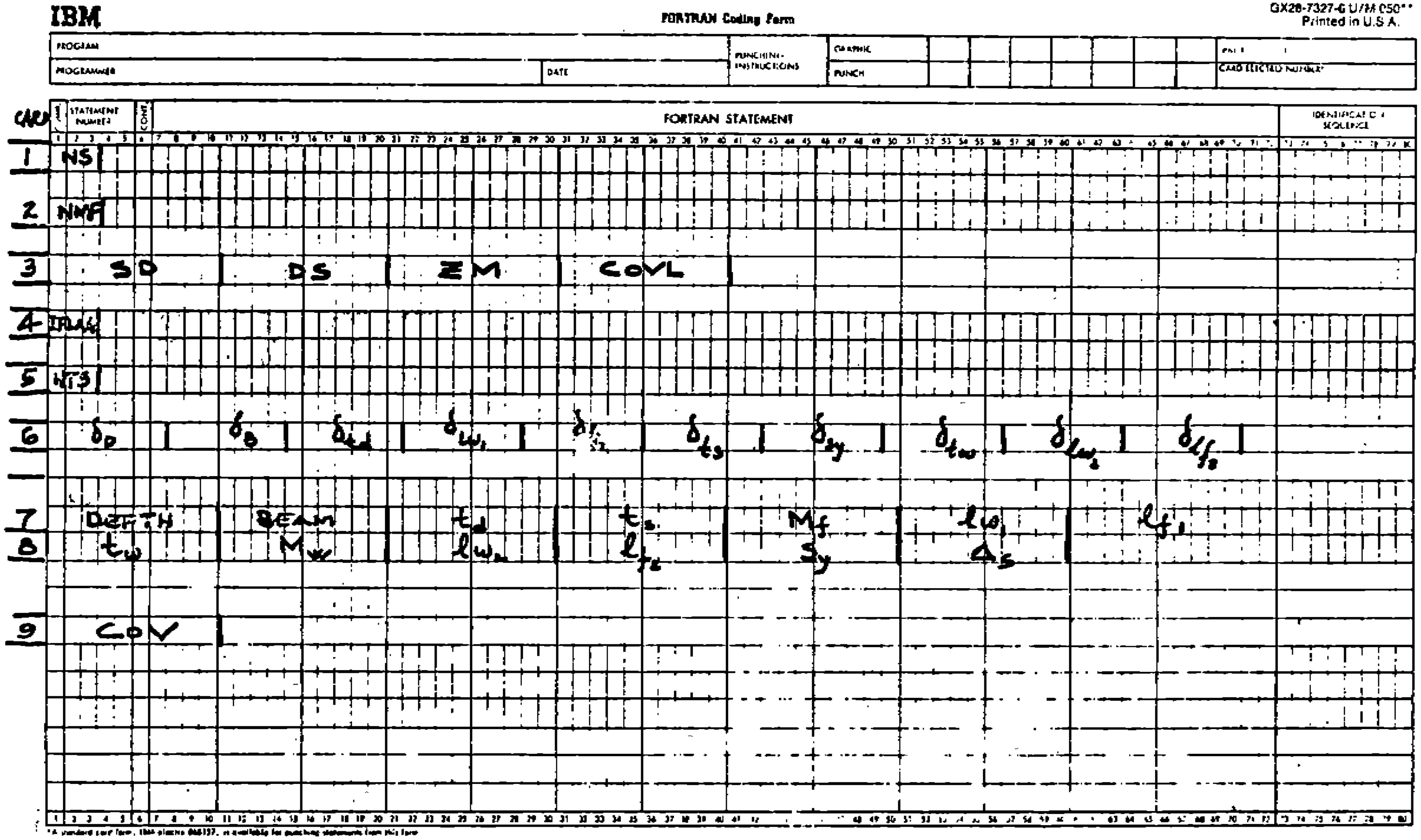
Tables B-1 and B-2 give the input cards and format for the program and Table B-3 a listing which should be self explanatory.

Table B-4 contains a listing of the data cards for the analysis discussed in Section 8.0, and Table B-5 presents the computer output.

TABLE B-1: INPUT CARDS AND FORMAT

CARD	FORMAT	NUMBER OF CARDS	SYMBOL	NOTES
1	I3	1	NS	Number of Ships
2	I3	1 per ship	NMF	Number of modes of failure, each ship must have this card repeated. Input all data cards for each ship before repeating.
3	4F10.4	1 per ship	SD, DS, ZM, COVL	SD = Actual Design MOD DS = Average Design Stress ZM = Mean of Load COVL = COV of Load
4	I3	1	I FLAG 0-Input strength COV 2-compute strength COV	Flag to indicate whether strength COV will be input or computed.
5	I3	1	NTS	This card to be filled out only if card 4 is "2". This directs card given the direction of which mode of failure is being analyzed and hence one is needed for every mode of failure. The next card should come after card 6. Currently only vertical bending is available in the program so that NTS = 1.
6	10F7.5	1	see notes	These cards to be filled out only if card 4 is "2". The following variables in order, as defined in Section 8.0, are required:
7	7F10.4	1	$\left[\text{23 quantities}\right]$	δD, δB, δt_d, δl_{w_1}, δl_{f_1}, δt_s, δs_y, δt_w, δl_{w_2}, δl_{f_2}, Depth, Beam, t_d, t_s, M_f, l_{w_1}, l_{f_2}, t_w, M_w, l_{w_2}
8	7F10.4	1		l_{f_2}, s_y, SUBCO(Subjective Uncertainties COV)
9	F10.5	1	COV	This card to be filled out only if card 4 is "0". In that case the strength COV is input and is given by: $COV = \sqrt{\delta_s{}^2 + \Delta_s^2}$ δ_s = objective uncertainty COV Δ_s = subjective uncertainty COV

TABLE B-2: INPUT CARDS



IBM FORTRAN Coding Form

GX28-7327-6 U/M 050**
Printed in U.S.A.

PROGRAM
PROGRAMMER
DATE
PUNCHING INSTRUCTIONS
GRAPHIC
PUNCH
PAGE
CARD ELECTRO NUMBER

CARD	FORTRAN STATEMENT									
1	NS									
2	NMA									
3	SD	DS	ZM	COVL						
4	IDRAG									
5	NTS									
6	δ_D	δ_B	δ_{td}	δ_{lw_1}	δ_{lf_1}	δ_{ts}	δ_{sy}	δ_{tw}	δ_{lw_2}	δ_{lf_2}
7	DEPTH	BEAM	t_d	t_s	M_f	l_{w_1}	l_{f_1}			
8	t_w	M_w	l_{w_2}	l_{f_2}	S_y	Δ_s				
9	COV									

*A standard card form, IBM electro 888157, is available for punching statements from this form

TABLE B-3: LISTING OF COMPUTER PROGRAM

```
C
C
C
COMPUTER ALGORITHM TO DETERMINE SHIP HULL LONGITUDINAL STRENGTH SAFETY
C INDICES BY THE PROBABILISTIC METHOD DISCUSSED IN SR-241
C
C READ IN NUMBER OF SHIPS TO BE CONSIDERED
C
      REAL MF,LW1,LF1,MW,LW2,LT2,MSI
      DIMENSION  A(10)
      N=2
      M=3
      READ (N,1)  NS
C
C DO LOOP FOR NUMBER OF SHIPS
C
      DO 200 NSFL=1,NS
C
C READ NUMBER OF MODES OF FAILURE TO BE CONSIDERED FOR SHIP 'I'
      READ(N,1) NMF
    1 FORMAT (I3)
C
C DO LOOP FOR MODES OF FAILURES
C
      DO 100 I=1,NMF
C
C READ ACTUAL DESIGN STRENGTH,SD,AVERAGE DESIGN FAILURE STRESS,DS,MEAN
C OF THE LOAD,ZM,AND COV OF LOAD,COVL
C
      READ (N,2)SD,DS,ZM,COVL
    2 FORMAT (4F10.4)
C
C SELECT BETWEEN INPUTTING STRENGTH COV OR COMPUTING IT BY SUBROUTINES
C AND INPUT UNCERAINTY COV'S AND ONE VALUE FOR SUBJECTIVE COV'S
C
      READ (N,1)IFLAG
      IF (1-IFLAG)   3,3,4
    3 CONTINUE
C
C READ IN TYPE OF STRENGTH TO BE CONSIDERED
C
```

TABLE B-3: LISTING OF COMPUTER PROGRAM (CONT.)

```
PAGE    2

      READ (N,1)  NTS
      GO TO (5),NTS.
    5 CONTINUE
C
C READ IN VERTICAL BENDING STRENGTH UNCERNTAINTY COV'S
C
      READ (N,6)     A
    6 FORMAT (10F7.5)
C
C READ IN SHIP PARAMETERS NEEDED TO COMPUTE STRENGTH COV
C
      READ (N,20)DEPTH,BEAM,TD,TS,MF,LW1,LF1,TW,MW,LW2,LT2,SUBCO
   20 FORMAT  (7F10.4)
C CALL SUBROUTINE TO COMPUTE STRENGTH COV FROM UNCERTAINTY COV'S
C
      CALL  VERTS (A,DEPTH,BEAM,TD,TS,MF,LW1,LF1,TW,MW,LW2,LT2,COVO)
      COV=SQRT(COVO**2+SUBCO**2)
      GO TO 7
    4 CONTINUE
C
C READ  THE GIVEN STRENGTH COV
C
      READ (N,8) COV
    8 FORMAT (F10.5)
    7 CONTINUE
C
C COMPUTE MEAN OF STRENGTH
C
      MSI=SD*DS
      THETA=MSI/ZM
C
C COMPUTE THE SAFETY INDEX
C
      SI=(THETA-1.0)/(SQRT(THETA**2*COV**2+COVL**2))
C
C OUTPUT
C
      WRITE (M,10) NSFL
   10 FORMAT (1X,'SHIP NUMBER = ',I3)
      WRITE (M,11) NMF
   11 FORMAT (1X,'MODE OF FAILURE NUMBER =',I3)
      WRITE (M,12) SD
   12 FORMAT (1X,'ACTUAL DESIGN  MODULUS =',F20.4)

      WRITE (M,13) DS
   13 FORMAT (1X,'AVERAGE DESIGN FAILURE STRESS =',F20.4)
      WRITE (M,14) MSI
   14 FORMAT (1X,'MEAN OF STRENGTH = ',F20.4)
      WRITE (M,15) ZM
   15 FORMAT (1X,'MEAN OF LOAD = ',F20.4)
      WRITE (M,16) COV
   16 FORMAT (1X,'COV OF STRENGTH = ',F10.4)
      WRITE (M,17) COVL
   17 FORMAT (1X,'COV OF LOAD = ',F10.4)
      WRITE (M,18) SI
   18 FORMAT (1X,'SAFETY INDEX = ',F10.4///)
  100 CONTINUE
  200 CONTINUE
      CALL EXIT
      END
```

```
C
C SUBROUTINE TO COMPUTE THE DUCTILE YIELDING STRENGTH COV FOR A SHIP IN
C VERTICAL BENDING.FORMULATION ACCORDING TO THAT GIVEN IN SR-241.
C
      SUBROUTINE  VERTS (A,DEPTH,BEAM,TD,TS,MF,LW1,LF1,TW,MW,LW2,LT2,
     1XOV)
      REAL  MF,LW1,LF1,MW,LW2,LT2
      DIMENSION  A(10)
      A1=BEAM*TD
      B=MF*TS*(LW1+LF1)
      C=DEPTH*TW
      D=MW*TS*(LW2+LT2)
      E=MF*TS*LW1
      F=MF*TS*LF1
      G=MW*TS*LW2
      H=MW*TS*LT2
      COV1=(3.0*A1+3.0*B+4.0*C+2.0*D)**2*A(1)**2/9.0
      COV2=A1**2*(A(2)**2+A(3)**2)
      COV3=E**2*A(4)**2
      COV4=F**2*A(5)**2
      COV5=(3.0*E+2.0*G)**2*A(6)**2/9.0
      COV6=9.0/(3.0*A1+3.0*B+2.0*C+2.0*D)**2
      COV7=COV6*(COV1+COV2+COV3+COV4+COV5)
      COV8=A(7)**2
      COV9=4.0*(C**2*A(8)**2+G**2*A(9)**2+H**2*A(10)**2)
      COV10=(3.0*A1+3.0*B+2.0*C+2.0*D)**2
      COV11=COV9/COV10
      XOV=SQRT(COV7+COV8+COV11)
      RETURN
      END
```

TABLE B-4: LISTING OF DATA CARDS

```
// XEQ LONST
 12
  1
  61200.      15.2     240500.      .1595
  0
 .11
  1
  81320.      15.2     371000.      .1400
  0
 .11
  1
  97000.     15.2      448000.      .1402
  0
 .11
  1
  87950.      15.2    406350.       .1444
  0
 .11
  1
  131600.   15.2        580000.     .1325
  0
 .11
  1
  143000.    15.2     671000.     .1229
  0
 .11
  1
  158000.   15.2        679000.     .1265
  0
 .11
  1
  366300.   15.2      1587750.     .1105
  0
 .11
  1
  437600.   15.2      1835700.     .1066
  0
 .11
  1
  493750.   15.2      2037900.     .1066
  0
 .11
  1
  493750.   15.2      2496400.     .1068
  0
 .11
  1
  493750.   15.2      2496400.     .1066
```

TABLE B-5: COMPUTER OUTPUT

```
SHIP NUMBER =   1
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =          51200.0079
AVERAGE DESIGN FAILURE STRESS =             15.2000
MEAN OF STRENGTH =          778240.0017
MEAN OF LOAD =          240500.0316
COV OF STRENGTH =     0.1100
COV OF LOAD =     0.1595
SAFETY INDEX =     5.7323


SHIP NUMBER =   2
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =          81320.0158
AVERAGE DESIGN FAILURE STRESS =             15.2000
MEAN OF STRENGTH =         1236064.0034
MEAN OF LOAD =          371000.0632
COV OF STRENGTH =     0.1100
COV OF LOAD =     0.1400
SAFETY INDEX =     5.9434


SHIP NUMBER =   3
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =          97000.0158
AVERAGE DESIGN FAILURE STRESS =             15.2000
MEAN OF STRENGTH =         1474400.0034
MEAN OF LOAD =          448000.0632
COV OF STRENGTH =     0.1100
COV OF LOAD =     0.1402
SAFETY.INDEX =     5.9015


SHIP NUMBER =   4
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =          87950.0158
AVERAGE DESIGN FAILURE STRESS =             15.2000
MEAN OF STRENGTH =         1336840.0029
MEAN OF LOAD =          406350.0633
COV OF STRENGTH =     0.1100
COV OF LOAD =     0.1444
```

TABLE B-5: COMPUTER OUTPUT (CONT.)

```
SAFETY INDEX =     5.8770


SHIP NUMBER =    5
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =          131600.0316
AVERAGE DESIGN FAILURE STRESS =                    15.2000
MEAN OF STRENGTH =         2000320.0034
MEAN OF LOAD =          580000.1264
COV OF STRENGTH =     0.1100
COV OF LOAD =     0.1325
SAFETY INDEX =     6.0939


SHIP NUMBER =    6
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =          143000.0316
AVERAGE DESIGN FAILURE STRESS =                    15.2000
MEAN OF STRENGTH =         2173600.0058
MEAN OF LOAD =          671000.1267
COV OF STRENGTH =     0.1100
COV OF LOAD =     0.1229
SAFETY INDEX =     5.9410


SHIP NUMBER =    7
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =          158000.0316
AVERAGE DESIGN FAILURE STRESS =                    15.2000
MEAN OF STRENGTH =         2401600.0068
MEAN OF LOAD =          679000.1267
COV OF STRENGTH =     0.1100
COV OF LOAD =     0.1268
SAFETY INDEX =     6.1997


SHIP NUMBER =    8
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =          366300.0633
AVERAGE DESIGN FAILURE STRESS =                    15.2000
MEAN OF STRENGTH =         5567760.0117
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TABLE B-5: COMPUTER OUTPUT (CONT.)

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MEAN OF LOAD =            1597750.2529
COV OF STRENGTH =      0.1100
COV OF LOAD =      0.1105
SAFETY INDEX =      6.2471


SHIP NUMBER =   9
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =         439600.0632
AVERAGE DESIGN FAILURE STRESS =             15.2000
MEAN OF STRENGTH =         6681920.0117
MEAN OF LOAD =            1836700.2534
COV OF STRENGTH =      0.1100
COV OF LOAD =      0.1068
SAFETY INDEX =      6.3691


SHIP NUMBER =  10
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =         493750.0633
AVERAGE DESIGN FAILURE STRESS =             15.2000
MEAN OF STRENGTH =         7505000.0136
MEAN OF LOAD =            2037900.2529
COV OF STRENGTH =      0.1100
COV OF LOAD =      0.1068
SAFETY INDEX =      6.4035


SHIP NUMBER =  11
MODE OF FAILURE NUMBER =  1
ACTUAL DESIGN  MODULUS =         493750.0633
AVERAGE DESIGN FAILURE STRESS =             15.2000
MEAN OF STRENGTH =         7505000.0136
MEAN OF LOAD =            2486400.5058
COV OF STRENGTH =      0.1100
COV OF LOAD =      0.1068
SAFETY INDEX =      5.7370


SHIP NUMBER =  12
MODE OF FAILURE NUMBER =  1



ACTUAL DESIGN  MODULUS =         493750.0633
AVERAGE DESIGN FAILURE STRESS =             15.2000
MEAN OF STRENGTH =         7505000.0136
MEAN OF LOAD =            2486400.5058
COV OF STRENGTH =      0.0840
COV OF LOAD =      0.1068
SAFETY INDEX =      7.3332
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The Ship Design, Response, & Load Criteria Advisory Group prepared the project prospectus, evaluated the proposals for this project, provided the liaison technical guidance, and reviewed the project reports with the investigator:

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SHIP STRUCTURE COMMITTEE PUBLICATIONS

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