SSC-354

STRUCTURAL REDUNDANCY FOR CONTINUOUS AND DISCRETE SYSTEMS

SHIP STRUCTURE COMMITTEE

1992

SHIP STRUCTURE COMMITTEE

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STRUCTURAL REDUNDANCY FOR CONTINUOUS AND DISCRETE SYSTEMS

The concept of design, inspection, and structural redundancy interactions was introduced at the Ship Structure Committee sponsored DIRT Symposium held in Williamsburg, Virginia in 1983. To date, most design practices address overall global response using linear elastic models and a subsequent examination of local component response using rational limit state methods. Hence, the true reserve strength inherent in the overall structure and the influence of redundancy on safety and reliability are not adequately considered.

The purpose of this preliminary study was to assess the role of redundancy in marine structures as it relates to reserve and residual strength. This report provides an excellent review of the basic terminology and definitions in the context of discrete and continuous structural topologies. It should prove to be a valuable reference with the continuing development of structural modeling techniques.

A. E. HENN
Rear Admiral, U.S. Coast Guard
Chairman, Ship Structure Committee

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16. Abstract

The design, inspection and structural redundancy interactions (DIRT) concept was first introduced at a Ship Structure Committee sponsored symposium in Williamsburg, Virginia, USA in 1983. In order to produce structures that are safer and more efficient in performance throughout their lifetime and in which a degree of damage tolerance can be planned, it is important that the full effects of redundancy are allowed for in the design validation process. The purpose of this study is to assess the role of redundancy in marine structures in the context of reserve and residual strength.

The study has included a review of the basic terms and definitions of redundancy and reserve and residual strength in the context of both discrete and continuous structural topologies and has then examined criteria for defining overall stability in such topologies. A series of numerical studies on simple structural forms, representative of both discrete and continuous structures, has been undertaken and used to illustrate the major aspects and problems involved.

The study concludes with recommendations for a range of future projects with the goals to examine and quantify more rigorously the role of redundancy on the reserve and residual strength characteristics of specific ranges of marine structures.

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METRIC CONVERSION FACTORS

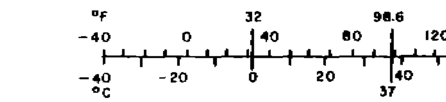
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Me

Symbol	When You Know	Multiply by
		LENGTH
mm	millimeters	0.04
cm	centimeters	0.4
m	meters	3.3
m	meters	1.1
km	kilometers	0.6
		AREA
cm^2	square centimeters	0.16
m^2	square meters	1.2
km^2	square kilometers	0.4
ha	hectares (10,000 m^2)	2.5
		MASS (weight)
g	grams	0.035
kg	kilograms	2.2
t	tonnes (1000 kg)	1.1
		VOLUME
ml	milliliters	0.03
l	liters	2.1
l	liters	1.06
l	liters	0.26
m^3	cubic meters	35
m^3	cubic meters	1.3
		TEMPERATURE (exa
°C	Celsius temperature	9/5 (then add 32)



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EXECUTIVE SUMMARY

In order to produce structures that are both more safe and efficient in performance throughout their required lifetime and in which a degree of damage tolerance is planned for, it is important that the full effects of redundancy are allowed for in the design development and validation process. Most conventional design practices address overall global response using linear elastic models and subsequently examine local component response using rational limit state methods, including the calculation of local collapse. Thus safety and reliability assessments of the overall structure are actually made at the component level. Hence the true reserves of strength inherent in overall structures where the effects of redundancy can have a major influence on safety and reliability, are not allowed for. Similarly the residual strength of the structure following some form of local damage and in which redundancy provides the mechanism for maintaining overall stability, is not normally considered.

Thus the purpose of this study, which is a precursor or pilot study to a subsequent more detailed program of investigations, is to assess the role of redundancy in marine structures (ships, mobile offshore drilling units and fixed offshore production platforms) in the context of reserve and residual strength.

The study which has been undertaken has included a review of the basic terms and definitions of redundancy, reserve and residual strength in the context of both discrete and continuous structural topologies and has then examined criteria for defining overall stability in such topologies. The categorisation of structures into either discrete or continuous forms is made in order to enable the features which create redundancy to be more clearly identified. This has also included the concepts of series and parallel systems where redundancy exists within a structure.

The study also includes both a brief review of the elastic and inelastic response of structures where such regimes of behaviour can effect the overall performance of redundant systems and also of the causes of damage.

In this study an attempt has been made to establish and review the role of redundancy applicable to both discrete and continuous structures using probabilistic as well as deterministic models. It is an accepted fact that in order to carry out a reliability analysis it is necessary to provide a deterministic physical framework and the general tools upon which the reliability models would be built.

A series of simple numeric examples based upon an elementary two-dimensional framework model, representative of discrete structures, was employed in order to explore the relationships between redundancy, reserve strength and residual strength. It was considered that the ranking, that is the placing into order of importance, of the diagonal and horizontal bracing members, for example, within a complex three-dimensional framework structure in a multi-directional wave environment will be difficult.

A series of both deterministic and probabilistic studies were undertaken on stiffened plate structures, of both flat panel and circular cylindrical form. Such structures are representative of low order continuous structures.

Several simple deterministic analyses were undertaken to examine the role of redundancy in ship hull girders, which are clearly in the category of continuous structures. For this study elementary rectangular section box beam models were employed and their ultimate strength, when subjected to sagging bending moments, was determined. It was found, for these models, that the ultimate strength decreased rapidly with failure of the upper deck flange part of the structure.

The potential scope for a follow-on research programme is clearly quite extensive, noting the considerable range of ship and offshore structure types and configurations, local and overall failure mechanisms, possible damage scenarios, etc. Thus to identify and select specific aspects that would have some merit and priority for study, within the auspices of the Ship Structure Committee's mandate, requires most careful deliberation and possibly cost-benefit assessments. The final chapter within the report reviews a broad range of possible research and development projects without, initially, placing any order of importance or priority against each. The Ship Structure Committee will need to decide upon a focus for further work, e.g. educational/instructional, methods development, background studies for code development, etc., and to select a priority group from within the range of structural families.

1.0 INTRODUCTION

This report is the outcome of a pilot study and from which it is intended that the Ship Structure Committee can then initiate a coordinated and cohesive plan for a subsequent 3-R program (i.e. redundancy, residual strength and reserve strength) of the design, inspection and redundancy triangle.

In the current design process of most forms of marine structures it is generally assumed that at the overall global response level the application of small deflection linear elastic theory provides the appropriate measure of structure performance. From such a global assessment of structural response the forces applied to individual members or components may then be compared with local capability assessed in terms of ultimate strength. Approaches to the determination of local ultimate strength may include non-linear, large deflection or plastic response type formulations. This combined global-local response approach is also implicit within many overall reliability based studies. Thus the general state of the art could be considered to be currently at a plateau.

The next major step in the state of the art and which has begun to be focussed upon in recent years, is the part played by redundancy at the global, or overall, structure strength level. This has been recognised in the evolution of the DIRT philosophy (design-inspection-redundancy triangle) and in the need for damage tolerant structures.

Structural redundancy plays many parts, for example:

- In the behaviour of the structure in the non-linear regime, including when local component buckling occurs.
- In the behaviour of the overall structure in the ultimate strength and post-ultimate strength regimes and
- in the response and capability of the structure following some form of damage.

Whilst redundancy can be allowed for in design development, for example by employing an appropriate analytical method (e.g. one of several finite element or finite difference based computer codes), it does however represent a complex and expensive study to undertake. The problem is further considerably compounded if a reliability based approach is to be employed and it would require careful study before a formal level 1 partial factor based code allowing for varying degrees of redundancy could be developed. Thus an important first developmental step must therefore be to identify, assess and quantify the role of redundancy in all forms of marine structures (ships and both mobile and fixed-site offshore structures). This SSC study takes a broad conceptual examination as related to the general nature of both reserve (intact) and residual (damaged) strength. To allow for the diversity of structural forms it is convenient to classify structures as being either of the 'discrete-framed' forms (e.g. typical fixed-site steel jackets) or of the 'continuous-semi monocoque' forms (e.g. ship hull girders, semi-submersible pontoons).

"NOTE: The reference lists given at the end of each section in this report contain some references not referred to in the text. However these have come from the results of an indepth literature survey and it was considered that for completeness they should be included in this report."

2.0 DEFINITION AND REVIEW OF TERMS

□ Redundancy

A more detailed review of Redundancy is given in Section 3. For simple discrete member structures such as trusses and beam-column frameworks the concept of redundancy is well defined and understood. It is associated with the concept of stability and determinancy. A stable structure is one which is in a state of static equilibrium and a discrete stable structure is statically determinate with respect to the applied forces if all the individual component forces can be completely determined by applying the equations of static equilibrium. If that is not the case, the structure is statically indeterminate or hyperstatic, and the degree of indeterminancy is the number of unknowns over and above the number of condition equations available for a static solution. The excess reaction components are called redundants because they are unnecessary for the overall stability of the structure. Within a redundant structure there is often no simple rule which qualifies a redundant element from a statically necessary element and often more than one statically determinate structural system can be identified. Individual elements of structures can also be internally redundant. The same concept cannot be readily applied to a continuous system, for example ship hull girders, which are in reality highly redundant unless they are made equivalent to a discrete like structure. Structures may be made to be redundant either by design (e.g. with collision safety in mind or for some operational requirements) or by the fabrication/production approaches taken.

□ Redundancy Index

A general measure of internal redundancy which has been postulated [2.1] is the redundancy index (RI) and is given by

$$RI = \frac{P_u - \bar{P}_u}{\bar{P}_u} \tag{2.1}$$

in which P_u is the ultimate load carrying capacity of the structure under consideration and $\bar{P}_u$ is the ultimate load carrying capacity of the parent structure. The parent structure is one in which all members that are not absolutely necessary for stability have been removed [2.1]. (It may be possible to identify in a complex structure more than one feasible parent form.)

Assuming that a structure remains stable above the maximum demand load (possibly including any required factor of safety) then that structure will have an excess capacity, i.e. a reserve of strength. The redundancy index proposed in [2.1] is clearly different than the 'classical' measure of redundancy as expressed via the degree of indeterminancy. The parent structure is obviously not unique and when applying the definition contained within equation (2.1) the parent with the smallest ultimate strength (load carrying capacity) should be used.

Excess capacity is generally realised when the full ultimate strength is realised and in the definition of the Redundancy Index excess capacity designed into members is fully considered.

□ Reserve Strength

The Reserve Strength, RS, may be defined as the difference between the ultimate load carrying capacity of the component or the system (the whole structure) and the actual maximum applied load, i.e. the design load.

The design loads are determined to be caused by the anticipated extreme environmental and operational events with possible load combinations together with the self weight etc. For example, for offshore structures a return period for maximum environmental events of 50 or 100 years is usually assumed. The current practice for the design of fixed platform structures is generally based on API RP2A[2.2] or similar rules published either by classification societies[2.3, 2.4] or other regulating bodies, e.g. DoE[2.5]. The structural design of these platforms is mainly governed by component strength checking procedures and they are based on a working stress approach using traditional factor of safety concepts which limits a stress value. The exceedance of this limiting stress in a particular member constitutes an unacceptable condition or failure for both the member and for the structure as a whole regardless of the degree of redundancy. In addition to the reserve strength of the individual members in, for example, a fixed platform structure, the structure as a whole is likely to be structurally redundant and hence the reserve strength of the whole system against failure due to the design loads is likely to be very high and certainly much higher than the component safety factor built into the code.

In fact this reserve strength which is inherent in a structural member will vary between different codes and will depend on the type of formulations adopted and the safety margin imposed in the code. There will also be a model uncertainty factor (X_m) which is defined as the ratio of the actual strength (determined from experiment) to the theoretically predicted strength. The predicted strength is the codified strength based on either rational formulations or empirically derived ones or a combination of both. Thus reserve strength can be defined for a single component involving one mode of failure:

$$RS = X_m P_u \left[1 - \frac{P_d}{X_m P_u}\right] \qquad (2.2)$$

in which P_u is the ultimate load carry capacity, X_m is the model uncertainty factor and P_d is the design load. The uncertainty in the design load, etc., although an important consideration, is not considered further in this report. (Clearly in determining the loads that are applied to a structure some uncertainties must exist in the models, methods and data that are employed. However whilst it is necessary to appreciate the existance of such uncertainties, and which can be quite considerable, examining such is not the purpose of this study.)

For components subjected to the simultaneous action of different loads, the reserve strength can be expressed in terms of some form of interaction equation.

$$RS = X_m P_u \left[1 - \sum \left[\frac{P_{di}}{X_{mi} P_{ui}}\right]^{\gamma_i}\right] \qquad (2.3)$$

in which P_{di} is the design load for type i, X_{mi} is the model uncertainty factor for type i, and P_{ui} is the ultimate load carrying capacity for type i.

Thus as can be seen from both equations (2.2) and (2.3) the reserve strength equals some fraction of $X_m P_u$.

The degree of interaction depends on the exponents γ_i. As a general note interaction curves are mathematically simple to express but are often found to be difficult to actually quantify. For a given class of structures (or problems) when the intent is to provide guidance or codes for use in the general design process then quantifying the exponent for use in the interaction curves can be done by

(i) controlled laboratory experiments, or

(ii) by undertaking detailed numerical studies with an appropriate rigorous technique that allows for all modes of ultimate collapse (e.g. the development of plastic hinges, elastic/inelastic interactions, buckling, etc.).

The Reserve Strength Index, RSI, may be defined as the ratio of the actual ultimate strength to the design load for the structure.

$$RSI = \frac{X_m P_u}{P_d} \tag{2.4}$$

Depending on the value of P_d, the RSI may vary considerably. A value of unity indicates no reserve strength. If the RSI is below unity, (i.e. $P_d > X_m P_u$) failure is considered to have occurred. This applies to both the 'component' and the system reserve strength.

□ <u>Residual or Damaged Strength</u>

The Damaged Strength Ratio, DSR, of the structure may be defined as the ratio between the load carrying ultimate capacity of the damaged structure (P_r) and the ultimate load carrying capacity of the intact structure.

$$DSR = \frac{X_m P_r}{P_u} \tag{2.5}$$

P_r is also associated with a model uncertainty factor X_m and which will be very difficult to determine because of the wide range possible in the character, position and extent of any potential damage (multiple damage locations and combinations could also be considered but will clearly make the problem more complex). Clearly the damage strength ratio should be qualified with regard to character, position and extent of any damage. From these it could be concluded that it is possible for a structure to have several damage strength ratios.

In [2.1, 2.6], the Residual (Damaged) Strength Index (RDI), which is identical to the damaged strength ratio DSR, (except for X_m), is defined as the ratio of the residual strength (P_r) to the ultimate strength of the intact structure:

$$RDI = P_r/P_u \tag{2.6}$$

between zero for a structure with no residual strength to near one for a structure with a high degree of residual strength.

□ Limit States

The above definitions of reserve and residual strength are made only from considering the ultimate strength limit state of the structure but in principle they could also be extended to include other limit states such as (i) Fatigue limit state, (ii) Serviceability limit state and (iii) Progressive collapse limit state (which itself infers the degree of local failure/damage) [2.3].

Fatigue Limit State, FLS, relates to the criteria associated with the effects of repeated cyclic loading. The aim of this design activity is to ensure adequate safety provisions against fatigue damage occurring within the planned life of the structure. Methods of analysis generally used are (i) those based on fracture mechanics, and (ii) those based on fatigue test data.

Fatigue damage criteria, in general, is as follows. The cumulative Damage Ratio D (according to the Miner-Palmgren hypothesis) is given by:

$$D = \sum_{i=1}^{S} \frac{n_i}{N_i} \leq \eta \tag{2.7}$$

where S is the number of idealised stress ranges, n_i is the number of actual or assumed experienced stress cycles in stress block i, N_i is the number of cycles to cause failure at constant stress range $(\sigma_r)_i$ and η is the design limit damage ratio ($\leq$ 1.0) which is effectively a factor of safety and which will depend on the maintainability, i.e. possibility of inspection and repair, as well as the importance of the particular construction detail considered.

The fatigue-based definition of Reserve Strength Index, RSI, should provide an indicator of strength, and may be expressed as:

$$RSI = (1/D)^{1/m} \tag{2.8}$$

where D is the cumulative damage ratio and m is log-log slope of the S-N curve. In this context the RSI is independent of the time taken to acrue the fatigue damage which constitutes failure, as defined by the Palmgren-Miner hypothesis.

Fracture mechanics procedures aim to give a more fundamental view of fatigue crack growth than does the test specimen based crack initiation S-N curve approach. However at the present time the main contribution of fracture mechanics to fatigue development is the Paris crack growth law[2.7], which is largely empirical itself. The simplest use of this law tends to give the same results as the S-N curve approach but it gives greater meaning to the various constants. The crack growth rate according to the Paris Law is given by:

$$\frac{da}{dN} = C(\Delta k)^m \tag{2.9}$$

where Δk is the range of stress intensity factor and is given by:

$$\Delta k = \alpha \; S \; \sqrt{a\pi} \tag{2.10}$$

where 'a' is a direct measure of crack size, S is the stress range and α is a constant dependent on the geometry of the case under consideration. α is a geometry correction function and which is given by

$$\alpha = F_E \cdot F_S \cdot F_T \cdot F_W \cdot F_G$$

where F_E is a basic shape factor which depends upon the aspect ratio of the crack; F_S is a front face factor and which depends upon the crack opening stress distribution, free surface shape, crack shape and position of the crack front; F_T is a back face or finite thickness factor and which depends upon the crack geometry and stress distribution position on the crack front; F_W is a finite width factor and which is important for a through thickness crack and F_G is a stress gradient factor which takes into account the shape of the stress distribution. C and m are empirical constants.

Hence for a constant stress range S and constant α values, the crack growth from $a = a_i$ to $a = a_f$ over N-cycles is given by:

$$\int_{a_i}^{a_f} a^{-m/2} \, da = C \; \alpha^m \; s^m \; \pi^{m/2} \int_{o}^{N} dn \tag{2.11}$$

Hence:

$$NS^m = \frac{1}{\left[\frac{m}{2} - 1\right] \; C\alpha^m \; \pi^{m/2}} \left[\frac{1}{a_i^{m/2-1}} - \frac{1}{a_f^{m/2-1}}\right] \tag{2.12}$$

This corresponds to an S-N curve, i.e. $NS^m = k$, and suggests that the constants k and m can be expressed as a function of more fundamental quantities.

For stress cycles of varying amplitude, it is possible to use equation (2.12) as the S-N curve equation and to incorporate the results obtained in equation (2.10) to calculate cumulative damage. Alternatively, as with the S-N curves, some crack growth data is available for stress cycles of varying amplitude. It appears that for a narrow-band stationary process the crack growth law, i.e. equation (2.10), may still be used with Δk as the root-mean-m^{th} value of stress intensity factor range.

In equation (2.10), the α factor, which is assumed to be constant for simplification, is of limited use for practical design purposes.

In applying equation (2.10) to the case of a finite width plate under different loading conditions and with different crack orientations and complex crack geometries, the designer normally has to decide which standard case is the nearest appropriate one, for the case under consideration, from standard tables, as given in [2.8, 2.9, 2.10].

Further discussions on residual strength as a function of crack size and brittle failure modes will be given in a later section. Fatigue and tensile fracture failure modes are also reviewed in detail in a later section.

The Serviceability Limit State, SLS, will normally include criteria related to displacements and vibrations. These limit states are to be checked for combinations of loads and masses and the displacements are to be evaluated for the characteristic values of these loads. This does not seem particularly relevant in the context of this study and hence is not considered further.

The Progressive Collapse Limit State, PLS, is considered for possible accidental loads against which sufficient local strength cannot be provided by reasonable means. This can be covered by calculating the residual strength of the overall structure.

A diagram showing structural capability in intact and damaged conditions is shown in Figure 2.1.

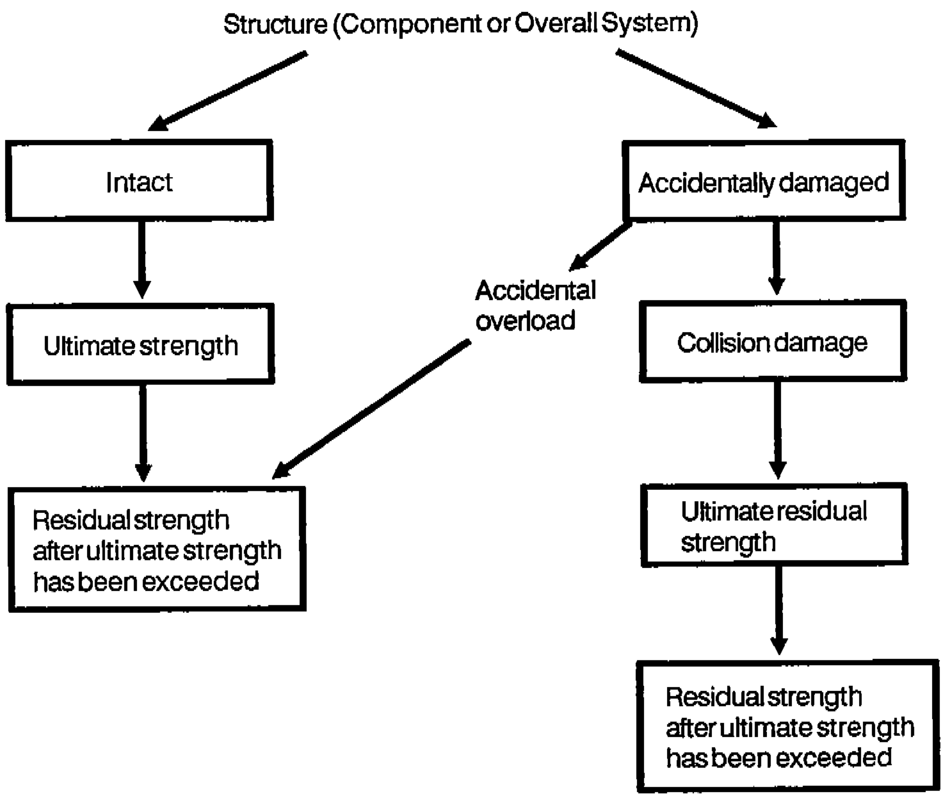

Fig.2.1 Structural Capability in Intact and Damaged Conditions

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3.0 CATEGORIZATION OF STRUCTURES

This study is made with reference to ships, mobile offshore drilling units and fixed site offshore platforms.

'Ships' is taken to infer conventional single hull configurations (merchant or naval types) and thus to exclude multi-hull and SWATH-type arrangements. (However as discussed later in this section this poses no real limitations of approach or considerations). 'Ships' could also include vessels that have been designed or converted to offshore operations, e.g. drill ships, diving support ships, etc.

'Mobile offshore drilling units' covers mainly semi-subersibles of various configurations and, possibly, steel jack-ups.

'Fixed site offshore platforms' covers, typically, steel jacket structures and tethered-leg configurations and could include the innovative articulated column forms.

The majority of ships and mobile offshore drilling units are fabricated from various grades of steel. However some designs have employed aluminium alloys in some regions, e.g. ship superstructures, topsides of offshore structures, etc. Some small vessels, particularly high performance types, have also employed aluminium alloys for the main hull structure. Considerations of ultimate strength, redundancy, reserve and residual strength are not particularly affected by whether the structure is fabricated from steel or aluminium except for, possibly, the effects of the generally higher ductility of aluminium alloys compared with steels. Both structures have potential problems vis a vis corrosion, fatigue and brittle fracture.

Another material employed for small vessels is fibre reinforced plastic, FRP. Noting both the complex anisotropic multilayered mechanical characteristics of FRP and their continuous integral forms of construction, assessments of redundancy, reserve and residual strength are quite complex issues and are not considered further within this report.

Historically some marine structures have been fabricated from reinforced concrete. This includes ships, mobile offshore units and fixed site so-called gravity platforms. Structures fabricated from concrete tend to have a robust low degree of indeterminancy when compared with welded steel structural forms. Additionally concrete components do not have the ductile nature under load that steel, aluminium and FRP structures exhibit. A concrete gravity platform typically consists of a large cellular caisson supporting 3 or 4 towers with a deck on the top. The upper part of the superstructure is generally of steel grillage construction, the main reason being to keep the self weight low during tow out and thus reduce offshore installation time. The basic concept of gravity platforms is to obtain stability in the permanent condition by its own weight without special anchoring. The main dimensions are governed by the requirements of possible oil storage volume, stability during tow out, foundation area and structural strength.

The ultimate strength of concrete structures may be referred to failure modes due to material weakness. Such failure modes are pertinent to flexural members, zones with abrupt changes in geometry and concentrated loads, etc.

In Ref.[3.1] comprehensive experimental results have been presented on the carrying capacity of plane corner specimens modelling the intersection between a cylinder and a dome which is a typiçal component of some gravity platforms. The test results were compared with analytical formulation and codified format and discrepancies were noted in shear strength predictions. Significant uncertainties are also associated with the "beam" strength of heavy tubular members, as for instance, found in the shafts of gravity structures. The fatigue strength of under-reinforced concrete structure depends primarily upon the fatigue properties of steel reinforcement which are strongly influenced by possible bending and welding of the reinforcement. The scatter in the fatigue strength is of the same order of magnitude as for other steel components. The compression failure in over-reinforced members and shear and bond failures are difficult to predict in particular because concrete failure involves multiple cracking rather than a single dominant crack as in steel.

The ultimate limit state often gives the dimensions of gravity structures. In the design of the concrete caisson, the serviceability limit state (SLS) may also play an important role due to possible oil leakage that may result from cracking and the difficulty in repairing offshore structure.

3.1 Ship Structures

In terms of structural topology merchant and naval ship designs are very thin-walled hollow non-prismatic box beams, having in cross-section usually more than one cell. Any superstructures, depending on their shape and design, may or may not contribute to the overall hull girder strength in bending, shear and torsion. Thin-walled internal full or partial transverse and longitudinal bulkheads, decks both continuous and partial, etc., sub-divide the ship into compartments, tanks, holds, machinery and equipment spaces. As well as satisfying operational and functional requirements such internal structures contribute to the strength and stability of the overall hull structure. The efficient performance of the structure is developed by providing stiffening/framing members either longitudinally or transversely orientated. Such stiffener/frame members may be either of relatively light or of heavy proportions, depending on local and overall functional requirements, e.g. girders, deepweb transverse frames, wash bulkheads, etc. Thus a typical ship's structure is a complex assembly of various shapes and types of stiffened panels, both flat and curved, and some deep web girder and frame-like members. Considerable bracketing is usually provided to ensure local stability and cross bracing between elements.

Structural redundancy is difficult to define in the context of typical ship structures, albeit implicit in such obviously multi-load path topologies.

3.2 Mobile Offshore Drilling Units

With the exception of jack-ups and converted ships, most mobile offshore drilling units are of the semi-submersible type, albeit having a wide range of possible configurations. In general semi-submersibles, in terms of structural topology, can be considered as multi-member shell-type relatively large enclosed volume forms, typically columns/towers and pontoons. Steel shell structures which enclose appreciable watertight volumes are designed to provide buoyancy and strength in addition to other functional and operational needs; for example work, equipment and storage spaces. Such shell structures are normally provided with suitable internal stiffening of both 'longitudinal' and 'transverse' type for both strength and stability purposes.

Semi-submersibles also frequently contain discrete structural elements of the external cross-bracing form - such elements being, typically, simple circular section struts interconnecting between the main shell-like members and the main deck structure. The decks of offshore structures may be either a deep web beam grillage, with a stiffened plated deck, or may be of a multi-deck multi-celled stiffened plated form.

An obvious degree of redundancy is immediately seen in many designs of semi-submersibles, e.g. at the 'primary' level, by the numbers of towers and pontoons and at a 'secondary' level the form and disposition of the major bracing arrangements.

In some, if not all, designs of semi-submersibles the tubular space frame so formed is an integral part of the overall performance of the structure and really cannot be regarded as secondary in the redundancy calculations - they are however of somewhat less importance than the overall pontoons and column members. Such bracing elements are also less likely to be exposed to damage, e.g. due to work boat collisions, than the main columns and pontoons. However these bracing elements also have, owing to their generally simple unstiffened form of construction, less 'internal' local redundancy than the main elements of the columns and pontoons.

In the design of jack-ups the 'legs' are usually skeletal multi-element frameworks fabricated from tubular elements and the 'decks' are of a variety of forms similar to those found in semi-submersibles.

In jack-ups two levels of redundancy can be visualised, one at a primary level and the other at a secondary level, namely

- the number of legs (clearly also reflecting overall rig and foundation stability, etc.), and

- the number of elements in each leg.

These are in addition to any redundancy within the deck structure.

3.3 Fixed Offshore Platforms

3.3.1 Steel Jacket Structures

Steel jacket structures consist of a basic framework and a topsides structural assemblage. The basic framework is generally a large complex three-dimensional skeletal-like construction formed out of relatively slender thin-walled unstiffened tubular steel elements meeting at, sometimes, complex joints. The topsides structure may be either of a complex stiffened flat panel form containing decks and bulkheads that create a complex multi-celled arrangement or may be an assemblage of deep girder-type grillage structures.

The redundancy within the framework is clearly recognisable by the large numbers of vertical, horizontal and diagonal elements and the complex sea-bed piling arrangements.

3.3.2 Tethered Leg Platforms

With the exception of the tethers and structural provisions for them, these fixed-site platforms have strong topological similarities with some forms of semi-submersibles.

Ascertaining the degree of redundancy (except for the tethers) in TLP's is possibly a more complex problem than for general semi-submersibles. Possibly owing to the way in which TLP's respond to wave actions and the forces provided by the tethers, the TLP designs produced to-date do not have or appear to need to have, the tubular framework system of bracing elements found in many designs of semi-submersibles. It is on this basis that the redundancy issues, or rather the obviousness of degrees of redundancy, tend to be more complex for TLP's compared with some semi-submersibles.

3.4 Discrete and Continuous Structures

It is clear from the above brief review of structures relevant to this study that there are two general groups of geometric topologies, i.e.

(i) discrete structures, (frameworks) and/or associated mathematical models, and

(ii) continuous structures, (stiffened three-dimensional shells)

and/or associated mathematical models.

It is relevant to introduce the difference between an actual structure and the mathematical model employed for overall response analysis purposes. For example most semi-submersibles are in reality continuous structures, however the mathematical model employed for overall response analysis may be a three-dimensional framework and which would normally be considered to be a discrete structure.

A most important feature of a discrete structure, possibly the most important in the context of this study, is that the degree of redundancy can be fairly readily identified in terms of indeterminancy. This is generally not so for continuous structures.

□ References

3.1 Lenschow, R. and Hofsøy, A. "Carrying Capacity of the Intersection between Dome and Cylinder Wall of a Reinforced Concrete Structure", Proceedings Int. Conference on the Behaviour of Offshore Structures, Trondheim, August 1976.

4.0 SERVICE FACTORS AND CAUSES OF DAMAGE

The forces imposed on ships and offshore structures are those due to:

(i) the environment (e.g. waves, wind, green seas, sea and air temperatures, ice, etc.)

(ii) operations (e.g. cargo masses, loading/unloading, wheeled vehicles, equipment operation, etc.), and

(iii) accidents (e.g. grounding, collisions, berthing, dropped objects, etc.).

It is not the purpose of this study to review environmental and operational forces and conditions, however clearly related to such will be some likelihood of structural degradation, e.g.

- corrosion (internal and external)
- fatigue cracks
- permanent deformation (due to overload or inadequate scantlings, for example due to wave impact and operational factors, e.g. grab and wheel damage, etc.).

Occasionally vessels suffer more serious structural damage, for example:

- major hull girder failure due to extreme wave conditions [4.1, 4.2]
- major hull girder failure due to temperature induced stresses and brittle fracture [4.3].

However in the context of serious damage to ships, in statistical terms collisions and groundings are the most likely causes and such data is amply available within the open literature. The damage statistics held by the classification societies and regulatory authorities provide the major source of such information. Also a considerable amount of data has also been reviewed and presented by various ISSC committees over the years.

Although some forms of damage and structural degradation are random events, (e.g. collisions and groundings), in both time and location within the structure, some forms such as corrosion (e.g. in internal tanks) and fatigue cracking show some approximate trends. Similarly green seas and slamming damage/wave impact will predominate at the fore end. Types of damage/ degration may also be combined, e.g. scantlings diminution due to corrosion and plate/stiffener buckling deformation due to overload or wheel damage. Similar statistics are also available for various families of offshore structures.

Clearly some forms of damage will require fairly immediate repair when it occurs or it is detected and other less severe consequential forms may be left until some convenient maintenance period.

If a design process is to produce a 'damage tolerant' structure, allowing for the aleviating effects provided by redundancy, then it will be necessary to define 'survivable damage' models. This may also relate to degree of local repairability.

Survivable damage models may be either:

- embodied within the design requirements (e.g. as may sometimes be found in the design of LNG/LPG ships), or

- the results of some statistical analysis from the service records of similar ships or offshore structures (e.g. attendant boat collisions with offshore platforms).

'Survivable damage' models for design purposes would need to relate to the shape and size of the most likely damage and the probable regions of occurrence within the overall structure, e.g. similar to defining extent of ship damage for associated flooding and damaged stability requirements.

Various simple damage models could be postulated and then in a form rising through stages to quite comprehensive damage, e.g.

- loss of one or more panel stiffeners due to, say, joint cracking,
- loss of a stabilising/anti-tripping bracket,
- loss of a floor or girder web,
- loss of a pillar or strut,
- loss of a light web frame,
- loss of a complete panel,
- size and extent of a massive indentation on a main deck or side shell member, etc.

In a 'failsafe' design context the local structure could or should be designed to have a residual strength capability at least equal to the maximum forces that may occur before repairs are undertaken. This would place demands on any redundancy within the structure and the associated redistribution through alternative load paths, etc.

In the context of normal ultimate strength design the maximum design loads are computed to be those which could occur, at the relevant level of statistical certainty, during the required life of a structure. Statistically the maximum loads could occur at any time during the life of the structure, e.g. in the first year or in the last year of required service. However as the likely severity of loads, due to the natural environment (e.g. waves and winds) increases with duration, the probability that the maximum lifetime loads will occur during a given interval of time within the overall lifetime diminishes slightly. If the time span is that between the initial onset of damage and the actual physical repair then the probability of the maximum lifetime loads occurring is less than 100% and the shorter the time span then the more likely the maximum forces that occur during that time period are to be a lower percentage of the maximum lifetime forces. This is recognised in the failsafe design philosophy for aircraft.

Thus when assessing the implications of the residual strength of damaged structure some analysis should be made of the maximum loads that could be imposed on the structure before repairs are made. However if the design procedures employ failsafe principles then an important problem is in determining the onset of the actual damage. This will relate to:

- degree of inspectability,
- confidence in the inspection process,
- inspection intervals, and
- repairability, etc.

An as-manufactured structure will also contain various flaws and imperfections, some of which may be acceptable within the inspection standards followed and others not detected by inspection. However such flaws and imperfections are likely to be relatively minor and affecting local strength and fatigue capability.

□ References

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4.4 Weber, P.F. "Structural Surveys of Oil Tankers", The Inst. of Marine Engineers, Transactions, 1984.

4.5 "Collisions of Attendant Vessels with Offshore Installations. Part 1: General Description and Principal Results". Offshore Technology Report OTH84.208, UK Department of Energy, 1984.

5.0 STATICALLY DETERMINATE AND STATICALLY INDETERMINATE (REDUNDANT) STRUCTURES

5.1 Statically Determinate Structures

A statically determinate structure is one in which the forces imposed on each component, as a result of a system of external forces or self-weight, can be analytically determined by a simple static balance of forces, i.e. equilibrium is maintained. From this it follows that a determinate structure is one which contains the minimum number of elements and external supports necessary to maintain stability. Thus it is clear that if one element fails due to overload or accidentally caused damage then the entire structure collapses (i.e. there is no redundant structure or alternative path to take up the load with the possible exception of the effects of large overall deflections and load line changes). This applies whether the structure fails in small deflection, large deflection, elastic, buckling or plastic modes.

Thus there is a direct linear relationship between member forces and overall system forces, assuming that the total 'pattern' of externally applied forces is unchanged.

A structure which contains equal strength elements in parallel, e.g. for fatigue failsafe reasons, but otherwise has a statically determinate configuration can be considered to be a special case, although not strictly a form of redundancy.

Typical failsafe designs employ the use of parallel interconnected elements, for example the simplest being two equal area elements in parallel forming a simple tie-bar. When one element fails, for example in the form of a crack completely through the area, then the other element carries the full load through that path. However when one element has failed the stiffness of that load path must change and thus some overall redistribution of forces within the structural system must take place. In aircraft structures this may not be important in that the two elements are likely to be continuously connected along their length by rivets or bolts and hence such failure is more in the form of a local stress concentration. The heavy reliance, albeit not solely, of multiple parallel elements in aerospace failsafe designs, and noting the methods of construction in that industry, would be impracticable in the marine field. However this, of course, does not mean that damage tolerance capability by the provision of multiple parallel load paths should not be considered.

5.2 General Response of Redundant Structures

When any stable structure is subjected to a fixed pattern of external forces and when such forces gradually increase in a monotonic manner two levels of response are eventually reached:

1 The limit of proportionality, followed by

2 The limit of elastic response.

The latter level allows for both material properties and for behaviour such as wholly elastic buckling. A more general, and possibly more useful way, of regarding the second level is to consider that each structure, when subjected to a specific pattern of forces will have a:

3 Limit of recoverable performance.

The limit of recoverable performance is passed when either:

(i) a brittle failure occurs, or

(ii) a component experiences strain beyond its elastic limit.

In many structural materials, at normal ambient temperatures, the elastic limit is somewhat greater than the true linear proportional limit - however the latter is often difficult to accurately measure. Similarly the elastic limit is somewhat less than the yield stress (or proof stress). However clearly the important factor being that there will be some strain limit above which some degree of permanent set remains when the external loads have been removed.

Most structures contain inbuilt stresses which have been developed during the fabrication process, e.g. due to welding and handling activities. These are in addition to the normal self weight/still water condition stresses. Thus the onset of inelastic conditions is to be associated with the most adverse combinations of externally imposed forces, still water forces and inbuilt/residual stresses.

Clearly any redundancy within the structure plays an important role once the limit of recoverable performance has been exceeded.

The mode of failure of a component and its post-failed performance has an affect on response of a structure. For example a brittle failure could:

- Cause the overall failure of a structure which has no redundancy, i.e. no alternative load transmission paths being available between the applied load and the reaction point (e.g. the foundation).

- Cause a redistribution of internal forces in a redundant structure, i.e. a multi-load path structure.

One particular consequence of a brittle failure (and some forms of ductile failure) is that the strain energy that was contained within the element prior to failure will be released and rapidly redistributed into the intact structure and other force/strain energy redistributions will most likely take place as a new condition of overall equilibrium is reached. This transient dynamic condition may result in transient component load magnification factors which will be a function of both the rate at which the brittle failure occurs and the amount of strain energy in the component prior to failure relative to the strain energy in the rest of the structure.

Fortunately in many design cases for steel structures brittle failure tends to be the least likely and ductile failure modes generally predominate. However, as will be discussed later, in some structural forms ductile collapse can occur in a similar precipitative manner.

Once the limit of recoverable performance has been passed it is still possible for the applied forces to be increased in magnitude until either a second failure site occurs or another region experiences stresses beyond the elastic limit. If no component failure has taken place and, at this stage, it is the elastic limit that has been exceeded at more than one site/component within the overall structure, then the performance may be assessed in terms of either local permanent deformation or on overall/global deformation. Depending upon the location of the local deformation and the degree of redundancy within the structure the magnitude of the deformation may not be the controlling factor.

The ability of a structure to withstand increases in applied forces above the limit of recoverable performance does not necessarily relate solely to 'overall' redundancy but could be due to, or imply, a form of 'secondary' or even 'tertiary' redundancy.

The categorisation of redundancy into 'overall', 'secondary' and 'tertiary' levels is, obviously, an artificial one - and is a means to relate to the topological levels of definition of structure fromthe global level down to the basic details and the associated implications. 'Primary', 'secondary' and 'tertiary' are terms which are often employed to relate to the summation of stresses within the hull girder from various component response sources, e.g. as discussed in [5.4]. It is possible that the Redundancy Index could be expressed down to the equivalent of 'secondary' and 'tertiary' levels - however computing such for other than skeletal frameworks could result in a large range of numbers and which would probably 'cloud' rather than clarify the issue.

In continuous stiffened plate based structures, e.g. multi-columned semi-submersibles:

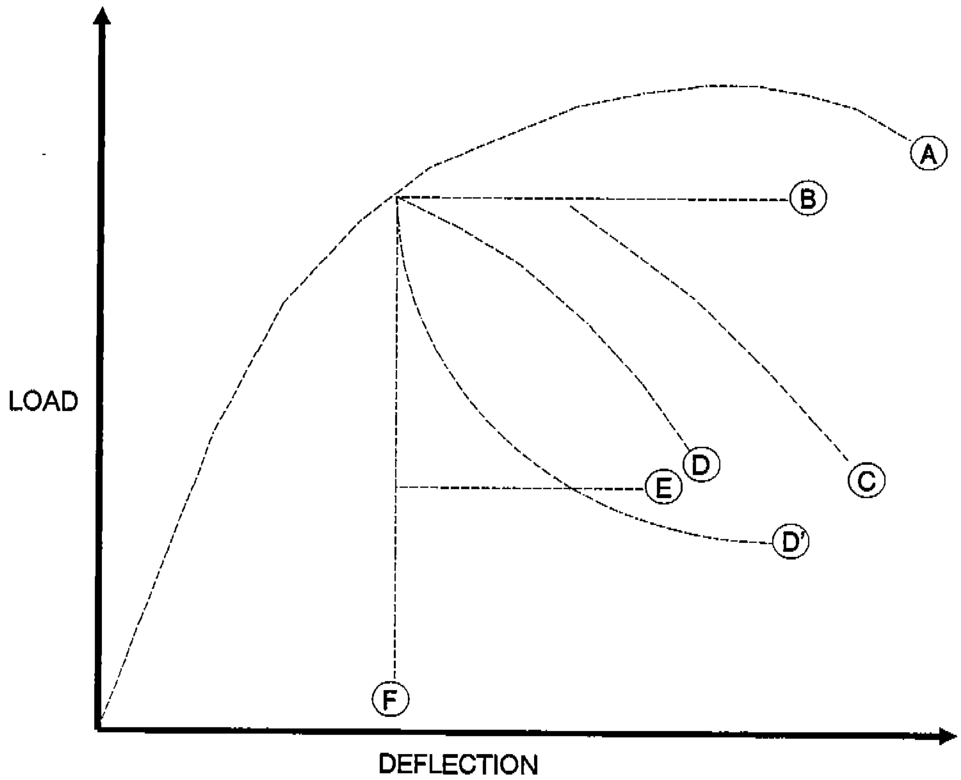
- 'tertiary' redundancy could relate to individual stiffeners on a panel,

- 'secondary' redundancy could relate to an individual panel on a column, and

- 'primary' redundancy could relate to an individual column.

Different structural topologies may be capable of exhibiting only one level of redundant performance, e.g. a structure constructed from unstiffened tubular members can only respond at the primary redundancy level and ship mono-hulls have only tertiary and secondary redundancy.

After the limit of recoverable performance has been passed and assuming that the structure remains intact (i.e. no brittle failure occurs) the response of the structure will entail an entity, or possible group of entities responding in a non-linear inelastic manner with the internal force distribution changing and redistributing within the structure depending upon the complexity of the structure and the alternative load paths available, commensurate with increases in the external force system applied. The degree by which the re-distribution of internal forces changes with further increases in the external load pattern depends upon the response of the entity when it is subjected to further increases in its boundary strains. The general nature of the various forms of response that an individual component can exhibit after exceeding its limit of capacity (ultimate strength) is illustrated in Figure 5.1.



KEY

A - Ductile - Some further capability to withstand higher loads
B - Ductile - Constant force response
C - Ductile - Constant force for limited strain and then unloading with further strain
D - Ductile - Immediate unloading with further strain
D' - Semi-Ductile (Typical)
E - Semi-Brittle
F - Brittle

Fig.5.1 Idealized Post Ultimate Strength Load Deflection Characteristics

Note that each element will have its own proportionality and recoverable (elastic) response. The character of the response after the ultimate capacity has been exceeded can affect the way in which the load is shed (energy released) into the remaining structure and the post-ultimate response curve (D) in Figure 5.1 will be similar effect to that of a sudden brittle tensile failure.

Elements can also exhibit brittle failure and clearly once such has occurred then that element plays no further part in the performance of a structural system. Some elements can fail in a form analogus to a semi-brittle mode, i.e. an instantaneous unloading to a lower strength plateau. Such a response could be due to a rapid buckling followed by the formation of a load carrying plastic hinge system. Some structures under the action of a progressively increasing load system can undergo a 'snap through' form of behaviour in which the response mode and deformation pattern changes.

The similarity of curve D to the brittle fracture curve depends upon the steepness of the unloading.

A structure which is 'redundant' in the conventional formal sense does not automatically contain reserves of strength when compared with the maximum values of applied forces. All structures whether redundant or statically determinate can be fully stressed at the maximum load conditions, i.e. where all elements are stressed to just below their ultimate capacity. In the 100% fully stressed condition any action which triggers entity failure will automatically cause overall failure. It is also possible to postulate a so-called redundant structure in which the primary members are stressed to their capacity but the lesser secondary members are stressed to well below their ultimate strength capacity. If the primary members are much larger than the secondary members, the structure may have no capacity to sustain the applied forces once a primary member has failed.

It is in this context that 'redundant' tends to be somewhat of a misnomer as the word 'redundant' tends to imply 'unnecessary' and clearly this is frequently not so. Reference [5.1] by Argyris and Kelsey provides the following definitions:

> "A structure is by common definition redundant if there are not sufficient conditions of equilibrium to obtain all internal forces (stresses or stress resultants) and reactions; the number of redundancies is the difference between the number of unknown forces (or stresses) and the number of independent equilibrium conditions. Strictly all actual structures are infinitely redundant but for practical purposes it is in general, necessary and justified to simplify and idealise the structure and/or stress distribution in order to obtain a system with a finite number (or even zero) of redundancies."

Since Argyris and Kelsey wrote the above in the mid-1950's the advent and subsequent widespread usage of finite element method based codes, for both linear and non-linear response, has clearly resulted in there being much less need to "simplify and idealise" the structure or to worry about whether a structure is statically determinate or statically indeterminate. Thus the term 'redundant' has itself in this context become somewhat less meaningful.

Thus with the possible exception of some simple skeletal steel frameworks a new definition of 'redundant' structural forms is desirable. Consider the ideal situation of a structure when subjected to the maximum fully factored design forces, with no margin on the capability, being in a fully-stressed condition, i.e. where every structural entity or component is at an applied stress level which equates to local capability. Thus in this condition the structure contains no un-utilised material, i.e. no 'redundant' material. However, for many reasons, structures contain material which is not fully utilised and it is only when previously fully-stressed material becomes overstressed that this material could be utilised more fully, subject of course to the load shedding being not too severe following component failure.

It may be possible to quantify the significance of 'redundant' or under utilised material on a strain energy basis. For a given mode of failure, and post-failure behaviour, a structural element will have a total strain energy capability. The total structural system will also have various energy states related to various degrees of progressive collapse. The value of a strain energy approach is that the volume of material of a component is involved and thus if a large element fails the energy release to be absorbed by the rest of the structure is also likely to be large.

Redundancy within a structure or component can be measured in several ways, e.g.

1 The ability to remain stable as a whole when one, or more, elements become unstable or become ineffective following damage.

2 The ability to progressively reform self-equilibriating internal force systems when one, or more, elements cease to function in a linear elastic manner.

If a structure can continue to sustain progressively higher external loading (of the same pattern) after one element begins to function in an inelastic non-linear manner then that element could be classed as having full functional primary redundancy.

A possible measure of redundancy/inelastic response is how low the secant modulus (of the system) can become compared with the elastic modulus (of the system) when the component reaches its overall ultimate strength, Figure 5.1.

Klingmuller Ref.[5.2], in the context of 'systems' reliability identified two types of redundancy

(i) active (hot) redundancy, and

(ii) stand-by redundancy.

'Active' redundancy refers to components which are additional, but not necessary components and that are fully active in normal response and can and would be used to maintain stability after failure of a parallel component. In 'stand-by' redundancy additional components, that are not used in normal response performance, have to replace components that have failed.

The redundancy of most statically indeterminate structures has to be put into the category of active (hot) redundancy, as all components are used in the daily performance to withstand imposed and self weight loads. "Stand by' redundancy tends to infer an element having some end rotational or axial slackness.

According to Klingmüller redundancy only results in additional safety if a redistribution is possible after some form of local failure has occurred. This is only possible in the context of plastic collapse in ductile material. Following the brittle failure of a component redundancy may not be of any benefit. There would appear to be diminishing returns re. the degree of redundancy and attendant increases in safety.

Klingmüller's 'active' and 'standby' redundancy classifications are possibly mainly useful in idealised mathematical systems reliability studies and as the analysis capability increases then the interpretations of standby redundancy begin to disappear.

Lloyd and Clawson [5.3] suggest a member redundancy hierarchy for indeterminate structures, as follows:

Member Redundancy Level	Member Classification
0	A member whose failure leads to progressive collapse for dead weight load conditions (a statically determinate structure would fall into this category).
1	A member whose failure leads to progressive collapse for dead plus some fraction of live weight load conditions.
2	A member whose failure leads to progressive collapse for a limited set of load conditions that include dead and live loads in combination with some fraction of the design environmental load.
3	A member whose failure leads to progressive collapse for a limited set of load conditions that include dead and live loads in combination with some multiple of the design environmental load.
4	A member whose failure has little effect on the design strength, but whose presence enhances the redundancy of nearby members, i.e., a normally lightly loaded member that provides an alternative load path when a nearby member fails.
5	A member whose failure has no bearing on the design, reserve or residual strength, i.e. a nonstructural member.

In comparing Klingmüller's terms with the classification by Lloyd and Clawson, possibly the closest is that 'standby redundancy' and 'member level 4' may have some similarity. This may be particularly so when considering the implications of accidentally caused damage. In the accurate response modelling of structures of the offshore steel jacket form the horizontals, in an X-braced tower, are active elements and not standby, i.e. they provide end bending moment capability and take part in the overall shear response, etc.

In the classification by Lloyd and Clawson the demand (load) and response (capability) are intermixed and whilst redundancy per se could be related to the form of the loading rather than the magnitude, the separation into dead, live and environment components, and permulations thereof, makes the degree of redundancy to classification relationship difficult to define.

The above classification translates directly and readily into discrete structures, e.g. steel jacket forms. In the context of continuous structures, for example ship hull girders, the interpretation is somewhat more difficult:-

- level 0 is very hard to envisage. The deadweight condition would equate to the lightship and the only form of such catastropic failure would be of the form of massive brittle fracture propagating in an unrestrained manner, similar to some of the early Liberty ships, exacerbated by thermal gradient induced stresses and weld flaws.

- level 1 is also hard to envisage, relating again to the still water condition. Any such failure could only come about following some form of massive damage, e.g. due to collision or grounding. This level of damage would not be designed for.

- level 2. Again for a ship this would require some fairly substantial damage followed by some low cycle high stress reversal events in order to propagate the damage to the size where the hull girder collapses. This has occurred, or has been postulated to have occurred, in some ship casualty/loss studies.

- level 3 begins to be meaningful in the context of a whole range of ship primary components, e.g. deep girders, parts of web frames, etc. and even major longitudinal stiffeners.

- level 4 relates mainly to the bracket level, tripping brackets, web supports, etc. which are very much supportive members enabling others to perform much more efficiently.

In the design of a conventional ship level 1 would imply, for example, complete failure of the bottom shell region, including double bottom if fitted. This infers the failure of several panels and noting the difficulty in quantifying redundancy for hull girders this infers lowering the degree of redundancy appreciably. That is to say that the failure of one single member could not lead to progressive collapse in the load level limitations given in level 1. In the classification of Lloyd and Clawson a single multi-stiffened panel could be considered to be at either level 2 or level 3 and a single double bottom girder could be taken as a level 3 member. Following the same example an individual stiffener on a multi-stiffened panel will generally be at about level 3 or where there are many stiffeners on a panel at level 4. A heavy hatch side girder could be visualised as a level 3 member. Clearly these classifications are somewhat subjective and may be dependent upon the position of the element (stiffener, panel or girder) within the overall cross section through the hull girder.

□ References

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6.0 SERIES AND PARALLEL SYSTEMS

In order to understand the role of redundancy in structural performance it is important to have some concept of series and parallel systems. In a series system, the failure of the system occurs if any of its members is subjected to forces which exceed its load carrying capacity, i.e. it fails. Such a system is also called a weakest-link system. A statically determinate system such as the simple pin-jointed truss shown in Figure 6.1 falls under this category.

In a series system, whether the member's failure mode is ductile or brittle does not make any difference. As soon as one member fails, there is no mechanism to redistribute the load and the entire system collapses.

Let R be the strength of the series system and let R_i be a random variable describing the strength of failure element 'i', ..., n. Further, let a load r on the series system result in a load effect r_i in failure element i, i=1 ..., n. If F_{R_i} is the distribution function for the random variable R_i, i=1, ..., n, then, the distribution function is given by [6.1].

$$F_R(r) = P(R \leq r) = 1-P(R > r) = 1-P(R_1 > r_1 \cap R_2 > r_2 \cap \ldots \cap R_n > r_n)$$

$$= 1-(1-F_{R_1}(r_1))(1-F_{R_2}(r_2)) \ldots (1-F_{R_n}(r_n))$$

$$= 1 - \prod_{i=1}^{n} (1-F_{R_i}(r_i)) \qquad (6.1)$$

in which F_{R_1}, F_{R_2} ... F_{R_n} are the distribution functions for the R_i of element 'i' and it is assumed that the strengths of the elements are independent of each other.

When the distribution function F_R for the strength R of the series system is determined, the probability of failure P_f can be calculated as for a single element by:

$$P_f = \int_{-\infty}^{+\infty} F_R(r)\ f_s(r)dr = 1 - \int_{-\infty}^{+\infty} \prod_{i=1}^{n} (1-F_{R_i}(r_i))\ f_s(r)dr \qquad (6.2)$$

where f_s is the density function for the load s on the series system.

The reliability index β_s for a series system can then be calculated by:

$$\beta_s = -\varphi^{-1}(P_f)$$

when φ is a standard normal distribution function [6.1].

The probability of failure for a series system with n equally correlated elements is given by [6.3]:

$$P_f(\rho) = 1 - \int_{-\infty}^{+\infty} \left[\varphi\left[\frac{\beta_e + \sqrt{\rho}\ t}{\sqrt{1-\rho}}\right]\right]^n \psi(t)dt \qquad (6.3)$$

where φ and ψ are the distribution and density functions for the standard

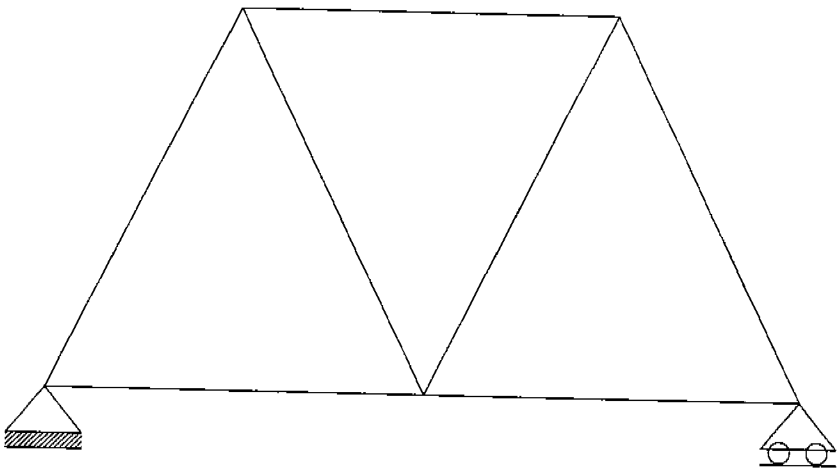

Fig.6.1 Simple Truss - Series System

Fig.6.2 Series System - Mathematical Model

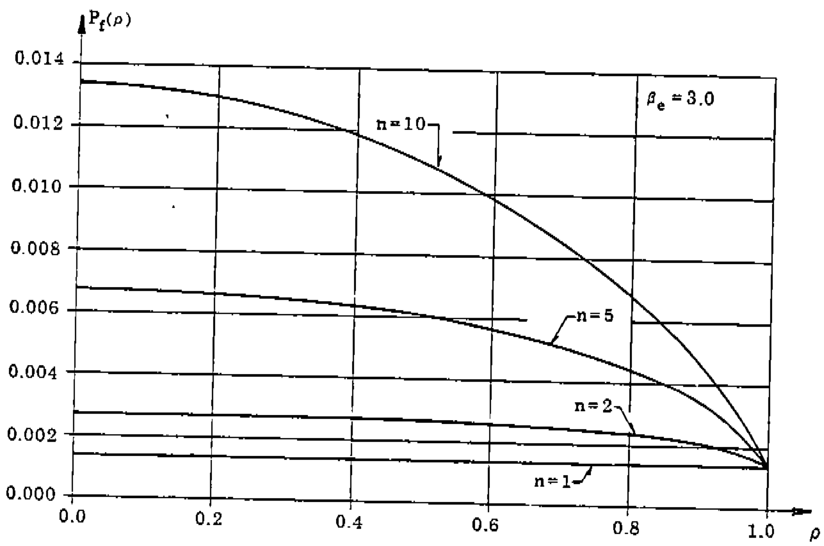

Fig.6.3 Series System - Typical Results [6.1]

Gaussian random variable and β_e is the reliability index of elements which are assumed to have the same value and ρ is the common correlation coefficient between pairs of elements.

A plot of P_f with various values of ρ assuming a fixed value of β_e=3.0 and different values of n is shown in Figure 6.3 taken from Ref.[6.1]. As seen from this figure, the probability of failure P_f decreases with increase in the correlation coefficient ρ and increases with the number of elements n.

In a parallel system, failure of a single element does not necessarily cause the failure of the entire system because of the load re-distribution which can take place among the remaining elements. A statically indeterminate structure, i.e. in which there is an element of redundancy, falls into this category. This type of structure can have a great number of failure modes where each failure mode can be modelled as a parallel system. These separate parallel systems are again combined into a series system. A parallel system as shown in Figure 6.4 will only fail when all elements in that system have failed. Clearly Figure 6.4 is only a mathematical model of a parallel system and in the real world true parallel structures are only infrequently found.

The strength R of a parallel system having n perfectly ductile elements (Chapter 7 of this report contains a discussion of the nature of response of ductile and brittle elements) is given by:

$$R = \sum_{i=1}^{n} R_i \qquad (6.4)$$

in which R_i is the strength of element 'i'. If R_1, R_2 ... R_i are independent and normally distributed, then R is also normally distributed $N(\mu,\sigma)$ and its expected value is given by:

$$E(R] = \mu = \sum_{i=1}^{n} \mu_i \qquad (6.5)$$

$$Var(R] = \sigma_R^2 = \sum_{i=1}^{n} \sigma_i^2 \qquad (6.6)$$

For a parallel system with n ductile members and in which the strength R_i, which is identical for each member and is normally distributed, the reliability index β_s for the system is given by[6.1]:

$$\beta_s = \beta_e \sqrt{\frac{n}{1+\rho(n-1)}} \qquad (6.7)$$

in which ρ is the common correlation coefficient.

A plot of probability of failure P_f [$P_f = \varphi(-\beta_s)$] with ρ for different values of n and a selected constant value of β_e is shown in Figure 6.5 [6.1]. Unlike the series system, here P_f increases with ρ and decreases with number of elements n.

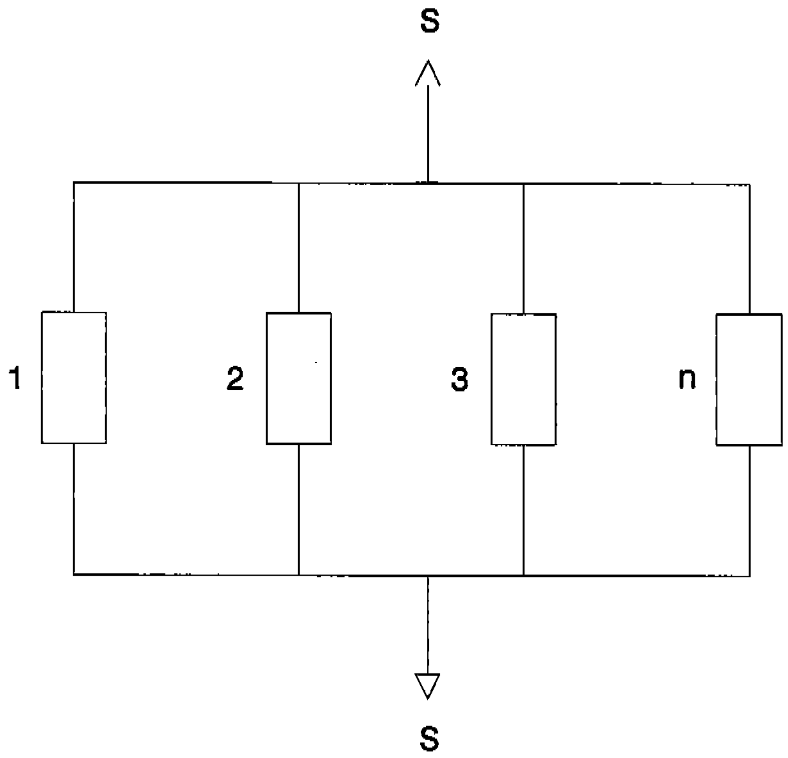

Fig.6.4 Parallel System - Mathematical Model

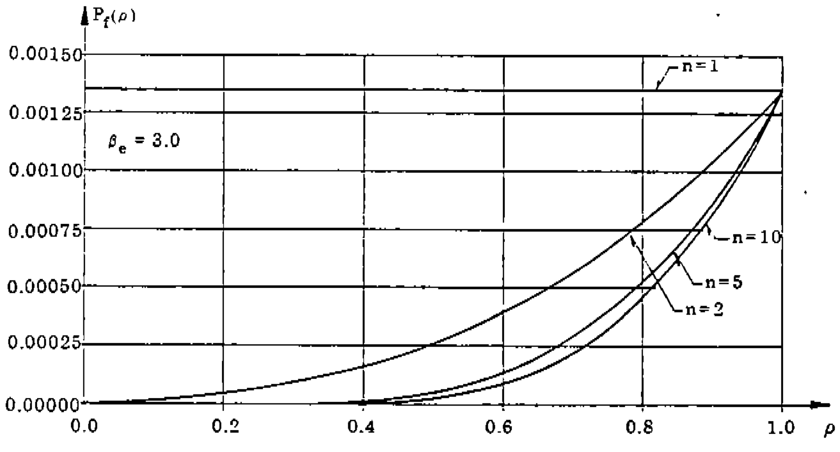

Fig.6.5 Parallel System - Typical Results [6.1]

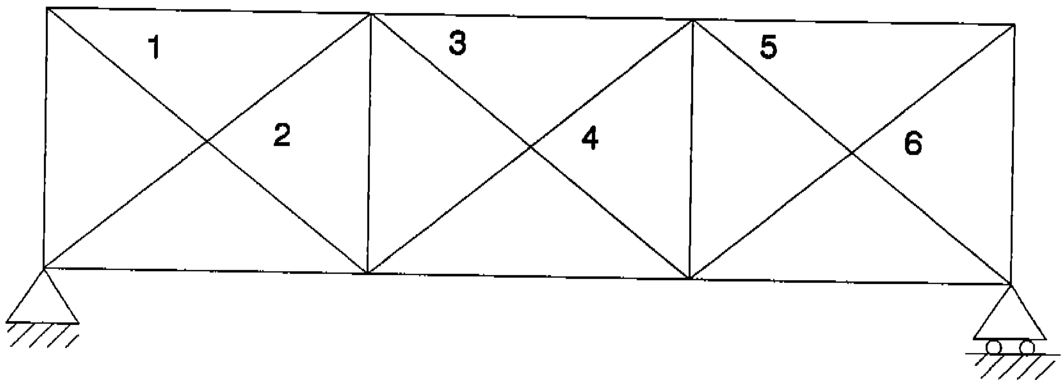

Fig.6.6 Complex Truss - Mixed Parallel and Series System

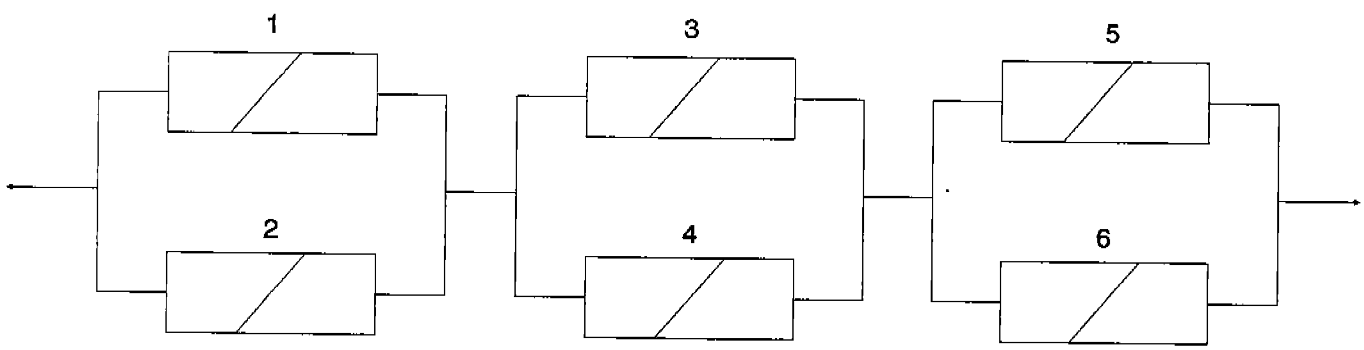

Fig.6.7 Mixed Series and Parallel System - Mathematical Model

The numerical calculation of the multi-normal distribution function is very time consuming and sometimes impossible, so P_f is evaluated using approximate or bounding techniques.

There are many bounding techniques to formulate upper and lower bounds of the exact probability of failure and the gap between these bounds determine their practical utility and are useful when the gap is narrow. There are simple bounds[6.2] and Ditlevsen bounds[6.5], which are not discussed here. They are very useful for both system reliability study and for practical application to marine structures.

Equations (6.3) and (6.7) are based on assumptions of common correlation coefficients between pairs of elements but they become more complicated for a system with unequal correlation coefficients. These are taken into account in these equations by using average values correlation coefficients. This has been investigated by Thoft-Christensen and Sørensen [6.1].

The following example problem illustrates how an actual structure can be broken up into a combination of series and parallel systems from which the probability of failure can be calculated for the whole structure.

Consider a statically indeterminate truss with 3 panels as shown in Figure 6.6. It is assumed that only the diagonals 1, 2, ... 6 can fail. This structure can then be modelled by the system shown in Figure 6.7. It is assumed here that the strength of the elements to be modelled by normally distributed random variables which are equally correlated with a common correlation coefficient $\rho = 0.6$. Further it is assumed that the loads are deterministic and constant in time and all elements are designed in such a way that they have the same reliability index $\beta_e = 2.68$.

The reliability index β_p for a single panel with two diagonals is, in accordance with equation (6.7), given by

$$\beta_p = \beta_e \sqrt{2/(1+\rho)}$$

The probability of failure P_f for the system can now be calculated from equation (6.3) with n=3, and β_e replaced by β_p, and ρ replaced by ρ_p, i.e. the correlation coefficient between the strength of the panels.

The correlation coefficient ρ_p can be determined in the following way. Let R_1 and R_2 be the strength of the diagonals identically distributed $N(\mu,\sigma)$ in a panel with the strengths $R_p = R_1 + R_2$.

$$\left.\begin{aligned} &\text{Then } E[R_p] = 2\mu \\ &Var[R_p] = \sum_{i=1}^{n} Var[R_i] + \rho \sum_{i,j=1}^{n} \left[Var[R_i]Var[R_j]\right]^{\frac{1}{2}} \\ &\qquad = n\sigma^2 + n(n-1)\ \rho\sigma^2 \\ &\qquad = 2\sigma^2 + 2\rho\sigma^2 \\ &\qquad = 2(1+\rho)\sigma^2 \end{aligned}\right\} \qquad (6.8)$$

and covariance

$$\left.\begin{aligned} Cov[R_1\ R_2] &= E[R_1\ R_2] - E[R_1]\,E[R_2] \\ \text{or} \qquad \rho\sigma^2 &= E[R_1\ R_2] - \mu^2 \end{aligned}\right\} \qquad (6.9)$$

Let R_p^1 and R_p^2 be the strengths of two panels. Then

$$E\left[R_p^1\ R_p^2\right] = 4\ E[R_1\ R_2] = 4(\rho\sigma^2 + \mu^2) \qquad (6.10)$$

Finally

$$\rho_p = \frac{E\left[R_p^1\ R_p^2\right] - E[R_p]^2}{Var[R_p]} = \frac{4(\rho\sigma^2 + \mu^2) - 4\mu^2}{2(1+\rho)\sigma^2} = \frac{2\rho}{1+\rho} \qquad (6.11)$$

Hence

$$\rho_p = \frac{2 \times 0.6}{1 + 0.6} = \frac{1.2}{1.6} = 0.75$$

The probability of failure, P_f, for the system with n=3 (i.e a series system composed of three parallel systems) is then

$$P_f = 1 - \int_{-\infty}^{+\infty} \left[\psi\left[\frac{\beta_p + \sqrt{\rho_p}\ t}{\sqrt{1-\rho_p}}\right]\right]^3 \psi(t)\ dt$$

$$= 1 - \int_{-\infty}^{+\infty} \left[\psi\left[\frac{3 + 0.866\ t}{0.5}\right]\right]^3 \psi(t)\ dt$$

$$= 33.01 \times 10^{-4} = 3.301 \times 10^{-3}$$

The probability of failure of individual members was assumed to be β = 3.0 i.e. $P_f \to 10^{-3}$ in this calculation in which a probability density function (PDF) was assumed to cover the uncertainty with a mean and spread values of design variables such as material properties, fabrication errors and load uncertainties. From the above it is seen that for the example problem the probability of failure of component and system are of the same order of magnitude.

□ <u>References</u>

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7.0 ELASTIC AND INELASTIC RESPONSE OF STRUCTURES

7.1 General

Clearly the failure of a minor component within an overall structure can have one of two general effects:

- The structure responds with a re-distribution of internal forces, the form of which depends both on the degree of redundancy of the structure and on the manner in which the component locally fails (the common forms of failure response characteristics are illustrated in Figure 7.1).

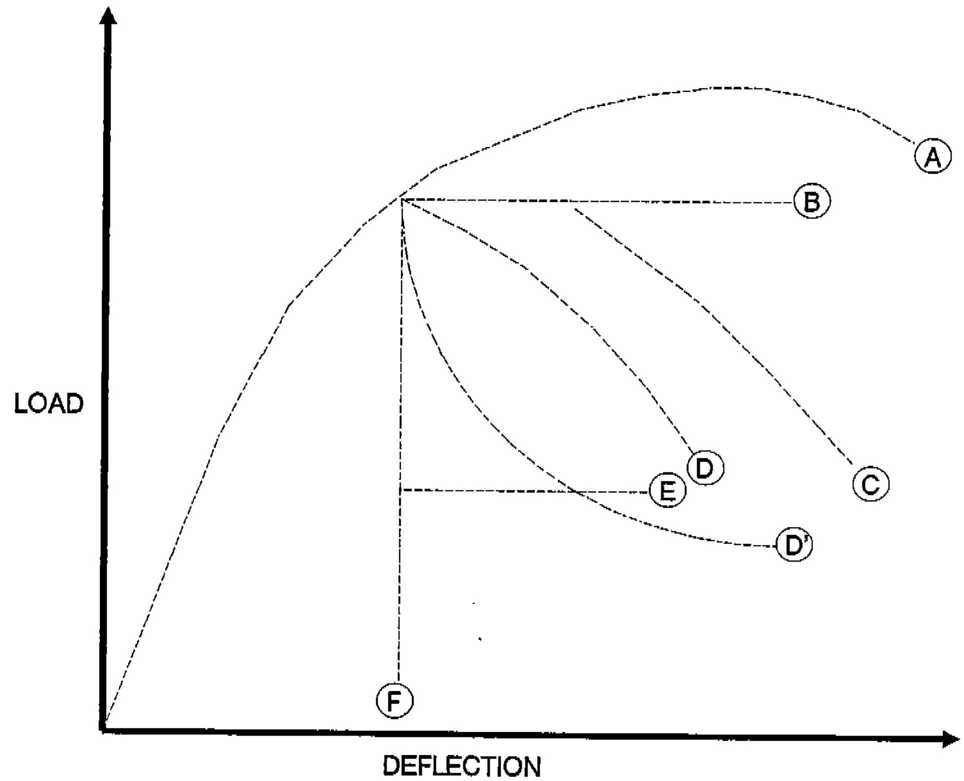
- Local failure precipitates progressive failure, at either constant overall applied force levels or at slightly higher levels. This progressive failure situation is discussed in the next part of this Chapter.

Thus local failure is only acceptable in the context of statically indeterminate structures in which multiple load paths enable such re-distributions to take place. This includes the design of 'fail-safe' structural arrangements.

When local structure has failed in any form of inelastic manner (as distinct from purely elastic behaviour) the overall structure has experienced some degree of permanent set or damage and will respond differently to any subsequent loading pattern; even for loads less than those which caused the damage. The damage remains 'locked-in' to the structure, and can affect also the stiffness and vibration characteristics of the structure. Hence local failure is important in that it may affect serviceability and operational performance. The acceptance of local failure is sometimes more amenable for situations where the applied loading is of a once-in-a-lifetime form, e.g. an over-pressurisation test case.

The most important exception to the above brief review of the effects of local damage is where due to some initial relatively high loading (e.g. due to pressure testing) pockets of localised permanent set are developed. Such would typically occur at regions of high geometric stress concentration factors, e.g. cut-outs, some joint details, etc. At this scale the localised pockets of permanent set would have little effect on the overall performance of the structure and in some cases could have a modest beneficial effect on the subsequent local fatigue life. There is also the possibility that appreciable service loads may 'shakedown' the structure and effect a release of the initial as-manufactured residual stresses, e.g. those due to welding and assembly activities. Shakedown thus also has a beneficial effect on the subsequent performance of the structure.

Local failure cannot generally be tolerated where the load level that caused the failure is regularly applied. Not only would there be a service-ability and repair problem but there would also be, most likely, a high stress low-cycle fatigue problem that caused the damage to rapidly spread. There are some exceptions to this, for example where the local failure was a purely elastic buckling. The elastic buckling of thin flat plates is tolerated in many structures.



KEY

A - Ductile - Some further capability to withstand higher loads
B - Ductile - Constant force response
C - Ductile - Constant force for limited strain and then unloading with further strain
D - Ductile - Immediate unloading with further strain
D' - Semi-Ductile (Typical)
E - Semi-Brittle
F - Brittle

Fig.7.1 Idealised Post Ultimate Strength Load Deflection Characteristics

Fatigue damage in ductile structure is possibly the most common example of local failure. A definition of what constitutes an upper limit of acceptable damage is difficult to form. A possible measure could be the transition between slow and fast crack propagation. This would involve a fracture mechanics study in which real structures, with imperfections and inclusions, are studied as well as the normal geometric stress concentration factors. Clearly the development of fatigue cracks can also affect structural performance, e.g. leaking tanks.

Medium rate loading, e.g. collisions and impact often results in local damage.

☐ Local Failure Criteria

There are two possible modes of local failure:

(i) Brittle failure, and
(ii) Ductile failure.

If the local failure is brittle then a rapid re-distribution of local internal forces is implied and there could be some transient dynamic effects which can provide the energy to propagate the extent of the local failure.

Ductile failure, for example yielding or some forms of buckling, allows for a more gradual re-distribution of local internal forces. In some cases the 'failed' component continues to respond effectively as a constant load spring (assuming no strain hardening effects) in the subsequent behaviour at increased overall applied load levels. However, some forms of buckling, while still ductile, rapidly reduce the local load carrying capability.

There is a range of formulae in regular employment in the design codes promulgated by various agencies. In each loading and response category, all are based on theoretical equations which assume a perfect structure (i.e. no geometrical or material imperfections) and either a purely elastic response or a simple plastic hinge failure mechanism.

Although the purpose of this study is to review and evaluate the nature and characteristics of redundancy and how it manifests itself within various forms of ship and offshore structures there is, vis a vis the overall analysis process, a considerable degree of uncertainty in determining the loads which are applied to the structure. There have been many studies, e.g. [7.1, 7.2, 7.3 and 7.4] aimed at quantifying in some statistical manner the uncertainties in determining applied loads, whether such are associated with still water conditions, wave induced effects, operational and cargo conditions, etc. With the current trends towards weather routing and heavy weather avoidance prudent operation can appreciably reduce the loading demands on the vessel.

In the context of offshore structures uncertainties in the calculation of fluid loading are most serious for slender structural members, that is one dominated by drag. It is necessary to improve the form of the Morison's equation to take account of the random nature of waves, in particular the directional spread of the spectrum, and to improve the choice of the force coefficients for use in these equations.

Uncertainties do exist at present (i) for treating the combined effects of the various environmental inputs to loading, (ii) in the presentation of the joint probabilities of currents and waves, (iii) in the prediction of extreme velocities and accelerations in random seas, and (iv) in estimating the environmental design parameters.

The actual response of normal structures is usually influenced by three factors:

(i) the effects of material inelasticity, and

(ii) the non-linear effects of the length of the element, affecting both local buckling modes and interactions with overall buckling modes, e.g. column failure.

(iii) the effects of geometric and material irregularities and imperfections,

There are also uncertainties in the applied loads as discussed above.

Each industry and its associated design agencies has employed the results of limited rigorous theoretical studies and experimental results to produce formulae which it uses, within bounds, and considers to provide acceptable service. However, as each industry also tends to employ fairly generous simple safety factors, to avoid the possibility of local failure, the real level of accuracy is difficult to ascertain in each approach. It is thus not possible, at this stage, to identify which is the most rational of the various approaches and formulae. The simpler forms are clearly the more useful ones for initial design studies and attendent analyses, and the more complex approaches would be employed when more of the structure's geometric parameters have been established, e.g. (L/r and r/t) ratios, etc.

For example, simple Euler or Perry-Robertson types of formulations could be employed to generate first estimates of stiffener and beam area properties. With actual properties of available or prefered fabricated or rolled sections, for example, more definitive calculations can be made allowing fully for slenderness ratios, element width to thickness proportions, statistical degrees of as-manufactured straightness and residual stresses, etc. Obviously such factors are allowed for either explicitly or implicitly within the codes that are promulgated by the design approval and regulatory agencies.

□ Progressive Failure

Progressive failure can be considered under three headings:

- Monotonically increasing loading
- Low-cycle fatigue
- High-cycle fatigue

• *Monotonically Increasing Loading*

Monotonically increasing loading is when, typically, a load pattern is gradually applied to a structure, rising progressively to the maximum magnitude. In a statically determinate structure, when any component reaches its failure load, (which may be at a load level which is less than the maximum design level), overall structural collapse or failure occurs. In a statically indeterminate structure internal load re-distribution takes place following the local failure of an individual component (if the structure is indeterminate in the region of the failure). The overall loading can then be further increased, with a different structural response, (e.g., deflection characteristics), until either the failure spreads from the original point or a new incident of local failure occurs. Depending upon the types of locations of the local failures and upon the degree of structural indeterminancy the above can be repeated until the structure has sufficient failures for complete collapse to occur. This is the most common form of progressive failure in terms of complete structural collapse.

- *Low-cycle Fatigue*

Low-cycle fatigue can often occur in structures which are frequently subjected to the maximum design stress and in which the margins of safety are rather low. A frequent characteristic of low-cycle fatigue is a crack which increases significantly in length with each applied load cycle. The rate at which the crack spreads depends upon the stress level and the material characteristics. Some structures, e.g. of riveted construction, may inhibit the spreading of the crack.

The occurrence of creep at elevated temperatures can also be considered as a form of progressive failure; although an infrequent if not rare problem in marine structures.

Clearly the design of 'fail-safe' structures (particularly 'fail-soft' ones which are designed to resist impact) requires an understanding of progressive failure.

- *High-cycle Fatigue*

High-cycle fatigue in ships and other marine structures is the most common source of local failure. If unchecked it can progress to more extensive, and expensive to repair, cracking and structural damage. In the extreme, fatigue cracking can lead to impairment of the structural function and then to catastrophic collapse, e.g. across the tension flange of a box beam, or around a major structural joint. During this process the overall structural response changes to a degree related to the level of indeterminancy in the basic design. Because there may be an appreciable time interval the structure may be exposed to corrosive environments, thus possibly exacerbating the situation.

7.2 Failure Modes

During structural design, care must be taken to ensure that all possible failure modes are considered for all design loads. Structural failures can be categorised into two general groups; those which involve fracture without significant deformation and those which involve gross deformations and which may or may not culminate in local fracture. Each of these categories can be sub-divided as illustrated in Figure 7.2.

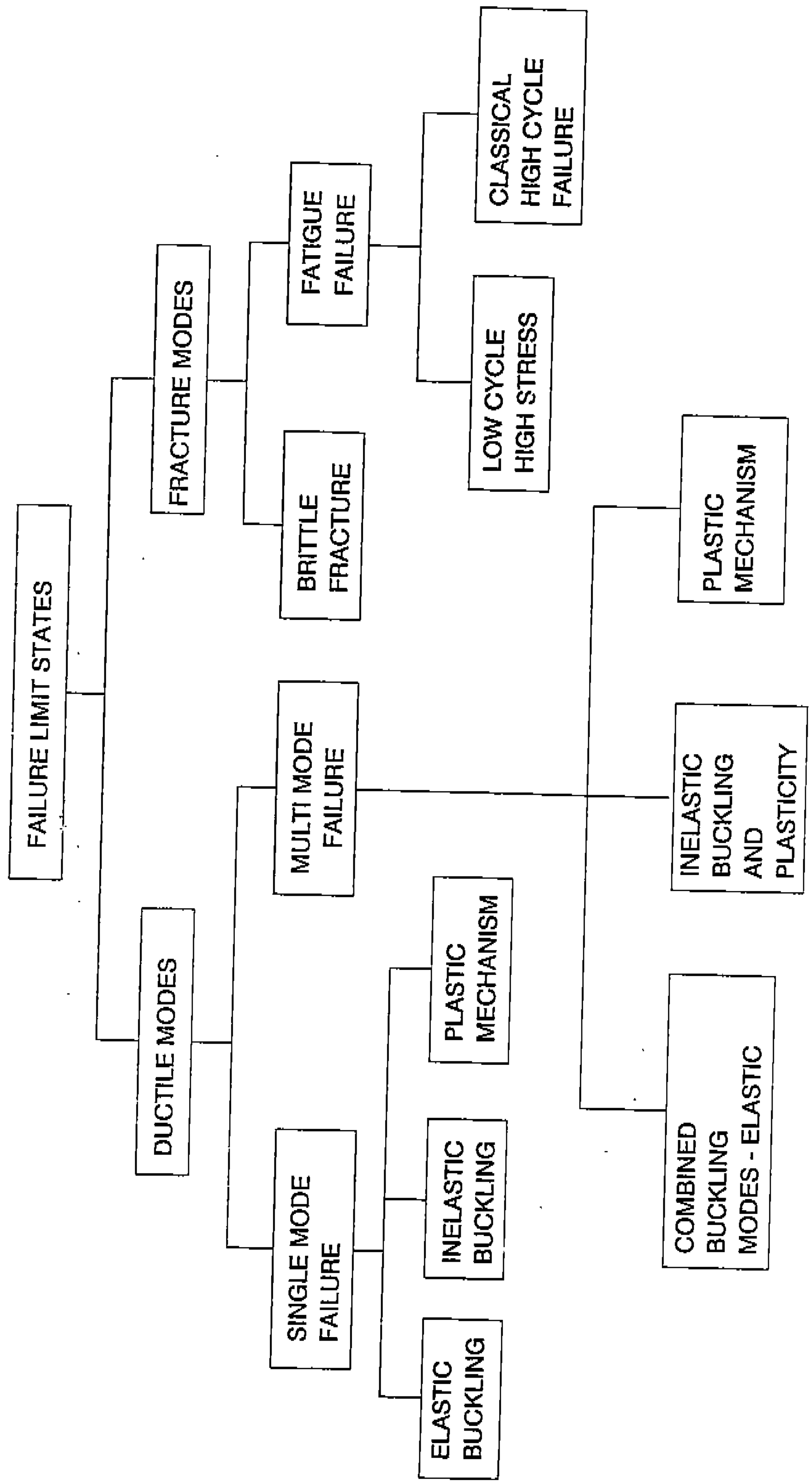


Fig.7.2 Various Structural Failure Modes

□ <u>Ductile Failure Modes</u>

Elastic Buckling. This may take many different forms depending upon the configuration of the component or structure, its restraints and the applied forces. It may be confined to the plating or the webs or flanges of certain stiffeners, it may occur by tripping or by flexure of longitudinal stiffeners between transverse stiffeners, or it may involve entire grillages or overall structural assemblies. Care must be taken during design to ensure that all possible buckling modes are considered. Most forms of buckling precipitate complete collapse of the structure in statically determinate structures and may lead to progressive collapse in statically indeterminate structures. (However there are a few modes in which strengthening mechanisms come into play after initial buckling to give some reserve of strength - notably plate buckling, where the development of membrane tension has this effect and the diagonal tension mechanism following shear buckling.) Furthermore, the initial deformations and residual stresses that result from fabrication usually lead to some loss of buckling strength and this may not always be taken into account when checking the strength of a structure.

In many structural designs buckling is acceptable as long as the buckle is purely elastic and disappears on removal of the applied loads. It is important that the range of stresses with repeated buckling and unbuckling cycles are low enough to prevent fatigue damage.

The elastic buckling of many slender structural components is often fully recoverable, for example thin plate elements in stiffened flat panels. However, for some structural components there is a fairly rapid transition from elastic buckling to permanent damage - typical examples are stiffened and unstiffened thin-walled circular cylinders.

In real structures, with their attendant manufacturing imperfections and residual stresses, purely elastic instability is rare. It is however a useful measure of performance for many structural components.

The elastic buckling of components within statically determinate structures is much more serious than similar behaviour within statically indeterminate ones. For example the elastic buckling of a single member within a statically determinate plane truss results in overall failure, whereas plate element buckling within a stiffened flat panel does not normally consitute failure.

Depending upon the complexity of the structure and the degree of load path redundancy it may be necessary to review:

- areas where local buckling is constrained and does not precipitate overall failure, and

- areas where local buckling can lead to or precipitate overall failure.

In areas where the consequences of buckling failure are significant most codes employ higher factors of safety for the particular structural elements concerned.

Most elastic buckling formulations are for simple structural elements, (typically beams, columns and plates) and for some unstiffened and stiffened shell elements. For complex structures finite element or finite difference methods are required. However inelastic analyses can be very expensive and even this technique often does not allow for changes in local forms of response following buckling, for example the diagonal tension mechanism which develops following shear buckling.

When employing buckling stress levels as design criteria (with appropriate factors of safety) the designer should have an appreciation of the characteristics of the 'post-buckled' behaviour of the particular elements of structure, see Figure 7.3. Such post-buckled behaviour is of particular significance in structures that are not statically indeterminate or fail-safe.

Figure 7.3 presents idealised end load shortening curves, for the pre- and post-buckled conditions for simple columns, flat plates and circular section cylinders. This figure illustrates the general nature of the post-buckled region and compares the predictions given by theory which assume perfect specimen with the typical performance given by real specimen which contain both geometric imperfections and residual stresses, etc.

When considering overall structural response into the fairly large deflection regime the elastic end-shortening component, whilst clearly present at all times, will be relatively small.

Of particular concern is the post-buckled response, to continued loading, of curved shell elements, for example cylinders in axial compression, Figure 7.3(c), or bilge plates with in-plane shear, where it is usual to find that there is a sharp fall-off in resistance to loading. Under such conditions unless the surrounding structure can accept loads being shed off from the collapsed structural element then total failure may be precipitated.

Considerations of post-buckled behaviour and overall structural redundancy affect the selection of factors of safety.

Most structures are so proportioned that buckling will probably occur in the inelastic regime and because of such, together with the effects of the manufacturing imperfections and residual stresses found in actual structures, some permanent deformation will remain in the element when the loading is removed.

Inelastic Buckling. In efficient structures theoretical elastic buckling stresses tend to be higher than yield stresses and therefore actual collapse usually involves inelastic elasto-plastic rather than purely elastic buckling. Since rigorous methods of analysis for elasto-plastic buckling are often only available in a few specific families of elements it is often necessary to employ approximate methods in order to ensure sufficient strength. These approximate methods sometimes include the use of reduced (tangent) moduli data (if available, Fig.7.4) or the design of structure to have hypothetical elastic buckling stresses well above the actual yield stress. Residual stresses and initial as-manufactured deformations can significantly reduce stability and they must be taken into account.

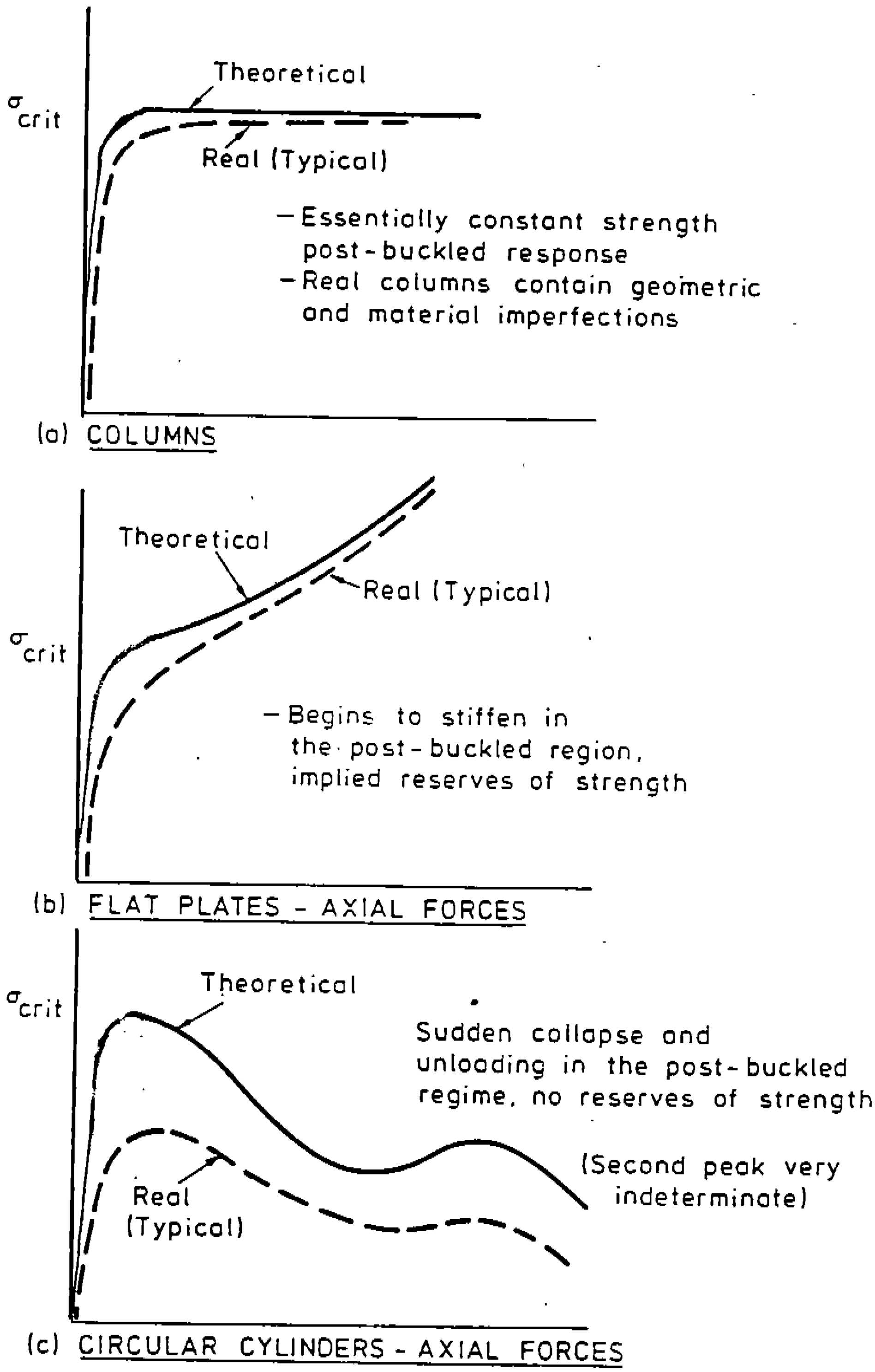


Fig.7.3 Idealised Load Shortening Curves – Pre and Post Buckled Condition

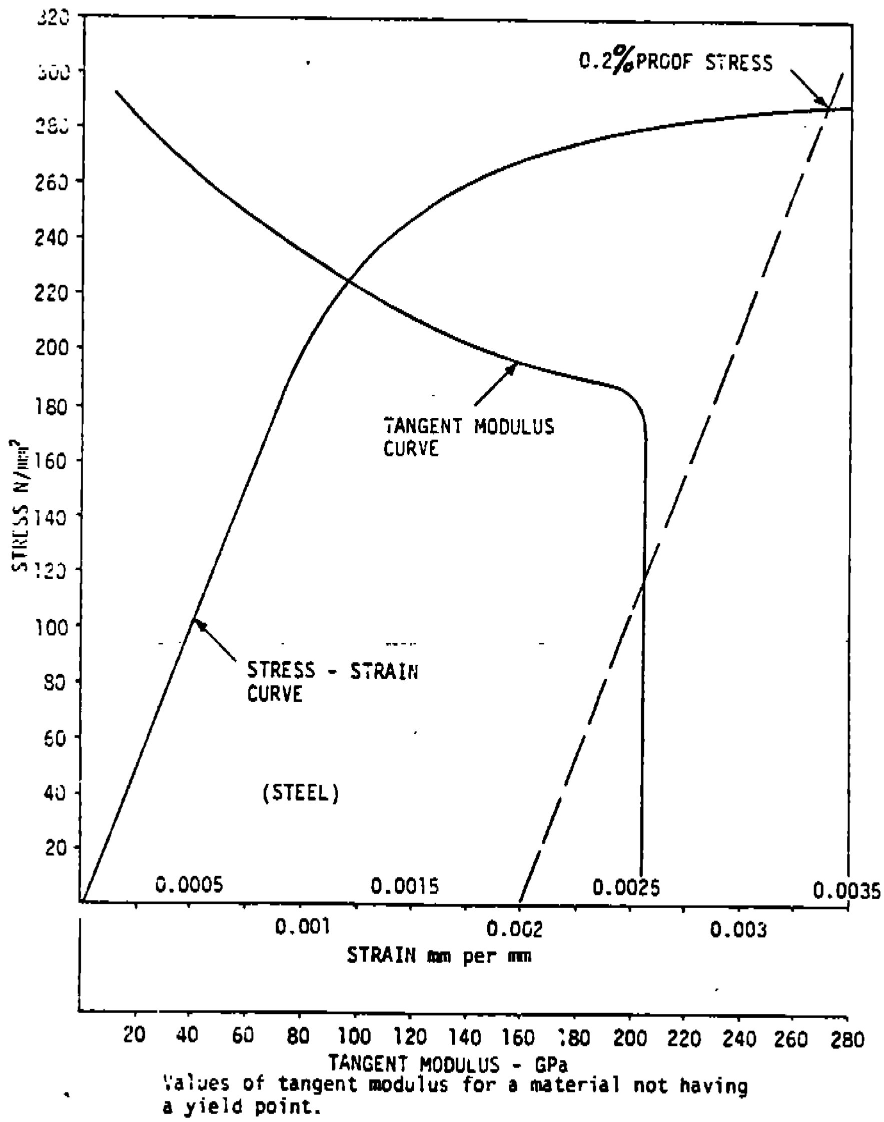


Fig.7.4 Typical Material Stress/Strain Behaviour and Properties (Ilustration only)

Inelastic instability stresses can be estimated, in some cases, fairly accurately from comprehensive material stress-strain data; e.g. Fig.7.5; otherwise a parabolic relationship with the material yield stress, as the maximum possible stress, is often assumed.

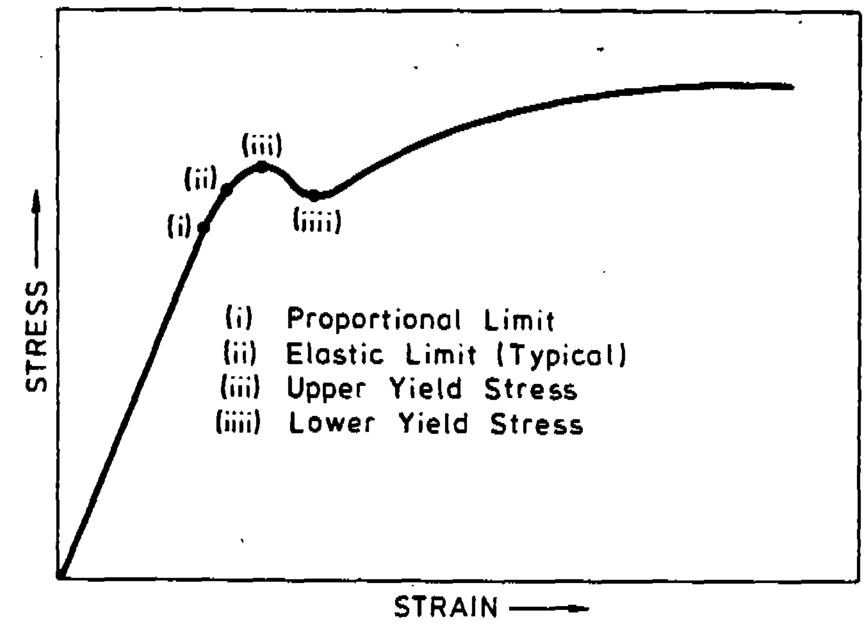
Even within small deflection theory, the transition from elastic response to inelastic response, as a function of geometric proportions and properties, cannot be simply defined in mathematical terms; hence the cases for the parabolic and straight line types of approximations in the inelastic zone. Thus elastic instability criteria are usually associated with the minimum of either the computed elastic instability stress or a specified percentage of the material yield stress. Various codes employ different stress levels for this transition, typically of the order of 50-60% of the material yield stress; this may vary between weld fabricated and rolled components. It is to be appreciated however that where codes employ a Perry-Robertson type of formulation this transitional effect is not so explicit, see Fig.7.6(b).

Although buckling does not usually constitute failure it is frequently employed as a measure of structural performance, with suitable factors of safety, in lieu of more realistic and rigorous failure studies. Most codes do not separate elastic from inelastic buckling and employ the same factor of safety with each. The damage that could be caused by inelastic buckling is not given particular consideration, however most inelastic buckling calculations are based on the yield stress and are thus slightly conservative.

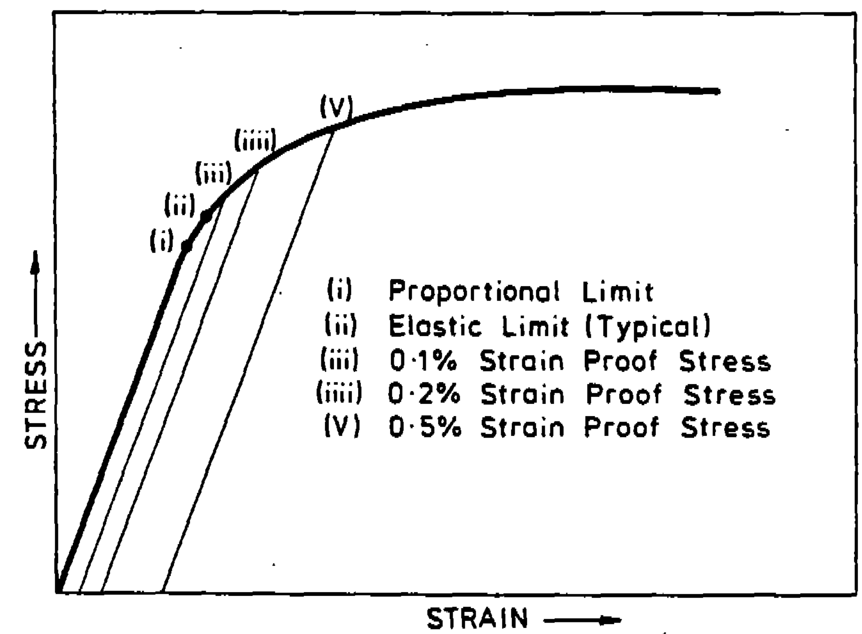
When the theoretical elastic buckling stress has been determined, making any allowances as necessary for the effects of geometric imperfections, by the employment of the previously given formulae for the various geometric proportions and loading conditions, then the effects of material inelasticity must be considered. Such effects are of concern when the theoretical elastic buckling stresses are determined to be, typically, above $(\sigma_Y)/2$. Two approaches commonly used, the Johnson-Ostenfeld (Figure 7.6(a)) and various versions of the modified Perry-Robertson (Figures 7.6(b)) approach.

In some structures localised buckling (whether elastic or inelastic) may occur at stress levels appreciably lower than the material yield. Typical examples could be the flanges and webs of fabricated columns or deep beams where the designer or the appropriate design code has not placed suitable upper limits on the breadth to thickness ratios of these elements in order to ensure that local buckling does not occur before, or precipitate, overall buckling. In this case the limiting material yield stress would be replaced by the local buckling or crippling stress, σ_{cr}, as illustrated in Fig.7.6. This affects both the maximum short column strength and the transition from the elastic column curve.

Other methods include the Merchant-Rankine type and various modified versions of the above approaches employed by the classification societies and API, etc.

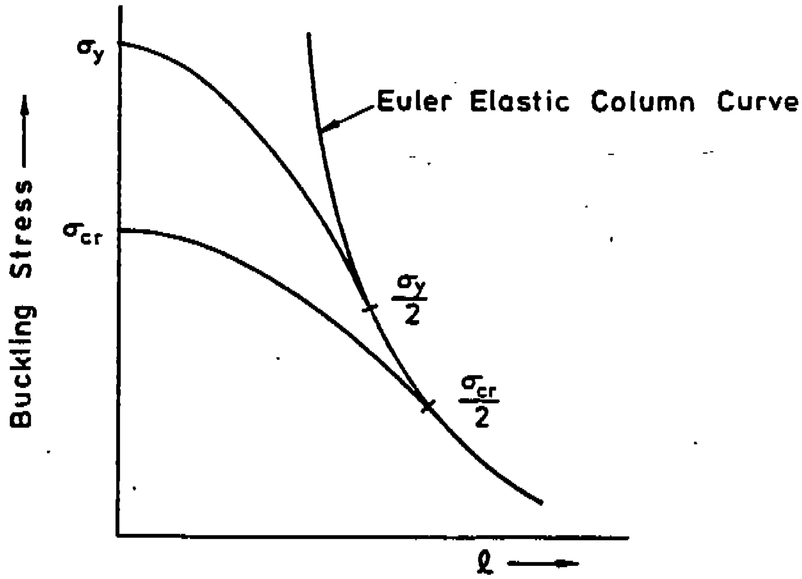


(a) Material Having Well-defined Yield Points

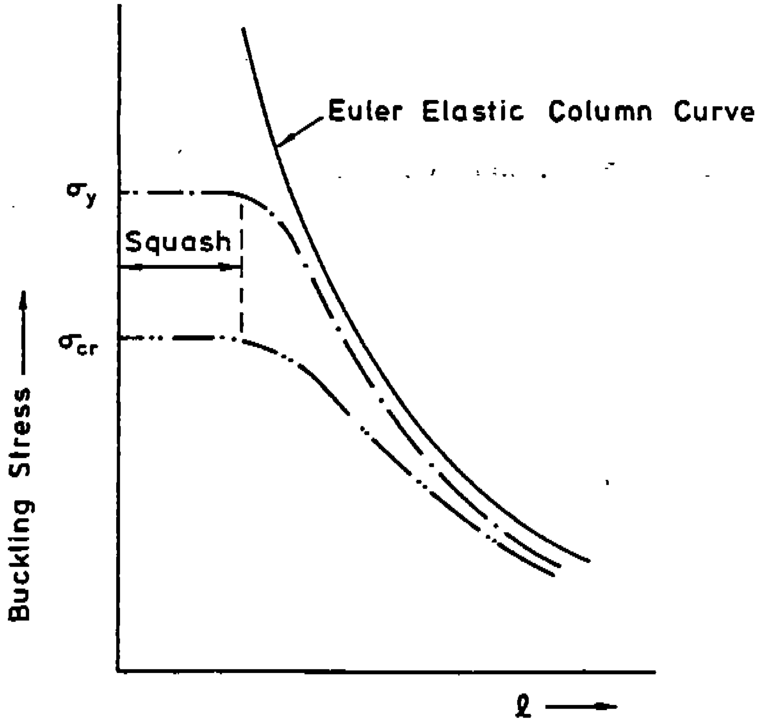


(b) Material Not Having Well-defined Yield Points

Fig.7.5 Material Stress Definitions Based on Stress-Strain Curves



(a) Johnson-Ostenfeld Approach



(b) Modified Perry-Robertson Approach

Fig.7.6 Column Curves - Approaches for Inelastic Response

Plastic Collapse. Although there are several collapse mechanisms, this basically takes place when yielding occurs in sufficiently large volumes of material that subsequent small increases in applied load cause very large increases in deflections. Such forms of failure may be caused by lateral loads, by in-plane loads or a combination of the two.

1 Lateral Loading. Plastic collapse under lateral loading involves the formation of sufficient plastic hinges to form a mechanism. It is not precipitated by yielding at a single point except in some very elementary structures. Plastic collapse load prediction methods which are readily applicable during design only exist for relatively simple beam or plate-like structures.

2 In-plane Loading. Plastic collapse takes place under in-plane loads when average direct and shear stresses (combined using a suitable theory - usually the Maximum Shear Strain Energy theory) reach an intensity corresponding to yield. In practice, however, this particular mode of failure is only important in special circumstances.

The simple plastic collapse mechanism is usually only found in simple, robust and otherwise stable structures, for example beams, flat plates and simple frameworks. Many fabricated structures fail in a complex pattern of plastic hinge mechanisms and elastic and inelastic buckles. Hence several failure modes are usually considered in order to find upper and lower bounds to the probable behaviour.

Small constrained zones of plastic flow in regions of high stress concentrations, such as joints, cutouts and notches, are normally of concern, only for fatigue crack propagation. Of more concern are larger areas of plastic flow which are unconstrained by surrounding elastically-responding material, for example the formation of plastic hinge collapse mechanisms in beams and frameworks.

Inelastic failure is usually more gradual, in ductile materials, enabling internal forces to be re-distributed and new load paths to develop, depending upon the degree of redundancy in the structure. The effects of strain hardening are usually ignored, giving some conservatism.

Material yielding is rarely defined precisely since the materials proportional limit (i.e., the point at which the stress-strain curve begins to depart from the linear elastic straight line) is difficult to measure precisely and consistently. Noting the statistical variation of properties, typically, 0.1%, 0.2% or 0.5% proof strain measurements ref. Fig.7.5(b) are used to represent the effective yield stress, σ_Y. Hence designing up to the material yield stress infers a small measure of permanent deformation. This is, however, insignificant in the vast majority of cases.

High rate loading, either from externally applied forces or from internal force systems, (for example during crack propagation), should be associated with material yield properties derived from dynamic tests in which the material may appear to be more brittle but have a higher effective yield stress.

☐ Combined Loading

The subject of combined loading is complex because of the uncertain consequences or results on the overall and local structural behaviour.

The typical approach is to use interaction equations to modify the critical results for a load type in the presence of others. In general the form of the equations varies with the different authors or design codes. These differences mainly involve the arrangements of the terms and the degree of interaction represented by the powers. In some cases different equations are used for different load combinations, while others quote single expressions covering all loads types. In some cases stresses from bending and axial compression are regarded as additive whilst in other cases they are assumed to interact linearly. The adequacy of the equations for the inelastic range is also questioned by some and regarded as acceptable by others.

For a multi-axial stress state a convex yield surface, as postulated by Von Mises, needs to be employed in order to define a relationship between various components which would lead to the initiation of plastic flow.

There is a wide variety of approaches and data is needed to substantiate any of the formulae proposed.

☐ Failure Modes Involving Fracture

Fracture is usually considered to be failure which takes place without any significant yielding of the structure. The two conventional modes of fracture are:

- Brittle fracture (implying tensile loading)
- Fatigue failure (implying reversing loads)

The important aspect of brittle fracture is that, because negligible yielding is involved, failure can be rapid and without the gradual load re-distribution effects often seen in statically- indeterminate ductile structures. Thus brittle fracture can be quite catastrophic. The transition from ductile failure to brittle fracture is usually both material and temperature related.

Although low cycle-high stress fatigue may involve appreciable local yielding, particularly in the region of stress concentration factors, such local yielding may not affect overall internal forces redistribution in a gradual manner. Given the formation of a large crack developed through low cycle-high stress fatigue then fairly rapid ductile fracture may follow, albeit not with the dynamic aspects of brittle fracture.

Fatigue. Because most of the loads imposed on ships and other marine structures are cyclic the possibility of fatigue failures always exists. Indeed, many, if not most, of the structural failures that occur in ships result from fatigue. Despite this, fatigue considerations often have little influence in general scantling development because fatigue failures do not usually endanger the structure even though they may be a nuisance and are often expensive to repair. Almost all fatigue cracks originate at severe stress concentrations; they are best minimised by keeping stress concentration factors low rather than by keeping general stress levels low. Conversely, higher standards of detail design could lead to the acceptance of higher fatigue stress levels causing other limit states to govern the structural design.

Brittle Fracture. Brittle fracture has caused many failures in merchant ships and in some offshore structures. The susceptibility of a structure to brittle fracture is crucially dependent on the material from which it is constructed, but it is also increased by the presence of stress concentrations, notches and imperfections, exposure to low temperatures, the sudden application of loads, and the use of thick sections. Fabrication processes, particularly welding methods, are also important. Because it is virtually impossible to avoid some degree of stress concentration, it is essential to select materials which are not susceptible to brittle fracture.

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8.0 STABILITY OF STRUCTURAL SYSTEMS

Modes of failure relevant to both component levels and overall structural response are reviewed and discussed in Chapter 7. However the possible implications of the re-distribution mechanisms provided by redundancy requires further consideration.

Structural systems lose their 'stability' (i.e. controlled response to monotonically increasing externally applied forces) through either the development of some form of instability (conventional buckling) or the forming of a sufficient number of plastic zones/regions (typically plastic hinges) for a mechanism to develop (complicated by the effects of both large deflection load-line changes and strain-hardening).

8.1 Ductile Behaviour

Components that fail in a ductile manner generally do so through a series of progressive stages:

- the onset of non-linear response, where the load carrying capability begins to increase at a rate which is less than the rate of straining or deflection,

- the development, in many cases, of a plateau of capability, that is an almost constant load carrying capability regardless of deflection (e.g. a typical plastic hinge with no strain-hardening effects), and finally

- a post ultimate strength load-deflection curve, which will generally be component-type dependent, and which will often show an unloading trend.

Within a redundant structure component ductile behaviour and subsequently failure results in a gradual re-distribution of internal forces when the system is subjected to a monotonically increasing overall force system. Many structures, particularly large and complex ones, will frequently show regions of buckling distress or excessive straining well before overall failure occurs. Overall failure may then, subsequently, be in the form of classical buckling (with or without the effects of inelasticity). This could occur in both discrete framework-like structures and continuous stiffened shell types of structures. Both of these topological problems can be examined for 'classical' instability by employing a suitable finite element method computer program.

Failure by the formation of local plastic hinges (regions of plasticity) is particularly, but not solely, related to skeletal framework type systems (both two- and three- dimensional). The number of hinges required for failure is very strongly related to the degree of redundancy within the overall system.

If factors such as manufacturing imperfections and residual stresses, material strain hardening and tangent modulus characteristics, etc., are allowed for, then loss of overall stability tends to be a fairly gradual effect, particularly in large continuous systems.

Ang and Ma [8.1] considered that in the case of structural systems composed of ductile components, the overall system resistance and eventual failure would be independent of the sequence of failure of the individual components. Klingmüller [8.2] considered that redundant structures only attain the re-distribution benefits of redundancy if the components fail in wholly ductile modes.

However, if the structure was fully stressed, or nearly so, then the first component to buckle or yield would precipitate overall failure. Even in a fully stressed design there will be some statistical variation in actual member capability compared with ideal theoretical capability and thus one item would fail before the others.

Moses [8.5] also notes that system reserve strength, if properly modelled and accounted for, may play a major role in reliability. It requires redundancy and a condition where the multiple parallel load paths are not simultaneously loaded to a similar proportion of local capability by the design case. Furthermore, according to Moses, two conditions must be present to achieve full system benefits:

1 Component failure must be ductile, and

2 Secondary members which come into play only when load path distributions are changed must have sufficient reserve capacity to carry any required additional loads (i.e. the structure must not be fully stressed).

8.2 Brittle Failure

Brittle failure, as the result of a rapidly propagating crack, may result in the loss of one single member, in the case of a discrete structure. In some structures, e.g. at a complex node point, a propagating crack may cause the loss of several members. In a complex continuous structure a rapidly propagating crack will cause considerable loss of effective load-carrying paths, until the crack is arrested. Classical brittle fracture introduces an instability in energy re-distribution which may be transient until the growth is arrested, e.g. by special arrester strips, regions of low stress, etc. However in mathematical modelling terms brittle failure is associated with sudden immediate loss in load-carrying capability and this may be due to tensile failure or precipitative buckling failure (however in most 'real' structures the latter is probably more accurately modelled as a brittle-ductile, or semi-brittle, failure process).

Transient behaviour implies that a new condition of overall equilibrium is formed both during propagation and when propagation is arrested. Depending upon the rate of crack propagation the associated internal strain energy release could involve short-term dynamic effects. This is a complex problem which is fortunately quite rare. According to Klingmüller [8.2], in failure due to brittle or fatigue cracks or precipitious local buckling, additional dynamic forces are made responsible for immediate total collapse after first reaching a strength limit. Dynamic brittle effects are analogus to a local transient force and thus will have their greatest effects near the brittle failed element. (This is reviewed in more detail later in this Chapter.)

With regard to overall system stability, Ang and Ma [8.1] considered that in the case of a brittle system (i.e. all elements fail in a brittle manner) the collapse load of the system depends on the sequence in which the individual components fail as the overall loading is increased. Each sequence of failure of a set of components constitutes a different mode of collapse of the overall system. Since a brittle system is defect sensitive, failure sequences for a given number of components can be permutated. Thus the number of alternative geometric modes of collapse of a system could be large. However if the resistances of the individual components were assumed, or designed, to be perfectly correlated with the imposed demand (and the structure was defect free) then the collapse of the system becomes independent of the component failure sequences, [8.2]. This would greatly simplify the collapse analysis of the system.

8.3 Semi-Brittle Failure

Several studies (e.g. [8.6] & [8.7]) relate to components that fail in what is termed a semi-brittle manner. This is illustrated in Fig.7.1. The term semi-brittle infers a rapid drop-off of capability after the ultimate strength is reached down to some residual strength. Physically this is difficult to realise in tension loading level except in the case of a crack which has propagated part of the way through the member (or an associated joint). In the compression sense another possible model, using perfect ideal components, as the components fail suddenly, then forms a series of local buckles or plastic hinge line mechanisms and thus provides a residual capability.

For overall systems capability it is probably better to regard the failure as ductile-brittle-ductile, with the overall performance being more ductile.

8.4 Mixed Mode Failure

In many real structures overall failure may involve some regions subjected to predominately compressive internal forces and other regions to predominantly tensile forces. Assuming that the external forces are applied through a number of cycles and the corresponding internal stress levels are high, system failure may be a result of mixed ductile, semi-brittle and brittle local failure modes.

8.5 Failure Related to Accidental Damage

The same ductile and brittle modes exist for a damaged structure as for an intact structure. The damage could be modelled by removed or deformed members.

A common design problem is to allow for accidental damage to structure where for safety or economic reasons such damage must be controlled, for example collision damage to liquefied gas carriers or offshore structures.

Reference [8.4] suggests that there are basically three approaches for controlling progressive failure.

These are:

(i) 'Event control', which can involve operational procedures and the provision of secondary barriers to guard against the possibilities of accidental damage. Clearly these factors are outside the control of the designer of the structure.

(ii) An 'indirect design' approach which assumes that the providing of structure having greater than minimum levels of strength, ductility and overall joint and member continuity etc., will provide for, in an indirect manner, adequate resistance to progressive failure without postulating the actual form of the damage and corresponding failure modes. (A statically indeterminate structure may be implied.)

(iii) The 'direct design' approach. This method is the explicit consideration of modes of damage and associated structural resistance to progressive failure at the actual design stage. It would involve full consideration of the response of statically indeterminate structures, alternative load paths and the ability to absorb damage both with and without local failure occurring. Such could involve large deflection inelastic non-linear considerations and with an involved deliberation of the magnitude and extent of the damage that could be tolerated as a qualification (for example the extent of penetration into a liquefied gas carrier hold space). The direct design approach must be based upon a fully-probabilistic method and should include an estimate of the energy absorbed by the damage.

For accidental damage response the loading is usually assumed to be extreme. However as the load occurrance is assumed to be only once, and probably unlikely, significant physical damage (e.g. buckling, tearing, limited penetration) is normally considered acceptable as long as the overall structure remains intact and survives the incident with no loss of life or serious risk of environmental damage. The damaged structure must also be repairable. Some forms of progressive failure, or tolerance of accidental damage, are considered by some civil engineering codes, e.g. large multi-story building, nuclear power stations.

8.6 Energy Release in Continuous Structures

When a component within a structure completely fails the strain energy that was in that component is redistributed (released) into the surrounding structure. Within a discrete structure, e.g. a skeletal framework, this behaviour is relatively readily seen. However within continuous structures, this redistribution of energy is somewhat less obvious.

The following conceptual approach, based upon an analogy with linear elastic fracture mechanics concepts, was postulated by Mr. P.W. Marshall, of the Shell Oil Co., at both a Project Technical Committee meeting and in a private communication to the authors of this report.

Consider a broad uniformly multi-stiffened flat (or singly curved) panel as representing a system of complex highly redundant continuous (parallel) systems. Assume uniaxial inplane edge forces aligned with the stiffeners.

Although normal design procedures assume all elements are identical in both geometry and material properties, in real structures there could be significant statistical differences. Therefore some stiffeners would fail before others.

Assume the structure is damaged, e.g. due to collision or some form of accident, and that the damage is in the form of either a crack or local failure of one or more stiffeners and plate material midway between supporting transverse structures, i.e. the failed/damaged structure may have some local residual strength. The assumed residual strength possibilities, per stiffener, are illustrated in Figure 8.1.

Assume that the overall panel topology is as illustrated in Figure 8.2. The effects of the region of damage are contained by an intact stiffener on each side of the damage, also as illustrated in Figure 8.2, i.e. the strain energy that is released from the damaged zone is contained solely by the same two stiffeners (ignoring considerations of shear flow and overall strain pattern compatibility).

Assuming uniform inplane tensile forces and employing the conventional concepts of 'linear elastic fracture mechanics', the Griffith's formula for a simple straight crack plane stress condition gives the energy release, U, as

$$U = \frac{\sigma^2 \pi a^2}{2E} \text{, per unit thickness} \tag{8.1}$$

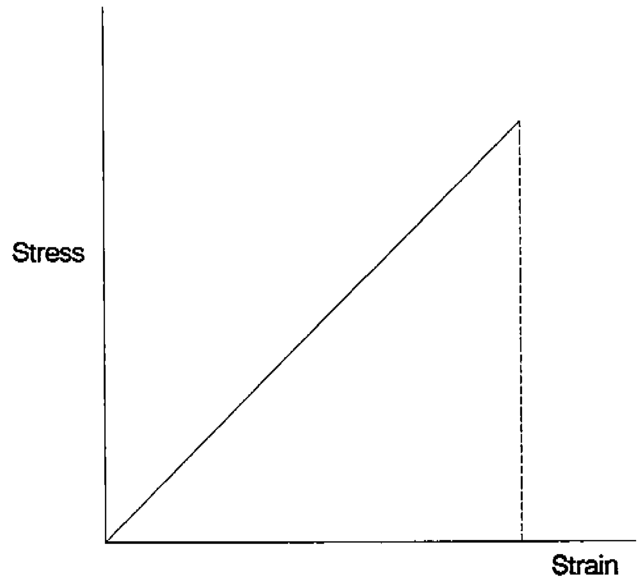
where σ is the uniformly applied field stress, a is the half length of the crack, and E is the modulus of elasticity.

Equation (8.1) is simply the strain energy which is released from a volume of material equal to a circle of the same diameter as the full length of the crack (2a), i.e.

$$\text{Strain energy} = \frac{\sigma^2}{2E} \text{ (volume)}$$

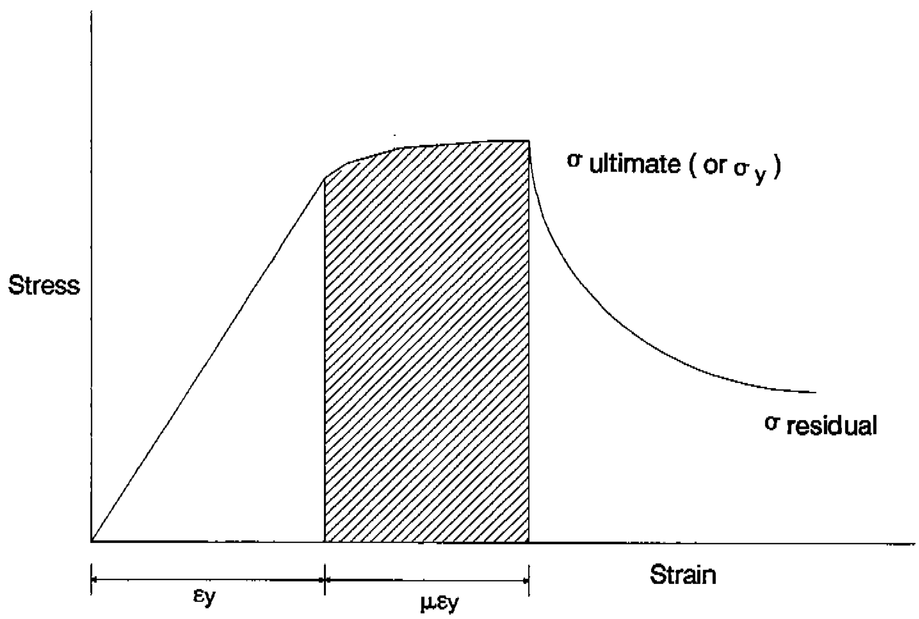
$$\text{Volume} = \pi a^2 t$$

$$\text{or} \quad \text{Volume} = \pi a^2 \text{ (per unit thickness)}$$



(a) - complete failure, 'brittle' system,

- structure either cracked through or complete compressive failure



(b) - ductile failure

- following collapse in compression, residual compressive strength

Fig.8.1 Failure of Brittle and Ductile Systems

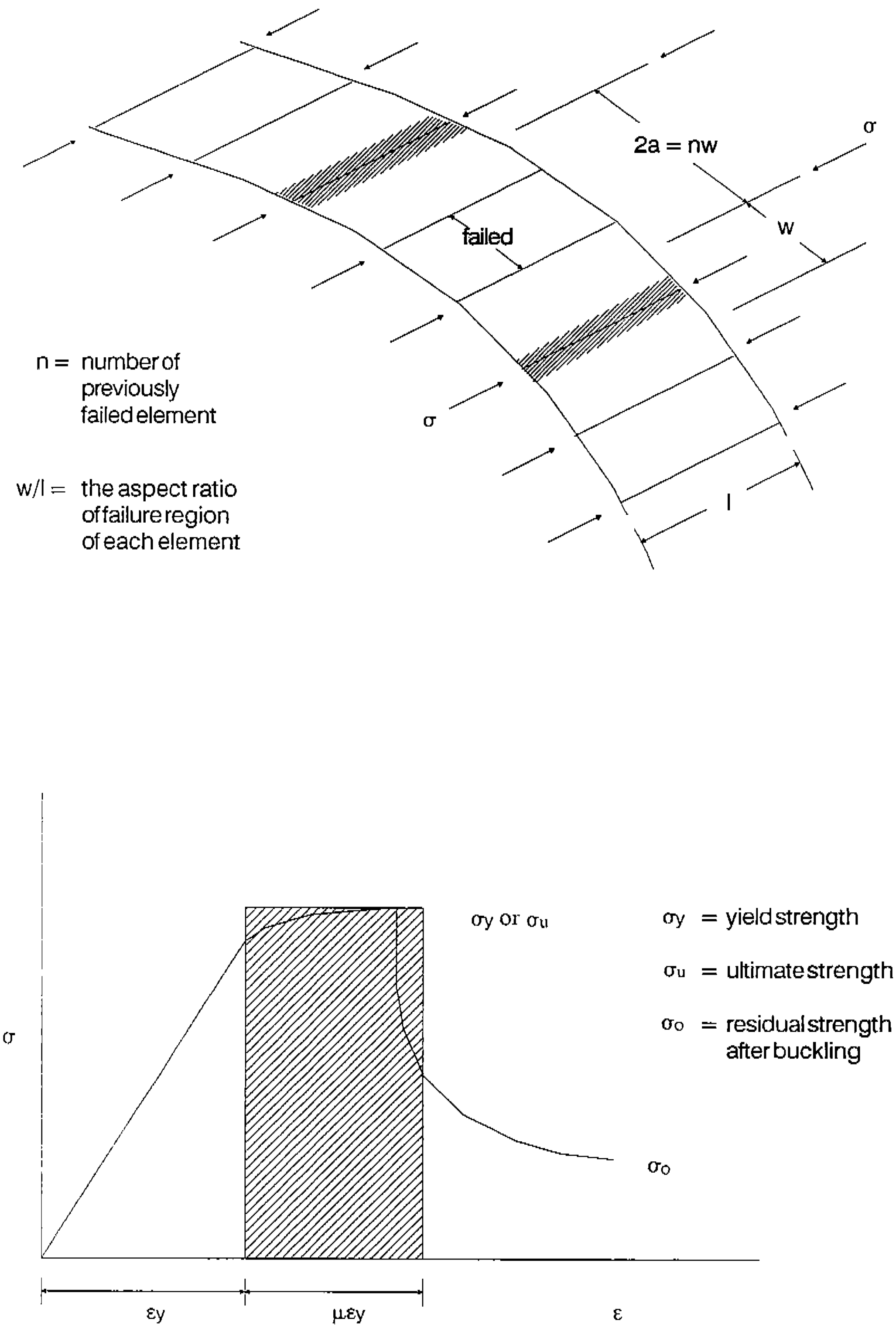


Fig.8.2 Conceptual Model of Structure in the Region of Damage

If the failed zone, instead of being cracked, has a residual capability, σ_r, as illustrated in Figure 8.1(b), then equation (8.1) could be rewritten

$$U = \frac{(\sigma - \sigma_r)^2 \ \pi a^2}{2E}$$

$$= \frac{\left[\sigma^2 - 2\sigma\sigma_r + \sigma_r^2\right] \pi a^2}{2E} \qquad (8.2)$$

If the residual strength, expressed by σ_r, is small compared with the field stress, σ, then equation (8.2) can be simplified to

$$U = \frac{\left[\sigma^2 - 2\sigma\sigma_r\right] \ \pi a^2}{2E} \qquad (8.3)$$

Equation (8.3) which ignores the '$+\sigma_r^2$' component thus under estimates the energy release. However where the residual stress is low compared with the field stress the difference may be small. There will also be a change in energy balance owing to the development of shear stresses within the plate material in front of and behind the region of the failure. It is possible that these two simplifying assumptions are partially self cancelling.

Consider the energy release rate, i.e. the rate at which energy is released with increase in area of damage (<u>not</u> a time related release rate in, say, the context of a fast tensile fracture). This will require that, in a stable system, more energy will need to be absorbed within the adjacent regions of undamaged structure.

For a brittle system, from equation (8.1) energy release rate

$$\frac{dU}{da} = \frac{\sigma^2 \pi a}{E} \qquad (8.4)$$

and for a ductile system with residual capability, from equation (8.3)

$$\frac{dU}{da} = \frac{\left[\sigma^2 - 2\sigma \ \sigma_r\right] \pi a}{E} \qquad (8.5)$$

Hence in both cases the simple energy release rate is direct linear function of width of damage, which, by inspection, is predictable.

From Figure 8.2, $a = nw/2$, where n is the number of stiffener elements within the full region of the damage and w is the stiffener spacing. Thus equations (8.4) and (8.5), respectively, can written

$$\frac{dU}{da} = \frac{\sigma^2 \pi \ nw}{2E} \quad \text{, for a brittle system} \qquad (8.6)$$

or

$$\frac{dU}{da} = \frac{\left[\sigma^2 - 2\sigma\ \sigma_r\right]\pi\ nw}{2E} \qquad \text{for a ductile system} \tag{8.7}$$

The assumption is made that the energy which is released from the failed region has to be absorbed by a width of plate equal to one stiffener spacing on each side of the failed region plus the effect of the associated stiffeners. Clearly the same widths of plate will already be subjected to the overall boundary stress, σ, which is likely to be a high fraction of either the yield stress of the material or the crippling stress, under compression, of the stiffener-plate combination. The capacity of the edge stiffener-plate elements to absorb further increases in loading is thus mainly represented by the ductile plateau of capability as illustrated by the shaded area in Figure 8.1(b).

The amount of transfered strain energy that can be absorbed, up to the point of failure of the edge strips, will be as illustrated in Figure 8.3. There will be two components - one in the remaining elastic regime, Figure 8.3(a) and one in the inelastic/plastic regime, Fig.8.3(b). If, as discussed above, the overall field stress is a high fraction of either the crippling stress or the yield stress of the plate plus stiffener combination, then conservatively the strain energy absorption characteristics are represented by the shaded area given in Figures 8.1(b) and 8.3(b).

Thus the amount of strain energy that can be absorbed, per edge strip, is given by, for ductile tensile yielding

$$U = \frac{\sigma_Y\ (\mu\sigma_Y)}{E} \times \text{volume} \tag{8.8}$$

where volume per 'unit' thickness = $w\ell$ (where unit thickness includes the sectional area of the stiffener).

Therefore

$$U = \frac{\mu\ \sigma_Y^2\ w\ell}{E} \tag{8.9}$$

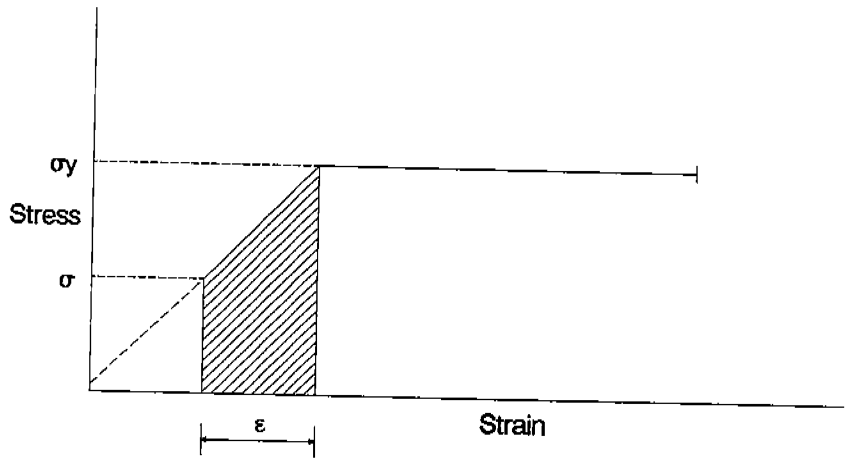
Thus total strain energy that can be absorbed by two sides

$$= \frac{2\mu\ \sigma_Y^2\ w\ell}{E} \tag{8.10}$$

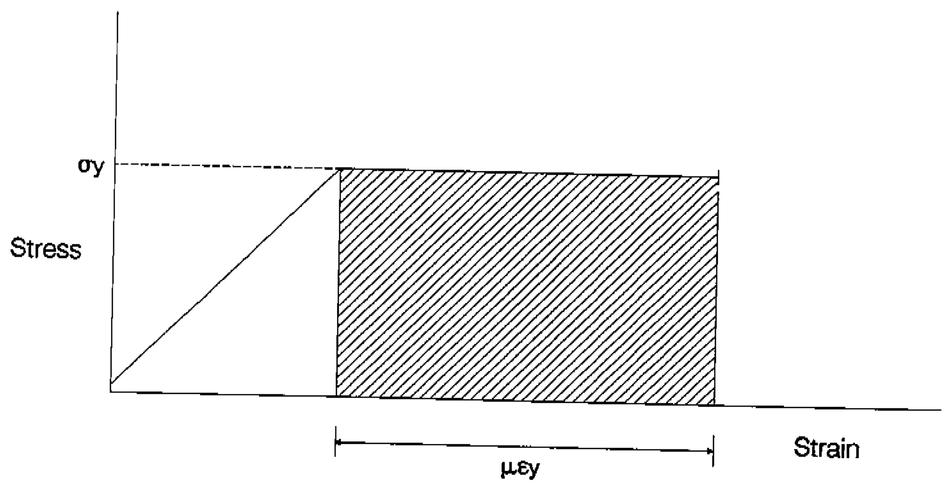
For a ductile compressive failure stress plateau equal to σ_{cc}, the total strain energy that can be absorbed by two sides

$$= \frac{2\mu\ \sigma_{cc}^2\ w\ell}{E} \tag{8.11}$$

It is to be noted that the assumption of only one element per side absorbing the energy from the damaged region is probably somewhat conservative, e.g. there would be an incompatible strain field.



(a) - elastic region



(b) - plastic region

Fig.8.3 Strain Energy - Idealised Stress Strain Curve

Thus the rate at which the strain energy is absorbed, given by du/dw

$$= \frac{2\mu\ \sigma_Y^2\ \ell}{E}\ , \text{ for the tension case,} \tag{8.12}$$

or

$$= \frac{2\mu\ \sigma_{cc}^2\ \ell}{E} \text{ for the compression case} \tag{8.13}$$

- depending upon the ultimate stress level capability.

Assuming compressive overall field stresses and that the damaged region has a residual capability then, it is postulated, that the overall system is stable if the energy absorption rate capability is greater than the energy release rate capability, i.e. on equating equations (8.13) and (8.7), and the condition is stable if

$$\frac{2\mu\ \sigma_{cc}^2\ \ell}{E} > \frac{\left[\sigma^2 - 2\sigma\ \sigma_r\right]}{2E}\ \pi\ nw \tag{8.14}$$

i.e. stable if

$$\mu > \left[\left[\frac{\sigma}{\sigma_{cc}}\right]^2 - \left[\frac{2\sigma\ \sigma_r}{\sigma_{cc}^2}\right]\right] \frac{\pi}{4}\ n\ \frac{w}{\ell} \tag{8.15}$$

For many stiffener sections the plateau region tends to be relatively small, i.e. 'μ' is small and it is thus postulated that for a large region of damage the system will only be stable if $(\sigma^2 - 2\sigma\ \sigma_r)$ is a negative value, i.e.

$$\sigma^2 < 2\sigma\ \sigma_r$$

$$\sigma < 2\sigma_r.$$

However this should be reconsidered in view of the simplification discussed earlier and the more precise requirement will be that the value of $(\sigma^2 - 2\sigma\ \sigma_r + {\sigma_r}^2)$ is found to be a negative number.

The squash, approximate plateau of capability, characteristics of stiffener-plate combinations is often dependent upon:

- the mode and direction of failure, e.g. pure flexural, lateral instability, mixed modes,

- whether the elastic or inelastic regimes predominate, and

- the levels of as-manufactured geometric imperfections and residual stresses.

The squash/plateau zone, in both level and extent, may also be affected by the presence of lateral forces in addition to the axial forces.

The mode of stiffener failure can relate to the type of stiffener section, e.g. flat bar, symmetrical T-section, unsymmetrical section, etc., and to any possible interactions with local instability modes, e.g. flange buckling. Whilst some well designed stiffener sections may have a reasonably well defined plateau of ultimate capability, other less well designed sections may fail in a precipitious manner.

Although they may reduce the actual ultimate strength level of stiffener sections, manufacturing imperfections may lead to a more appreciable "plateau" of capability, i.e. strain range between the limit of elastic behaviour and the sharp drop to residual capacity, although without the plateau being particularly flat.

Clearly when damage occurs in stiffened plate ship and offshore structures some effects can develop which alleviate the situation. For example, if the damaged region of the stiffened plate forms part of a ship's outer bottom then water ingress could occur. Depending upon the magnitude of the water head that could build-up within the structure then the lateral forces applied to the panel will accordingly diminish. In this case the stiffened panels which will have been designed to withstand the most adverse combinations of inplane and lateral loadings will have the ability to withstand higher than design assumed axial forces and thus more capacity to withstand damage. However, depending upon the magnitude of the lateral forces in the intact condition and the assumptions of multi-bay symmetry, the designer may have employed fixed-end beam/column conditions in his analyses - thus developing a higher axial load carrying capability than if he had assumed no lateral loadings and assumed simple end supports for each stiffener span. Thus a rigorous damage assessment study of this type really needs to be on a case by case basis and the above mathematical illustration serves to indicate a feasible approach.

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9.0 RELIABILITY ASSESSMENT OF REDUNDANT STRUCTURES

In the following chapters, several applications of reliability analyses have been made to examples of stiffened flat panel structures and to both stiffened and unstiffened cylinders. They are all made on a component level. Most of the recent design codes are also based on studies of component level reliability analyses [9.42]. However it has been recognised that an accurate estimate of the reliability of a structure must be based on a full system reliability analysis. In the case of a statically determinate system it is sufficient and reasonable to base overall reliability on the reliability of the individual components, because the failure of a single component will result in the failure of the entire system. However this is not the case of statically redundant structures. Failure of a single element will redistribute internal forces among the remaining elements if they have the capacity. Failure of a redundant structure requires more than one discrete component to fail. Continuous structures such as floating platforms and ship structures have much inherant redundancy and should be analysed as a structural system in order to have a uniform and consistent level of safety within the overall structure.

During the past few years a considerable amount of work has been carried out on overall system reliability, but most of the work has concentrated on discrete structures, such as jacket platforms. Very little has been done on continuous structures such as semi-submersibles, TLP's, ships, etc., because of difficulties in identifying the possible failure modes and redundant load paths in these types of structure.

An estimate of the reliability of a structural system on the basis of the failure of a single structural element, i.e. the element with the lowest reliability index of all the elements within the system, is generally called the system reliability at level 0 [9.1]. Thus if a structure consists of n failure elements and if the reliability index for element class i be denoted β_i, then at level 0 the system reliability index β is,

$$\beta_s = \min_{i=1,n} \beta_i \tag{9.1}$$

A more realistic estimate of the reliability of a structural system can be performed at the so-called, Level 1 where the probability of failure of any failure element is taken into account by modelling the structural system as a series system. At Level 1, system failure is still defined as failure of one element. At Level 2, the next higher notional level, the system's reliability is estimated as the reliability of a series system where the elements are themselves parallel systems each with two failure elements - so-called critical pairs of failure elements. The β-unzipping method [9.1] is a rational method to identify these critical pairs of failure elements. This method is quite general - it can be used for two-dimensional and three-dimensional framed and trussed structures, with elements having ductile or brittle modes of failure, for a number of different failure mode definitions. This method can also be used to efficiently identify significant plastic mechanisms. First the fundamental mechanisms are automatically generated and the corresponding reliability indices are calculated. Reliability analysis based on overall mechanism failure is called the system reliability analysis at mechanism level. The validity of this approach largely depends on the assumptions that can be made regarding the structural behaviour of the elements, in particular whether they display brittle or ductile failure characteristics.

The probability of failure, P_f, for the Level 1 reliability of a system composed of normally distributed and linear safety margins is given by [9.1]:

$$P_f = 1 - \int_{-\infty}^{\beta_1} \int_{-\infty}^{\beta_2} \cdots \int_{-\infty}^{\beta_n} \psi_n(\overline{x},\overline{\overline{\psi}})\ dx_1 dx_2 \ldots dx_n \qquad (9.2(a))$$

As shown in Ref.[9.1], this equation can also be written:

$$P_f = 1 - \varphi_n\ (\overline{\beta},\overline{\overline{\psi}}) \qquad (9.2(b))$$

where:

ψ_n and φ_n are the non-dimensional density and distribution functions for n standardised normal variables $\overline{X} = (X_1 \ldots X_n)$

$\overline{\overline{\psi}}$ is the matrix of correlation coefficients and

$\overline{\beta}$ are the reliability indices $(\beta_1, \beta_2, \ldots \beta_n)$ for the safety margins $\overline{M} = (M_1 \ldots M_n)$.

When all correlation coefficients are equal, i.e. $\psi_{ij} = \psi > 0$, then from [9.1]

$$P_f = 1 - \int_{-\infty}^{+\infty} \psi(t)\ \prod_{i=1}^{n} \varphi \left[\frac{\beta_i - \sqrt{\overline{\psi}}\ t}{\sqrt{1-\overline{\psi}}}\right] dt \qquad (9.3)$$

Conversely, when the correlation coefficients, ψ_{ij}, are unequal a simple approximation for P_f can be obtained from (9.3) by putting $\varphi = \overline{\psi}$ where $\overline{\psi}$ is the average correlation coefficient defined by:

$$\overline{\psi} = \frac{1}{n(n-1)} \sum_{i,j=1,\ i\neq j}^{n} \psi_{ij} \qquad (9.4)$$

Usually for a structure with n failure elements the estimate of the failure probability for the series system can be calculated with sufficient accuracy by only including some of the failure elements, namely those with the smallest reliability indices. One way of selecting the smallest is to include only failure elements with β values in the interval $[\beta_{min}, \beta_{min} + \Delta\beta_1]$ where β_{min} is the smallest reliability index of all failure element indices and where $\Delta\beta_1$ is a positive number. The failure elements chosen to be included in the system reliability analysis at Level 1 are called critical failure elements. If two or more critical failure elements are perfectly correlated, then only one of them is included in the series system of critical failure elements.

Another approach suggested by Moses [9.2], for reliability analysis of limit state design problems is to find the probability distribution of a linear combination of random variables, where

$$Z_j = \sum_i a_{ji} M_i - \sum_k b_{jk} P_k \quad j=1, \ldots m \text{ collapse modes} \tag{9.5}$$

in which:

Z_j represents the reserve strength in a particular collapse mode,

M_i is the structural resistance of the i^{th} member,

P_k is the load acting on the member,

a_{ji} is the resistance coefficient determined by the position and condition of the i^{th} member related to the j^{th} failure mode,

k is the number of loads,

b_{jk} is the load coefficient determined by the position and condition of the k^{th} member related to the j^{th} failure mode and

m is the number of failure modes.

The failure probability of any collapse mode is the probability that $Z_j < 0$. The overall reliability analysis requires the probability that any $Z_j < 0$ and involves the correlation between Z_j terms which result because some of the same random variables are embedded in the different Z_j terms. This leads to the equation for a frame having n collapse modes as:

$$\begin{aligned} P_f = {} & P_r[Z_1 < 0] + P_r[Z_2 < 0, Z_1 > 0] \\ & + P_r[Z_3 < 0, Z_1 > 0, Z_2 > 0] + \ldots \\ & P_r[Z_n < 0, Z_1 > 0, Z_2 > 0 \ldots Z_{n-1} > 0] \end{aligned} \tag{9.6}$$

A reliability analysis to assess the safety of a redundant structure must include the effects of redundancy. The reliability index is related to the reserve strength of the structure and a similar index due to residual strength of the structure needs to be evaluated. This evaluation is extremely complex for a practical structure, and many approximations are required even to derive this figure for a simple structure.

The methods described above, and also shown in references [9.6-9.9], are based on the failure path approach and are very popular. The method is very efficient from a computational point of view since only the dominant failure paths are considered in the analysis. Because of this the results obtained from this procedure may lie on the unconservative side. This approach can be applied to structures with ductile or brittle component behaviour.

Work on the applications of system reliability to floating platforms was carried out in Ref.[9.15] using a linearised failure equation approach. Component failure was based on plastic collapse analysis.

Application to TLP structures was reported in Ref.[9.34], in which the incremental load method was extended to a multi-loading case. The incremental load method is a procedure in which the collapse of the structure progresses in a pre-defined failure sequence as the load increases, by a set of load increments. These load increments are defined in terms of the strength of the failed components. The total load at a particular component failure stage is the sum of the load increments up to that stage, and represents the system resistance, expressed in terms of the strengths of the failed components.

A computer program [9.34] is available based on the above approach for structural system reliability analysis, especially for floating offshore structures such as TLP's and semi-submersibles. This program can calculate the environmental loading coupled with reliability analysis. The reliability analysis consists of the component reliability, identifying the important failure modes and evaluating the bounds of the probability of system failure, in which Ditlevsen bounds [9.36] are calculated.

Extensive research has been performed during the last decade and several methods have been developed. Reference [9.32] provides a good review of various methods of system reliability.

Redundancy is useful in system reliability to increase safety. A study was conducted in Ref.[9.37] in which the coefficient of variation of the load factor was related to the degree of redundancy by

$$V_\lambda = V_F \frac{1}{\sqrt{\gamma+1}} \tag{9.7}$$

in which V_F is the coefficient of variation of the yield limits (uncorrelated) and γ is the degree of redundancy. This equation is an approximation and was found to be applicable for parallel models in which sidesway failure of frames with nearly equal cross sections is the dominant mode of failure. A functional relationship of the loads factor and the degree of redundancy for a given allowable probability of failure was derived and it was shown that redundancy has significant influence on the safety of the structure only for small values of the degree of redundancy and relatively high coefficients of variations of strength constants. Beyond a certain degree increasing redundancy will not give additional safety.

The purpose of this chapter is to give a brief overview of the various system reliability methods and their practical applications to redundant structures. Research in this field is at its infancy, and whilst there has been some work for the analysis of particular structures from a systems reliability view point however it appears that no particular work is directed towards studying rigorously the effect of redundancy on overall system reliability and this may be due to both complexity and computation time that is involved in a system reliability analysis procedure. With more progress on system reliability work, probably on simplified approaches if possible, this work can be directed towards parametric studies on the effect of damage on various redundant members in a structure on the system reliability.

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10.0 GENERAL PERFORMANCE OF STRUCTURAL COMPONENTS

Throughout the pursuance of the programme of work within this project the concepts and definitions of redundancy have been examined in order to ascertain if an alternative more useful definition, or definitions, could be postulated. Whilst classical definitions of indeterminancy are appropriate to discrete member skeletal types of framework structures such definitions are more difficult to apply to continuous structures and, in particular, to their primary components, e.g. unstiffened and stiffened shells and panels.

The 'classical' definitions for the quantification of redundancy within structures have generally been those of the degree of indeterminancy. A statically determinate structure, one in which it is normally assumed that there is no redundancy, is one in which the forces in all of the members can be determined from the fundamental equations of statics. From this it is also found that if the applied force system remains constant and any one member is removed then the structure becomes a mechanism and collapses. A statically indeterminate structure contains a larger number of elements (and constraints) than can be handled by simple balance of force statics and additional equations based upon the elastic response of the structure must be involved to obtain a solution. In a statically indeterminate structure, subjected to a given external applied force system, some members or constraints can be removed and the structure will still remain stable. Thus the degree of redundancy in the simple 'classical' indeterminate sense is the number of elements (and element capabilities) and/or constraints above the minimum necessary to ensure that the structure was indeed a stable structure and not a mechanism. In some design studies idealisations are made which ignore some element capabilities as being of second order and are thus taken to negligible (this also simplifies the method of solution). Thus some actual frameworks are idealised as simple trusses by ignoring element bending capability and bending transfer continuity at joints. Thus even a statically indeterminate truss, and the 'classical' degree of indeterminancy as a number, will not be a true measure of redundancy in the actual structure.

In complex structures the degree of redundancy, expressed in simple numbers, relating to total degrees of system freedom and equations of static equilibrium, is generally very hard to quantify.

A problem with some simple numeric descriptions of indeterminancy is that no indication is given of the relative significance/importance of the various members (and their load carrying capabilities) within a structure. Clearly some members owing to both their position and relative properties may be of profound importance to the integrity of the overall structure, whereas other elements may be of little consequence (and/or quite redundant). Some 'redundant' capability may be there but not by deliberate design. The differences between significant and secondary elements in a redundant structure will also be important to resolve in the instance of the occurrence of accidental damage. Lloyd and Clawson [5.3] addressed this in their classification and in the context of complex continuous structures Chapter 5 of this report considers 'primary', 'secondary' and 'tertiary' levels.

As discussed elsewhere in this report redundancy manifests itself in the ability to change the significance of the various load paths within a complex structure when one or more elements begin to perform less than their linear elastic predictions when the overall structure is subjected to an increasing external force system. From this it also follows that if one or more members fail, i.e. they have either no or a greatly reduced load carrying capability, then a redundant structure will still have an overall load carrying capability.

Thus the numeric degree of indeterminancy, used to indicate/quantify redundancy is only really useful for simple structures (or actual structures that have been modelled by the analyst as a simple equivalent structure) - hence the postulation of more general 'system' level methods developed by various researchers, i.e. system performance is quantified rather than attempting to relate overall degrees of freedom (or similar) to the limitations solvable by simple statics, etc. This study has involved a review of this situation to see if there are any other potential definitions which will provide some quantification of the measure or degree of redundancy within a structure by inspection of the character and complexity of the structure rather than by invoking system collapse analyses.

If one cannot quantify redundancy in some manner, numeric or otherwise, however crudely, without invoking complex overall system strength analyses then it becomes difficult to deliberately design for, or to mandate for in codes of practice, in the context of target levels.

Paliou, et al [ASCE 1990] proposed to define redundancy by a probabilistic measure and uses the definition that the probability P_r is that the structure will eventually survive given the simultaneous failure of one or more of its members.

The purpose of this section is to review some aspects and possible interpretations of redundancy at the structural component level. In consideration of the wide potential spectrum of ships and offshore structures such components, of most general application, are taken to be:

- unstiffened and stiffened circular section cylinders, and
- unstiffened and stiffened flat plates.

When considering the performance at the component level it is also relevant to consider the differences between theoretical 'ideal' structures and as-manufactured 'actual real' structures with their imperfections and statistical variances.

Redundancy within a component can be assessed or measured in several ways, e.g.

1 The ability to progressively reform self-equilibriating internal force systems when one, or more, elements cease to function in a linear elastic manner, and
2 The ability to remain stable as a whole when one, or possibly more, elements become themselves unstable or when an element, or elements, become ineffective following damage.

If a component on forming a new self-equilibriating internal force system, following one element beginning to function in an inelastic non-linear manner, can continue to sustain progressively higher external loading (of the same pattern) then the component could be classed as having full functional primary redundancy.

A possible qualitative measure of redundancy/inelastic response is how low the secant modulus (of the system) can become compared with the elastic modulus (of the system) when the component reaches its overall ultimate strength. Figure 10.1 shows two alternative systems having the same overall strengths.

□ Unstiffened Cylinders

It is reasonable to consider a simple circular section unstiffened cylinder to have no quantifiable redundancy. Clearly, however, such a component can have reserve strength (depending upon the radius/wall thickness ratio), associated with axial forces and bending, etc., (compared with design strength, safety factors and analysis assumptions) and residual strength following damage (e.g. heavy denting due to ship to platform collision).

□ Ring Stiffened Cylinders

Ring stiffeners are added to otherwise thin-walled cylinders to increase their resistance to buckling (e.g. due to axial compression and bending) or to increase their external pressure capability. Whilst increasing the number of rings, effectively decreasing the length of unsupported cylinder, has the result of increasing the load carrying capability of the cylinder, albeit not in a direct proportion, this does not in practicable terms add to or produce redundancy. Indeed a thin-walled ring stiffened cylinder is more likely to fail, under say compression or bending, in a precipitative manner compared with the inelastic 'squash' type response of an equivalent thick walled cylinder. As a thin-walled cylinder is likely to be relatively imperfection sensitive compared with a thick walled unstiffened equivalent it is probable that the former is likely to have lower reserve and residual strength capability compared with the latter.

□ Longitudinally Stiffened Cylinders

Longitudinally stiffened thin-walled cylinders are particularly employed in many offshore structures, and are efficient when longitudinally aligned compressive axial forces predominate.

Initially the axial compressive forces are equally shared by both the plating and the stiffeners. However, invariably, in the final loading stages the plate between stiffeners buckles out leaving the stiffeners, each with an associated attached width of plating, to withstand the compressive forces. Each stiffener, plus attached plate flange, withstands the applied forces in a column mode and subsequently failing in such a manner (this could include both Euler-bending type failure and torsional instability).

Thus the only possibility of affording some form of, or measure of, redundancy lies in the number of stiffeners.

This could be considered in the light of each of the various separate force systems that need to be evaluated when designing a longitudinally stiffened cylinder. For shear loading the diagonal tension mechanism can invoke a form of redundancy, as discussed later in this section and for overall bending the ultimate strength analysis approach can be of the form discussed for hull girders in another chapter of this report.

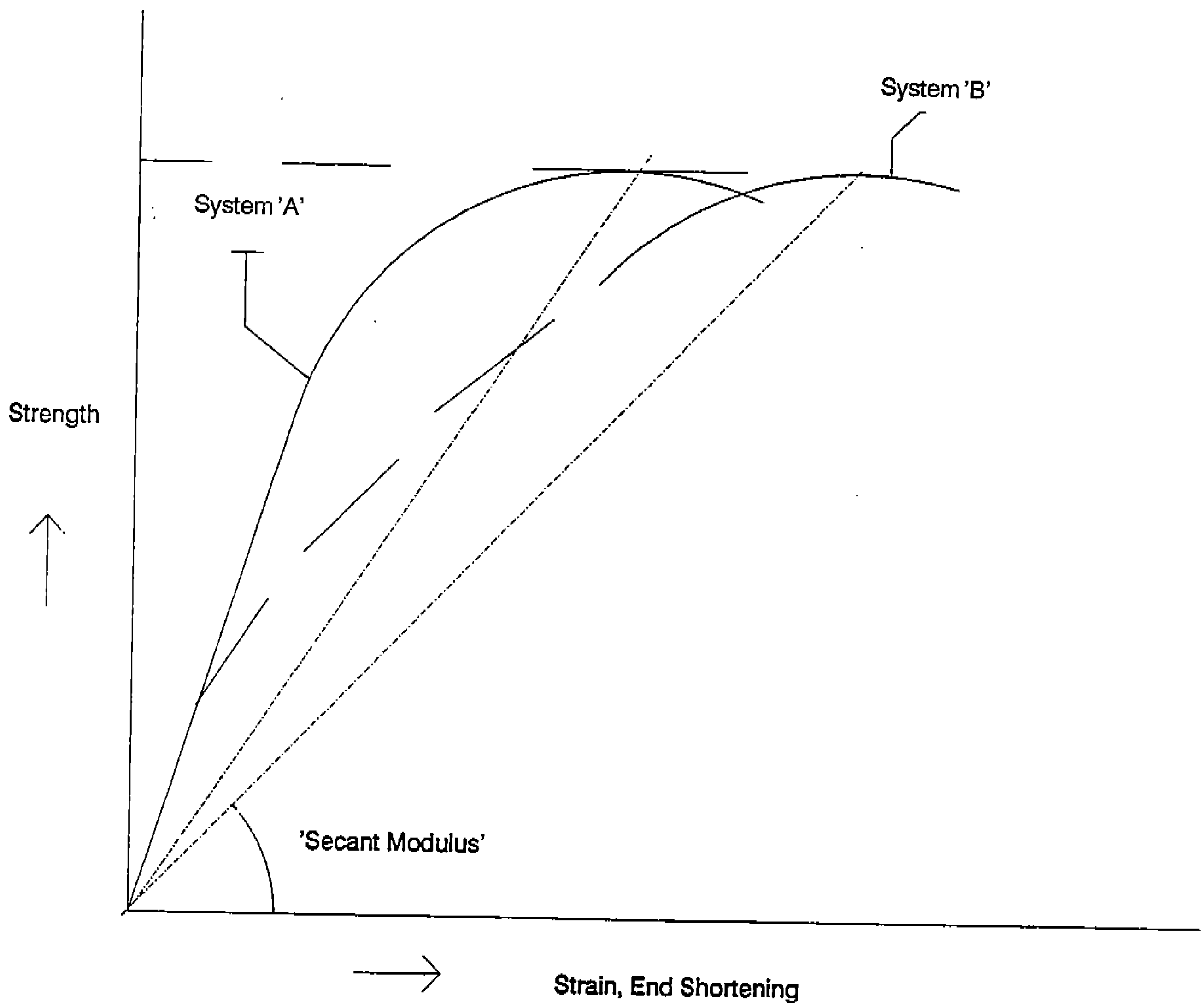

Fig.10.1 Two Alternative Systems with the Same Overall Strength

For inplane axial compressive loading and the 'redundancy' provided, or potentially provided, by multiple stiffeners the following discussion for longitudinally stiffened flat and slightly curved panels is equally appropriate to longitudinally stiffened cylinders.

The presence of longitudinal stiffeners should make a cylinder more damage resistant and damage tolerant.

□ Unstiffened Flat or Slightly Curved Plates

Clearly a flat or slightly curved plate has boundaries at which there is some form of support, e.g. at bulkheads, frames, deep girders, etc. Again, for this type of structural member there is no real quantifiable measure of redundancy for association with the various force systems that may be applied.

As is the case for the analysis of most structural components there will be apparent reserves of strength provided by the differences between the generally used small deflection response analysis methods and the more complex often large deflection response of such elements as they approach their true failure conditions.

A simple example of this is given by the response regimes for flat rectangular constant thickness plate elements subjected to uniform pressure loading. There are four regimes for which there are 'strength' formulae:

(i) small deflection, elastic response
(ii) small deflection, plastic hinge type response,
(iii) large deflection, elastic response, and
(iv) large deflection inelastic response.

(The upper levels of strength can only be realised if the supporting sub-structure remains stable and adequately stiff.)

One significant difference however between flat or slightly curved panels and unstiffened or ring stiffened cylinders is clearly that provided by the boundary supports to the flat/slightly curved panels to which load can be shed as panel failure begins to be approached. A particular example is the diagonal tension mechanism which developes after shear buckling capability has been exceeded and which results in both axial and bending forces being imparted on the elements bounding the plate, Figure 10.2. However, again, this is not redundancy in the degree of indeterminancy sense, although it does represent the formation of a new self-equilibriating internal force system and which was postulated earlier as being indicative of a measure of redundancy.

□ Longitudinally Stiffened Flat and Slightly Curved Panels

Consider uniform inplane compressive loading applied to a panel of uniform proportions. In the design process leading to the spacing and sizing of longitudinal stiffeners it is usually assumed that all stiffeners are equally loaded and that all stiffeners have equal capability. Thus in the 'classical' sense there is no real redundancy and the entire panel could fail simultaneously when the appropriate external force was applied.

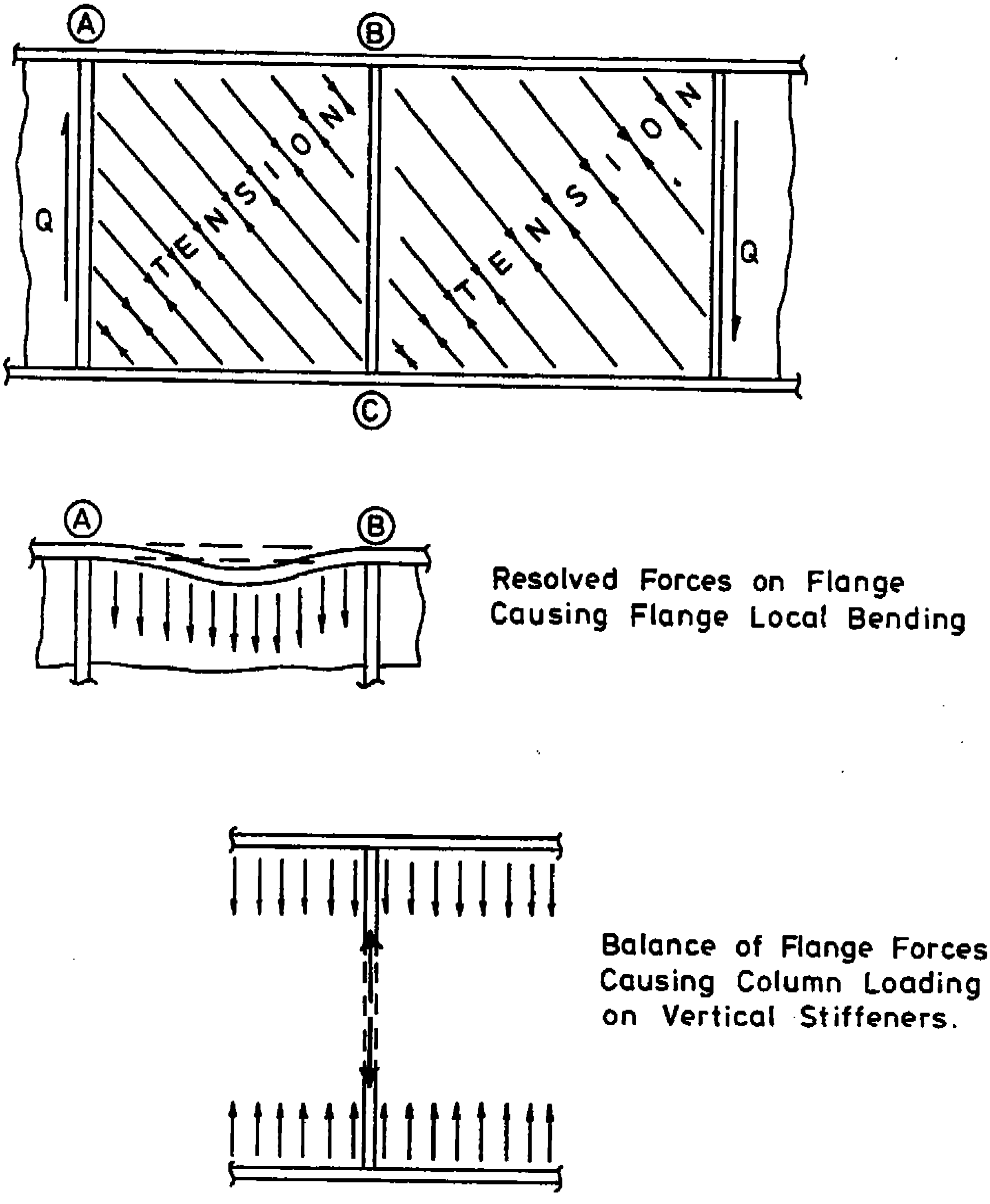

Fig.10.2 Diagonal Tension Behaviour in Girders

However in 'real' structures, in general, this would not happen for several reasons:

- The boundary supports along the longitudinal edges of the panel will influence the adjacent panel stiffeners and generally increase their load carrying capability.

- Some stiffeners will, statistically, fail before other notionally identical stiffeners fail owing to:

    - slight variations in geometry, scantlings, material properties, etc.

    - variations, (slight to modest) in geometric imperfections, weld induced residual stresses, weld induced distortions, etc.

However, it is also appropriate to note that some of the above effects could be, in a multi-stiffened panel, ameliorated by the gradual inelastic non-linear response that generally developes as individual stiffeners begin to approach their ultimate strength level. This produces somewhat of an 'averaging-out' of capability across the breadth of the panel type of effect. This is a beneficial effect, in a form of redundancy, that occurs in multi-stiffened panels compared with ones that have only a small number of stiffeners.

In many structures, e.g. frameworks, the effects of redundancy can be examined by selecting and removing individual elements, whilst keeping the rest of the geometry unchanged. However, in following conventional design practices, the removal of a single stiffener from a multi-stiffened panel would immediately reduce the overall strength of the whole panel, in at least a direct proportion and generally more so. The strength of the panel would reduce to at least (N-1/N)% of the original, where N is the original number of stiffeners. Localised redistribution effects around, say, damaged stiffeners to the immediately adjacent stiffeners, as illustrated in Figure 10.3, would probably result in reducing the overall panel strength by a greater degree.

Clearly even if a stiffener is not removed and it fails under axial loading in a precipitative manner (i.e. rapidly unloading) at a high fraction of the failure load of the adjacent stiffeners then their failure could be anticipated, subsequently cascading the failure to the other stiffeners. However again this effect is diminished in many well proportioned stiffener designs in that some modest inelastic plateau effects develop at the local ultimate strength level and thus avoiding the precipitative unloading actions.

Thus for uniformly stiffened flat panels although there is no redundancy in the classical indeterminancy sense owing to the modelling assumptions that have been made there is a form of redundancy in an internal response equilibriating effect provided by the numbers of parallel stiffeners. Clearly if there was only one stiffener then failure of that stiffener equates to panel failure, i.e. there is no redundancy, however in panels in which there are several stiffeners when the first stiffener to begin to fail does so it is likely to be supported by the other stiffeners, hence there is a form of redundancy. Similarly a multi-stiffened panel is much more likely to have an appreciable residual strength capability than a single element stiffened

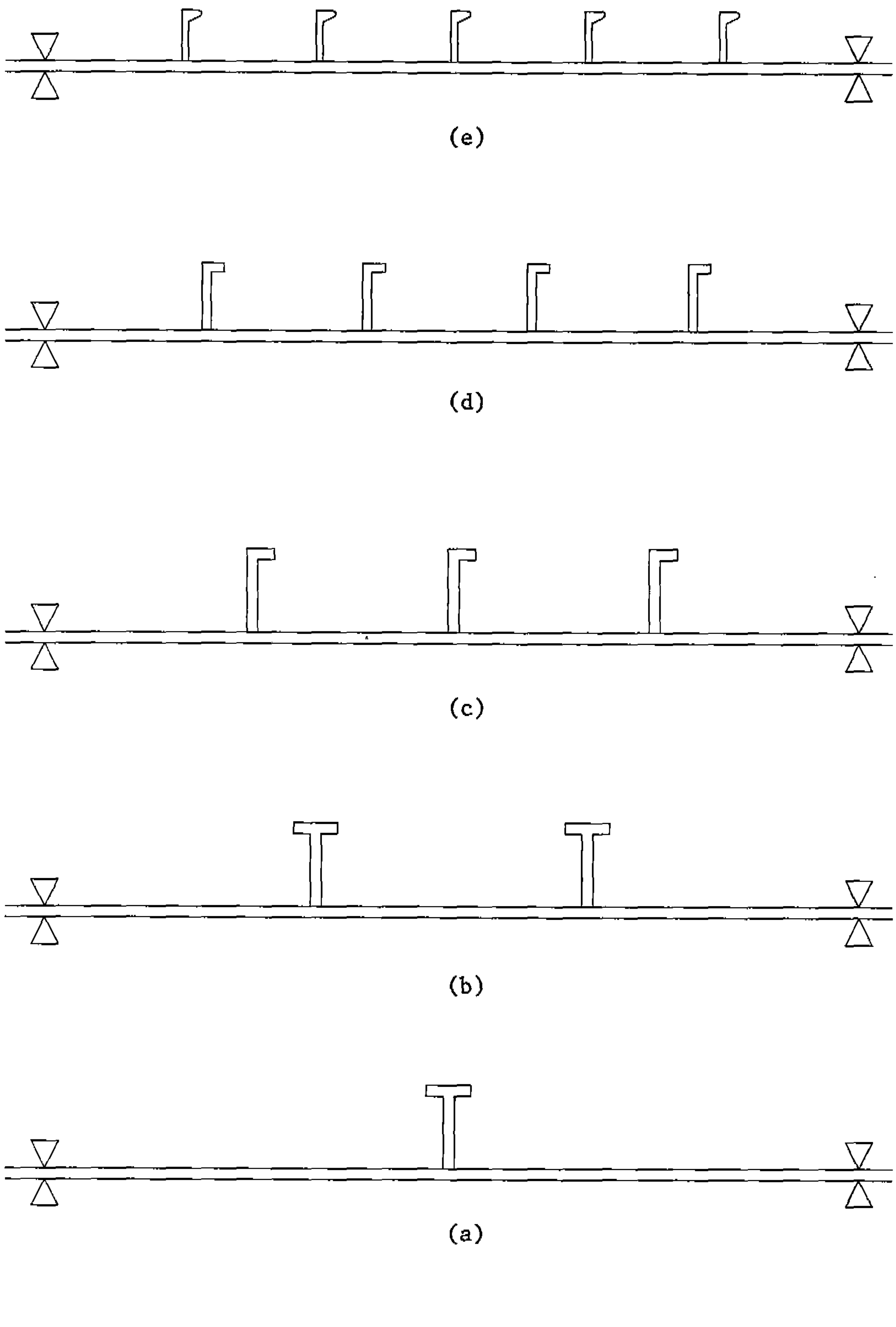

Fig.10.3 Uniaxial Panel Stiffening Alternatives

panel. Damage tolerance should thus increase with increase in number of stiffening members.

This form of redundancy is not readily numerically quantifiable nor is it a direct function of the numbers of stiffeners on a specific panel. However it will only become of significance on panels which have an appreciable number of stiffeners, e.g. 5 or greater.

Clearly the above discussion re possible views on redundancy within stiffened flat panels is not the same as designing minimum weight panels. For a given overall width of panel between supports and for a given design axial compressive loading and an associated factor (or factors) of safety calculations would need to be made to determine the number and size of stiffeners and panel thickness to give the least overall weight (assuming no other criteria or loading conditions also need to be satisfied).

For a given width of panel, that is between major longitudinal supporting structure, the designer's task is to identify the basic plate thickness and number and size of longitudinal stiffeners which efficiently maintain integrity (for the appropriate limit state). In most real design cases practical requirements impose both minimum and maximum limits on the various dimensions. Where the panel is part of, for example, hull girder primary structure the designer also has to allow for the effects of changing panel scantlings on applied inplane loading (although the relationship is not a direct one obviously - indeed if all cross-sectional scantlings change by the same percentage then the inplane axial forces, e.g. lbs or Newtons/unit width, will not change). Increases in numbers of longitudinal stiffeners will increase the load carrying efficiency of the plate material (a function of local buckling). However, as each stiffener adds area (and hence weight) there will be diminishing returns past a certain number of stiffeners.

The range of panels shown in Figure 10.3 could each be proportioned to carry the same applied forces. However by inspection, and allowing for the differences between 'real' and theoretical/ideal structures:

(i) intuitively (e) possesses a higher level of implied redundancy than (a), and

(ii) (c) is probably much more damage tolerant and has higher residual strength than (a), however

(iii) (a) is likely to be the more stable and may have higher reserve strength (i.e. could function closer to the material yield stress level as far as the central region is concerned but not the average panel-wide stress).

□ Multiple Bay Uniaxial Loading - Compression

Transverse connected structure, for example in the form of deep beams/girders, web frames, etc., support the areas of stiffened plate surfaces in a manner which results in the 'length' dimension, which is a prime factor in stability/strength determination, being reduced to a direct function of the spacing or distance between the transverses.

Assuming uniaxial inplane panel loading and simple unbracketed connections between the longitudinal and transverse structures the controlling length for stability and strength calculations becomes the distance between adjacent transverse structures. The stiffened panel deflection mode becomes a simple symmetrical pattern of inwards then outwards deflections between adjacent spans. (As this may involve one span deflecting in an opposite direction to its normal single span "preferred" failure direction this can affect an improvement in the overall panel performance, averaged between two adjacent bays.)

If the connection between the longitudinal structure and the transverse structure involves heavy brackets and a torsionally stiff transverse structure, then this can reflect in an effective pin-ended panel length somewhat less than the distance between adjacent transverse structures, with a possible attendant increase in load carrying capacity.

(The case of combined inplane and lateral loading provides the third possibility where, subject to the relative magnitude of the lateral loading, the stiffened panel responds with zero slope across the supporting transverse element giving effectively a fixed-ended column model and with the equivalent pin-ended column length being equal to 50% of the distance between adjacent transverse structures.)

The above assumes that the transverse structure is 'stiff' in its own plane. If the transverse structures are only modest beams, having, say, cross-sectional second moment of area values only a few multiples of that of the longitudinals plus attached plate flange then the overall structure will function as a large plated grillage supported at its boundaries by appropriately stiff structure. This, for example, could represent a whole deck between bulkheads.

Clearly failure of a transverse member, or connection between longitudinal and transverse members will result in a considerable reduction of inplane strength. Apart from shedding load to adjacent panels there are no alternative load paths and hence no redundancy at this level of general performance within the normally assumed definition of redundancy.

However the structure should have some capacity to withstand damage with load redistribution taking place around the region of the damage, subject of course to the severity and extent of the damage. There is thus some inherent redundancy within the structure.

□ <u>Damaged Single Span Stiffened Panels</u>

Damage may be from either direction, i.e. from the stiffener side or from the plate side and may be to either or both the plate material or one or more stiffeners. The damage may be either in the central region of the panel or local to one of the long or short edges. The most adverse location, with regard to simple uniaxial loading, will be in the central region of the panel's breadth, Figure 10.4.

Clearly damage to a stiffener, e.g. to the free flange/bulb, will appreciably lower that stiffener's axial load carrying capability and damage to a plate element between, say, two stiffeners could appreciably lower the strength of the two adjacent stiffeners.

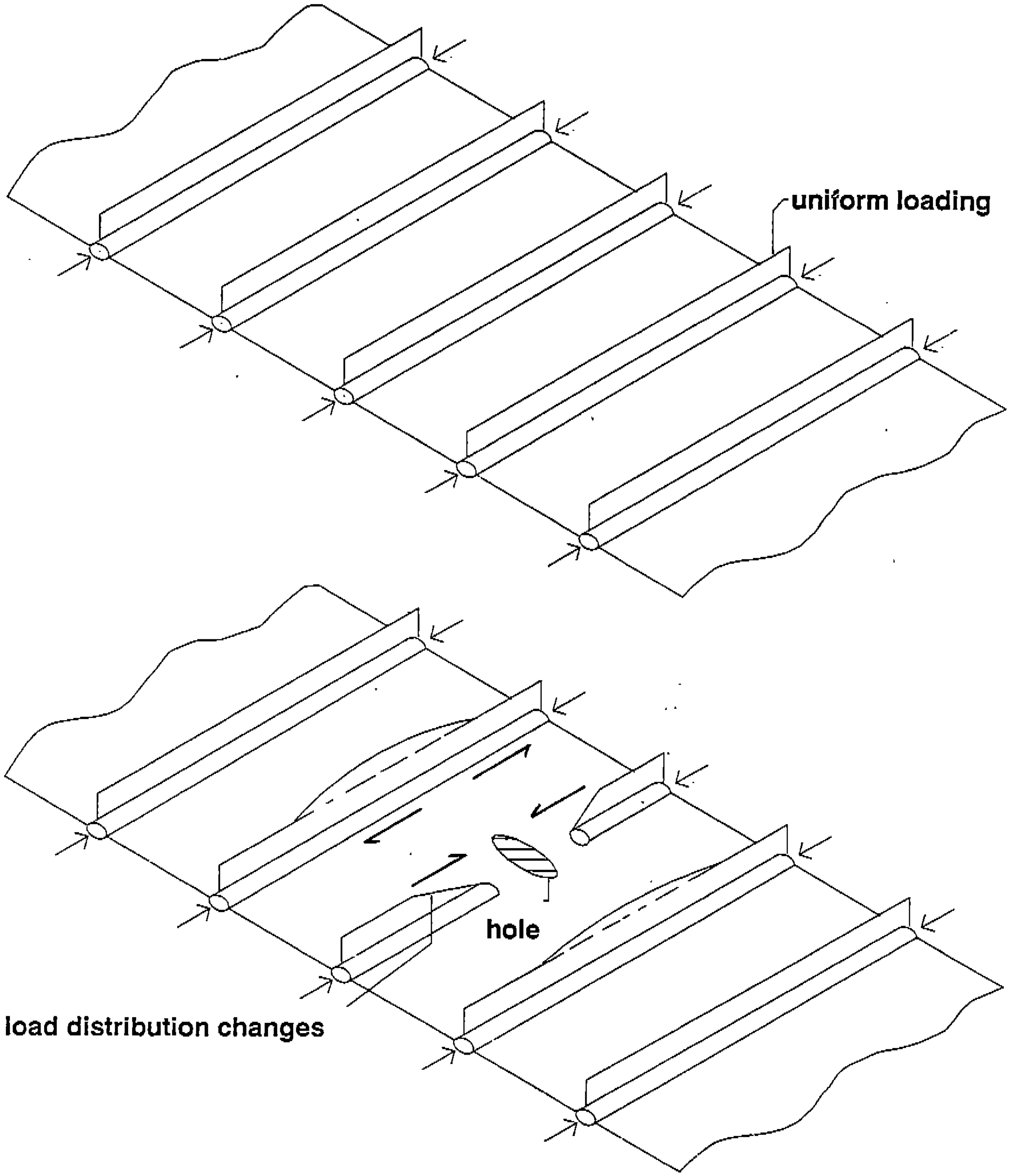

Fig.10.4 The Effect of Local Damage on Stiffener End-Load Variation

The larger the number of stiffeners on a panel the higher is likely to be the residual strength following the occurrence of damage to the panel. Due to 'stress concentration' type effects, i.e. placing a higher burden on the undamaged structure immediately bounding the zone of damage, the residual strength will not be simply proportional to the number of undamaged stiffeners, unless all stiffeners have a significant 'strength' plateau.

□ Components in Tension

For the range of component types reviewed and discussed earlier in this section there is no form of internal redundancy related to overall tensile loading systems. Tension failures will relate to excessive yielding, fatigue cracking and fracture (either ductile or brittle).

Many as-built components will contain local stress concentration features. Local yielding, failure or damage will result in stress pattern re-distributions around the particular sites. For simple stress concentrations local yielding (i.e. exceeding the material's elastic limit) diminishes their effect and a state of more uniform strain/stress gradually develops. An exception to this is when large cracks, or damage with sharp corners, exists and where tensile loading, particularly if of a cyclic nature, may cause crack propagation - leading to either ductile or fast brittle failure.

Local damage, e.g. accidentally caused, involving structural deformation will result in both residual stresses and local bending effects when overall tensile or compressive loading is applied to the component. However tensile loading may tend to diminish the distortions and compression loading to increase the magnitude of the distortions and hence the latter will be the more unstable situation.

Hence again there is no form of direct local component internal measure of redundancy that can be reflected in the design-analysis process. All of the components when in tension will have some degree of residual strength after damage. Any reserve strength, of undamaged structure, compared with the maximum design demand, will be based upon either the material's ultimate tensile strength or some limiting strain condition, depending upon the stress-strain characteristics of the material.

However for point forces, as distinct from uniformly distributed forces, some intangible form of redundancy may be attached to the shear lag response behaviour, e.g. in multi-stiffened panels.

□ References

10.1 Paliou, C., Shinozuka, M. and Chen, Y-N. "Reliability and Redundancy of Offshore Structures", ASCE, Journal of Eng. Mech., Vol.116, No.2, February 1990.

11.0 DISCRETE STRUCTURES - ILLUSTRATIVE MODELS

11.1 General Redundancy Considerations

For discrete member structures such as trusses and beam-column frameworks the general traditional concept of redundancy is well defined and understood. It is associated with the concepts of overall stability and member force determinacy. A stable structure [11.5] is one which is in a state of static equilibrium and a discrete stable structure is statically determinate with respect to the applied forces, including reactions, when all the individual component forces can be completely determined by applying the equations of static equilibrium. If that is not the case then the structure is considered to be statically indeterminate or hyperstatic and the degree of indeterminacy is equated to the number of unknowns over and above the number of condition equations available. The excess reaction components are called redundants because they are unnecessary for the stability of the structure. This traditional view of quantifying redundancy becomes difficult to apply to complex ship and other floating type structures.

Individual elements of structures can also be internally redundant. Structures may be made to be redundant either by design (e.g. with collision safety in mind or for some particular operational requirements) or by the fabrication/production approaches taken. However in the case of continuous structures, e.g. hull girders, any considerations of redundancy need to be related to the characteristics of failure. For example any assessment of redundancy vis a vis panels having multiple stiffeners and failing in compressive buckling will be quite different from failure due to brittle fracture. A fracture propagating across a stiffened panel may be momentarily arrested by each stiffener as it is approached however this is clearly not indicative of any form of redundancy.

The current practice for the scantlings design of fixed platform structures is generally based on API RP2A [11.1] or similar rules published either by classification societies [11.3, 11.4] or regulatory bodies, e.g. DoE [11.4]. The structural design of these platforms is mainly governed by component strength checking procedures and they are based on the working stress approach using traditional factor of safety concepts which limits a stress value. The exceedance of this limiting stress in a particular member constitutes an unacceptable condition for both the member and for the structure as a whole, regardless of the degree of redundancy and associated reserves of strength. In addition to the reserve strength of the individual members in, for example, a fixed platform structure, the structure as a whole is likely to be structurally redundant and hence the reserve strength of the whole system against failure compared with the design loads is likely to be very high depending on the degree of redundancy and efficient material utilisation. For example the reserve strength could be the same for a highly optimised design and certainly much higher than the simple component level safety factor built into the code.

A general measure of internal redundancy which was postulated earlier has been assumed in this study and this is the redundancy index (RI) and which is given by

$$RI = \frac{P_u - \overline{P}_u}{\overline{P}_u} \tag{11.1}$$

in which P_u is the ultimate strength of the structure under consideration and $\overline{P}_u$ is the ultimate strength of the parent structure. The parent structure is one in which all members that are not absolutely necessary for stability have been removed [11.5].

However this definition introduces some uncertainties. For example:

- is the geometry otherwise unchanged
- are the scantlings otherwise unchanged
- are the members removed on the basis that they have failed and have themselves no residual strength or stiffness
- are there any relationships, geometry or scantlings, etc., to be maintained between the actual structure and the parent structure.

The reference to a so-called 'parent' structure could cause some difficulties in complex structures. It is possible that more than one 'parent' form could be identified in some complex arrangement with correspondingly different results, viz a viz redundancy index and ultimate strength of the 'parent'. The identification of a minimum practicable 'parent' structure needs to be undertaken with regard to the full functional requirements of the overall structure (e.g. the support of a deck) and the pattern, or patterns, of the applied forces. For example if for the selected analysis example employed in the following study there were no deck support requirement and there was only the simple singular applied force then the parent model could be simplified to a single vertical beam element. However, such a geometric form would have little practicable value in most design situations. Alternative forms of 'parents' could be postulated having less redundancy than the selected parent in the following numerical analysis. For some forms of structure, the concept of 'parent' structure may be useful in comparative assessment but however this is not universally applicable to complex continuous structures.

The 'parent' structure thus can be taken to establish the most practicable base line structure, (vis a vis the overall functional and operational requirements) although not necessarily the minimum stable configuration and against which the effects of increases in redundancy as a result of adding further discrete elements can be assessed.

In the simple model employed in the following study each element adds a bending moment and an axial end load capability, thus potentially numerically increasing the degree of redundancy by the same amount - although not necessarily adding to the system strength in a similar proportion. Clearly whilst adding a member increases the degree of indeterminancy, redundancy, the location and dimensions of a member may be such that, for a given application of external forces, the member may be either critical or non-critical to the overall response of the structure. Thus for a complex offshore structure the concepts of a parent structure and a single redundancy number for the entire structure are not easily grasped.

The parent structure approach is thus clearly most useful for making illustrative comparisons, rather than as a direct design tool. However a series of numeric studies could be proposed, as a continuation of this initial study, and in which families of real structures could be examined, representative of existing inservice structures of various types. These studies could employ design variations to give a minimum practicable parent configuration, for each structural family, as well as various degrees of redundancy. The comparative results from such studies would then provide a measure of the implications of redundancy in full sized real structures.

It is to be remembered that the degree of redundancy, however quantified, relates to the overall structure and not to individual components within that structure. Hence the significance of a component, i.e. whether or not it plays a critical role in the performance of the overall structure, is unrelated to its contribution to 'degree' of redundancy.

It is part of this overall study to devise and expound upon a more useful definition (and quantification) of redundancy and thus the definitions and the associated numeric evaluations used in this section are for expediency until such new definitions can be postulated.

11.2 Analysis Process

In order to determine the maximum overall load of a given distribution which a structure can carry safely up to point of failure, one method to calculate this is to simply perform an incremental numerical analysis using the relevant non-linear response formulations. In a collapse analysis, the equations of equilibrium are satisfied for each increment of monotonically increasing loads, or time steps, using, for example, the total Lagrangian (TL) formulation.

$$(t_o\ K_L + t_o\ K_{NL})\ \Delta U^{(i)} = (t + \Delta t\beta)\ \Delta t_R - t + \Delta_o\ tF^{(i-1)} \qquad i=1,2 \qquad (11.2)$$

where

$\Delta U^{(i)}$ = Vector of increments in the nodal point displacements in iteration i,

$$t{+}\Delta t\ U^{(i)} = t{+}\Delta t\ U^{(i-1)} + \Delta U^{(i)}$$

Δt_R = Load vector for the first load step

$t{+}\Delta t\beta$ = Variable which scales Δt_R to obtain loads corresponding to time $t{+}\Delta t$

$t_o K_L$ = Linear strain incremental stiffness matrix

$t_o K_{NL}$ = Non-linear strain incremental stiffness matrix

$t{+}\Delta t_o\ F^{(i-1)}$ = Vectors of nodal point forces equivalent to the element stresses at time $t{+}\Delta t$ and iteration (i-1).

The Lagrangian formulation usually represents a more natural and effective analysis approach than the Euleran formulation usually used in the analysis of fluid mechanics problems. The formulation and development of this equation is quite complex and may be seen in ref[11.7]. This method is the basis of the non-linear analysis program SOLVIA [11.6] used to solve the illustrated example problems later.

Non-linear structural analyses involve, generally, large displacements of the elements within a structure, compared that is with the assumption of small displacements in conventional linear static elastic analyses. Large displacement non-linearities may also be associated with material stress-strain non-linearities. Various formulations have been developed by different researchers for the numerical method based analysis of structure discretised into assemblages of finite elements and in particular Lagrangian formulations have been employed.

The problem of non-linear response is to ensure that equilibrium is maintained throughout the monotonically increasing applied load history. The mathematical methods use the ploy of assuming that a force system is applied in a time dependent manner even though the actual non-linear problem being solved may itself be time independent, which is the general case. According to Bathe [11,11] the use of the time variable to describe the load application and history of results represents a very general approach and corresponds to the assertion that a 'dynamic analysis is basically a static analysis including inertia effects' - this applies whether or not the response is linear or non-linear. The response calculation is carried out using a stepwise incremental process with a number of iterations necessary to reach a condition of equilibrium for a given level of applied load. A typical widely used iteration process is a derivation of the Newton-Raphson method for the solution of a set of simultaneous non-linear equations.

In the determination of the response of a structure in finding a new position of equilibrium when the external forces have been increased, the analysis process, [11.11] is to follow all the particles of the structure in their motion from the earlier condition of equilibrium to their new position. For this a Lagrangian (material) formulation is employed. This is in contrast to the fluid mechanics problem of the motion of material through a stationary control volume (implying fixed boundaries), [11.11], and suitably adapted versions of finite element method based analysis procedures can be employed. Eulerian formulations of a non-linear problem could be employed, however the approach is more difficult than for a Lagrangian formulation which for the analysis of deformable solids and structures represents a more natural and effective approach.

The collapse state of the structure is reached when for a small additional load increment the displacements become relatively disproportionately large. Physically this means that the overall stiffness of the structure becomes small compared with the loads.

11.3 Selection of Models

As this overall study is perceived as being a pilot to a potentially much more detailed and exhaustive study it was considered at the onset that any numeric assessments should use simple illustrative examples of discrete systems (structures) - indeed this was specified within the project's workscope.

Real offshore steeljacket structures, for example, are generally quite complex multi-element three dimensional frameworks having very high degrees of redundancy. Thus in such structures the sensitivity to degrees of redundancy and to the effects of failure of individual members, whilst calculable, will generally show only small numeric differences in the global measures of performance being assessed. Thus any results and associated trends will most likely be somewhat obscure.

On the other hand, when using simple models the results and any trends are much more graphic and obvious - even though the analytical processes and principles are the same as those for the more complex structures.

On this basis the following numeric examples employ a very simple two-dimensional beam element framework arrangement.

11.4 Simple Two-dimensional Plane Frame Examples

In order to provide an illustration of the significance of redundancy, a series of simple plane frame structures, as shown in Figure 11.1, have been considered. These structures were developed and made progressively more redundant by incorporating additional members into the original parent structure. The parent structure is itself however redundant by virtue of the assumed external fixing/support conditions and full connectivity between the three beam elements. Initially these structures were considered on an equal total volume of material basis. An equal volume basis was chosen in order to keep the total weight of each frame constant. For the same weight of the structure, a study of redundancy is thus made in all the frames. Clearly this is an assumption for the purpose of developing comparative measures of performance and other modelling assumptions could also be employed. Thus the exercise was repeated with all elements in all models having the same sectional geometry as an alternative assumption.

The Frame No.1 as shown in Figure 11.1 has uniform section members and is made up of a single grade of steel material having a yield strength, σ_Y, of 36000 lbs/in^2. The geometrical dimensions of the frame are as shown in the same figure. Each member is assumed to be a tubular section of 12" external diameter and 0.25" wall thickness. The total length of the members in the whole frame is 30ft and hence the volume of material is 3322 in^3 (neglecting the effects of intersections). The details of the frame are given in Table 11.1(a). This frame can be regarded as the parent form and the other models were developed from this.

For this study, a simple elastic-plastic response mode analysis was considered (i.e. local section instability and overall instability are ignored). It is to be noted that plane frame action is only included in the models i.e. out-of-plane displacements are suppressed. For example the model for frame No.2 employs only one element for the diagonal bracing member. For the diagonal buckling is then not possible and bending is not described with any accuracy. Axial load action is then predominant in the collapse behaviour of the diagonal. The frame was analysed using the non-linear finite element program SOLVIA [11.6], which is basically the ADINA 84 program [11.7]. The ultimate point load P_u is given as the load at which the overall structure becomes a plastic mechanism and thus at which the deflection of the structure increases in an unrestrained manner at a constant load. (The effects of strain hardening are ignored.)

For Frame No.1, the collapse load was found to be 42.84 kips (1 kip = 1000 lbs) as shown in Table 11.1(a) (the model uncertainty factor X_m, was assumed to be unity in all the frames analysed). The model for frame No.1 is shown in Figure 11.1 and the corresponding load deflection plot is shown in

Table 11.1(a) : **Design and Collapse Loads for Discrete Framed Structures (on an equal volume of material basis)**

Frame No.	Total Length of Members (ft)	Volume of Material (in^3)	Area of Cross Section (in^2)	External Diameter (in)	Wall Thick-ness (in)	P_u (kips)	RI	P_d (kips)	RSI	X_m
1	30	3322	9.228	12	0.25	42.87	0	17.67	2.43	1
2	44.14	3322	6.272	12	0.169	184.7	3.31	82.39	2.24	1
3	58.28	3322	4.750	12	0.128	252.3	4.89	112.68	2.24	1
4	51.21	3322	5.405	12	0.145	164.8	2.84	59.50	2.77	1
5	51.21	3322	5.405	12	0.145	174.8	3.08	53.03	3.29	1
6	51.21	3322	5.405	12	0.145	174.0	3.06	49.67	3.50	1
7	51.21	3322	5.405	12	0.145	155.50	2.63	62.11	2.50	1

(1 kip = 1 thousand lbs)

NOTE: The RSI column shows that on the basis of equal volumes of material there is a change in RSI value in these frames. The frames are not designed sequentially to show improvements in RSI but only to show the change in RSI whilst maintaining the total volume of material in the overall structure.

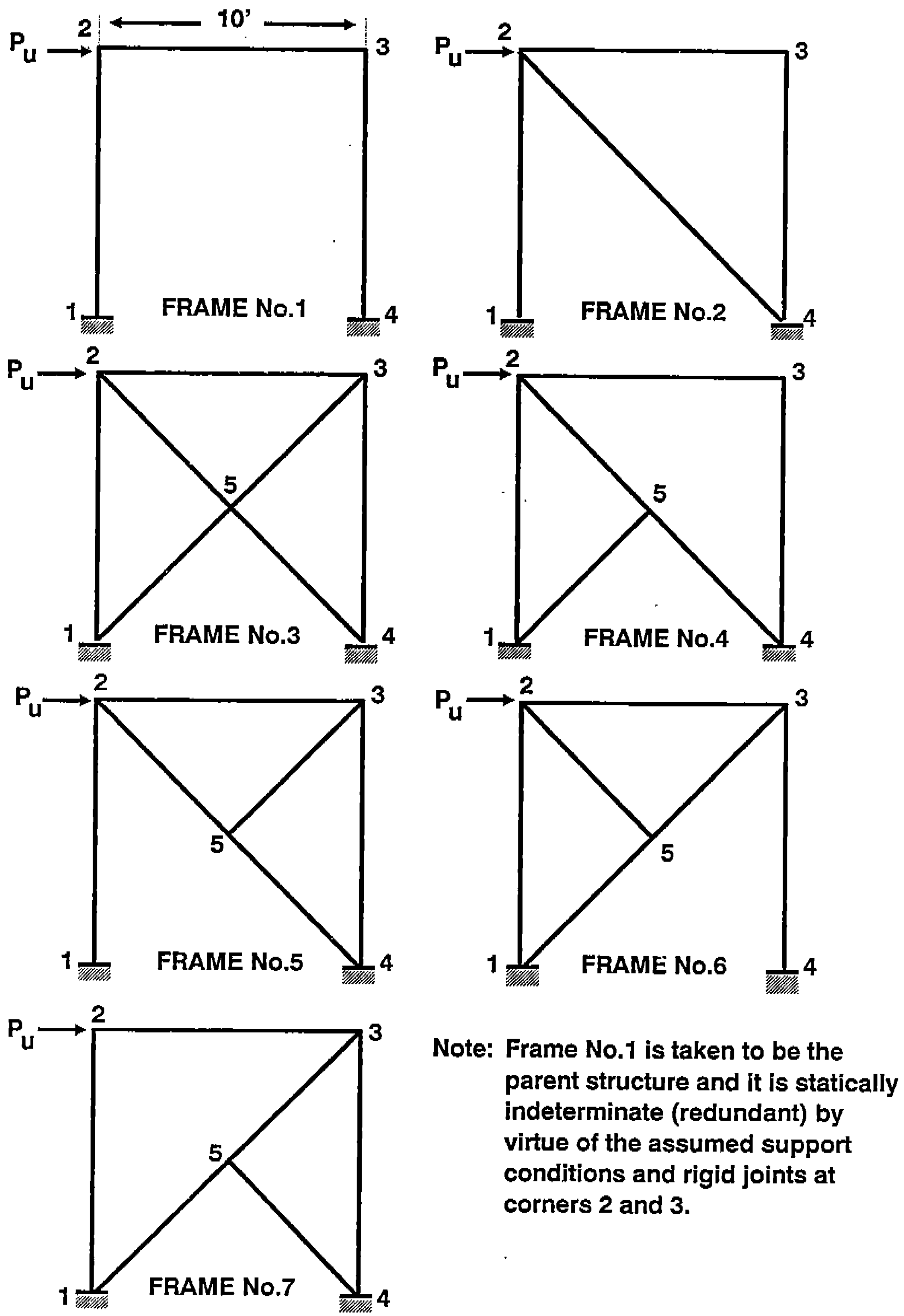

Fig.11.1 Discrete Frame Structures

Figure 11.2. The load multiplier 'lamda' is the actual collapse load since the initial load was unity in the assumed input data. The automatic load increment procedure, a feature within this computer program, is used in this analysis.

The pure plastic moment capacity, M_p, of each individual thin walled tubular member in Frame No.1 is given by:

$$\begin{aligned} M_p &= D^2 t\, \sigma_Y \\ &= 12^2 \times 0.25 \times 36 \times 10^3 \\ &= 12.96 \times 10^5 \text{ lbs in.} \quad (14.96 \times 10^5 \text{ kgcm}) \end{aligned}$$

and the theoretical collapse load based upon a four plastic hinge mechanism can be calculated as follows:

$$P_u = 4\,\frac{M_p}{\ell} = \frac{4 \times 12.96 \times 10^2}{10 \times 12} = 43.20 \text{ kips} \quad (19.63 \text{ tonnes})$$

This value compares well with the results obtained from using the SOLVIA program. (The above simple 'hand' analysis assumes a pure plastic hinge without any modifications for combined shear and axial forces.)

The overall design load is generally calculated from simple elastic analysis by satisfying adequacy for the critical members according to some interaction formula, typical of which is

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leqslant 1 \qquad (11.3)$$

in which f_a and f_b are the stresses due to axial loads and bending moments respectively in the structural elements. The allowable axial compression (F_a) and bending stress (F_b) should be determined from the appropriate qualification codes, e.g. [11.1, 11.2, 11.3]. A recommended formula [11.1] for tubular members is as follows

$$\left.\begin{aligned} F_a &= 0.6\, F_{a'} \\ \text{where} \quad F_{a'} &= \sigma_y \quad \text{for } D/t \leq 60 \\ &= F_{xc} \quad \text{for } D/t > 60 \\ F_b &= 0.66\, F_{b'} \\ \text{where} \quad F_{b'} &= \sigma_y \quad \text{for } D/t \leq 60 \\ &= F_{xc} \quad \text{for } D/t > 60 \end{aligned}\right\} \qquad (11.4)$$

where $$F_{xc} = \sigma_Y \left[1.64 - 0.23\,(D/t)^{0.25}\right] \leqslant F_{xe} \qquad (11.5)$$

and $$F_{xe} = 2C\,E\,t/D \qquad (11.6)$$

where C is the elastic buckling coefficient and E is the modulus of elasticity. The theoretical value of C is 0.6, however a reduced value of C = 0.3

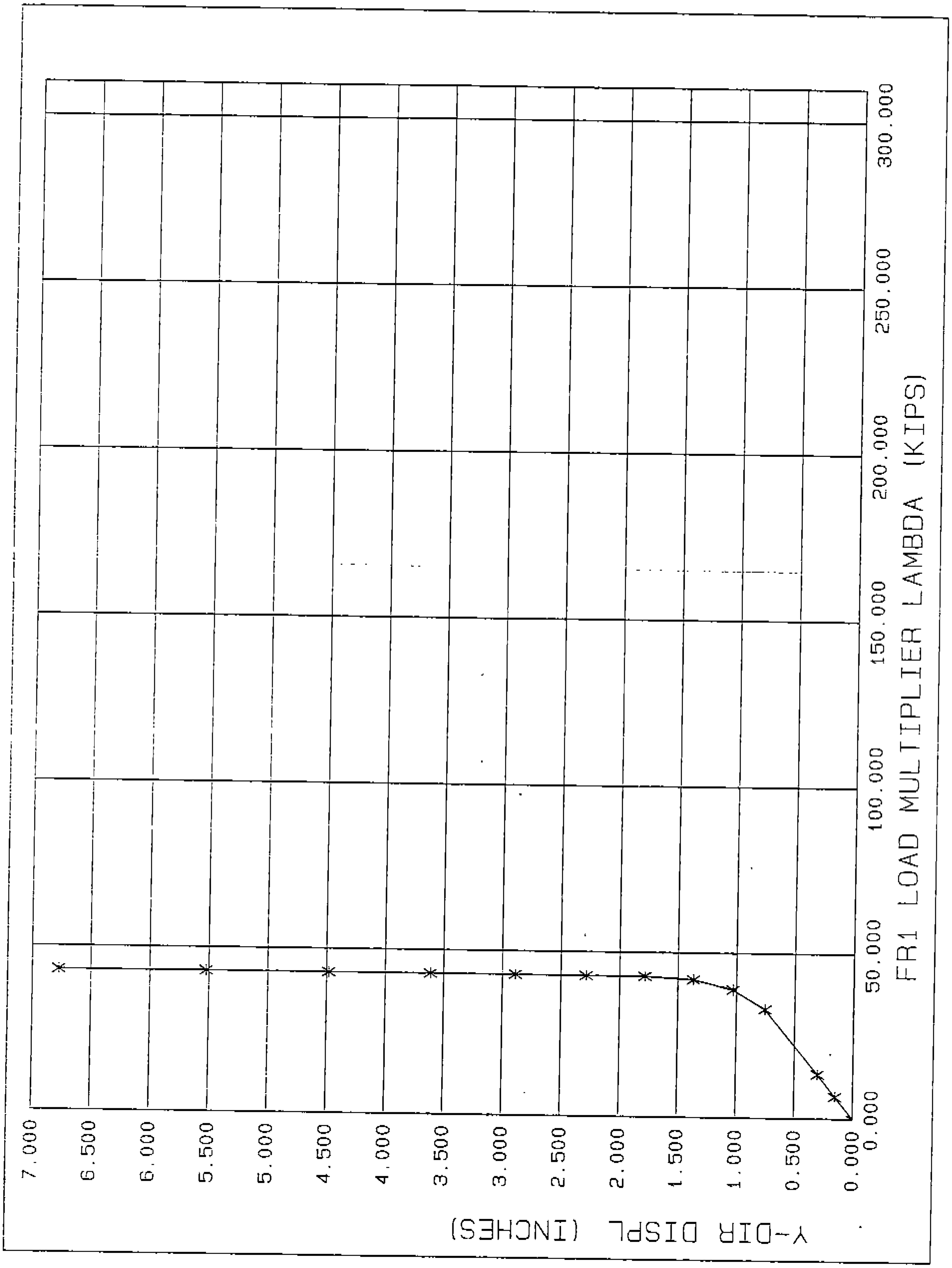

Fig.11.2 Side Load - Deflection Plot for Frame No.1
(Wall Thickness = 0.25 in)

is recommended for use in equation (11.6) to allow for the effects of initial geometric imperfections within API Spec 2B tolerance limits. In the case of axial tensile loads, F_{xc} is replaced by σ_Y, for all values of D/t.

Clearly equation (11.4) allows for a simple factor of safety and thus the appropriate numeric value of which will be contained within the following results. Equations (11.5) and (11.6) allow for local compressive instability of the thin walls of the tubular members in the form of crippling.

For Frame No.1, D/t = 12/0.25 = 48 and hence F_{xc} is equal to σ_Y, i.e. 36 ksi, for use in equation (11.4), (a robust stable section and thus compatible with the assumption that the beam elements in this section are capable of developing full plastic hinges). The maximum design load is calculated from equation (11.4) when the right-hand side is equated to unity, i.e.

$$\frac{f_a}{0.6\ F_{xc}} + \frac{f_b}{0.66\ F_{xc}} = 1 \qquad (11.7)$$

In using equation (11.7), the design load for Frame No.1 is found to be 17.67 kips. The reserve strength index (RSI), which is the ratio of the collapse load to the design load, is found to be 2.43. For the remaining frames, i.e. Frame No.2 to Frame No.7, the collapse loads and design loads were calculated using a similar procedure and they are as shown in Table 11.1(a). The load-deflection curves for all these frames are shown in Figures 11.3 to 11.8. It may be noted that when calculating the design load the critical member was subjected to either axial compression or tension and hence the appropriate interaction equation was used. The reserve strength index is seen to vary from 2.24 for Frame Nos.2 and 3 to 3.40 for Frame No.6).

It should be noted, however, that in Table 11.1(a) with the exception of frame No.1 the D/t ratios of the elements in the various frame models are greater than 60, i.e. some of them may fail locally by the onset of compression crippling before the material yield stress level is reached. However for the purposes of the overall frame collapse analyses the sections were assumed to be stable up to the material yield stress level, thus enabling the development of full plastic hinge capability.

For the calculation of the Redundancy Index, RI, the parent structure is taken to be the reference and hence its RI value is taken to be zero. The RI value is found to vary from 2.50 for Frame No.7 to 4.87 for Frame No.3.

The ultimate strength of Frame No. 3 was reevaluated assuming all the diagonals to have failed and was found to be 22.39 kips. This was about one-half of the ultimate strength of Frame No.1.

The 2D frames were again analysed, however this time with a constant section size of 4.75 sq.in. rather than constant total weight and the dimensions and results are shown in Table 11.1(b). The effect of member removal can now be seen from the values of P_u (kips) as shown in Table 11.1(b). The values of RI, RSI and RDI are also shown in the same Table 11.1(b). A typical load deflection curve for frame No.1 is shown in Fig.11.9.

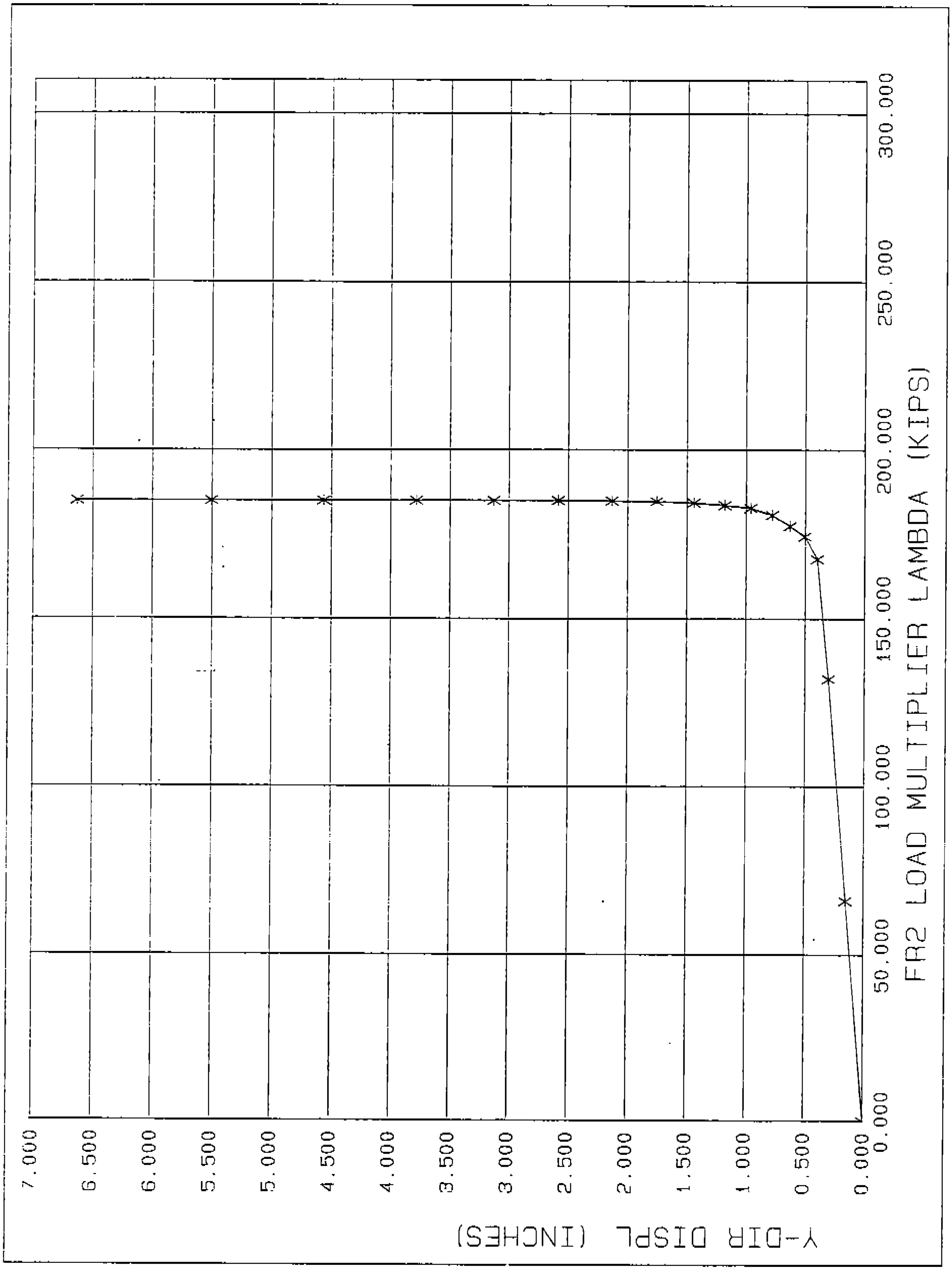

Fig.11.3 Load Deflection Curve for Frame No.2
(Wall Thickness = 0.169 in)

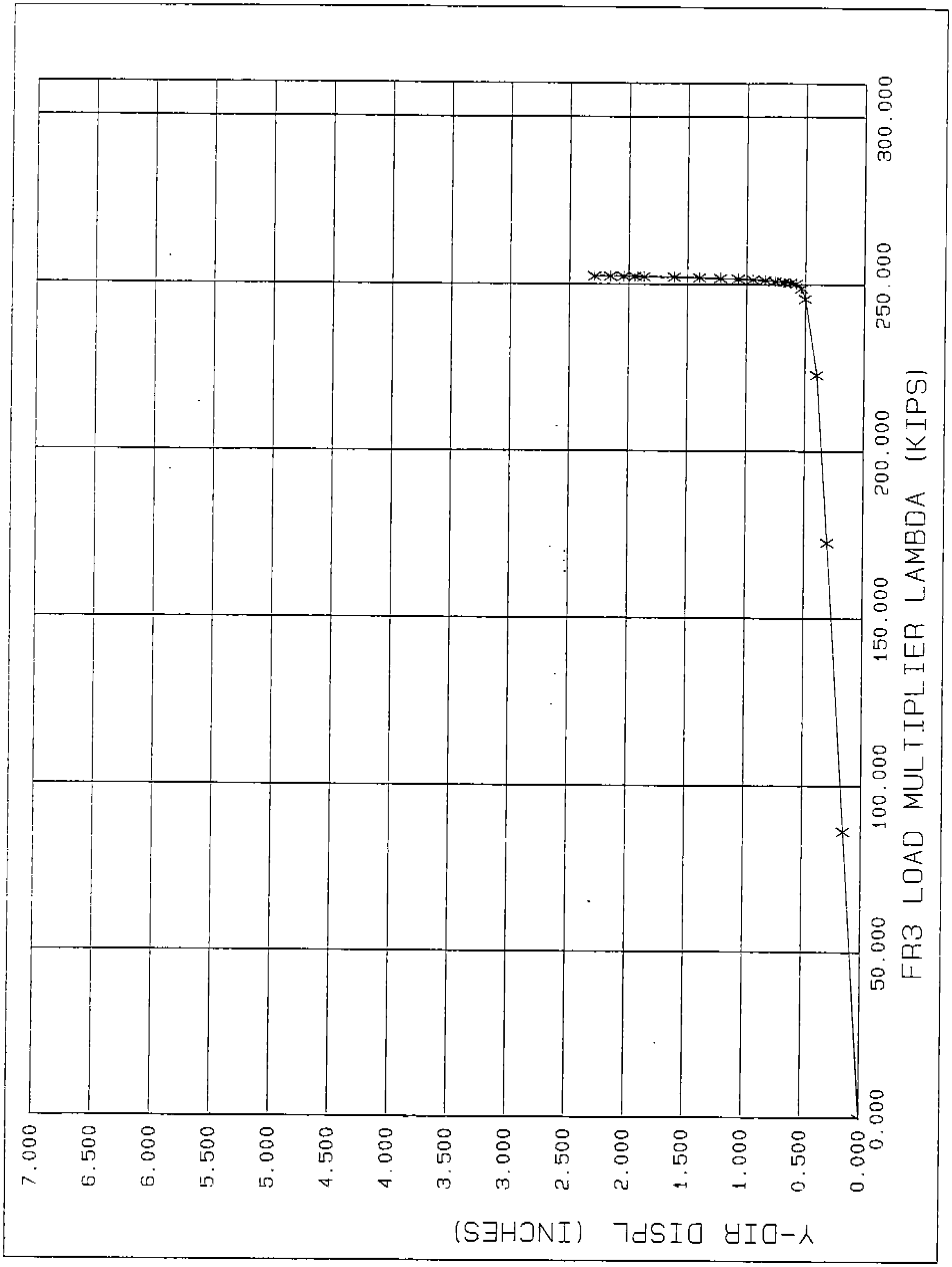

Fig.11.4 Load Defletion Curve for Frame No.3
(Wall Thickness = 0.128 in)

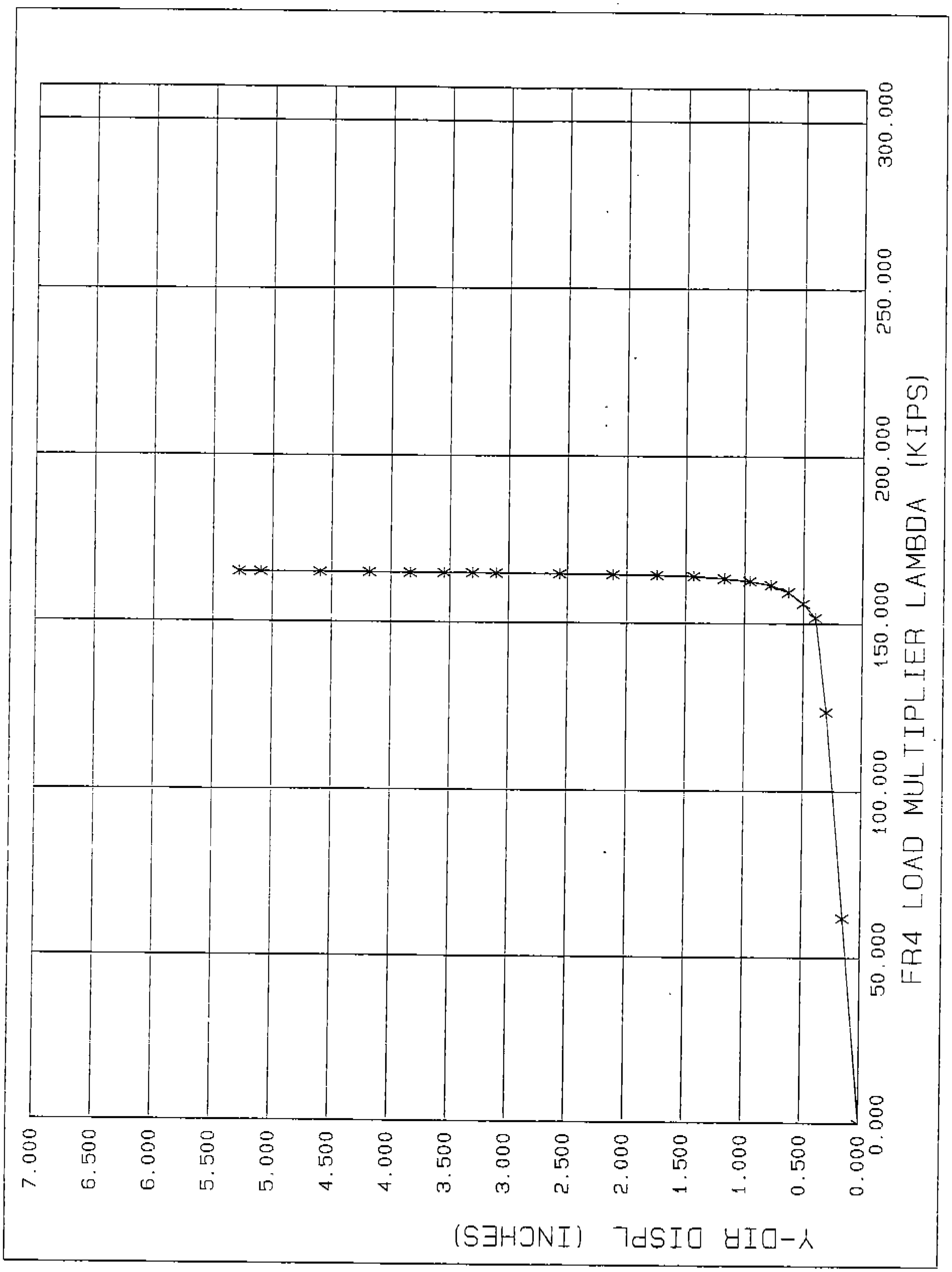

Fig.11.5 Load Deflection Curve for Frame No.4
(Wall Thickness = 0.145 in)

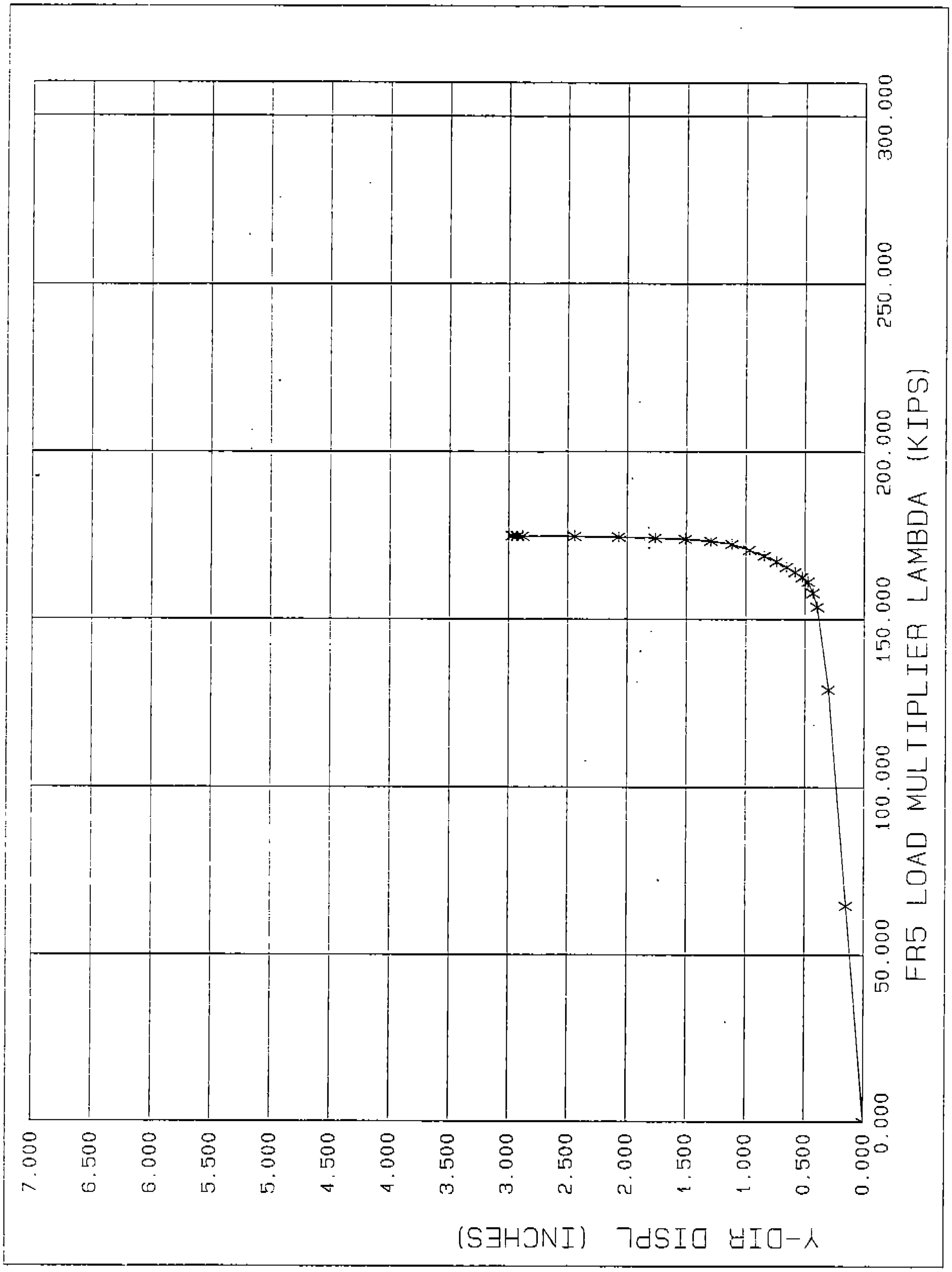

Fig.11.6 Load Deflection Curve for Frame No.5
(Wall Thickness = 0.145 in)

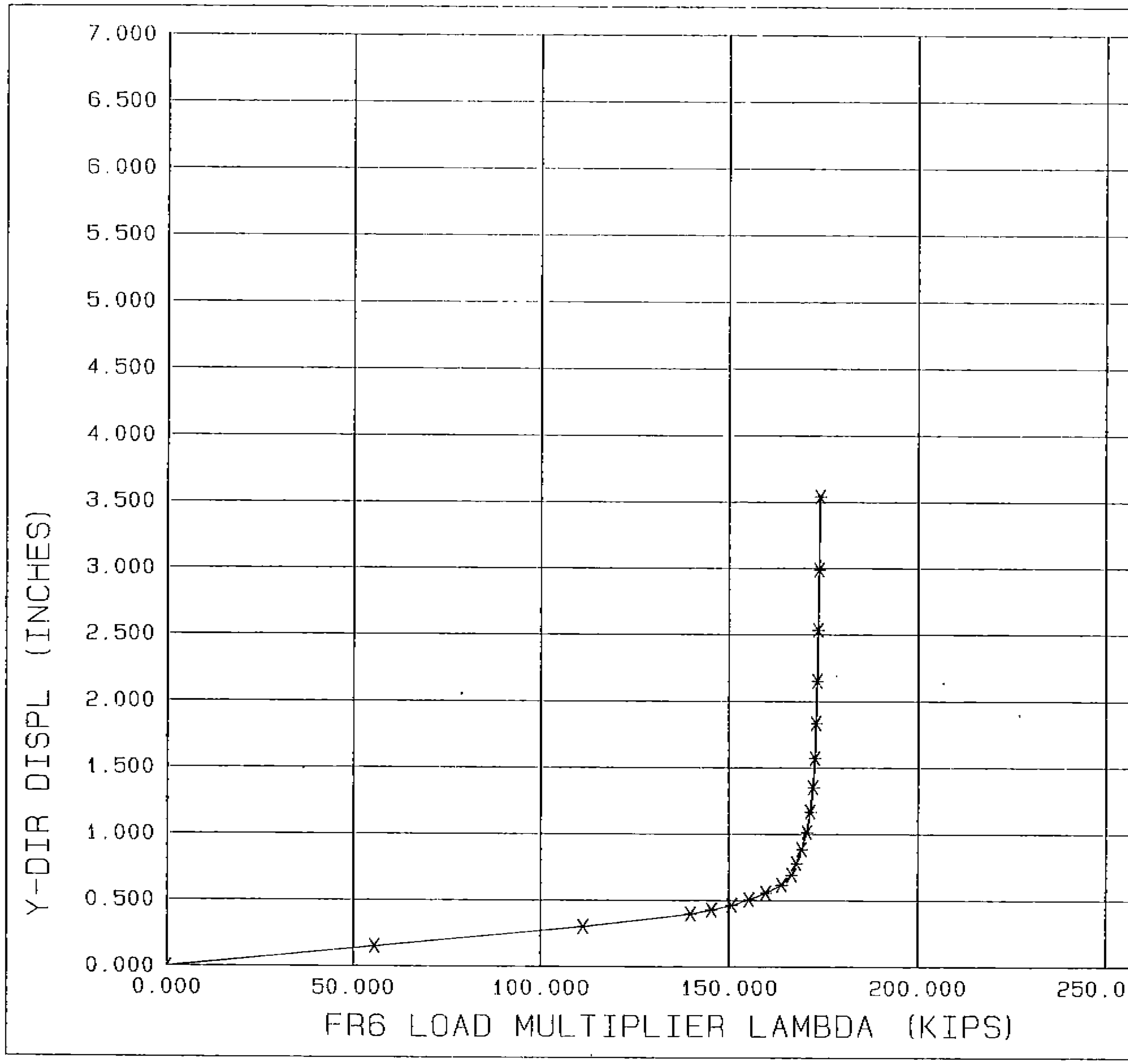

Fig.11.7 Load Deflection Curve for Frame No.6
(Wall Thickness = 0.145 in)

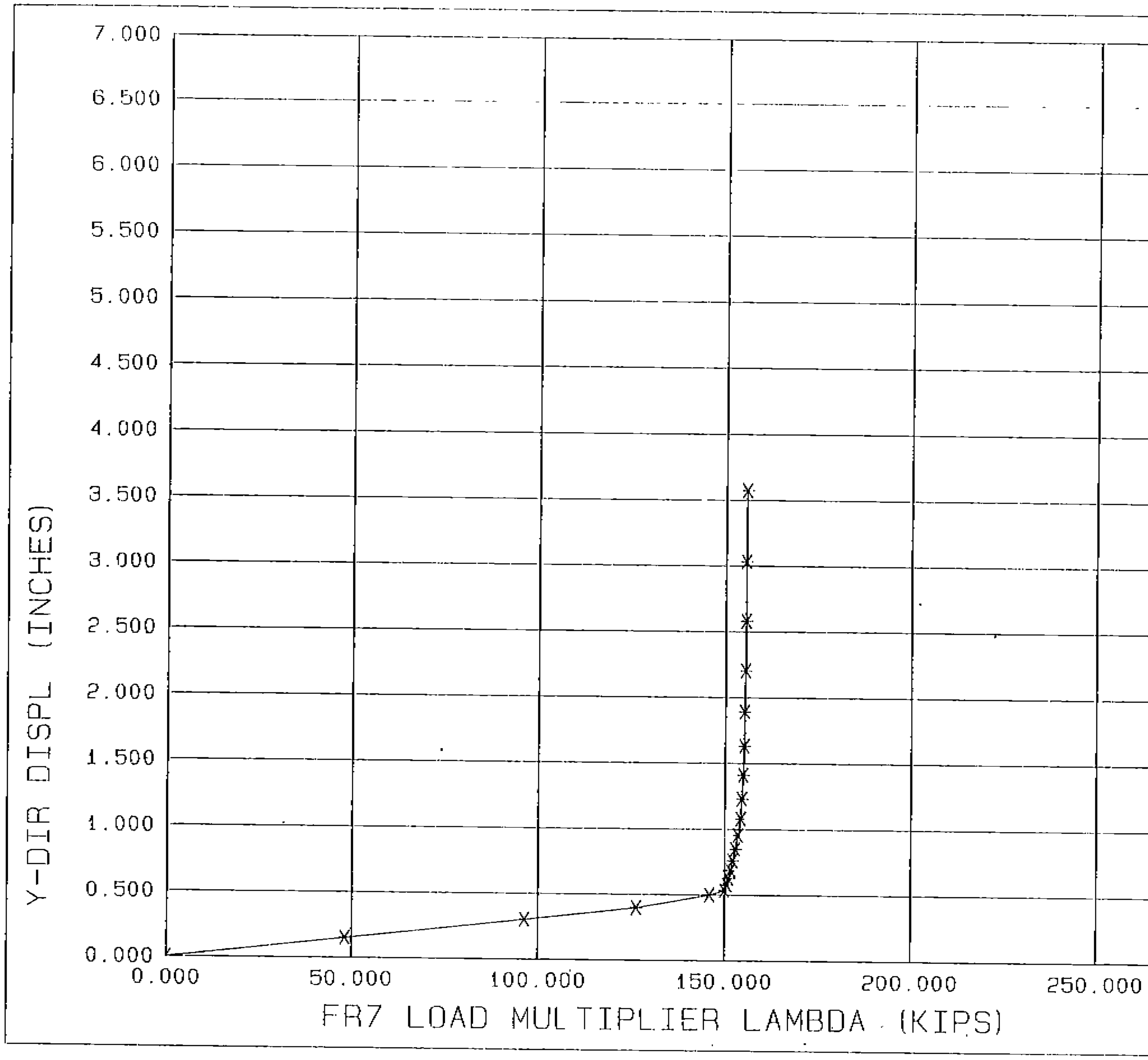

Fig.11.8 Load Deflection Curve for Frame No.7
(Wall Thickness = 0.145 in)

Table 11.1(b) : Design and Collapse Loads for Discrete Framed Structures (all members having the same cross section)

Frame No.	Total Length of Members (ft)	Volume of Material (in^3)	Area of Cross Section (in^2)	External Diameter (in)	Wall Thick-ness (in)	P_u (kips)	RI	P_d (kips)	RSI	X_m	RDI
1	30	1710	4.75	12	0.128	22.39	0	17.67	1.27	1	0.088
2	44.14	2516	4.75	12	0.128	140.4	5.27	82.39	1.70	1	0.556
3	58.28	3322	4.75	12	0.128	252.3	10.27	112.68	2.24	1	1.000
4	51.21	2919	4.75	12	0.128	145.5	5.50	59.50	2.45	1	0.577
5	51.21	2919	4.75	12	0.128	152.3	5.80	53.03	2.87	1	0.604
6	51.21	2919	4.75	12	0.128	150.8	5.73	49.67	3.04	1	0.598
7	51.21	2919	4.75	12	0.128	136.3	5.09	62.11	2.19	1	0.540

(1 kip = 1 thousand lbs)

NOTE: The same argument concerning RSI value holds as was used in Table 11.1(a). The RSI column shows the change in RSI value from that of the parent structure. In this case it is difficult to establish a trend because of different volumes of material in the seven frames considered.

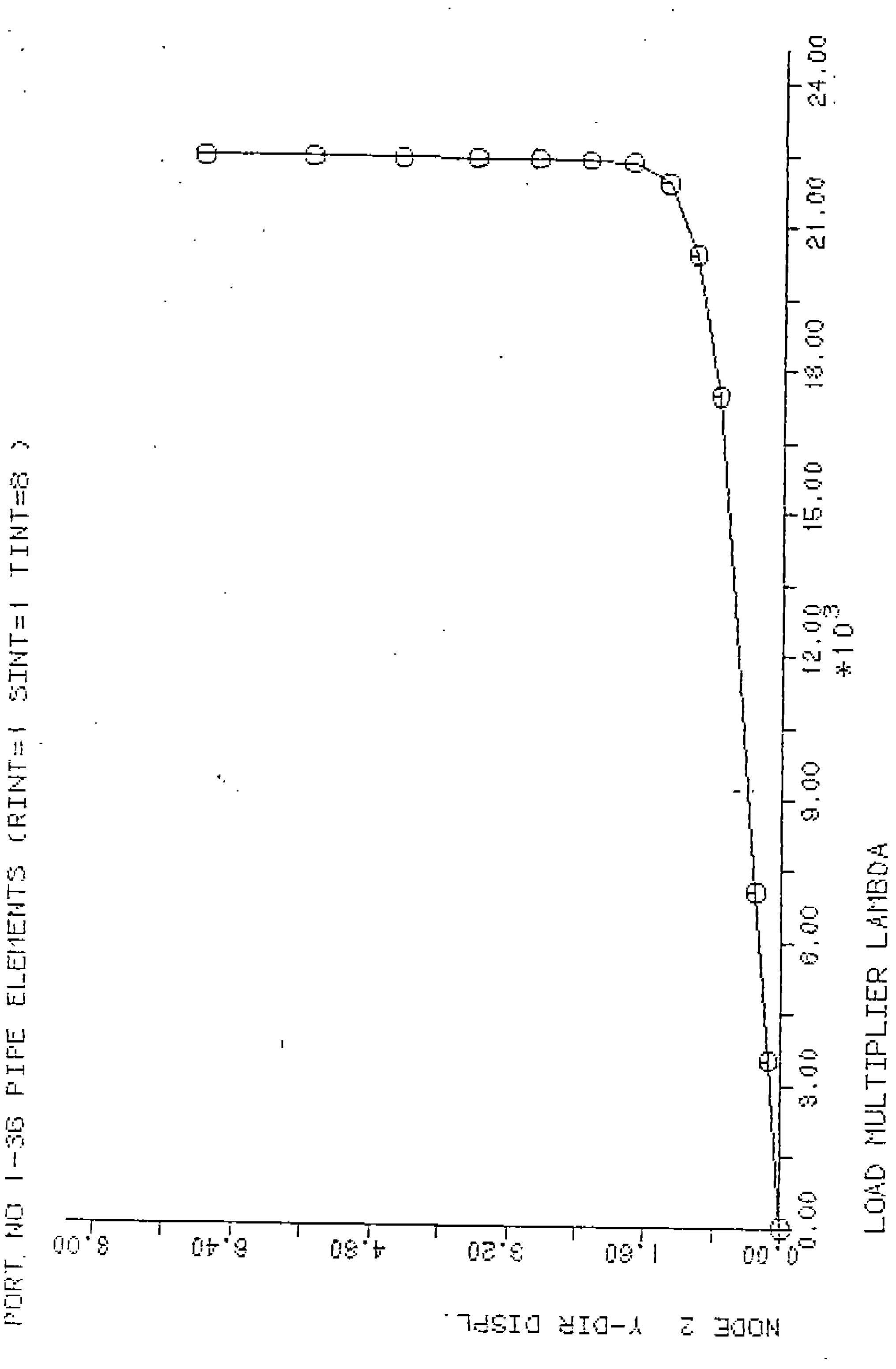

Fig.11.9 Load Deflection Curve for Frame No.1
(Wall Thickness = 0.128 in)

It should be noted in Table 11.1(b) that the D/t ratios of all of the elements are equal and greater than 60. However the global analysis has assumed stable sections to enable the full plastic hinge moments to develop. A rigorous analysis should allow for the effects of local buckling due to combinations of bending and axial forces that develop in each element.

The frames shown in Figure 11.1 were again analysed assuming the same external diameter of 12" and with the constant thickness of 0.25" in all of the frames i.e. the ultimate strength of each local member was kept constant. The results of these analyses are shown in Table 11.2. Again the model uncertainty factor (X_m) was kept unity in all the models. As before the parent structure was assumed to be Frame No.1 for which the redundancy index is thus taken to be zero. The maximum value of redundancy index (RI) was found to be 10.36 i.e. for Frame No.3 and the Reserve Strength Index (RSI) varied from 2.23 to 3.53 for the models. The load deflection curve for node No.2 is shown in Figures 11.10 to 11.16 for all the models.

In Table 11.2, RDI values are shown along with RI and RSI values. The RDI value ranges from 0.088 to 1.0. It is found that higher RSI values were obtained for the structure where half the diagonal was retained in the structure. As defined RSI is the ratio of the design load (P_d) to the ultimate load (P_u). The design load in this case was obtained using conventional code formulae which uses elastic analysis for the calculation of axial load and bending moment in the most critical member in the structure. The ultimate load for models 5 and 6 are of the same order and their design loads, also of the same order, are less than for the other models (with the exception of model 1). Hence models 5 and 6 have higher values of the RSI.

In Table 11.2 it is to be noted that the D/t ratio for all elements is 48 and hence full plastic hinge capability commensurate with the material yield stress can be developed.

The concept of Residual strength may be examined in reference to Table 11.2 and is as follows:

Consider Frame No.3, the most complex structure and for which the collapse and design loads are 488.20 kips and 218.59 kips respectively. If, in this frame, member 3-5 is completely damaged and is no longer effective, then the collapse load of the remaining structure is the same as that for Frame No. 4 i.e. 280.8 kips and hence the residual strength minus the design demand, now in Frame No. 3 is (280.8 - 218.59) i.e. +62.21 kips and the structure remains safe. Alternatively, if both members 1-5 and 5-3 are damaged in Frame No.3, the residual strength minus demand would have been (271.1 - 218.59) i.e 52.51 kips and thus still safe. However, if all the diagonals in Frame No.3 fail, then the residual strength minus demand becomes negative i.e. is (42.84 - 218.59) = - 175.75 kips. That means the structure is unsafe, i.e. the residual strength is less than the design demand.

An attempt was made to generate similar results for the seven frames in Figure 11.1, in which the external diameter and the Reserve Strength Index (RSI) were kept constant. In order to carry out this task, at first Frame No.1 was considered and in which thicknesses of the tubular members (constant for all frame elements and external diameter being 12") was varied from 0.06" to 1". The results of these analyses are shown in Table 11.3 and the same are plotted in Figure 11.17. It may be seen from Figure 11.17 that both P_d and P_u

Table 11.2 : Design and Collapse Loads for Discrete Framed Structures (On Equal Member Ultimate Strength Basis)

Frame No.	Total Length of Members (ft)	Volume of Material (in^3)	Area of Cross Section (in^2)	External Diameter (in)	Wall Thickness (in)	P_u (kips)	RI	P_d (kips)	RSI	X_m	RDI
1	30	3322.0	9.228	12	0.25	42.84	0	17.67	2.43	1	0.088
2	44.14	4887.9	9.228	12	0.25	271.20	5.33	121.06	2.24	1	0.555
3	58.28	6453.7	9.228	12	0.25	488.20	10.4	218.59	2.23	1	1.0
4	51.21	5670.8	9.228	12	0.25	281.3	5.57	104.82	2.68	1	0.576
5	51.21	5670.8	9.228	12	0.25	298.8	5.97	92.98	3.21	1	0.612
6	51.21	5670.8	9.228	12	0.25	297.0	5.93	84.25	3.53	1	0.608
7	51.21	5670.8	9.228	12	0.25	265.4	5.19	109.27	2.43	1	0.543

NOTE : The results for RDI are very similar to Table 11.1(b) which also is for equal member ultimate strength.

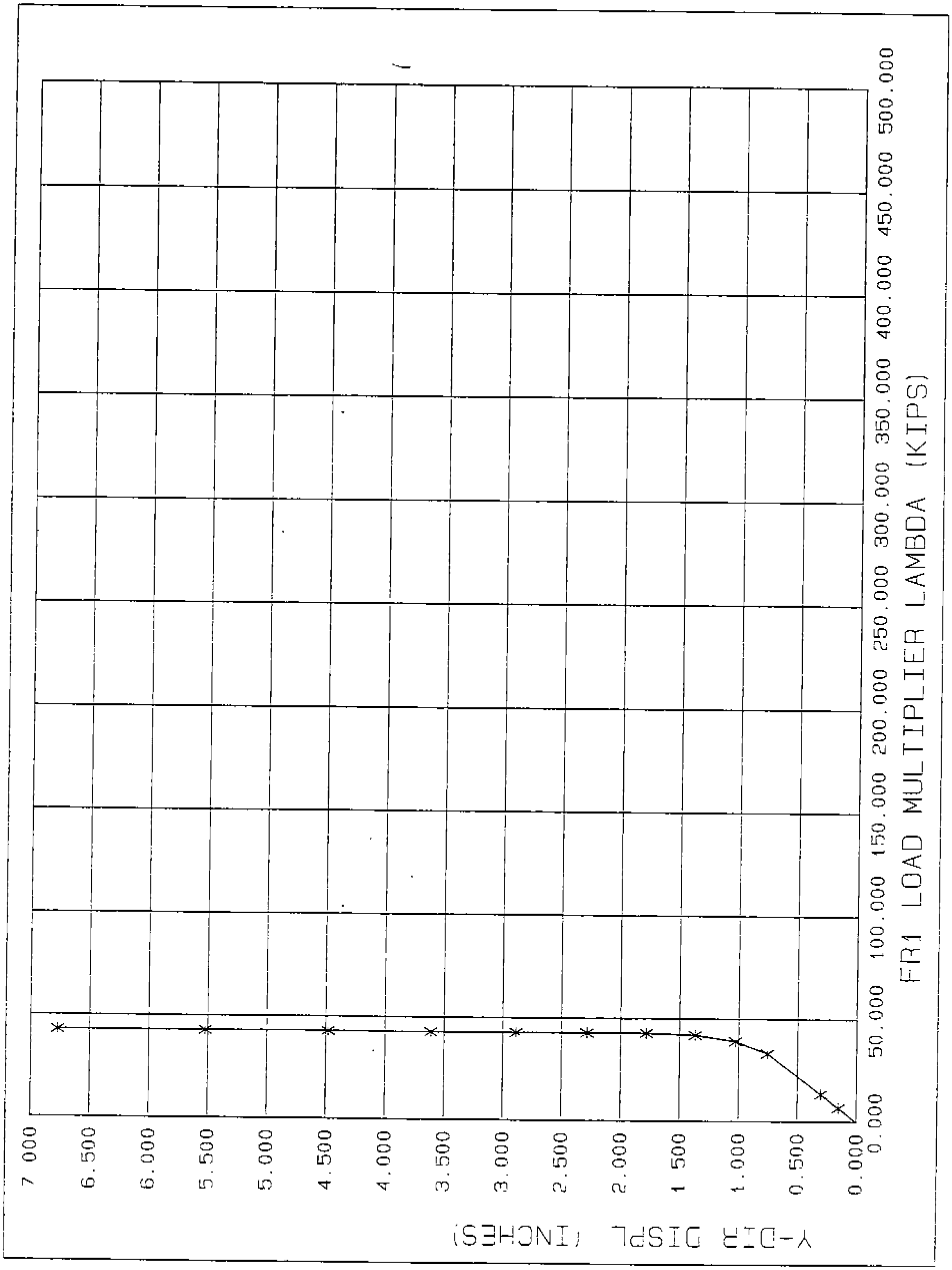

Fig.11.10 Load Deflection Curve for Frame No.1
(Wall Thickness = 0.25 in)

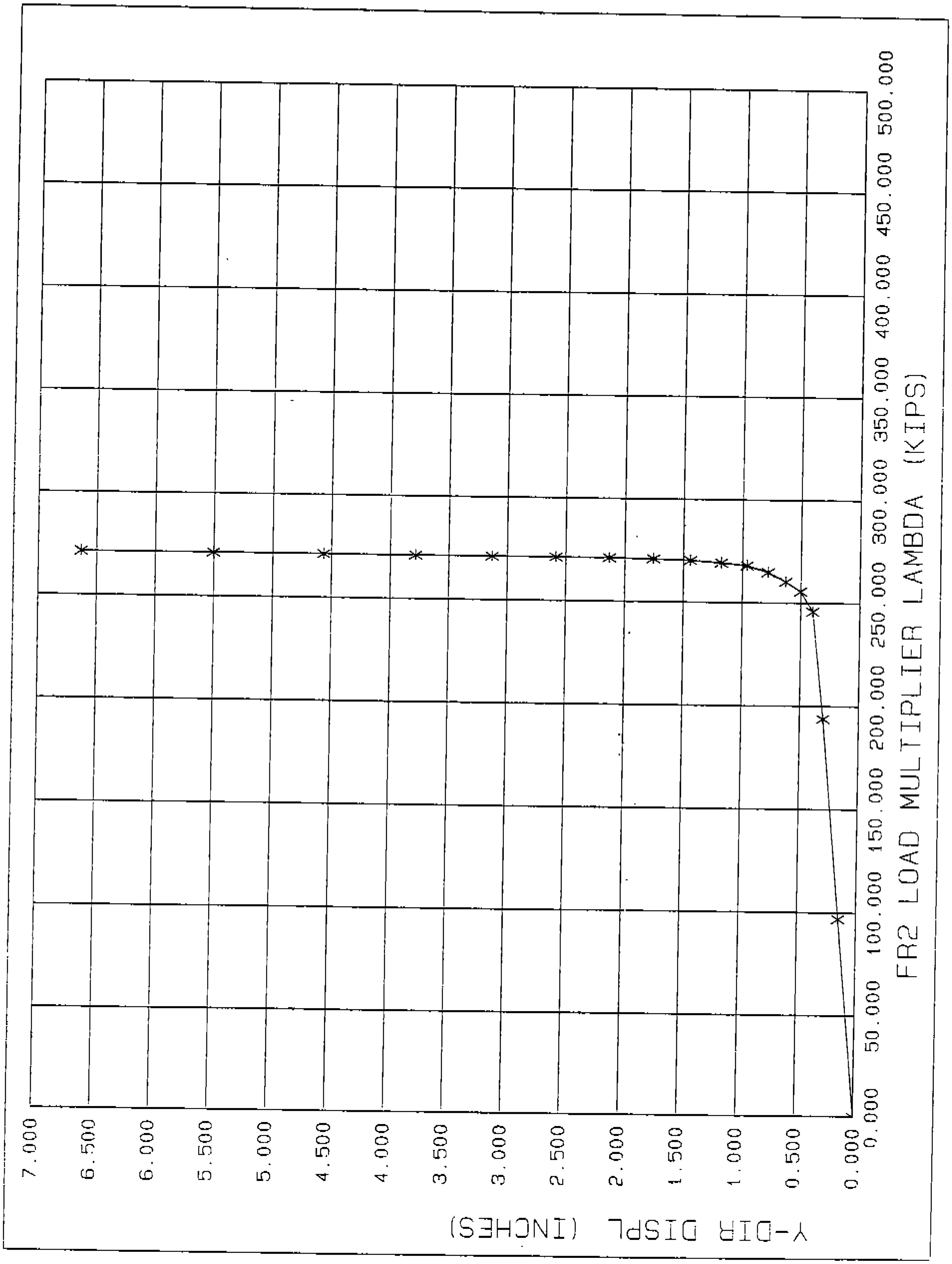

Fig.11.11 Load Deflection Curve for Frame No.2
(Wall Thickness = 0.25 in)

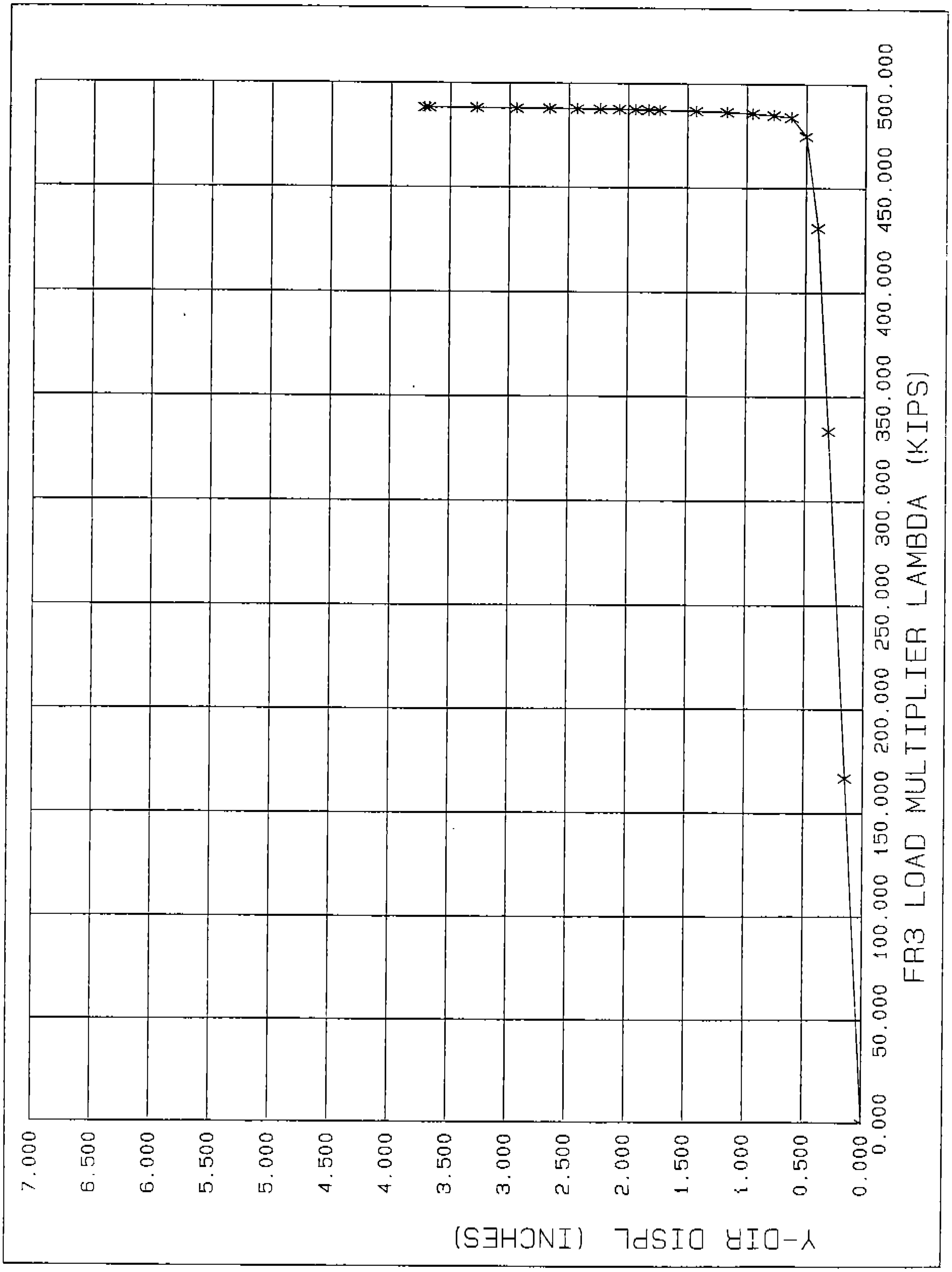

Fig.11.12 Load Deflection Curve for Frame No.3
(Wall Thickness = 0.25 in)

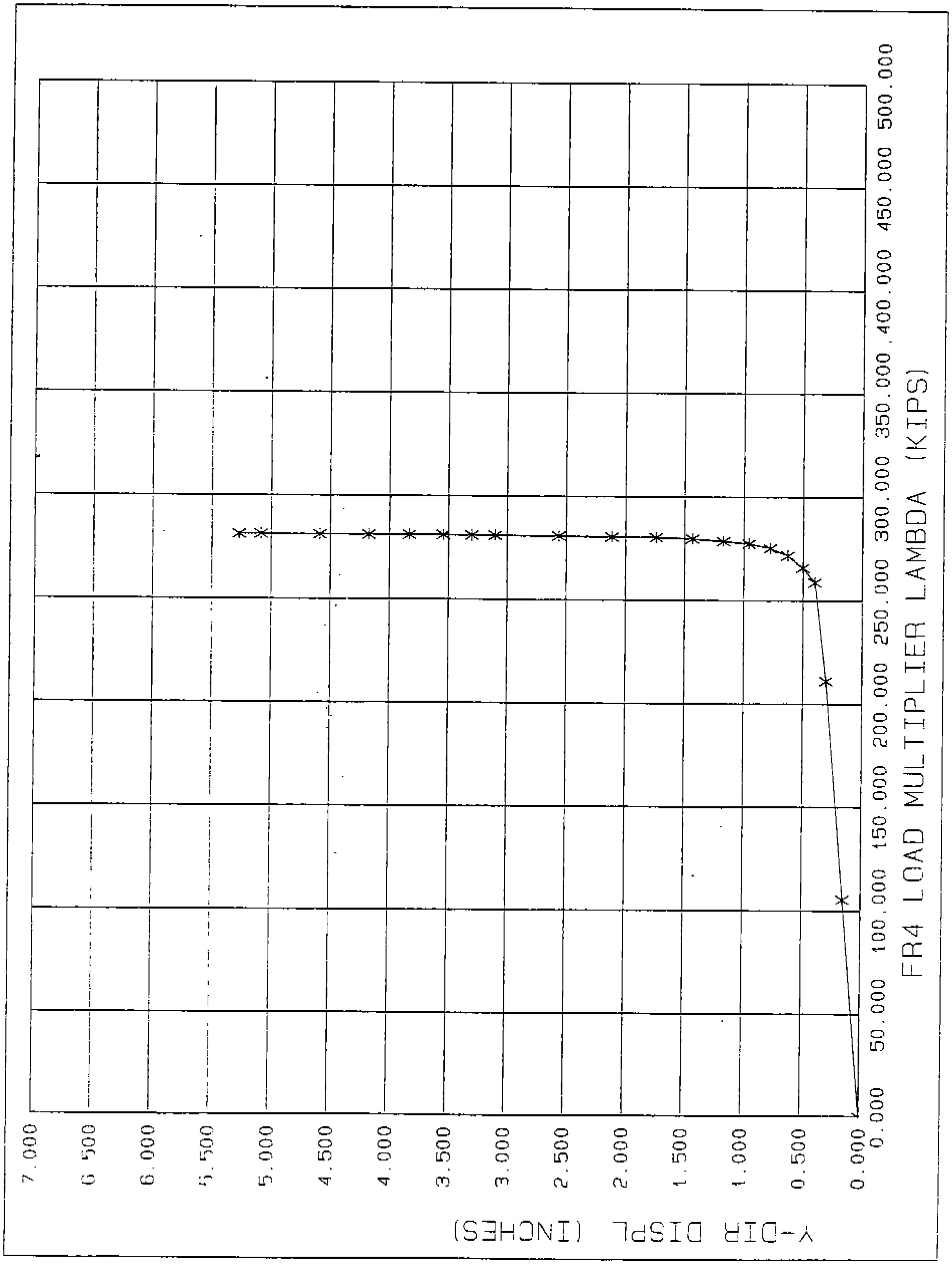

Fig.11.13 Load Deflection Curve for Frame No.4
(Wall Thickness = 0.25 in)

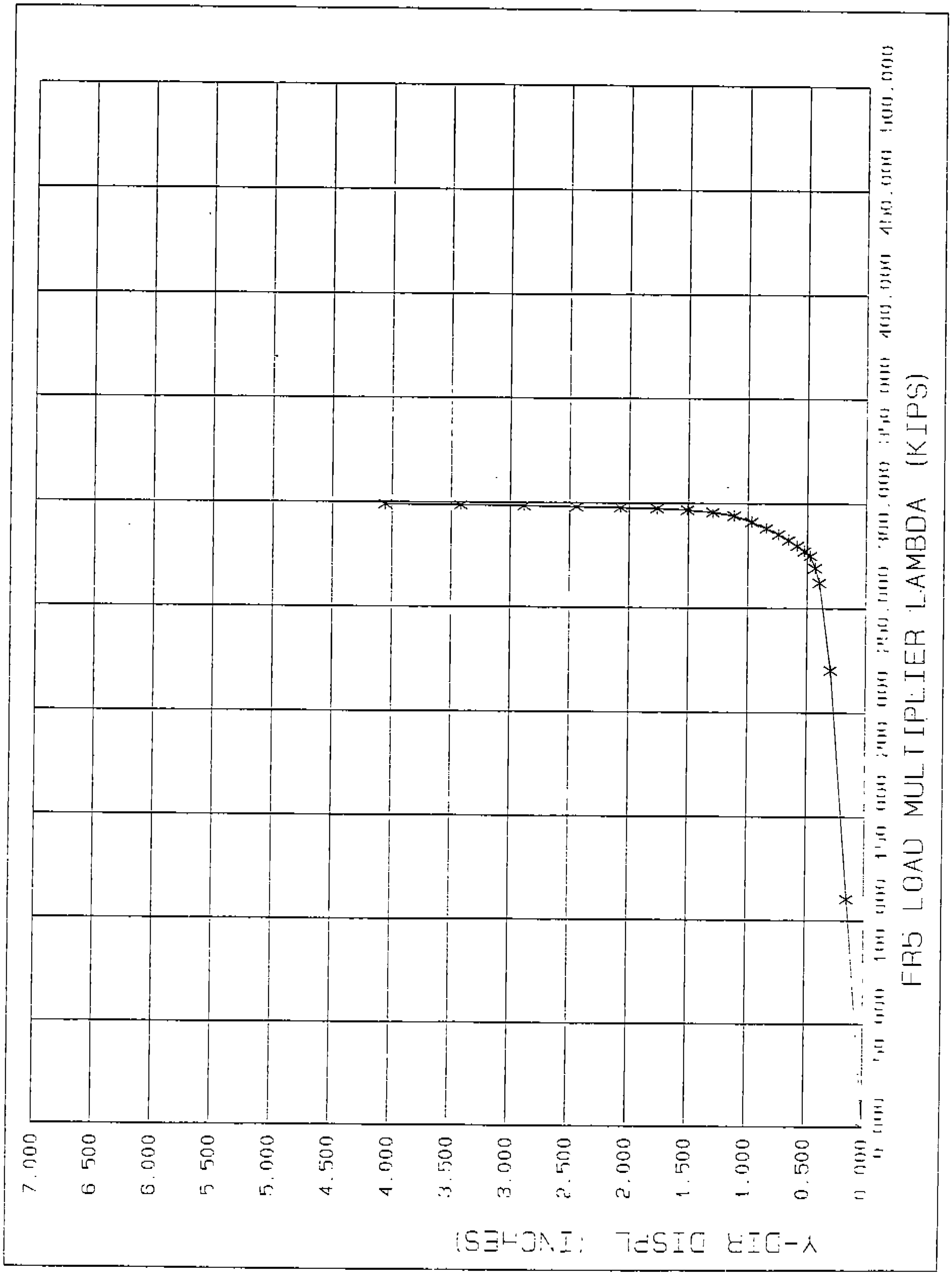

Fig.11.14 Load Deflection Curve for Frame No.5
(Wall Thickness = 0.25 in)

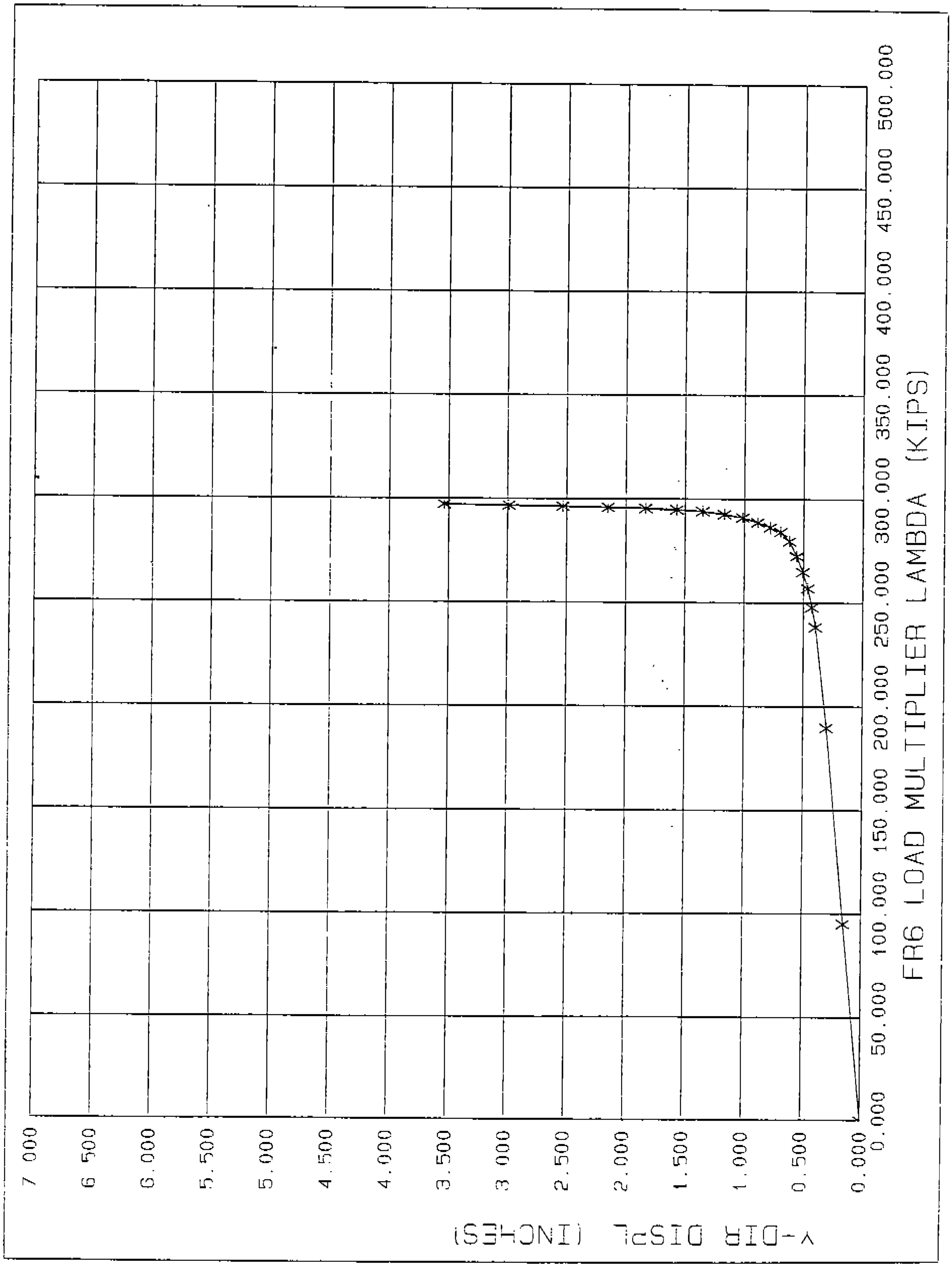

Fig.11.15 Load Deflection Curve for Frame No.6
(Wall Thickness = 0.25 in)

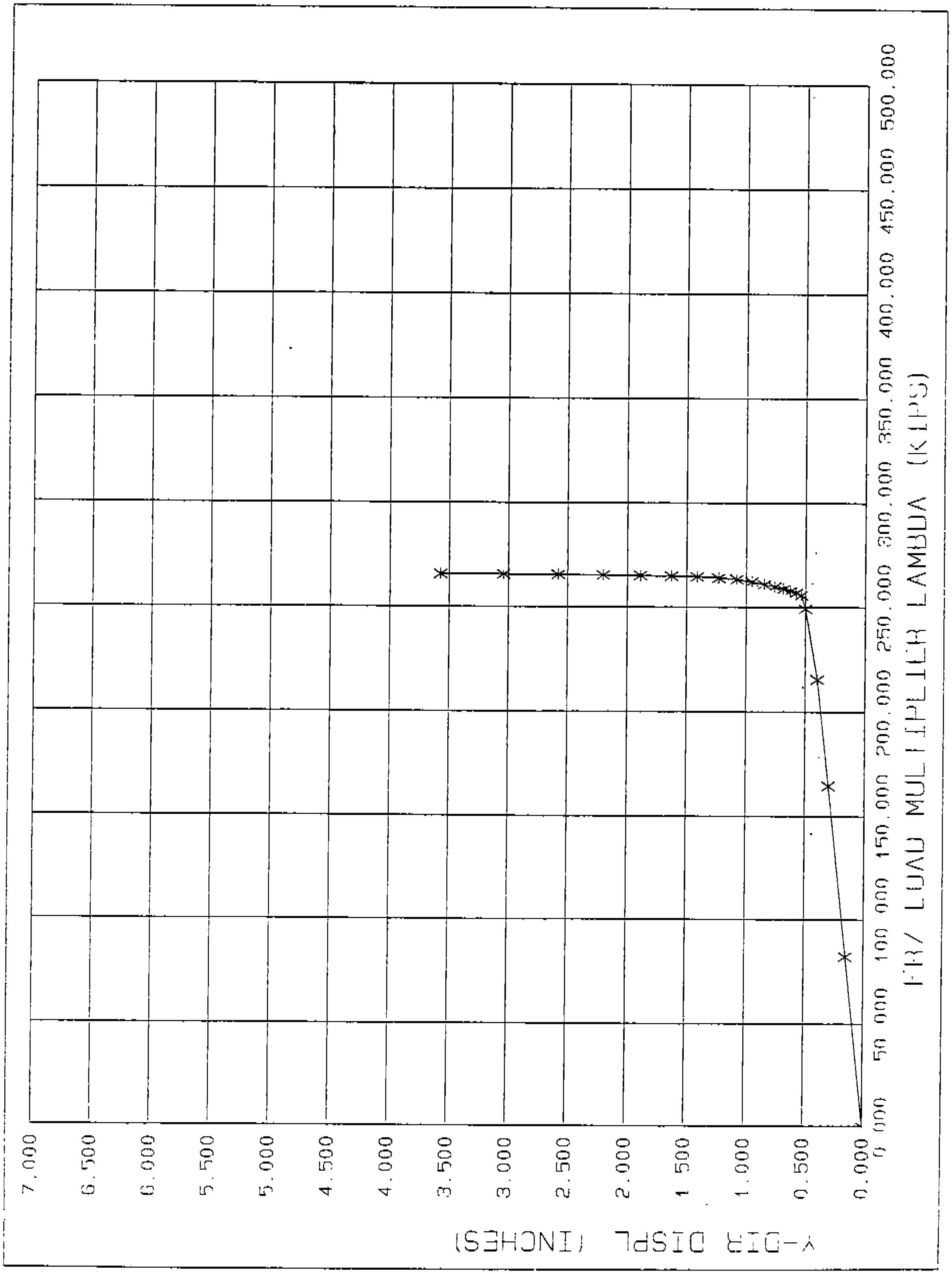

Fig.11.16 Load Deflection Curve for Frame No.7
(Wall Thickness = 0.25 in)

Table 11.3 : Ultimate and Design Load of Frame No.1 for Various Thicknesses of Tubular Members

t (inches)	P_u(Kips)	P_d(Kips)	RSI
0.06	10.62	4.448	2.39
0.128	22.39	9.33	2.40
0.25	42.84	17.67	2.43
0.50	82.07	33.27	2.47
0.75	117.80	46.90	2.51
1.00	150.20	58.77	2.55

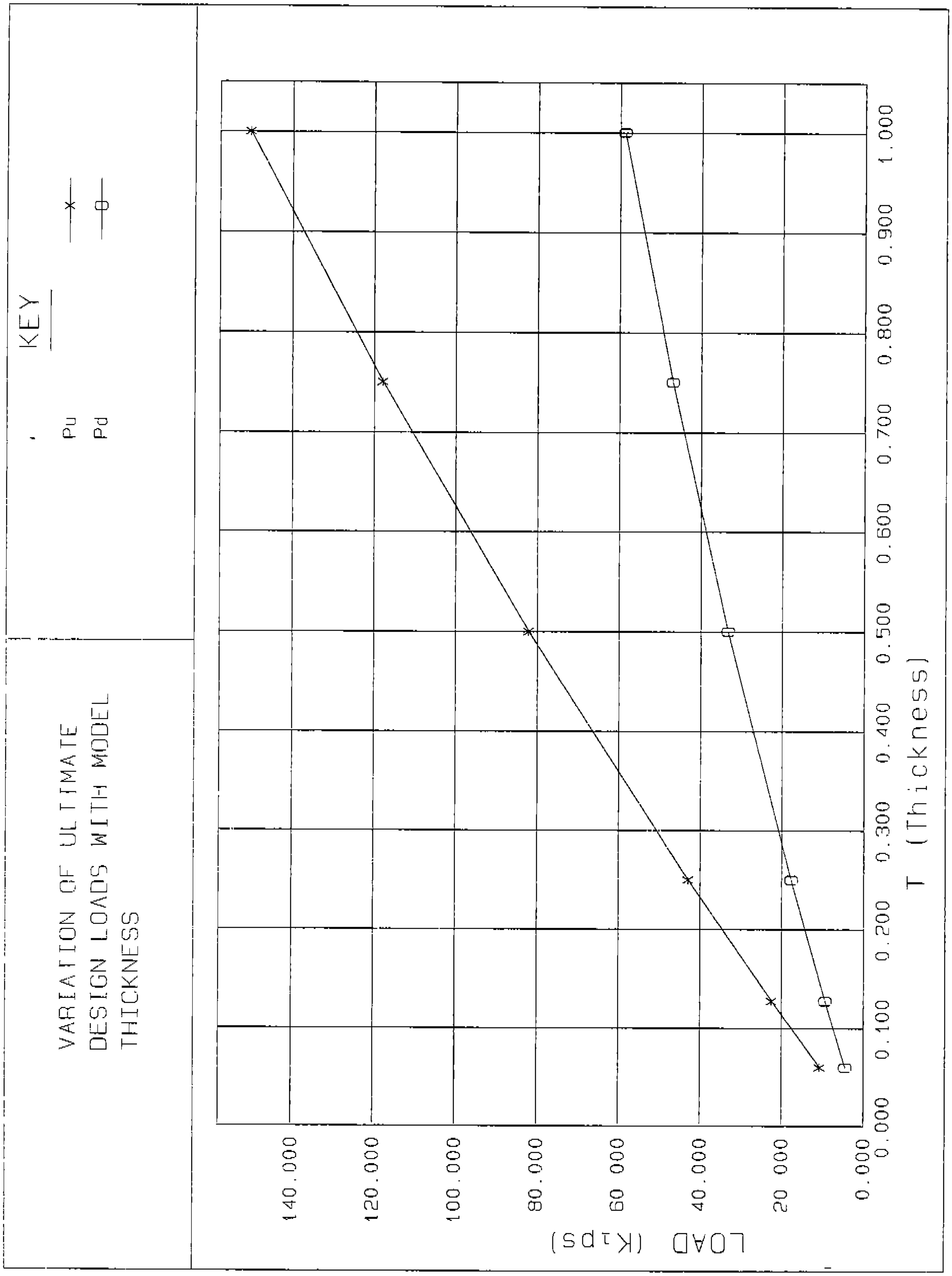

Fig.11.17 Variation of Ultimate and Design Loads with Model Wall Thickness

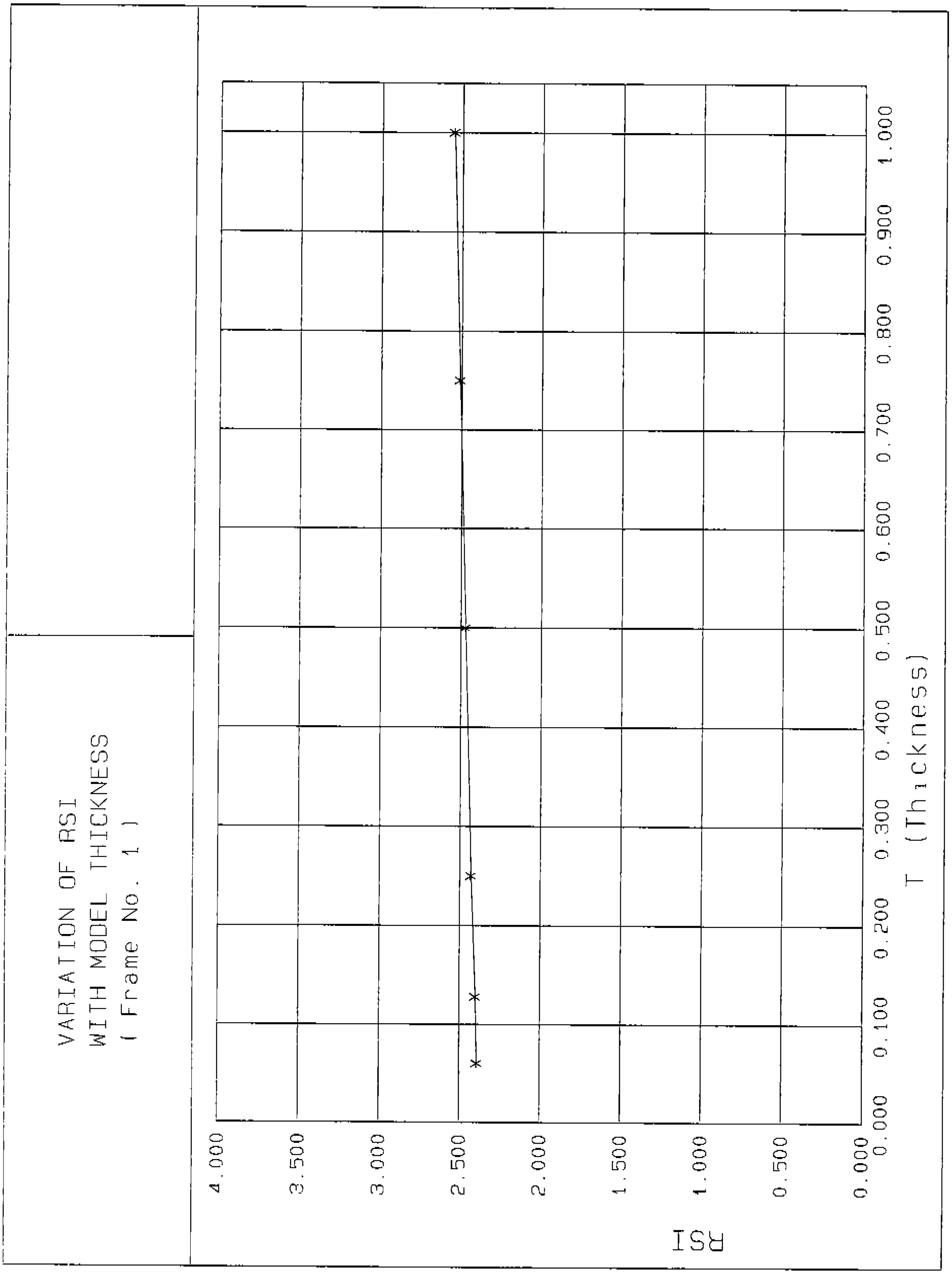

Fig.11.18 Variation of Reserve Strength Index (RSI) with Model Wall Thickness

varies almost linearly with the thickness. (Again, this assumes that full yield stress level plastic hinges can develop.) A plot of variation of RSI with thickness is shown in Figure 11.18 and it may be seen that the change in RSI with variation in thickness is not appreciable. It was concluded from these runs that for the other models i.e. Frame No.2 to Frame No.7, there would not be any appreciable change in RSI by changing the thicknesses of the tubular members. It was thus not possible to obtain similar results for all these frames keeping both the external diameter and the RSI constant. Perhaps one could try to vary the diameter also in order to obtain these results but this was not tried in this exercise.

This example problem, although simple in nature, has given useful insights into the nature of the role of redundancy in discrete structures. Tables 11.1(a) and 11.1(b) are generated based on different assumptions i.e. one is on an equal volume/weight basis whilst the other is based on constant ultimate strength of individual components. Table 11.2 is similar to Table 11.1(b) i.e. they are based on constant ultimate strength but of different values.

Frame No.1 in both the Tables 11.1(a) and 11.2 is the same, i.e. having same diameter and same thickness of the members. Frames No.2-7 of Table 11.1(a) differs from Frames No.2-7 of Table 11.2 by virtue of the different thickness of the tubular members in each case. It is obvious that the Ultimate Strength (P_u) and Design load (P_d) of Frames 2 to Frame 7 in Table 11.2 will be more than that of Frames 2-7 in Table 11.1(a) because of higher thickness of the tubular members. It is interesting to note that there is practically no difference in the RSI values obtained in Table 11.1(a) and Table 11.2. Also it is seen that for the models considered, RSI is not sensitive to model thickness.

These examples illustrate how the failure of a member in a redundant structure reduces the strength of the overall structure and what it means with reference to its original design capability, i.e. the loads for which it was designed.

The numerical example analyses carried out in this study illustrates one possible way to quantify the effect of redundancy on the reserve and residual strength of the structure. It is noted that the analyses could be extended to take into account component buckling (both overall and local) on overall ultimate strength and large deformation effects. Further examples with slightly higher degrees of redundancy could be studied in order to explore the variation of Redundancy Index in case of varying degrees of damage in single and multiple members on a more realistic structure. Also the effects of the relative strength and stiffness of the redundant members on these indices could be studied (e.g. where the diagonal bracing elements have smaller proportions and scantlings compared with the main members, etc.). However it was not intended within the scope of this pilot project to undertake such detailed studies on progressively more complex arrangements and it is accepted that the results obtained from simple models should suffice to illustrate the role of redundancy in general response.

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12.0 CONTINUOUS STRUCTURES - ILLUSTRATIVE MODELS

12.1 Stiffened Flat Panels

12.1.1 Simple Unidirectionally Stiffened Flat Panels

In order to limit the scope of the following analytical study it is assumed that the panels are simple rectangular planform constant thickness uniaxially stiffened members and that uniform edge uniaxial compressive loading is applied in a direction aligned with the stiffener direction.

A number of approaches have been proposed by various researchers and agencies for the ultimate strength analysis of nominally flat stiffened plating, uniformly stiffened in one direction and subjected to a longitudinally aligned compressive load. Some of these methods make allowances for manufacturing imperfections and weld induced residual stresses. Comparisons of results obtained from four of these methods[12.1-12.10] with existing test data were made in report[12.12] and a modified approach was developed for use by Lloyd's Register of Shipping in their direct calculation methods[12.13]. The present method employed in this study is similar to the Imperial College method adopted within Ref.[12.12].

All the four methods examined in Ref.[12.12] employ a beam-column idealisation for the analysis of a single bay (span) panel. These theories do not account for localised stiffener flange buckling, stiffener tripping or for web buckling between plate and flange members but all other failure modes are incorporated. (Design codes are usually formulated to constrain stiffener section proportions such that local buckling is avoided.) No allowance is made for the strengthening effects of the support along the panels' longitudinal edges or the full interactions between adjacent bays, both of which can be appreciable depending upon the panel's design.

A brief outline of the method adopted in these calculations is given in Appendix A. The analysis method followed assumes a simple single eccentrically loaded column, using a Perry-Robertson type formulation and in which the effective width of plating between stiffeners is allowed for and the effects of load line eccentricities owing to both loss of plate effectiveness and manufacturing imperfections are included.

Following the procedure as described in Appendix A, the ultimate theoretical strengths of 42 actual experimental test models, from a test programme undertaken in Glasgow, Scotland[12.12], have been calculated. The details of these models are shown in Table 12.1. All of these models were pin ended.

Both the test results and the theoretical values of ultimate strength for these 42 panels are shown in Table 12.2. In the case of the theoretical values it is to be noted that all results have been non-dimensionalised with respect to average yield stress for the whole panel and which is calculated as follows:

$$\sigma_{ym} = (\sigma_{yp} \cdot A_p + \sigma_{ys} \cdot A_s)/(A_p + A_s) \qquad (12.1)$$

(This is necessary in order to allow for the cases tested where the stiffeners are made from a grade of steel which has a different yield strength from the plate material.)

Table 12.1 : Details of Test Models

Sr. No.	Model No.	b (mm)	t (mm)	hw (mm)	tw (mm)	bf (mm)	tf (mm)	Le (mm)	B (mm)	N	σ_{yp} N/mm^2	σ_{ys} N/mm^2	E N/mm^2	ec (mm)
1	P1	88.4	3.07	17.4	4.88	12.7	6.17	244.0	412.5	5	250.0	283.0	190000	1.9
2	P2	147.0	2.62	30.4	4.83	12.7	6.22	384.0	686.0	5	250.0	262.0	190000	-0.9
3	P3	221.0	2.54	54.1	4.90	12.7	6.10	638.0	1031.3	5	256.0	247.0	190000	-2.6
4	P4	236.0	2.01	43.6	4.80	12.7	6.25	523.0	1101.3	5	221.0	250.0	190000	-2.7
5	P5	88.4	3.07	17.4	4.88	12.7	6.17	488.0	412.5	5	225.0	259.0	190000	+1.9
6	P6	147.0	2.62	30.4	4.83	12.7	6.22	767.0	686.0	5	239.0	259.0	190000	-0.9
7	P7	221.0	2.54	54.1	4.90	12.7	6.10	1275.0	1031.3	5	270.0	246.0	190000	-2.6
8	P8	236.0	2.01	43.6	4.80	12.7	6.25	1046.0	1101.3	5	247.0	259.0	190000	-2.7
9	P9	88.4	3.07	17.4	4.88	12.7	6.17	732.0	412.5	5	230.0	283.0	190000	+1.9
10	P10	147.0	2.62	30.4	4.83	12.7	6.22	1151.0	686.0	5	239.0	258.0	190000	-0.9
11	P11	221.0	2.54	54.1	4.90	12.7	6.10	1913.0	1031.3	5	239.0	252.0	190000	-2.6
12	P12	236.0	2.01	43.6	4.80	12.7	6.25	1570.0	1101.3	5	249.0	266.0	190000	-2.7
13	P13	88.4	3.10	26.4	3.10	0.0	0.0	262.0	412.5	5	253.0	261.0	190000	+0.11
14	P14	177.0	3.05	17.5	4.85	12.7	6.15	244.0	826.0	5	242.0	269.0	190000	-0.4
15	P15	265.0	3.07	34.0	4.95	12.7	6.20	422.0	1236.7	5	227.0	267.0	190000	-1.5
16	P16	295.0	2.57	30.5	4.90	12.7	6.12	384.0	1376.7	5	244.0	273.0	190000	-2.1
17	P17	88.4	3.10	26.4	3.10	0.0	0.0	523.0	412.5	5	229.0	256.0	190000	+0.11
18	P18	177.0	3.05	17.5	4.85	12.7	6.15	488.0	826.0	5	229.0	246.0	190000	-0.4
19	P19	265.0	3.07	34.0	4.95	12.7	6.20	843.0	1236.7	5	253.0	266.0	190000	-1.5
20	P20	295.0	2.57	30.5	4.90	12.7	6.12	767.0	1376.7	5	261.0	247.0	190000	-2.1
21	P21	88.4	3.10	26.4	3.10	0.0	0.0	785.0	412.5	5	258.0	262.0	190000	+0.11
22	P22	177.0	3.05	17.5	4.85	12.7	6.15	732.0	826.0	5	242.0	262.0	190000	-0.4
23	P23	265.0	3.07	34.0	4.95	12.7	6.20	1265.0	1236.7	5	244.0	262.0	190000	-1.5
24	P24	295.0	2.57	30.5	4.90	12.7	6.12	1151.0	1376.7	5	239.0	267.0	190000	-2.1
25	F1	229.0	2.54	38.1	9.53	0.0	0.0	348	1069.0	5	222.0	238.0	190000	2.3
26	F2	229.0	2.54	38.1	9.53	0.0	0.0	653	1069.0	5	227.0	262.0	190000	2.3
27	F3	229.0	2.54	38.1	9.53	0.0	0.0	958	1069.0	5	195.0	250.0	190000	2.3
28	F4	229.0	2.54	38.1	9.53	0.0	0.0	1262	1069.0	5	188.0	208.0	190000	0.2
29	FL1	136.0	4.93	63.5	3.02	0.0	0.0	577	634.7	5	321.0	321.0	190000	0.0
30	FL1S	136.0	4.93	63.5	3.02	0.0	0.0	577	136.0	1	321.0	321.0	190000	0.0
31	FL2	136.0	4.93	63.5	3.02	0.0	0.0	577	634.7	5	247.0	219.0	190000	0.0
32	FL2S	136.0	4.93	63.5	3.02	0.0	0.0	577	136.0	1	247.0	219.0	190000	0.0
33	T1	203.0	1.98	28.6	4.95	13.0	6.35	1224	947.3	5	190.0	208.0	190000	0.0
34	T2	169.0	1.98	19.0	4.95	13.3	6.35	874	788.7	5	188.0	278.0	190000	1.6
35	T3	202.0	1.91	28.4	4.95	13.3	6.35	986	942.7	5	184.0	184.0	190000	2.6
36	T4	166.0	2.08	19.0	4.95	13.2	6.35	704	774.7	5	196.0	287.0	190000	1.6
37	T5	159.0	2.41	29.3	5.08	13.3	6.35	1019	742.0	5	201.0	267.0	190000	0.0
38	T7	157.0	2.41	29.3	4.95	13.3	6.25	775	732.7	5	247.0	262.0	190000	2.6
39	T8	116.0	3.09	19.1	4.95	13.2	6.25	546	541.3	5	250.0	267.0	190000	2.3
40	T9	173.0	3.07	38.2	4.90	12.7	6.25	673	807.4	5	259.0	293.0	190000	3.0
41	T10	115.0	3.10	19.1	4.95	12.7	6.25	376	536.7	5	292.0	279.0	190000	0.0
42	T11	82.0	4.32	19.1	4.95	12.7	6.25	409	382.7	5	281.0	286.0	190000	0.0

NOTATION:

- b spacing of stiffeners
- t plate thickness
- h_w height of stiffener web
- t_w web thickness
- b_f breadth of flange
- t_f flange thickness
- e_c load line eccentricity
- L_e effective span
- B width of stiffened panel
- N number of stiffener
- σ_{yp} yield stress of plating
- σ_{ys} yield stress of stiffener
- E modulus of elasticity

(See Fig.A8 in Appendix A for clarification of details.)

Table 12.2 : Comparison of Theoretical and Experimental Values of Stress Ratio at Collapse (Average Stress/Average Yield Stress)

Sr. No.	Model No.	$(b/t)\sqrt{\sigma_{yp}/E}$	Theory	Test	X_m
1	P1	1.044	0.694	0.976	1.406
2	P2	2.035	0.705	0.733	1.040
3	P3	3.194	0.619	0.713	1.152
4	P4	4.004	0.578	0.567	0.981
5	P5	0.991	0.647	0.824	1.273
6	P6	1.990	0.668	0.750	1.123
7	P7	3.280	0.583	0.621	1.065
8	P8	4.233	0.554	0.515	0.929
9	P9	1.002	0.545	0.716	1.314
10	P10	1.990	0.582	0.660	1.134
11	P11	3.086	0.481	0.494	1.027
12	P12	4.250	0.466	0.448	0.961
13	P13	1.041	0.847	0.988	1.166
14	P14	2.071	0.668	0.764	1.144
15	P15	2.984	0.565	0.569	1.007
16	P16	4.113	0.560	0.506	0.903
17	P17*	0.990	0.735	0.822	1.118
18	P18	2.015	0.652	0.656	1.006
19	P19	3.150	0.535	0.563	1.052
20	P20	4.254	0.548	0.455	0.830
21	P21*	1.051	0.430	0.696	1.618
22	P22*	2.071	0.531	0.515	0.969
23	P23*	3.093	0.446	0.491	1.100
24	P24*	4.071	0.409	0.384	0.939
25	F1	3.08	0.472	0.566*	1.199
26	F2	3.12	0.428	0.577*	1.348
27	F3	2.89	0.385	0.459*	1.192
28	F4	2.84	0.415	0.339*	0.817
29	FL1	1.13	0.853	0.779†	0.913
30	FL1S	1.13	0.853	0.752†	0.882
31	FL2	0.995†	0.776	0.787†	1.014
32	FL2S	0.995†	0.776	0.723†	0.932
33	T1	3.24	0.416	0.390*	0.937
34	T2	3.33	0.348	0.352*	1.011
35	T3	3.29	0.404	0.416*	1.030
36	T4	2.56	0.392	0.403*	1.028
37	T5	2.15	0.512	0.619*	1.209
38	T7	2.35	0.478	0.558*	1.167
39	T8	1.36	0.614	0.744*	1.212
40	T9	2.08	0.538	0.634*	1.178
41	T10	1.45	0.796	0.879*	1.104
42	T11	0.394	0.849	0.820*	0.966

* Plate failure mode — Mean → 1.094

† Stiffener failure mode — COV → 14.9%

The model uncertainty factor X_m which is the ratio of actual strength (determined by testing) to the theoretically predicted strength is calculated for all the models and these are also shown in the same Table 12.2. The average values of the mean and the coefficient of variation (COV) of the model uncertainty factor is thus found to be 1.094 and 14.9% respectively.

An advanced First Order Second Moment reliability analysis procedure, as described in Appendix B, was then used to calculate the safety index of two selected typical examples, specifically models P1 and P17, as presented earlier in Table 12.1.

Model P1 has its lowest ultimate strength in the plate induced buckling mode and which is calculated as 79.168kN. This is relevant in order to calculate the equivalent design load corresponding to different values of the RSI. Reliability analyses where carried out for this model for four assumed values of the design load, corresponding to Reserve Strength Indices, RSI's, of 1.5, 2.0, 2.5 and 3 respectively. These are shown in Table 12.3. A typical result corresponding to a RSI of 1.5 (design load = 52.778 kN) is shown in Table 12.4. This table lists the basic variables pertinent to the strength model, and the statistical distribution type, mean and COV values assumed for each. From this table it can be seen that the model is most sensitive to both the modelling coefficient (X_m) and to the applied load and which is then followed by the material yield stress. The other variables were found to be not so sensitive.

The selection of the design load relates to the basic capability of the panel factored down to give the probable design level, that would be compatible with a normal factor of safety, for a panel of that dimensions, i.e. as if the designer had started with a maximum design load, added the required factor of safety and then designed a stiffened flat panel that just met those requirements. The assumed 10% coefficient of variation on the design load, as shown in Table 12.4, is purely notional and employed solely for the purposes of this illustrative study. In a real design study the appropriate COV would represent the combined effects of all uncertainties in load modelling (e.g. still water and wave induced forces and their relevant combinations) and in assessing, for example, overall hull girder response (i.e. leading to inplane forces being applied to the edges of the panel).

A safety index of $\beta = 2.496$ was found from this analysis and this corresponds to a probability of failure of $p_f = 0.627 \times 10^{-2}$. The resulting safety indices for all assumed design loads are plotted in Figure 12.1 and from which it can be seen that the safety index β increases rapidly beyond a RSI value of about 2.

Another model, P17, was examined. This panel's ultimate strength is dictated by the failure of the stiffener (stiffener induced mode) in compression and calculated to be 61.5kN. (The corresponding ultimate strength based upon plate buckling is 65.1kN.)

A reliability analysis for the failure mode representing the stiffener in compression was carried out for four design load levels corresponding to RSI values of 1.5, 2.0, 2.5 and 3.0 and the safety indices and probability of failure results are shown in Table 12.5. The reliability analysis results

Table 12.3 : Design Loads for Various Reserve Strength Index (RSI)
Model P1 (Panel Buckling)

$P_u = 79.168$ kN

RSI	P_d (kN)	β	P_f
1.5	52.778	2.496	0.627×10^{-2}
2	39.584	4.048	0.259×10^{-4}
2.5	31.667	5.262	0.766×10^{-7}
3	26.389	6.26	0.267×10^{-9}

Table 12.4 : Reliability Analysis of Stiffened Plate
(Model P1 - Panel Buckling)

Design Variable	Mean	COV %	Distribution Type	Sensitivity Factor	Partial Factor
L	244 mm	2	Normal	-0.0053	1.0003
b	88.4mm	2	Normal	0.0605	0.997
t	3.07mm	2	Normal	0.085	0.996
P	52778 N	10*	Normal	-0.484	1.12
σ_{yp}	250 N/mm^2	7	Log-normal	0.350	0.938 (1.066)
X_m	1.094	14.9	Log-normal	0.795	0.737 (1.36)

$\beta = 2.496$ $\qquad P_f = 0.627 \times 10^{-2}$

The results on the right side of the double vertical lines are the output from the program. L is the length, b is spacing, t is thickness, P is the axial compressive load and σ_{yp} is the plate yield stress.

* Notional COV

Table 12.5 : Design Loads for Various RSI Values
Model P17 (Stiffener Buckling)

$P_u = 61.50$ kN

RSI	P_d (kN)	β	P_f
1.5	41.0	2.676	0.372×10^{-2}
2	30.75	4.346	0.701×10^{-5}
2.5	24.60	5.658	0.886×10^{-8}
3	20.5	6.740	0.141×10^{-10}

Table 12.6 : Reliability Analysis of Stiffened Plate
(Model P17 - Stiffener Failure by Compression)

Design Variable	Mean	COV %	Distribution Type	Sensitivity Factor	Partial Factor
L	523 mm	2	Normal	-0.0917	1.005
b	88.4mm	2	Normal	0.0575	0.997
t	3.10	2	Normal	0.0553	0.997
P	41000 N	10*	Normal	-0.510	1.137
σ_{yp}	229 N/m^2	7	Log-normal	0.0095	0.996
X_m	1.094	14.9	Log-normal	0.851	0.706 (1.41)

$\beta = 2.676$ $P_f = 0.37221 \times 10^{-2}$

* Notional COV

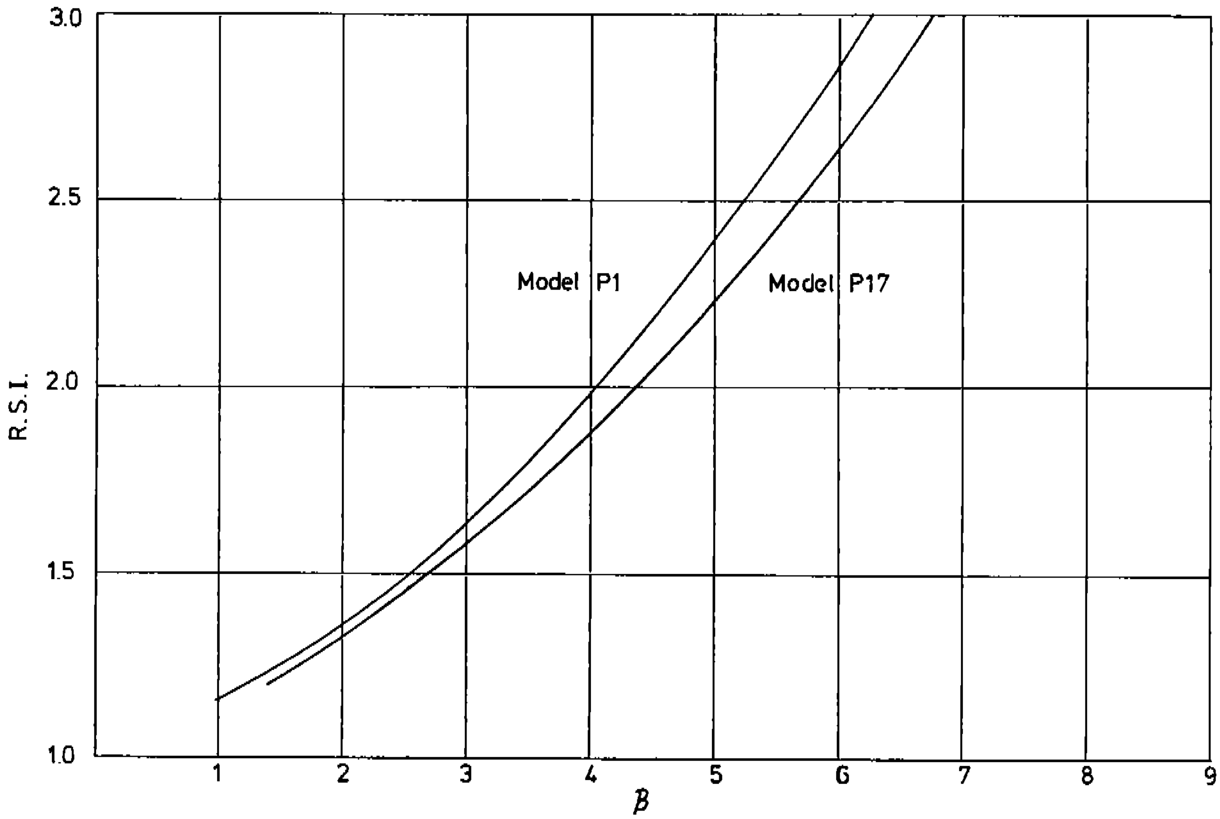

Fig.12.1 Variation of β with Reserve Strength (Model P1 and P17)

with the corresponding sensitivity and partial factors are shown in Table 12.6. for the single case of the RSI having a value of 1.5. These β values are also plotted and shown in the Figure 12.1.

12.1.2 Orthogonally Stiffened Flat Panels

In the case of the buckling mode of failure in plated structure, the critical forces are proportional to the flexural rigidity of the plate. For a simple rectangular plate with given edge boundary conditions and for a given length to width ratio, the magnitude of critical stress at which instability occurs is proportional to the square of the ratio of thickness to width. The stability of the plate can always be increased by increasing its thickness, however such a design will not be economical in respect to the total weight of the material. A more efficient solution is obtained by keeping the thickness of the plate as small as possible and by increasing the stability by introducing discrete reinforcing elements, stiffeners, and thereby increasing the redundancy in the plated structure. The unstiffened plate may be considered as the parent structure and the stiffened plate is considered to be the as-designed structure. The redundancy index (RI), as defined earlier, is then:

$$RI = \frac{P_u - \overline{P}_u}{\overline{P}_u} \qquad (12.2)$$

in which P_u is the strength of parent structure considering a specific mode of failure and $\overline{P}_u$ is the strength of as-designed structure considering the same mode of failure. (This is an assumption for the purposes of this study. However for some structures the mode of failure may differ to some extent between the as-designed structure and the reference parent.)

A simply supported rectangular constant thickness plate was considered for this study. The plate was gradually strengthened by adding, in increments a number of longitudinal and transverse stiffeners. The stiffeners were added located symmetrically with respect to the plate boundaries. Seven models were thus developed and they are shown in Figure 12.2 The plate-stiffener assembly was subjected to an initial longitudinal compressive force and this was increased until the buckling of the overall panel occurred. Initially only the wholly elastic buckling mode of failure was considered.

It was considered that in order to accurately assess the response of the panels to end loads that a finite element method based procedure would be the most appropriate approach.

The particular finite element program which was used for the solution is SOLVIA[12.14]. In this study it was assumed that a symmetrical mode of buckling would occur and therefore only one quarter of the overall model was necessary for numerical analysis. For all these models 4-node shell elements and 2-node isoparametric beam element were used. Buckling symmetry was ensured by selection of the appropriate boundary conditions. A typical finite element mesh for Model-2 is shown in Figure 12.3. The option of large displacements was specified. Full Newton Iteration with line searches was used.

For a given load, the displacement response of the stiffened plate was calculated. The structure was in a state of uniaxial compression using the deformed geometry corresponding to the external load and the resulting eigenvalue problem was solved to obtain the critical load.

The parent structure, i.e. Model-1, was an unstiffened plate of dimensions 102cmx54cm and of 5mm thickness and was subjected to an uniaxial compression as shown in Figure 12.4. The stiffeners used in Models 2 to 7 are of simple flat bar section with depth 4cm and thickness 0.54cm. The details of these models with their critical loads are shown in Table 12.7.

An initial starting load was then specified in order to enable SOLVIA to calculate the critical failure load. For example, Model-2 was subjected to an initial total edge load of p = 14580 kgs, on the quarter model, and the critical load was found to be p_{crit} = 18159kg. For the full model, the critical load is thus p_{crit} = 18159x2 = 36,318kg. Hence the average critical stress

$$\sigma_{cr} = P_{crit}/\text{total sectional area of the stiffened plate}$$

$$= 2\times 18159/29.16 \approx 1244 \text{ kg/cm}^2. \quad (17656.7 \text{ lbs/in}^2)$$

The static deformed element mesh and the mode shape are shown in Figure 12.4.

If the linearised buckling solution is performed with the plate subjected to a smaller total initial compressive load, e.g. p = 1458kg, the same buckling load is predicted, i.e. p_{cr} = 18159kg.

If there is no apriori knowledge of the buckling mode shape, the full structure must be discretized to include the possibility of a non-symmetric buckling mode shape occurring. It may also be noted that it is necessary to specify large displacements in the buckling analysis within the computer program employed.

Reference [12.6] gives an analytical solution for the critical load of a simply supported stiffened plate with one longitudinal stiffener. For this example:

a = length of plate = 102 cm

b = width of plate = 54 cm

β = a/b = 1.888

t = thickness of plate = 0.5 cm

μ = Poisson's ratio = 0.3

E = Modulus of elasticity = 2.1×10^6 kg/cm^2

$$D = \frac{Et^3}{12(1-\mu^2)} = 0.024\times 10^6 \text{ kg.cm}$$

Table 12.7 : Details of Various Models

Structure	Volume of Plate (cm^3)	Total Volume of Stiffeners (cm^3)	Total Volume (cm^3)	Area of Cross-Section * (cm^2)	P_{cr} (kg)	RI
Model-1	2754	-	2754	27	26756	0
Model-2	2754	220.32	2974.32	29.16	36318	0.36
Model-3	2754	440.64	3194.64	31.32	42729	0.60
Model-4	2754	660.96	3414.96	33.48	35509	0.33
Model-5	2754	336.96	3090.96	29.16	98508.4	2.68
Model-6	2754	557.28	3311.28	31.32	115,536	3.32
Model-7	2754	777.60	3531.60	33.48	117,699	3.40

(* i.e. transverse section through plate and longitudinal stiffeners only.)

NOTE: The trend in RI value can be justified if one looks at the detailed buckling behaviour of longitudinally stiffened plate. In this case the response behaviour of the plate depends largely on how closely the stiffeners are positioned. In this particular case models 2 and 3 are probably governed by discrete stiffened panel theory whereas model 4 may be governed by orthotropic plate theory.

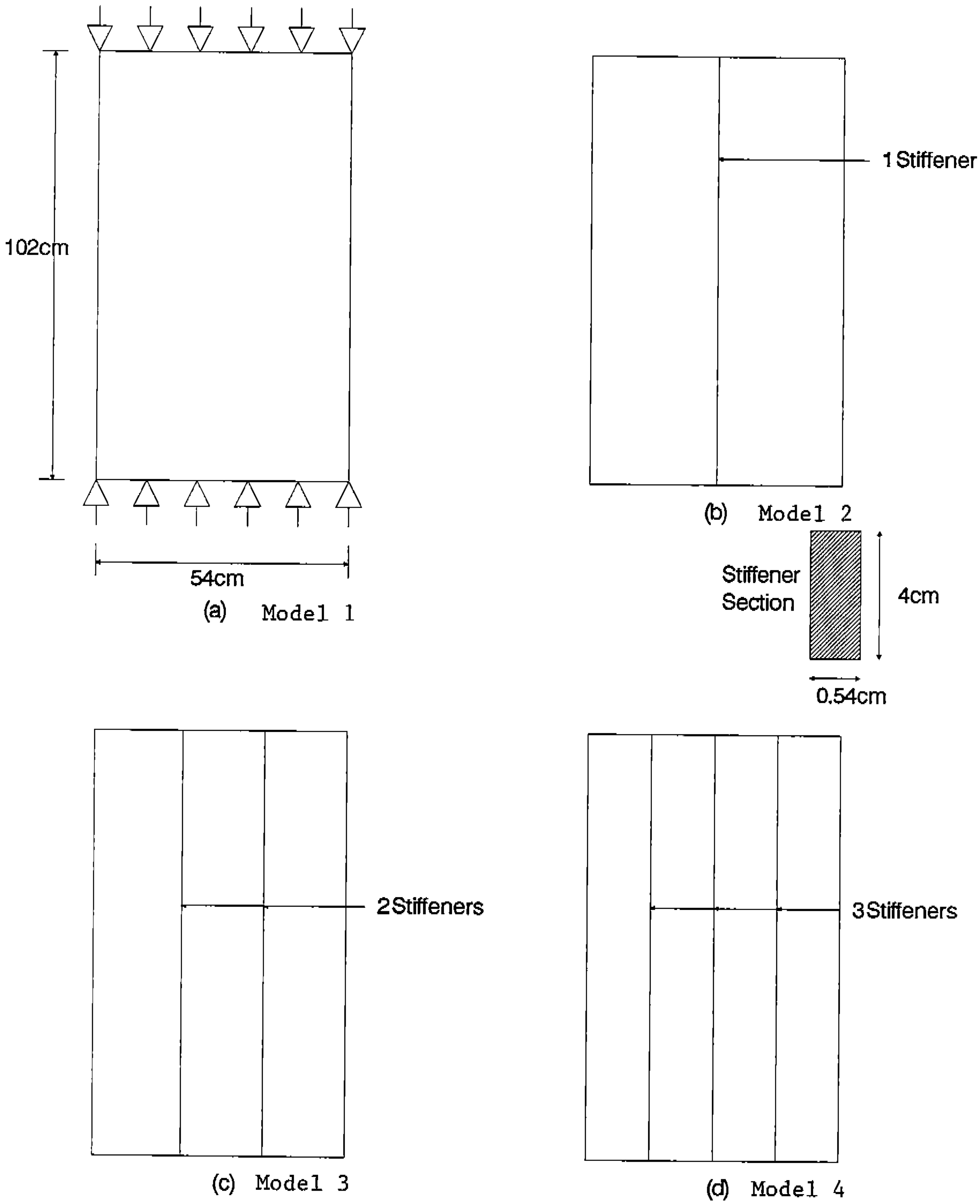

Fig.12.2(a) - (d) Selected Panel Geometries

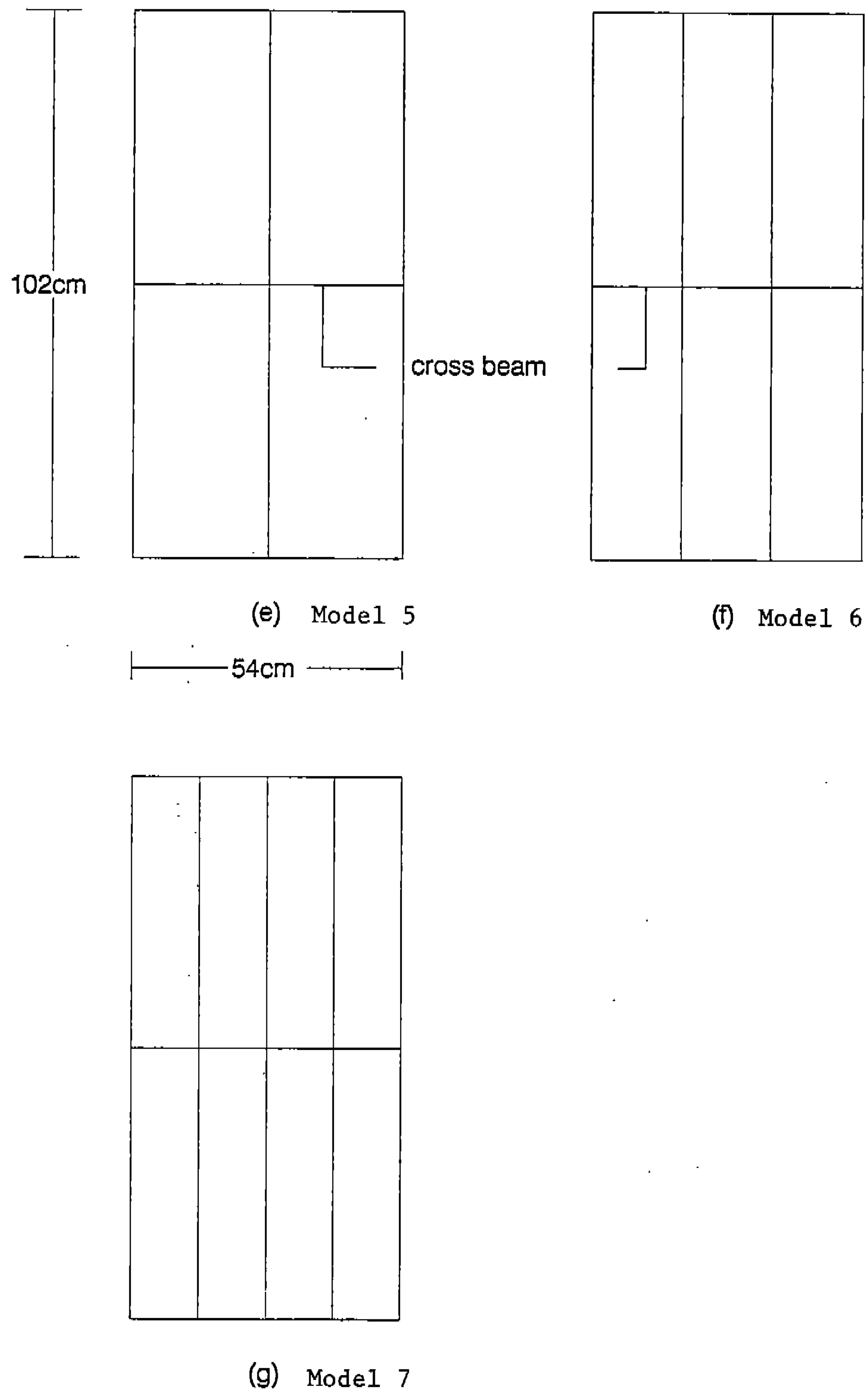

Fig.12.2(e) - (g) Selected Panel Geometries

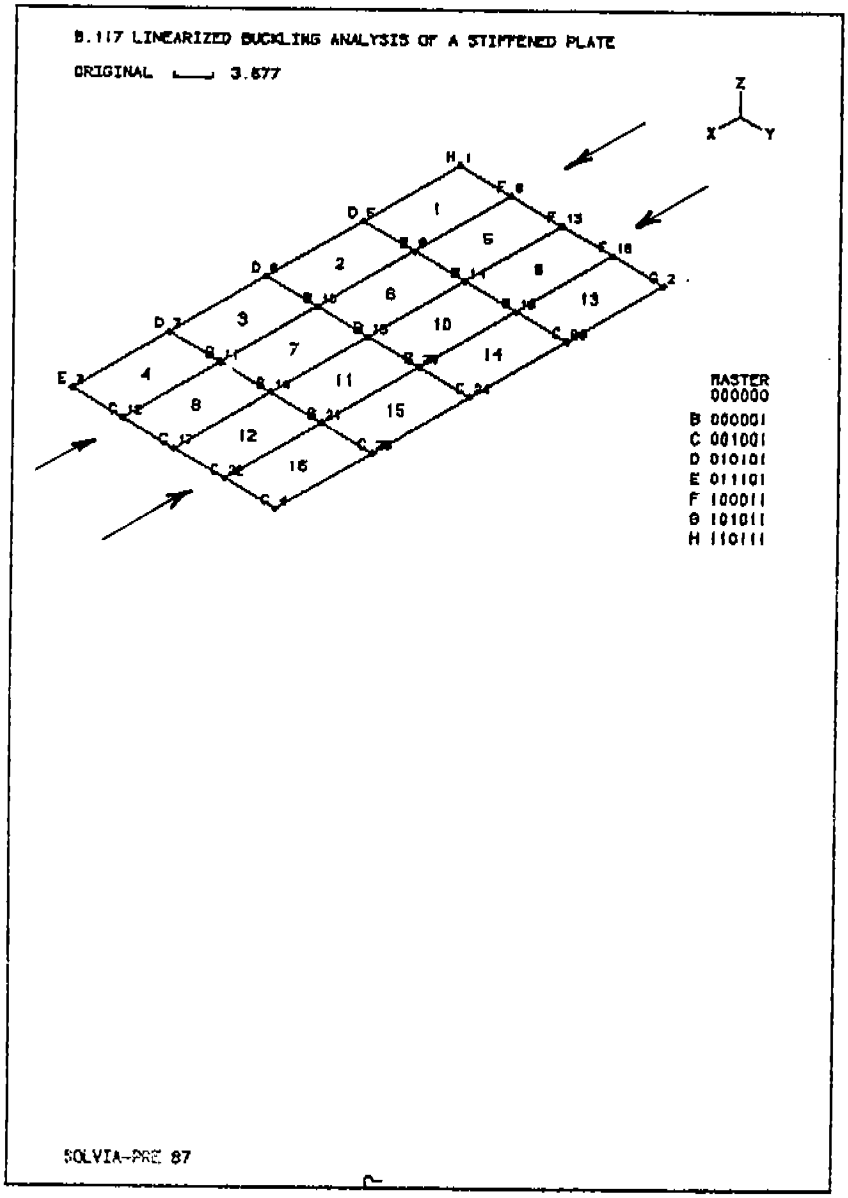

Fig.12.3 Finite Element Model : Quarter of Panel

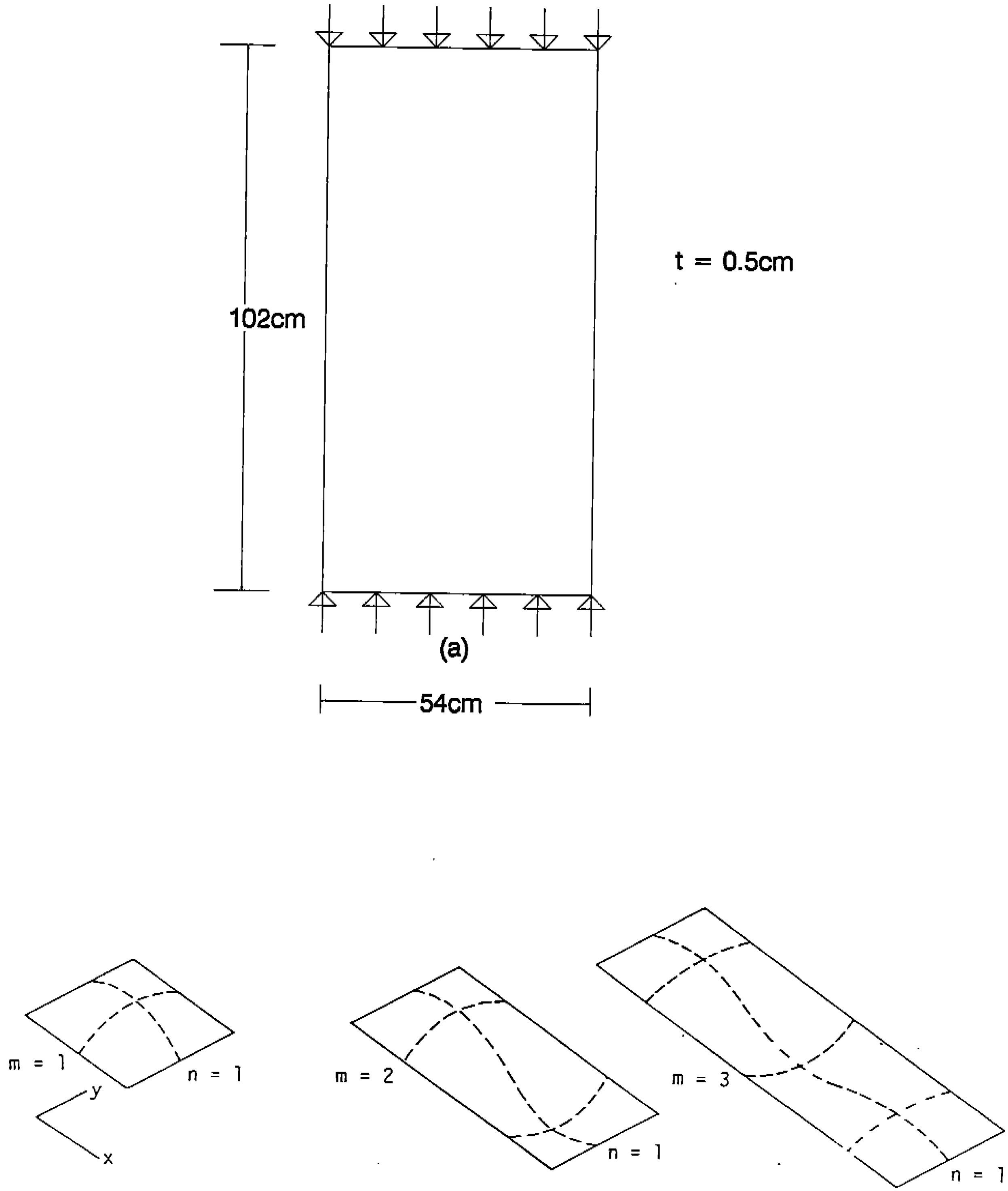

Fig.12.4   Basic Panel and Modes of Buckling

Stiffener dimensions: depth = 4 cm
: width = 0.54cm

Area of cross section of stiffened plate = 29.16 cm^2

I = Moment of inertia (second moment of area) of stiffener including an effective width of plate = 3.03 cm^4

$$\gamma = \frac{EI}{bD} = \frac{2.1^6 \times 10 \quad \times 3.03}{54 \times 0.024 \times 10^6} = 4.91$$

$$\delta = A_s/bt = \frac{4 \times 0.54}{54 \times 0.5} = 0.08$$ when A_s is the area of stiffener

From Table 9.16 of [12.16] the coefficient K was found to be 7.636.

Hence the buckling stress is given by:

$$\sigma_E = \frac{K\pi^2 D}{b^2 t} = \frac{7.636 \times \pi^2 \times 0.024 \times 10^6}{54 \times 54 \times 0.5}$$

$$= 1240 \text{ kg/cm}^2.$$

This compares well with the results obtained from using the SOLVIA program.

Table 12.7 shows the results of the critical load analysis of all the 7 models considered in this study. Using equation (12.2) as shown earlier, the redundancy indices (RI) were calculated for all these models.

RI for Model-1 is zero since this is considered to be the parent structure and in which the capability of the structure will reduce greatly in case any damage occurs in the plate. For other models, RI varies from 0.33 to 3.40. The lowest value of RI is with Model-4 and the greatest value is for Model-7, showing considerable design of redundancy to exist within this model.

However it should be noted that the panels were not optimised to give minimum total sectional area for given design load, etc.

12.1.3 Use of the Finite Element Method

Chapter 10 of this report contains a brief description of the possible forms and characteristics in stiffened flat panels. Redundancy in such structures clearly is a function of the numbers of the stiffeners involved (albeit not a linear relationship). Within most formulae driven methods to determine panel strength the methods analyse on a stiffener by stiffener basis with the panel strength being a simple multiple of the number of stiffeners on the panel - no allowance is made for edge support stiffening and strengthening effects. A finite element method approach that allows for large deflections clearly can determine the capability more accurately. Additionally, as discussed elsewhere within this report an FEA method can be much more readily used in order to determine the effects of local damage, including the removal of one stiffener whilst keeping the rest of the representation of the panel unchanged.

12.2 Circular Section Stiffened Cylinders

In this section some numerical calculations of the reserve and residual strength of 21 actual experimental combined ring and stringer stiffener test models, referred to in Appendix C, have been made. The reserve strength is based on a limit state design formulation as reviewed in Appendix C. and also given in [12.17] and the equivalent design load is based on API RP2A[12.24] and DnV[12.19] procedures. For the purpose of residual strength calculations, there is no specific procedure adopted in the various rules for the ultimate strength calculation of damaged structure. However some results based on a crushing mechanics procedure as shown in [12.20] have been developed.

The average axial crushing load for a thin walled ring only stiffened cylindrical shell of radius R and thickness t, made from a perfectly plastic material with an effective yield stress σ_Y, is given by, [12.21]:

$$P_{cr} = 2.286\ n^2\ t^2\ \sigma_Y \qquad (12.3)$$

where n is the number of diamond shaped buckles that form around the tube's circumference.

At low values of n the tendency will be for plastic collapse to occur . Theoretical analyses based on small deformations give, [12.21]:

$$n = 0.91\ \sqrt{R/t} \qquad (12.4)$$

However experimental observations have indicated that elastic buckles of only about half of this number will be formed.

It is suggested in [12.21] that for the usual practical range of R/t values, n may be taken as 5 and thus equation (12.3) gives:

$$P_{cr} = 57.16\ \sigma_Y\ t^2 \qquad (12.5)$$

This equation is more reasonable to use for practical analysis purposes.

For a stringer stiffened cylinder, which includes a reasonable number of longitudinal stiffeners, the average crushing load can be computed on the basis of results that have been found to be valid for simple geometric shapes representing the stiffener sections, following the approach proposed by Gerard [12.22] and applied in [12.20].

Following work by T. Wierzleickis, W. Abramowicz and J. deOliveira [12.23], the mean crushing strength of a ring and stringer stiffened cylinder can be estimated by:

$$P = 1.5\ \sigma_o\ t^{1.5}\ L^{0.5}\ g^{0.5}$$

$$g = m_f + m_c \qquad (12.6)$$

where

L = Total length of the plates and developed stiffeners in the plane of the cross section

$\sigma_o = 0.8\ \sigma_u$ = average crippling stress
t = wall thickness
m_f = number of flanges
m_c = number of cuts (between any corner points), (see [12.22]).

The behaviour of the two groups of cylindrical shells were then compared. The basic 21 models of combined ring and stringer stiffened shells were examined first, the geometry of which are given in Table 12.8. In this Table the predicted ultimate stress is calculated based on equation C.44 of Appendix C. The ring-stringer stiffened shells were then compared with equivalent ring only stiffened shells on the basis of equal material, volume or weight. This means that for each ring-stringer stiffened shell, the equivalent ring stiffened shell thickness was obtained by calculating the effective thickness of the shell which resulted from adding an equivalent thickness increment due to the stringer areas being uniformly distributed into the original shell thickness.

The results of the calculations for the Residual Strength Index (RDI) for both the 21 models and the equivalent ring only stiffened cylinders are shown in Table 12.9. The ratio σ_{us}/σ_{ur} shown in Table 12.9 indicates that the ultimate stress ratio for the ring and stringer to ring only stiffened shells ranges from a minimum value of 1.052 for model GU2 to a maximum value of 1.799 for the model UC9. The ultimate stress increases with the increase of the number of stringers. The crushing stress ratio σ_{cs}/σ_{cr} varies from 3.144 to 7.314 from model UC8 to model UC2 respectively.

The Residual Strength Index (RDI) given in Table 12.9 for the ring stiffened cylinders varies from 0.093 to 0.221 for the models UC2 and UC8 respectively. For the ring and stringer stiffened cylinder the RDI varies from a minimum of 0.309 to 0.499 for models GU2 to UC6 respectively.

With reference to the results presented in Table 12.9, the residual strength is taken to be the crushing strength and which is assumed to be independent of the amount of end load shortening. For a simple ring stiffened cylinder the axial load crushing stress is a function of the diameter and thickness of the shell and with the ring stiffeners having little or no influence (except for possibly effecting the formation of the buckle pattern, particularly when the rings are closely spaced). As noted above, equation (12.5) was employed to estimate the crushing, residual, stress.

For the combined ring plus longitudinally stiffened cylinders equation (12.6) was employed to estimate the crushing strength and again it was assumed that the ring stiffeners had little or no influence (again possibly slightly incorrect for closely spaced rings).

Thus the residual strength of the cylinder is much larger for the ring and stringer stiffened models than for the ring only stiffened models.

For the calculation of the Reserve Strength Index, (RSI), the feasible maximum design load P_d was determined by employing offshore codes API RP2A [12.24] and DnV [12.19]. Generally the design load is governed by partial (safety) factors on both the load and resistance components. For this exercise, the load factor was assumed to be unity whilst using the DnV Rules and only the resistance factor inherent in both the cases was taken into account in calculating the design loads.

Table 12.8 : Data on Combined Ring and Stringer Stiffened Cylinders Tested under Axial Compression

Model No.	r (mm)	t (mm)	ℓ (mm)	d_w (mm)	t_w (mm)	N	E (kN/mm²)	σ_{yc} (KN/mm²)	σ_u/σ_{yc} Test	Pred.	X_m
UC1	160.7	0.81	64.3	6.48	0.81	20	210	0.324	0.82	0.778	1.054
UC2	160.0	0.81	177.6	6.48	0.81	40	210	0.320	1.03	0.894	1.152
UC3	161.4	0.81	179.1	12.96	0.81	20	210	0.322	0.76	0.787	0.966
UC4	159.8	0.81	177.4	12.96	0.81	30	213	0.320	0.96	0.911	1.053
UC5	159.6	0.81	177.1	12.96	0.81	40	203	0.338	1.04	0.953	1.092
UC6	226.6	0.81	251.6	6.48	0.81	40	211	0.311	0.65	0.744	0.873
UC7	226.5	0.81	251.4	12.96	0.81	40	211	0.311	0.86	0.840	1.023
UC8	289.2	0.81	321.0	12.96	0.81	20	201	0.309	0.51	0.494	1.033
UC9	288.2	0.81	319.8	12.96	0.81	40	211	0.340	0.66	0.662	0.997
B1	226.8	0.81	353.8	12.96	0.81	40	210	0.313	0.82	0.785	1.045
B2	226.8	0.81	353.8	12.96	0.81	20	210	0.324	0.54	0.587	0.919
B3	226.8	0.81	251.7	12.96	0.81	20	210	0.284	0.60	0.660	0.909
B4	226.8	0.81	176.9	12.96	0.81	20	210	0.281	0.61	0.692	0.881
B5	226.8	0.81	353.8	8.67	1.22	40	210	0.318	0.82	0.730	1.123
GU1	571.7	2.0	890.0	32.0	2.0	20	191	0.234	0.57	0.655	0.870
GU2	572.1	6.0	760.6	95.0	6.0	8	197	0.300	0.89	0.901	0.988
GU3	588.5	3.0	650.0	45.0	3.0	30	204	0.420	0.69	0.835	0.827
IC1	160.0	0.84	65.0	6.72	0.84	40	201	0.348	0.96	0.964	0.995
IC2	160.0	0.84	65.0	6.72	0.84	20	201	0.348	0.96	0.766	1.253
IC3	160.0	0.84	180.0	13.44	0.84	40	201	0.348	0.95	0.954	0.995
IC4	599.2	3.53	666.0	48.0	3.53	20	205	0.289	0.87	0.860	1.011

Table 12.9 : Comparison of Combined Ring and Stringer Stiffened Cylinders with Equivalent Ring Only Stiffened Cylinder

Model No.	Cross Sectional area (mm^2)	RING-STRINGER STIFFENED			RING STIFFENED			RATIOS	
		N	σ_{us}/σ_y	RDI	t (mm)	σ_{ur}/σ_y	RDI	σ_{us}/σ_{ur}	σ_{cs}/σ_{cr}
UC1	924.9	20	0.778	0.406	0.914	0.570	0.130	1.365	4.263
UC2	1026.3	40	0.894	0.474	1.018	0.623	0.093	1.435	7.314
UC3	1033.4	20	0.787	0.380	1.017	0.616	0.093	1.277	5.220
UC4	1130.3	30	0.911	0.384	1.123	0.669	0.095	1.362	5.504
UC5	1234.2	40	0.953	0.406	1.228	0.667	0.105	1.429	5.525
UC6	1365.3	40	0.744	0.494	0.957	0.461	0.163	1.614	4.891
UC7	1574.7	40	0.840	0.407	1.105	0.522	0.122	1.609	5.368
UC8	1683.8	20	0.494	0.474	0.925	0.337	0.221	1.466	3.144
UC9	1888.7	40	0.662	0.473	1.042	0.368	0.175	1.799	4.862
B1	1576.2	40	0.785	0.436	1.104	0.517	0.123	1.518	5.382
B2	1366.3	20	0.587	0.443	0.957	0.444	0.169	1.322	3.465
B3	1366.3	20	0.660	0.394	0.957	0.492	0.153	1.341	3.454
B4	1366.3	20	0.692	0.375	0.957	0.496	0.151	1.395	3.454
B5	1579.4	40	0.730	0.447	1.106	0.512	0.125	1.426	5.098
GU1	8470.1	20	0.655	0.393	2.356	0.519	0.141	1.262	3.517
GU2	26240.7	8	0.901	0.309	7.262	0.857	0.086	1.052	3.777
GU3	15171.2	30	0.835	0.424	4.093	0.545	0.116	1.532	5.600
IC1	1072.5	40	0.964	0.446	1.064	0.592	0.102	1.628	7.120
IC2	959.6	20	0.766	0.420	0.952	0.545	0.099	1.405	5.963
IC3	1298.3	40	0.954	0.410	1.288	0.671	0.109	1.422	5.348
IC4	16717.9	20	0.860	0.377	4.427	0.711	0.094	1.210	4.851

σ_{us} = ultimate stress, ring plus stringer stiffened cylinders

σ_{ur} = ultimate stress, ring only stiffened cylinders

$\frac{\sigma_{us}}{\sigma_{ur}}$ = ultimate strength ratios $\qquad$ $\frac{\sigma_{cs}}{\sigma_{cr}}$ = crushing strength ratios

In the DnV Code [12.19] the partial factor format for control of instability in the rules is under single loading:

$$P_d \leq R_d = \frac{R_k}{\gamma_m} \frac{\psi}{\kappa} \tag{12.7}$$

where P_d is the design loading, R_d and R_k are the design and characteristic resistances, and γ_m, ψ and κ are as defined in equation C.35 of Appendix C.

Using equation (12.7) and equation C.32 of Appendix C, which correspond to DnV and API RP2A requirements respectively, the design load (P_d) is calculated for the ring stiffened cylinder and hence the Reserve Strength Index (RSI) values and these results are as shown in Table 12.10. In an actual structure where P_d will frequently be less than R_d, the reserve strength index will be somewhat higher than the values based on R_d.

An advanced first order second moment reliability analysis procedure, as reviewed in Appendix B, has been used to calculate the safety index of a typical example, selected to be model UC1 as given earlier in Table 12.8, and which is a combined ring and stringer stiffened cylinder.

Table 12.11 presents the results of these analyses. This table lists the basic variables pertinent to the strength model, and the statistical distribution type, mean and COV values assumed for each. A discussion on the selection of appropriate distributions can be found in [12.17], although in the case of the dynamic loading component a log-normal distribution is preferred in order to introduce a degree of conservatism on the most critical of the load components. The design load of 155.89 KN assumed in the analysis corresponds to a Reserve Strength Index (RSI) of 1.5. In addition to quantifying the probability of failure, p_f, and, more usefully, the safety index β, the analysis method also determines the design point, the sensitivity factor and the partial safety factor for each of the variables. These are listed in Table 12.11.

From Table 12.11, it can be seen that the model is most sensitive to both the modelling coefficient and to the applied load, then followed by the material yield stress and the shell thickness. The partial safety factors demonstrate a similar variability. The safety index is β=2.498 with a corresponding probability of failure of $p_f = 0.625 \times 10^{-2}$. The analysis was repeated for levels of design loads corresponding to RSI values of 2, 2.5 and 3 and a plot of the safety index β varying with RSI is shown in Figure 12.5 As can be seen from Figure 12.5 the β value increases with increase of RSI and when the RSI is less than unity, β becomes a negative value indicating that there is no reserve strength in the model.

Table 12.12 gives the corresponding results for an equivalent ring only stiffened cylinder based upon the original model UC1. The equivalent wall thickness is 0.914mm based upon equal material volume compared to the original thickness of 0.81mm of the stringer stiffened cylinder. The design load of 113.87KN is determined from the assumption of having a typical Reserve Strength Index of 1.5 in this model.

Table 12.10 : Reserve Strength Index, RSI, for Equivalent Ring-stiffened Cylinders

Model No.	σ_{ur}/σ_y	σ_{dr}/σ_y		RSI (σ_{ur}/σ_{dr})	
		APIRP2A	DnV	APIRP2A	DnV
UC1	0.570	0.386	0.381	1.476	1.495
UC2	0.623	0.403	0.417	1.545	1.495
UC3	0.616	0.401	0.412	1.535	1.495
UC4	0.669	0.417	0.447	1.603	1.495
UC5	0.667	0.429	0.446	1.552	1.495
UC6	0.461	0.340	0.308	1.354	1.495
UC7	0.522	0.363	0.349	1.438	1.495
UC8	0.337	0.294	0.225	1.147	1.495
UC9	0.368	0.314	0.246	1.169	1.495
B1	0.517	0.363	0.346	1.426	1.495
B2	0.444	0.340	0.297	1.305	1.495
B3	0.492	0.340	0.329	1.447	1.495
B4	0.496	0.340	0.332	1.459	1.495
B5	0.512	0.363	0.342	1.410	1.495
GU1	0.519	0.336	0.347	1.544	1.495
GU2	0.857	0.495	0.639	1.731	1.340
GU3	0.545	0.416	0.364	1.310	1.495
IC1	0.592	0.409	0.396	1.447	1.495
IC2	0.545	0.393	0.364	1.387	1.495
IC3	0.671	0.436	0.449	1.538	1.495
IC43	0.711	0.424	0.477	1.677	1.491

Table 12.11 : Reliability Analysis Results - Model UC1 (Ring-Stringer Stiffened Cylinder)

Basic Variables	Distribution Type	Mean	CoV(%)	Sensitivity Factor	Partial Factor
t (mm)	N	0.81	4	0.341	0.966
r (mm)	N	160.7	4	0.100	0.990
t_w (mm)	N	0.81	4	0.043	0.996
d_w (mm)	N	6.48	4	0.055	0.995
ℓ (mm)	N	64.3	3	-0.0107	1.0008
P (KN)	N	155.89	10	-0.565	1.141
E (KN/mm^2)	LN	210	3	0.060	0.995
σ_{yc} (KN/mm^2)	LN	0.324	8	0.351	0.929
X_m	LN	1.003	10	0.649	0.845

N = normal

LN = Log-normal

β = 2.498

$P_f = 0.625 \times 10^{-2}$

Actual test results for the equivalent ring-stiffened cylinders are obviously not available hence the mean and COV values of the model uncertainty factor are assumed to be 1.10 and 10% respectively for the purpose of this illustrative study. All other dimensions of the model and the material property are the same as that of the original ring and stringer stiffened cylinder. From Table 12.12 it can be seen that the model is most sensitive to applied load, then followed by the wall thickness.

In Table 12.11, a value of 1.003 for the model uncertainty factor (X_m) was used and this was derived from the set of experimental values which was discussed earlier. In Table 12.12, an assumed value of 1.10 was used as the model uncertainty factor for the equivalent ring stiffened cylinder in the absence of any experimentally derived model uncertainty factor. This value of model uncertainty factor was also varied for the ring stiffend cylinder from 1.05 to 1.20 and the results of these calculations are shown in Table 12.12. Since these results are based on an equal material volume basis, they are also thus valid for equal weight design.

The safety index was found to be β = 3.08 with a corresponding probability of failure $p_f = 0.10363 \times 10^{-2}$. As before the analysis was repeated for design loads which corresponded to RSI values of 2, 2.5 and 3 and the variation of safety indices with RSI for this model are as shown in Figure 12.6.

The corresponding safety indices for two other assumed values of the model uncertainty factor X_m, i.e. 1.05 and 1.20 (with the COV remaining the same, i.e. 10%) are shown in Table 12.13.

From Figure 12.5 it can be seen that for any particular value of RSI, the ring-stiffened cylinder yields a greater safety index than for the stringer-stiffened cylinder. However these results are in the context of the design load for the particular cylinder and the results are not to be construed as indicating that ring stiffening is the most efficient stiffening arrangement.

Another illustration of the variation of safety index (β), is shown in Figure 12.6, however in this case compared with the design load, and from which for a selected design load, the corresponding safety index can be determined. It can be seen that a ring and stringer-stiffened cylinder has a much higher intrinsic safety index than does the equivalent ring only stiffened cylinder for the same design load thus indicating that the probability of failure in the case of the stringer-stiffened cylinder is much less than that of an equivalent ring-stiffened cylinder, i.e. that the ring and stringer stiffened arrangement is the more efficient.

Table 12.12 : Reliability Analysis Results for an Equivalent Ring Stiffened Cylinder to Model UC1

Basic Variables	Distribution Type	Mean	CoV(%)	Sensitivity Factor	Partial Factor
t (mm)	N	0.914	4	0.485	0.940
r (mm)	N	160.7	4	0.056	0.993
ℓ (mm)	N	64.3	3	0	1.00
P (KN)	N	113.87	10	-0.552	1.17
E (KN/mm^2)	LN	210	3	0.134	0.987
σ_{yc} (KN/mm^2)	LN	0.324	8	0.155	0.959
X_m	LN	1.10	10	0.644	0.816

N = normal

LN = Log-normal

β = 3.08

$P_f = 0.10363 \times 10^{-2}$

Table 12.13 : Equivalent Ring Stiffened Cylinder (based upon Model UC1) - Variation of Safety Indices (β) with Model Uncertainty Factor

RSI	X_m		
	1.05	1.10	1.20
1.5	2.78	3.08	3.641
2	4.645	4.949	5.50
2.5	6.074	6.372	6.956
3.0	7.253	7.555	8.117

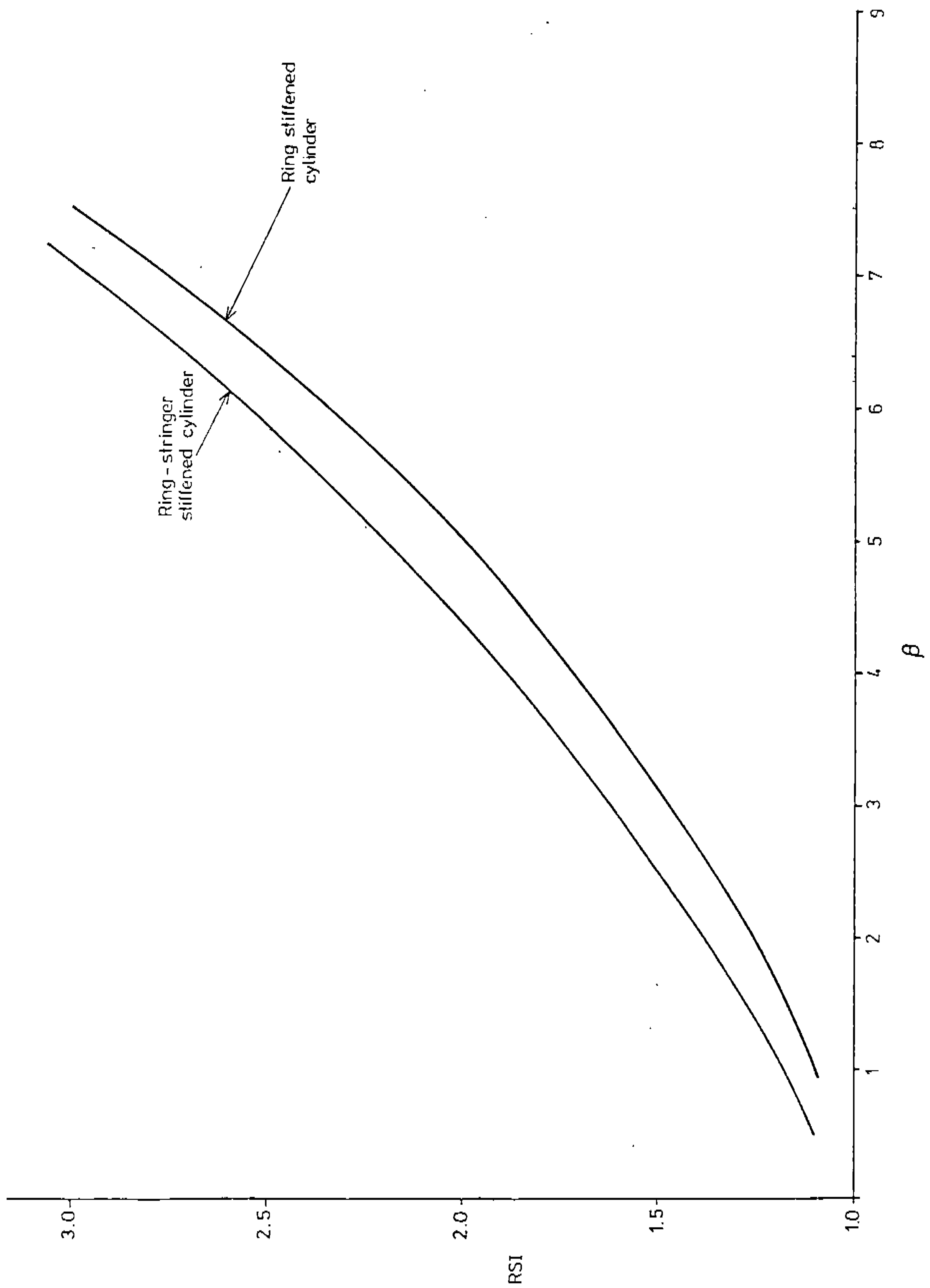

Fig.12.5 Variation of β with Reserve Strength Index RSI (Model UC1)

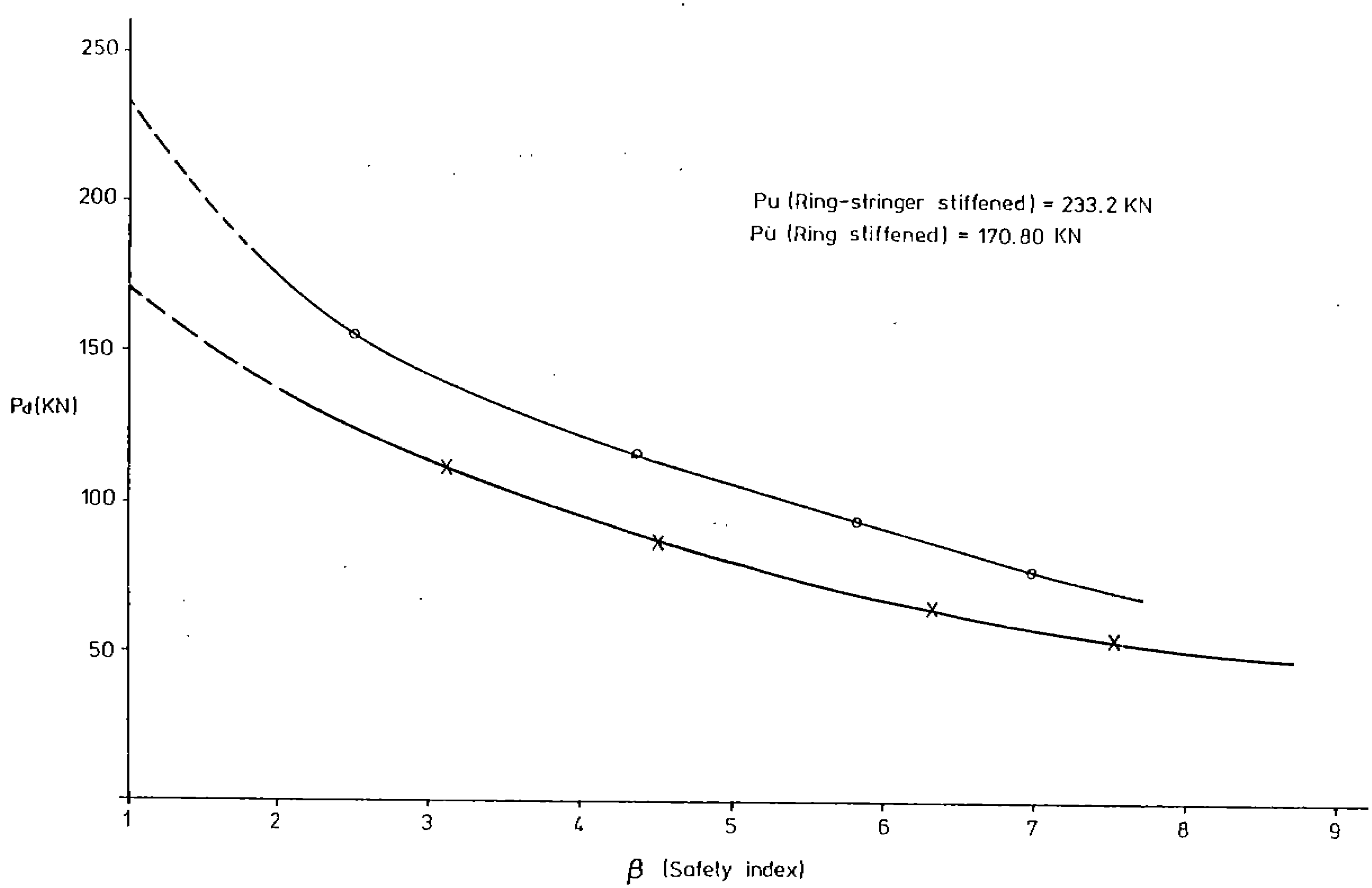

Fig.12.6 Variation of β with Design Load (Model UC1)

□ <u>Residual Strength of Damaged Ring Stiffened Cylinders</u>

In addition to the operational, environmental and nominal loads, for which offshore structures are designed, they may also be subjected to accidental loads due to, for example, impact of ships during material transfer or from heavy falling objects. The question which often requires an answer in the context of residual strength is whether the damaged platform is able to sustain the extreme loads that may subsequently occur before repairs are affected. The structure may sustain the load if it is found that the reduction in strength is not significant or, due to the degree of redundancy in the structure as a whole, there is not a substantial reduction in the overall ultimate strength of the platform due to the provision of alternate load paths. Knowledge of the post-damage strength of both the member and the platform is thus necessary in order to arrive at some decision about the degree of risk the platform is exposed to after such an incident has occurred and thus whether or not there is a need for platform abandonment. It has been reported in [12.25] that dents with a depth of 10% of the diameter or a permanent deflection of 0.4% of the length of the member occur in offshore platforms almost every two years in the North Sea and there have been a total of 560 various reported similar accidents around the world between 1970 and 1981 [12.26].

One of the earliest studies on the effects of damage on the axial compression buckling strength of an unstiffened tubular member was reported by Smith etc.[12.27]. The experimental work carried out in this study consisted of 16 specimen organised in 4 groups labelled A, B, C and D. Specimen in the A, B and C groups correspond to bracing members with small, medium and top end values of D/t ratios respectively. At larger D/t ratios, local buckling might be expected to influence post-collapse behaviour. The specimen D group consisted of thin-walled tubes of high strength steel. The nature of the damage considered in these specimens was in the form of lateral bending and/or local denting. These results indicated that the percentage loss of strength caused by damage was of the order of 15% to 48%. The results are shown in Table 12.14 (taken from [12.27]). However, these results are for relatively compact unstiffened tubular bracing members, clearly not in the same class as stiffened shells.

It was shown in [12.27] that there was a good correlation between the experimental results obtained and theoretical calculations made, using an elasto-plastic beam-column analysis when the damage was due to overall bending without denting (i.e. a column having an initial lateral misalignment rather than being straight).

In [12.25] the results of experimental studies undertaken on 24 specimen with almost no overall deflections were reported. However localised damage in the form of discrete dents were formed carefully in the specimen. The diameter/thickness ratios of the specimen varied from 40 to 62 and were tested under simply supported end conditions. The percentage of depth of dent to diameter varied from 2% to 20%. In this study a computer program called DENTA was then used to predict the load deflection behaviour of dented tubular members under axial compression loading. There was a good correlation found between the theoretical predictions and other experimental work.

Table 12.14 : Test Conditions and Collapse Loads [Ref.12.27]

Specimen No.	$\frac{D}{t}$	$\frac{L}{r}$	$- \frac{L}{r\pi}\sqrt{\frac{\sigma_y}{E}}$ (static σ_y)	Eccentricity of Axial Load	Damage Condition: Lateral Deformation δ_p/L	Damage Condition: Depth of Dent	Damage Condition: Max Lateral Load $PL/4D^2t\sigma_y$	Max Tube Ovality $(D_{max} - D_{min})/D_{mean}$: at Collapse	Max Tube Ovality $(D_{max} - D_{min})/D_{mean}$: post-Collapse	Experimental Collapse Load σ_u/σ_y: Estimated Static	Experimental Collapse Load σ_u/σ_y: Dynamic	% Loss of Strength Caused by Damage
A1	29.2	98.9	1.06	0	-	-	-	0.002	0.003	0.84	0.78	-
A2	29.0	98.9	1.04	0.16 D	-	-	-	0.003	0.006	0.49	0.45	-
A3	29.2	98.9	1.06	0	0.0055	1.4 t	0.95	*	*	0.48	0.43	43
A4	29.0	98.9	1.09	0	0.005	-	1.11	0.001	0.024	0.50	0.47	40
B1	44.7	78.2	0.77	0	-	-	-	0.001	0.046	1.00	0.93	-
B2	45.5	78.2	0.73	0.13 D	-	-	-	0.003	0.038	0.60	0.58	-
B3	45.2	78.2	0.76	0	0.005	3.7 t	0.81	*	*	0.52	0.47	48
B4	45.8	78.2	0.80	0	0.005	0.5 t	1.17	0.021	*	0.61	0.56	39
C1	60.2	60.9	0.63	0	-	-	-	0.004	*	1.10	0.98	-
C2	57.8	60.9	0.72	0.10 D	-	-	-	0.002	*	0.58	0.58	-
C3	58.1	60.9	0.67	0	-	2.0 t	-	*	*	0.76	0.73	31
C4	57.8	60.9	0.68	0	-	0.9 t	-	*	*	0.84	0.82	24
D1	87.3	68.4	1.02	0	-	-	-	0.002	*	0.75	0.75	-
D2	88.1	68.4	0.93	0.17 D	-	-	-	0.034	*	0.50	0.50	-
D3	86.3	68.4	0.98	0	-	3.2 t	-	*	*	0.53	0.52	29
D4	84.8	68.4	1.01	0	-	1.9 t	-	*	*	0.64	0.63	15

* affected by dent or local buckling

In [12.25] Taby & Moan employed the DENTA program to calculate the load-deflection characteristics of axially compressed, pin-ended, unstiffened cylinders. The damage considered was in the form of both a local dent and combined lateral deflection of any size and location. The theory employed could calculate (i) the behaviour of the model up to first yield point, (ii) behaviour from first yield up to elastoplastic overall instability and ultimate load, and (iii) full plastic post-collapse behaviour.

Based on experimental work on both large-scale and small-scale tests carried out at AMTE, England [12.27, 12.29] and in NTH, Tondheim, Norway, [12.25] an empirical method was suggested by Smith and which gave a good correlation with these experimental results.

Another simplified method proposed by Aanhold & Taby, referred to in [12.30], assumed that the dented length of a member could be replaced by an equivalent eccentrically placed (with regard to the axis of the undamaged cylinder) circular tube. The advantage of this model is that this could be easily implemented within a non-linear finite element program. However, a dent could also be modelled within a large deflection/non-linear capable finite element program.

Walker & Kwock [12.31] studied the mechanics of denting in thin walled cylinders. Their approach was based on a plastic hinge mechanism representing the shape of the dent and has provided an analytical relationship between applied denting load and residual dent depth and this theory was applied to experimental results from tests on five nominally identical models, having a R/t, (radius to wall thickness), ratio of 190 and found to be in good agreement.

The assumptions of a rigid boundary surrounding the dented region as a simplification was found to be at variance with evidence from the test results where bulging of the shell circumference was noted. With high L/t (length to wall thickness) and low R/t ratios this assumption will result in a gross simplification and as such may not be applicable to these ranges of tubular members.

In [12.33] Onoufriou established a relationship between the reduction in strength of a ring-stiffened tube caused by a panel dent between rings and this is shown in Figure 12.7. This procedure used a large-deflection elasto-plastic finite element package and as can be seen from the figure, demonstrated that a dent size of equal to only five times the plate thickness can reduce the axial strength capability by over 50 per cent. However, it seems that there is no test data available for comparison.

In [12.33] Onoufriou, et al, have used a numerical model to predict the residual state after impact damage and which can then be used as an initial condition for axial load analysis. A finite element method using a 'gap/contact' element was used to model the denting process. Correlations between the experimental results and the calculated numerical values was found to be good.

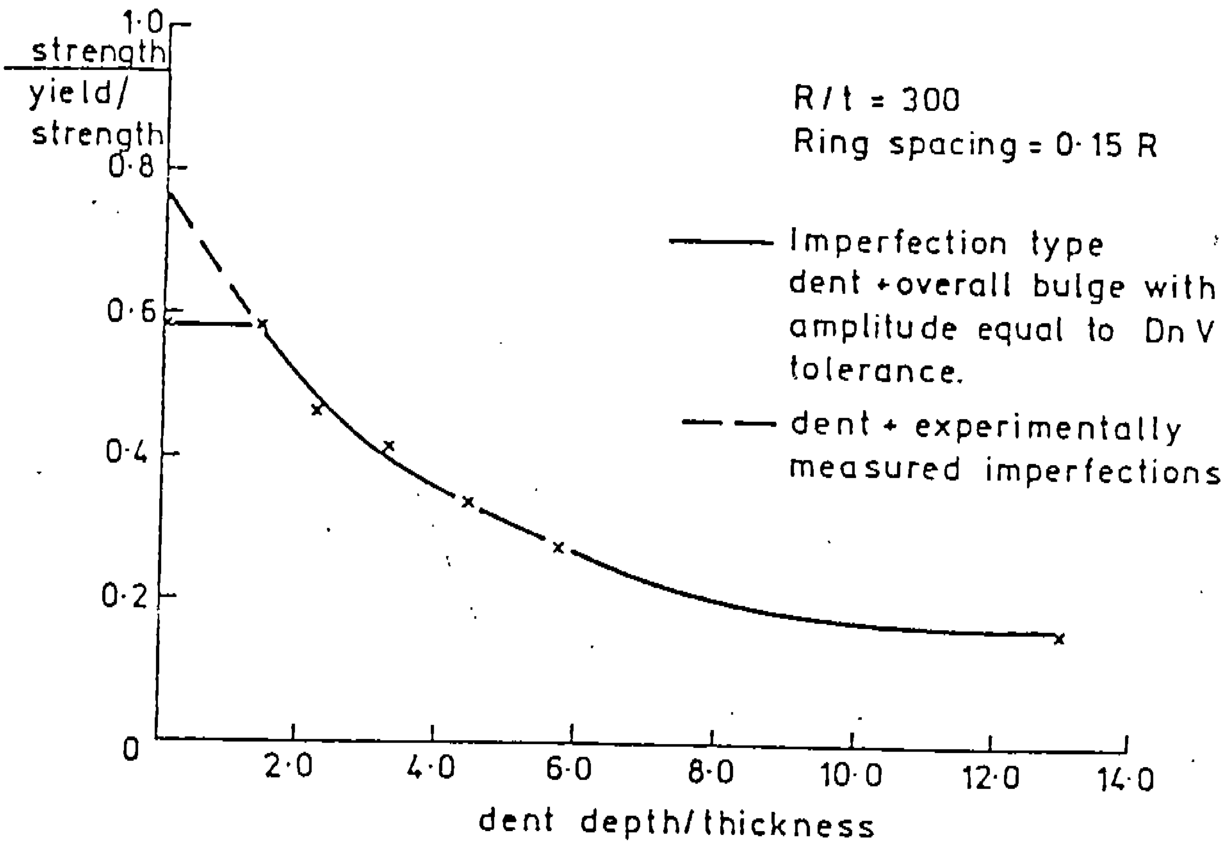

Fig.12.7 Reduction in Strength of Ring-Stiffened Tube Caused by Panel Dent

In [12.34], some damaged ring stiffened cylinders were analysed assuming external pressure loading and using available general purpose non-linear finite element programs. Numerical studies were made to establish the process of damage development and the subsequent residual strength of the damaged structure under external pressure. Various forms of damage were considered and they included (i) general and overall indentation of the shell plating between frames, (ii) localised damage caused by impact of sharp concentrated objects, and (iii) bending deformation of ring frames. It was concluded that (i) can cause substantial loss of strength, whilst (ii) in the form of short-wave length indentation of the shell between frames has a relatively small effect, and that (iii) may reduce the collapse pressure by between 40 and 50% of the strength of the undamaged structure having only characteristic as-manufactured imperfections.

Ellinas in [12.35] used an analytical model to evaluate the strength of axially compressed unstiffened tubular members allowing for both general bending deformation and local denting forms of damage. The method under-estimated response compared with other formulations, the results for increasing dent depth and decreasing D/t ratio.

Results of tests carried out at Lehigh University [12.36] on a 40 inch diameter ring stiffened cylinder revealed virtually no reduction in stiffness or ultimate strength under axial load due to indentation of a member. For another specimen of 60 inch diameter, the stiffness was also unchanged but the ultimate strength was somewhat reduced. The rather unexpected lack of the reduction in strength of the 40" diameter speciman due to the indentation was, in the opinion of the researchers who carried out the work attributed to the end conditions of the specimen during testing.

□ Residual Strength of Ring/Stringer Stiffened Cylinders

Available literature on the prediction of the behaviour of combined ring/stringer stiffened cylinders after damage is very limited. The results of one such study was reported by Ronalds & Dowling [12.37] and in which a simple analytical procedure was suggested. The predictions were then compared with some tests in which a lateral wedge induced loading was applied at mid-length to cause local damage.

The models were of a small-scale and with a diameter of 320mm. The ratios of radius to wall thickness were within the range $190 \leq R/t \leq 267$ and the stringers were either twenty or forty in number and were uniformly spaced. The geometric details of the models are as given in Table 12.15 [12.37].

Lateral deflections were induced by slowly raising the models against a rigid wedge-like indenter aligned parallel to the rings. Displacement transducers and strain gauges were used to monitor axial movements of the cylinders surface. The models experienced planar end shortening and rotation around most of the circumference, with additional non-linear pull-in within the dent zone. Strains were measured in the dent zone in order to calculate the membrane stresses.

Table 12.15 : Geometries of Models Ref.[12.37]

Cylinder	R/t	L/R	N	$A_1 = h_1 t_1$ mm^2	$A_r = h_r t_r$ mm^2	L_D/L
1A1	190	0.42	40	6.7 x 0.84	-	1
1A2	190	0.42	20	6.7 x 0.84	-	1
1B1	254	1.08	40	3.8 x 0.63	-	1
1B2	254	1.08	20	3.8 x 0.63	-	1
3A1	190	0.48	40	13.4 x 0.84	24.0 x 3.0	1
3A2	190	0.33	40	6.7 x 0.84	24.0 x 3.0	1
3A3	190	0.33	40	6.7 x 0.84	24.0 x 3.0	1
3B1	267	0.60	40	4.8 x 0.60	6.5 x 0.82	3
3B2	267	0.60	40	4.8 x 0.60	6.5 x 0.82	3
3B3	267	0.60	20	4.8 x 0.60	6.5 x 0.82	3
3B4	267	0.60	20	4.8 x 0.60	6.5 x 0.82	3

A_1 = Stringer cross sectional area = $h_1 t_1$.

A_r = Intermediate ring frame cross sectional area = $h_r t_r$.

L_D = Length of dent.

R/t = Ratio of radius of cylinder to wall thickness.

L/R = Ratio of length of cylinder to radius of cylinder.

L = The length of cylinder between adjacent ring frames

A plastic mechanism type analysis of the indenting process was developed by regarding the shell as a series of longitudinal beam-like elements supported by ring frames.

The load carrying capability of a single discrete longitudinal beam element at any stage in the denting process was shown to be given by:

$$\frac{P_j}{N_p} = \frac{4(\delta_j - \delta_{rj})}{L} \tag{12.8}$$

in which P_j is the lateral load capacity of beam element j, δ_j is the mid-span lateral deflection of beam element j, δ_{rj} is the lateral deflection of ring at beam element j, L is the bay length between ring frames and $N_p = \sigma_y A$, where σ_y and A are tensile yield stress and cross-sectional area of the beam element respectively.

Equation (12.8) is termed the 'membrane' solution.

The total lateral load supported by the shell is the sum of the load carrying capacities of all dented beam elements. A beam element can be thought of as being composed of an individual stringer and the associated width of shell plating and for the case where there are no stringers, thin longitudinal strips of the shell may be regarded as simple rectangular section beams. No edge connections between adjacent beam elements are assumed within this method.

When equation (12.8) is integrated across these individual elemental beam strips, this results in a dent force to dent depth relationship, which for single bay dents is given by:

$$\frac{P_t}{\sigma_y Rt} = 8 \frac{R}{L} (\sin\eta - \eta \cos\eta) \tag{12.9}$$

where $\cos\eta = 1 - \delta_o/R$, δ_o is the central dent depth, R is the radius of cylinder, t is the shell thickness and P_t is the total applied lateral load on the cylinder to cause the damage.

The energy absorbed by denting is given by the area under the curve of lateral load against central dent depth. In this analysis other secondary modes of response were not taken into account but this could be justified for its application to columns of buoyant offshore platforms where local denting is the important damage deformation mode. Various other studies have been made into the axial load carrying capability of a damaged/dented cylinder. These studies have ranged from simple stress concentration factor types of approaches, analogus beam-column models allowing for varying area properties through to complex finite element or finite difference based methods.

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13.0 MULTI-CELLED BOX BEAM STRUCTURES

13.1 General Multi-cellular Structures

Section 12.2 of this report reviews some aspects of the predominantly compressive loading of circular section stiffened cylinders. Whilst such elements are essentially continuous structures, if viewed locally, they are often modelled as discrete elements in some global analyses, e.g. in semi-submersibles which may be modelled as three-dimensional beam-element frameworks.

Circular section cylinders are single celled members. The next level of topological (and loading) complexity is a multi-celled box beam type of structure, i.e. a section which is essentially thin walled cylindrical but of any general cross-section shape and which has internal longitudinally orientated divisions which sub-divide the cross-section into more than one closed-cell (e.g. a watertight space). The external and internal surfaces may then be stiffened longitudinally, transversely or both.

The provision of such longitudinal internal structural surfaces, which must be bounded by either/or other internal surfaces or the external shell, provide load paths for shear forces, more appropriately described "shear flows", that are due to either direct overall shear or to torsional moments on the cylinder. These internal load paths provide means where the re-balance of equilibrium of the structure can take place following, for example, damage to one of the outer surfaces. Hence the multi-celled arrangement contains a form of primary/intermediate level of redundancy.

In the marine field the most complex multicellular forms are found in ship sections, e.g. tankers, bulkcarriers, multideck freighters, etc. which may contain double bottom and inner side shell arrangements as well as decks and longitudinal bulkheads. A simple tanker having a single bottom and one centreline longitudinal bulkhead thus has two cells - this is possibly the simplest multicellular ship type form.

The pontoons and towers of typical semi-submersible vehicles are often multicelled structures. It is to be noted that the internal divisions/surfaces may be there for one or more reasons, typically cargo support and/or separation, watertight subdivisions for flooding control, discrete tanks, etc.

Thus in terms of structural topology many marine and offshore structures are comprised of multi-celled box beams, of a non-prismatic form in the case of many ship types and in assemblages with complex joints in the case of many forms of semi-submersibles.

The loading on these structural elements is, in general, quite complex. Of most concern with regard to ships are longitudinally distributed vertical bending moments and shear forces. In some structures however the loading may be much more complex involving bending moments and shear forces about both 'horizontal' and 'vertical' axes and torsion about the longitudinal axis of the cylinder.

An illustration of redundancy at the primary level can be given with reference to overall shear and torsional loading. Figure 13.1 illustrates configurations of progressively increasing complexity in which the degree of redundancy, with regard to shear and torsional loading, also clearly increases.

Section (i), in Figure 13.1, represents a simple open beam section having good vertical bending and shear capability - but with, however, only minimal torsion capability, influenced by warping restraints that may be imposed at each end of the span of the element. A deep I-section beam could be a primary component of the main cross structure of, for example, a semi-submersible. In terms of pure vertical bending and shear a simple deep I-section is probably the most efficient in material usage.

Section (ii) could relate to a section through a very simple ship or barge-like vessel, inway of a cargo hatchway. This section could have good vertical and horizontal bending and shear capability but with, again, only minimal torsional capability and calling upon restrained warping mechanisms where possible.

Sections (iii) and (vi) are more representative of typical ship-like transverse sections having various degrees of efficiency vis a vis bending and shear about both vertical and horizontal axis and torsion about the longitudinal axis.

13.2 Ship Hull Girders - Bending Strength Analysis

□ Traditional Approach

The traditional approach for the determination of hull girder scantlings has been to assume that the simple theory of elastic bending is applicable and with all material being fully effective. This assumption has been applied to bending, shear and torsional response analyses and simple allowable stress criteria have been employed, although in some case simple component buckling checks have been called for or recommended, e.g. on deck plates and longitudinal stiffeners.

There are two variations in the traditional approach:

- the rules formulae, and
- the application of rational theory based formulae (e.g. based upon classically derived methods) for the stress and strength analysis of components.

In the latter of the above two approaches, allowances are generally made for the assessment of locally effective material in component level analyses. However, again, the overall response is generally based upon the simple theory of linear elastic bending applied to thin walled box beams and with all material being taken to be effective in determining the hull girder section's overall area properties.

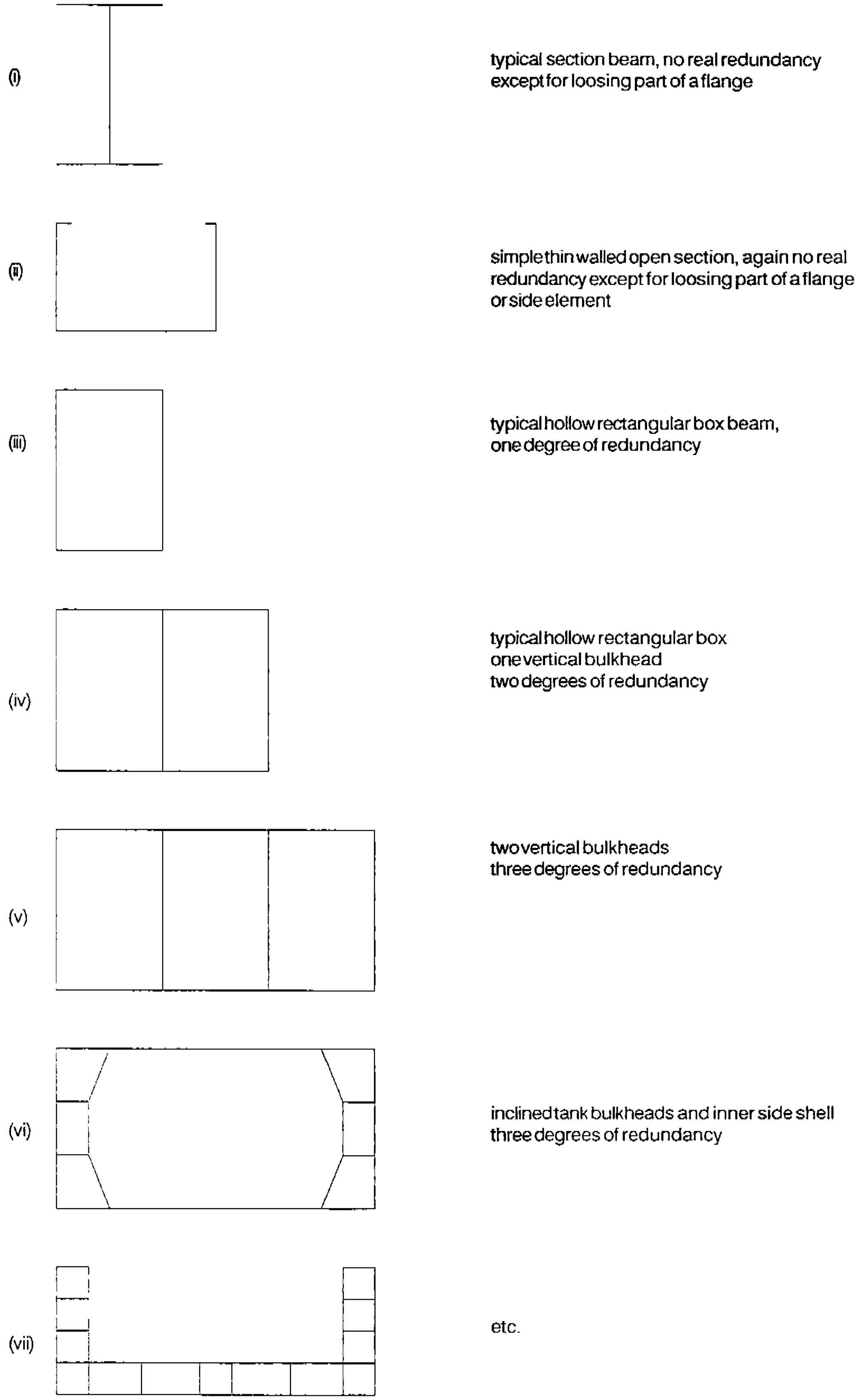
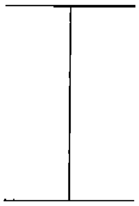
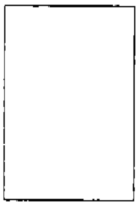


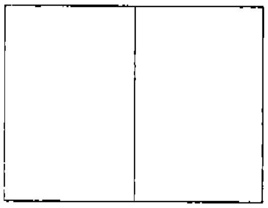
Fig.13.1(a) Simple Beam Sections : Re. Vertical Shear Capability

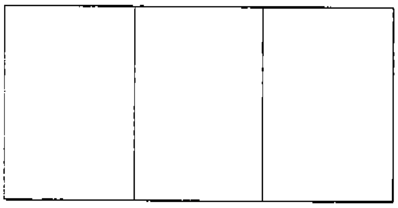
(i)

typical section beam, no real redundancy except for loosing part of a flange etc.

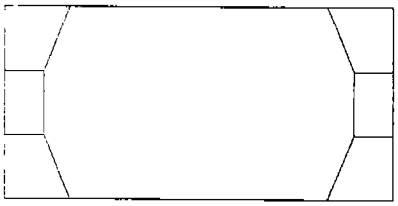
(ii)

simple thin walled open section

(iii)

some redundancy, but the percentage loss of capability is so large that it should be ignored

(iv)

two cells, redundancy to one level
level of performance reduction depends upon which surface (including horizontal) fails

(v)

three cells, higher redundancy
follows on from above

(vi)

multicellular, high level of redundancy

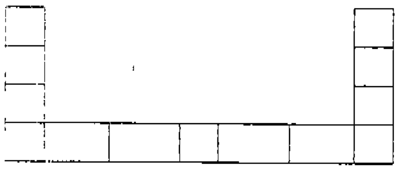
(vii)

etc.

Fig.13.1(b) Simple Beam Sections : Re. Torsional Capability (plus Warping etc.)

Neither the rules formulae based approach nor the alternative more rational approaches, at this level, reflect in any way a measure of the true ultimate strength capability of a hull girder section. Both the simple allowable maximum stress criteria and the design being based upon a single component reaching its failure point approaches, fail to allow for the large reserves implicit in the understressed material within a beam's cross-section.

However several aspects must be appreciated in judging the rules based and simple rational formulae based methods that have been employed to date:

(i) both hull stiffness, dynamic behaviour and fatigue considerations are involved in sizing a hull girder section (albeit not explicitly determined in all design development studies) and

(ii) the notional factors of safety, used in association with such approaches, have been arrived at following the accumulation of a great many ship-years of service.

In recent years several advanced analytical studies have been made on the levels of safety inherent in designs that have been developed by the traditional methods, for example studies made by Mansour, Faulkner, Sadden[13.1, 13.2] and others. Such studies have generally demonstrated quite high values of safety indices, but with considerable scatter, when full statistical variables have been allowed for, as illustrated in Figure 13.2. Although the general trends of safety related to ship length shown in Fig.13.2 are valid some caution should be attached to the relatively higher degrees of safety implied for the then 'recent' merchant ships compared with the military vessels. It would appear from [13.41 & 13.42] that for the merchant vessels the analyses were based on the long-term distribution characteristics rather than the extreme value distribution.

Other reliability method based assessment studies have been made on the performance of hull girders, allowing for the full statistical nature of all relevant geometric, material and loading variables, e.g. the study reported on by Hart, et al in 1985 [13.4]. However most of these studies, including the one by Hart, et al, assumed that the simple theory of elastic bending was satisfactory to determine the boundary forces applied to each of the stiffened flat panels which collectively formed the hull girder at the transverse section being analysed.

Amongst the first studies to develop a more rational representation of the overall hull girder response to vertical bending were the cooperative parallel studies undertaken at Registro Italiano Navale [13.5, 13.6] and by Committee V2, Applied Design, ISSC 1988. These studies allowed for the effects of ineffective plate material between longitudinal stiffeners and resulted in a somewhat hybrid combination of non-linear elastic-inelastic response behaviour of the overall hull girder and associated with rigorous analysis at the individual stiffened flat panel level.

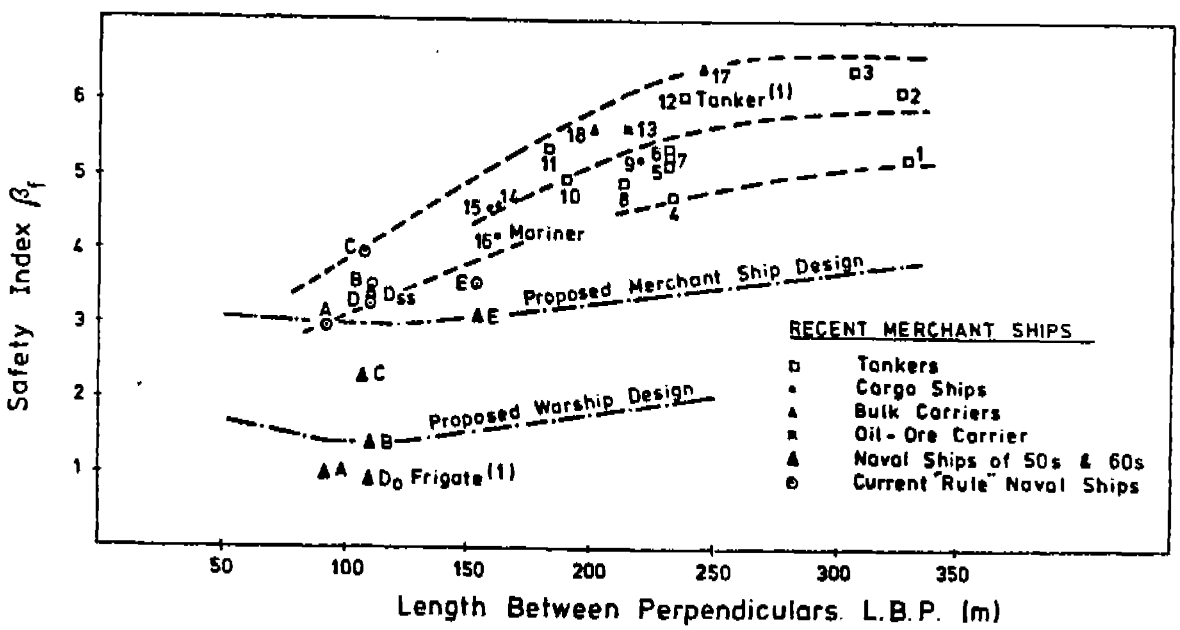


Fig.13.2 Safety Indices for Naval and Merchant Ships [13.1]

□ Simple Ultimate Strength Approaches

The study of hull girder ultimate strength, assuming simple prismatic box beam models, has been undertaken for many years. One of the earlier major studies was that reported on by Caldwell[13.7] which developed from simple fully effective material plastic hinge models to models in which effective material only was considered, e.g. representing the effects of material between longitudinal stiffeners buckling under inplane compression resulting from overall bending. Caldwell's method is readily adaptable to allow for regions of the structure which buckle at average stress levels well below the material yield, i.e. the simple plastic hinge calculation can readily be modified to allow for both locally effective material and different levels of limiting stress throughout the section, Figures 13.3, 13.4 and 13.5. (Similar methods have also been employed within the aerospace industry for many years.)

Several researchers, e.g.[13.11, 13.27], have further developed and extended the methods outlined by Caldwell. One particularly useful study is the US.SSC report compiled by Mansour[13.8].

The main failings of the methods developed by Caldwell, Mansour and other researchers, based upon the simple plastic hinge concept, is that no allowance is made for the effect of continued straining on member's once their ultimate capacity has been reached, i.e. a plateau of strength at either the material yield stress, or a some lesser average level, is assumed. Whilst this is acceptable for solid stable structures (although in such both strain hardening and strain limits are encountered) this method is unacceptable for elements which exhibit post-ultimate strength unloading as the axial straining increases, as typically found in stiffened shell construction.

□ Rigorous Ultimate Strength Approach

Whilst still assuming prismatic box-beam models more accurate non-linear elastic-plastic-buckling response analysis studies can now be undertaken.

The following method was outline by Smith in [13.9] and allows for the more accurate representation of the local behaviour of each major element within the box beam cross-section. The method can be applied to a broad range of box beam shapes, including ones with internal structural planes, e.g. a typical hull girder section. Pure bending only is allowed for, i.e. no allowances for the effects of overall shear, torsion, axial forces or local forces, e.g. external/internal hydrostatics.

The box beam cross-section is divided into elements, each one of which is either:

- a longitudinal stiffener, plus full associated width of plate.

- a plate element of full span (if unstiffened in a longitudinal direction), e.g. a girder web, hatch side girder web, shell plate in a transversely framed region.

- a 'hard' spot.

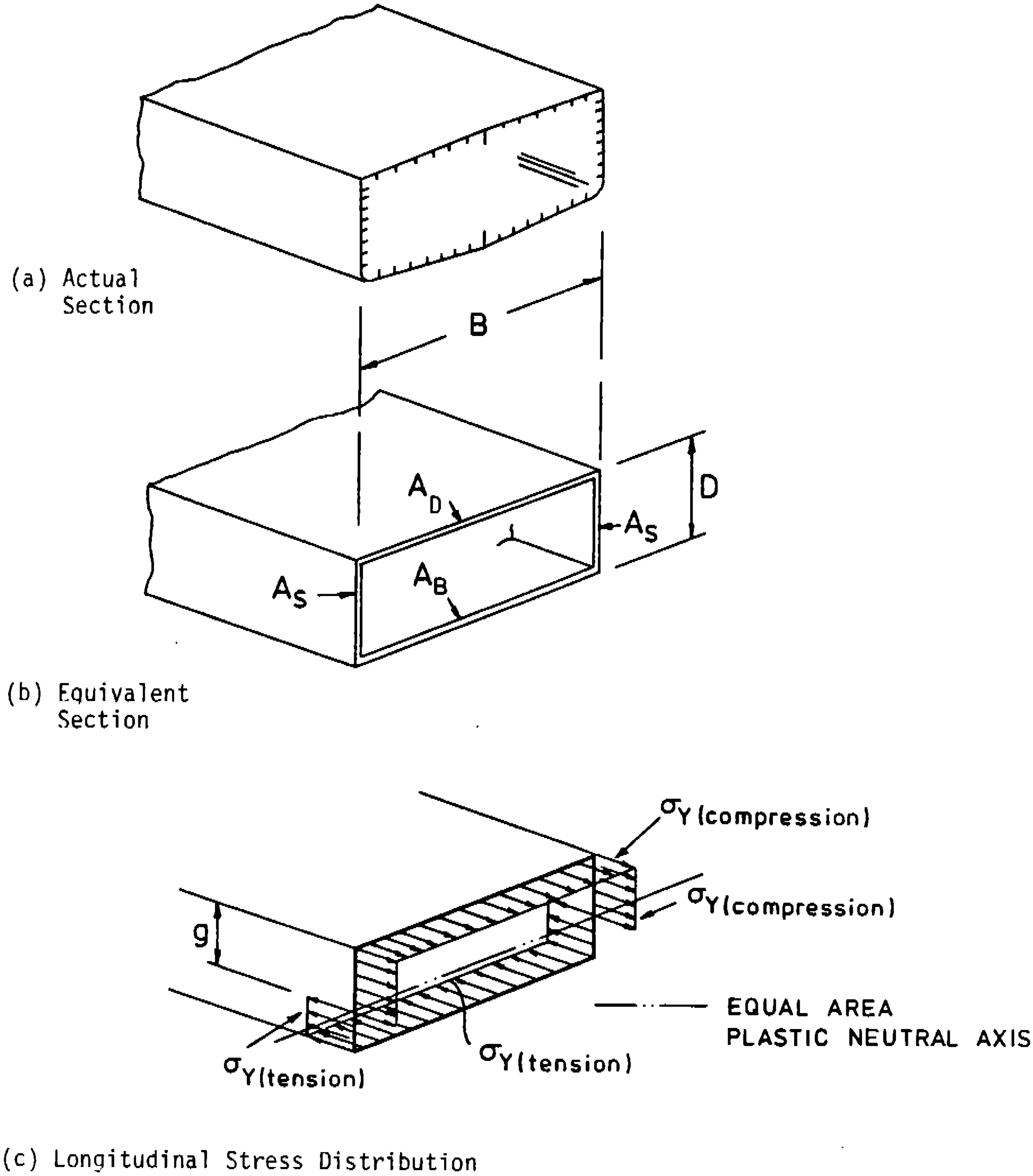
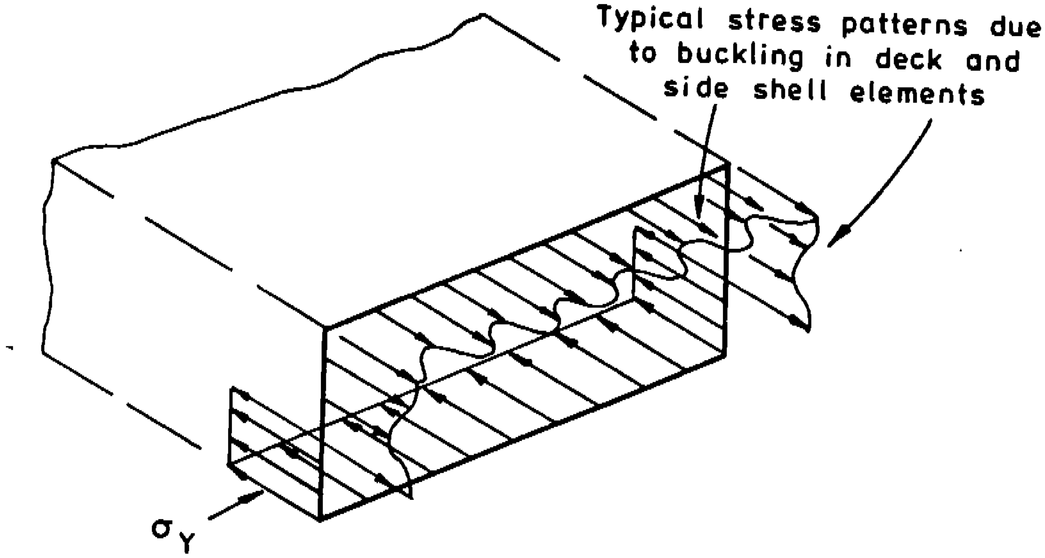
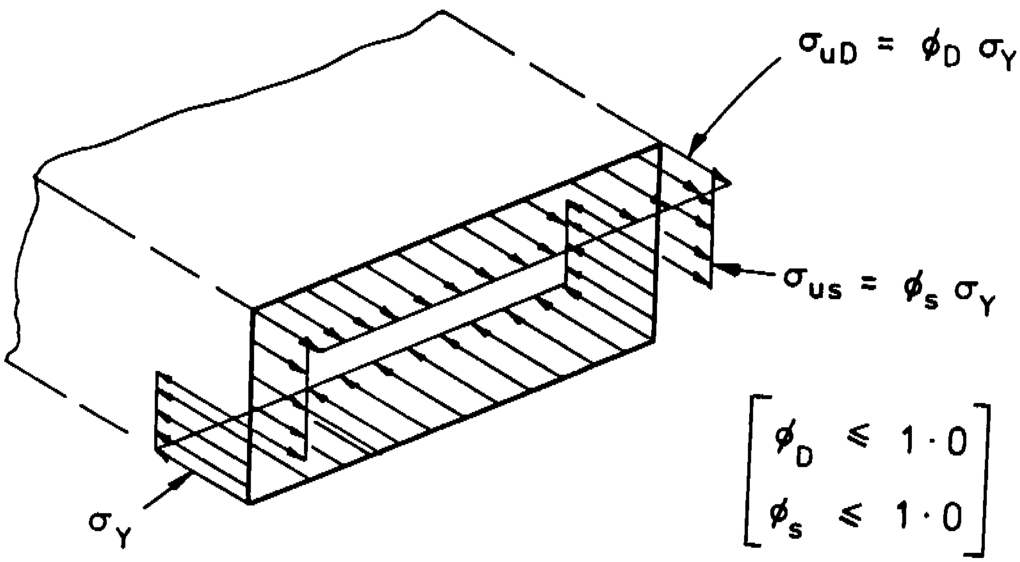


Fig.13.3 Fully Plastic Bending of Box Beam

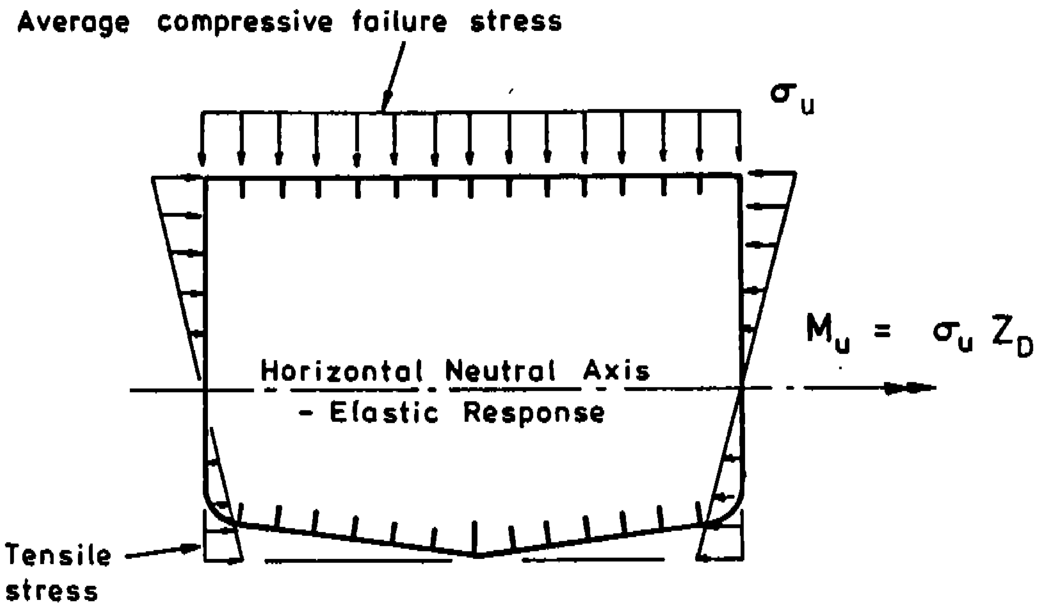


(a) Actual Longitudinal Stress Distributions

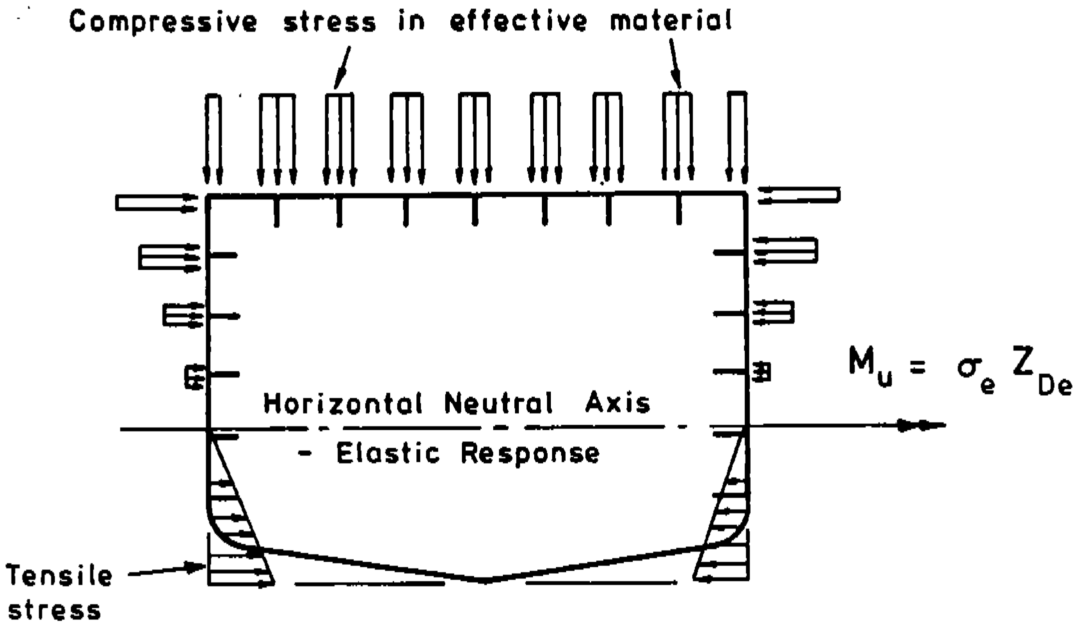


(b) Equivalent Stress Distributions

Fig.13.4 Ultimate Bending Strength Allowing for Buckling



(a) All Material Assumed Effective



(b) Due to Plate Buckling Reduced Effective Material on Compressive Surface

Fig.13.5 Ultimate Moment Limited by Buckling of Compressive Surface

A 'hard' spot is an area of structure where, due to the particular structural detail or arrangement, it is assumed that any form of buckling under the action of compressive forces is completely resisted. Thus a 'hard' spot is an item of local material capable of responding in the same elastic-perfectly plastic way as the tensile material in the overall cross- section, see Figure 13.6. Typical 'hard' spots are as follows:

- deck edge details, either due to plate joints or plate curvature (i.e. gunwhale radius),
- intersections of major plate elements, e.g. decks/bulkheads to shell plate, girders to shell plate,
- hatch side girders/continuous coamings to deck plate,
- the junctions of upper and lower tank shelf plates to shell plate and sloping bulkheads (longitudinal).

These examples are illustrated in Figure 13.6.

The load-shortening curves for each element are then created, noting that there may be many geometrically identical elements. Such curves will reflect both the tensile and compressive response of each element as, in general, it will be required to determine both the maximum hogging and maximum sagging strengths of the box beam section. The tension - compression load shortening curve for a typical longitudinal stiffener plus full width element of attached plate is of the form shown in Figure 13.7(a). It is generally found that it is possible to determine or estimate, via several methods, in adequate detail such a load shortening curve and then such would be subsequently employed in the following analysis method. However, in some cases, the data and methods whereby such curves can be determined may not be available and thus the idealised form, for both tension and compression response, as illustrated in Figure 13.7(b), would be employed.

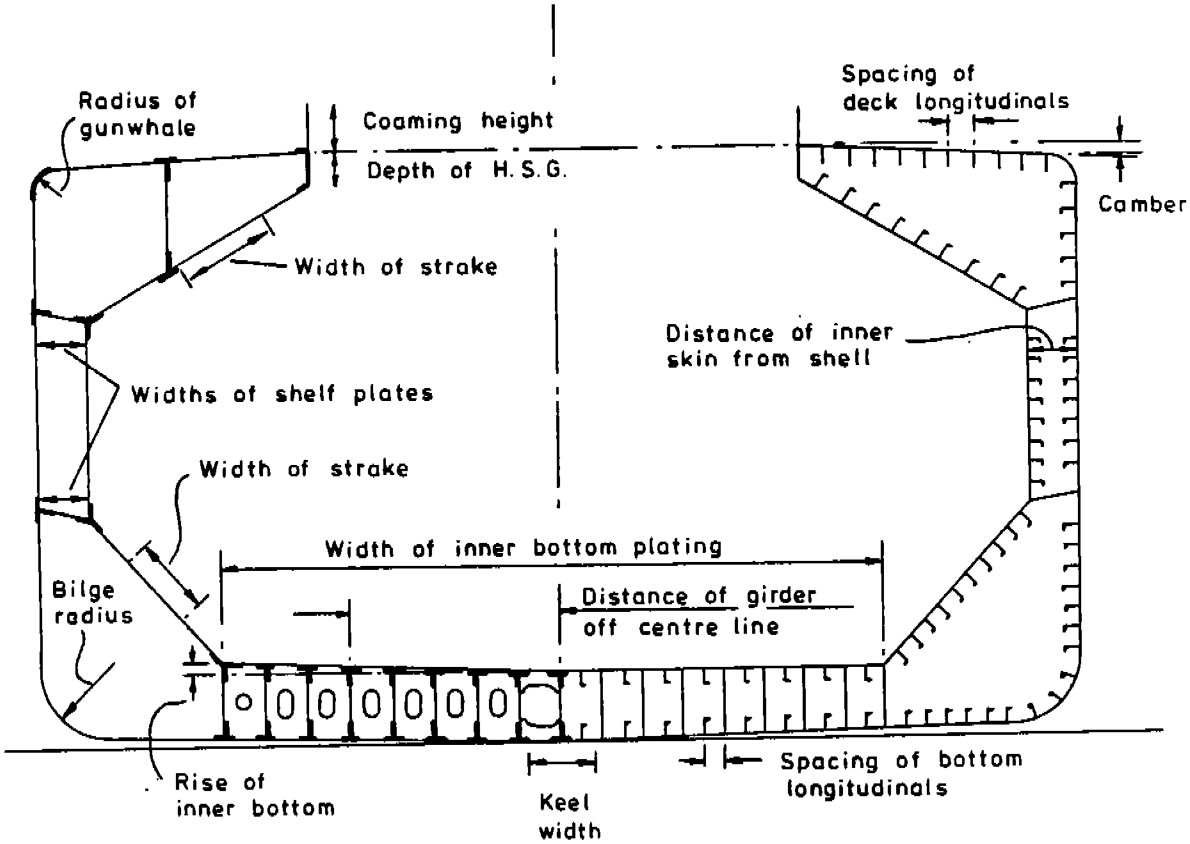
It is found that these end load shortening curves can have a range of shapes, some having an appreciable plateau of ultimate capability whereas others can, theoretically, fail in a most precipitative manner analogus to a semi-brittle mode, as discussed in Chapter 8. Clearly these extremes of response will have a significant effect on the resulting hull girder overall behaviour vis a vis ultimate bending strength.

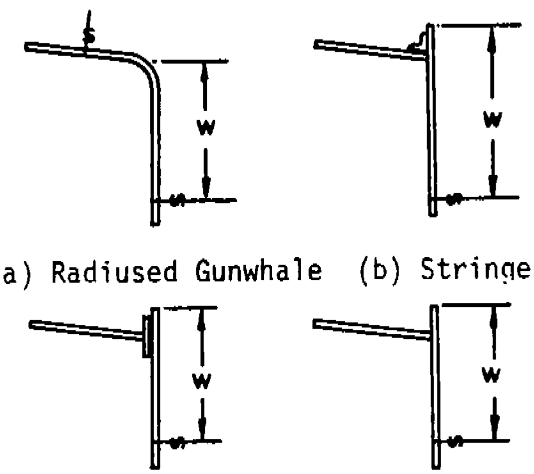
The procedure assumes that the local curvature of the beam develops from zero bending moment to the collapse bending moment in an incremental manner. According to the simple theory of bending:

$$\left[\frac{1}{R}\right] = \frac{M}{EI} = \frac{\sigma}{EY}$$

By definition (1/R) is the curvature of the beam section. For any given box beam section, assuming all material to be fully effective the maximum value of curvature corresponding to the elastic limit anywhere within the section can be readily developed. This is done by:

- determining the properties of the section, the second moment of area (I), and position of the elastic neutral axis.

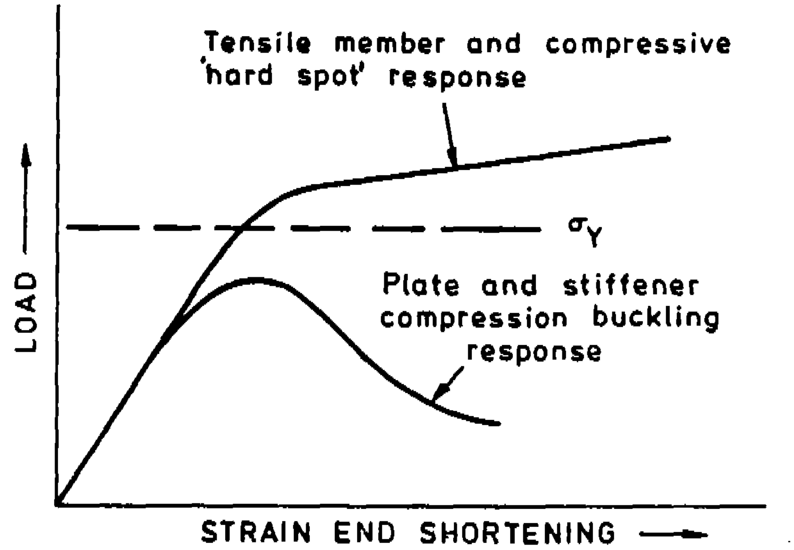




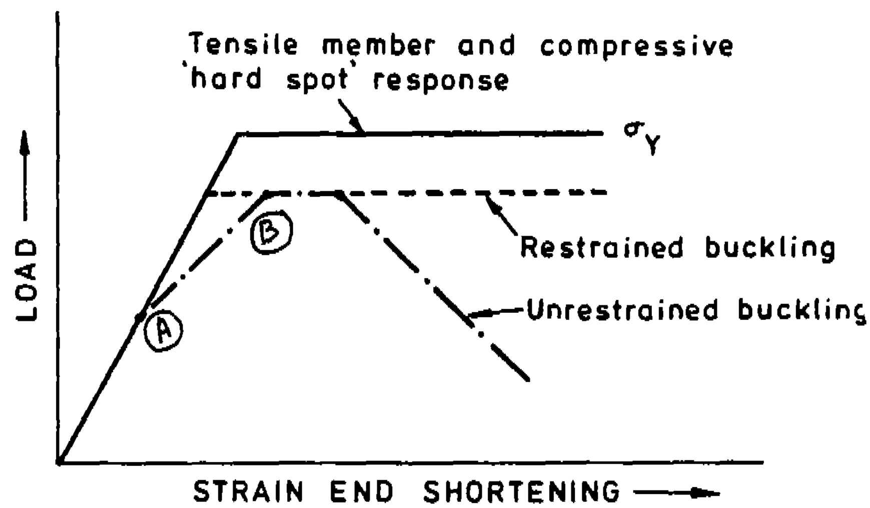
(a) Radiused Gunwhale (b) Stringer Angle

(c) Stringer Bar (d) Stringer Plate Welded Directly to Sheerstrake

Fig.13.6 Typical 'Hard' Spots



(a) Typical Axial Loading Response



(b) Idealised Axial Loading Response

Fig.13.7 Stiffened Plates Under Axial Load

- applying a unit moment and determining the corresponding stresses throughout the section.

- determining which element will reach its elastic limit first, i.e. via Figure 13.7(b), and

- then the corresponding maximum elastic moment is then readily determined and the associated value of the radius of curvature.

This value of the elastic limit radius of curvature then forms the starting point for the incremental process. The magnitude of the subsequent increments would be arbitrarily selected to represent only a small percentage of the elastic limit curvature.

The assumption is made that plane sections remain plane and that at each incremental change of the curvature that the strain imposed on any element is determined from:

$$\text{Strain} = \bar{Y} \times \left[\frac{1}{R}\right]$$

where $\bar{Y}$ is the distance of the element from the current, or instantaneous, position of the neutral axis.

Given the strain imposed on each element, the actual load-shortening curves are then used to determine the corresponding elemental forces. At this stage at least one element will have reached point 'A' on its compression load-shortening curve (unless the section is tension critical) and further increases in strain will result in a more flexible local structure, as represented by the slope from 'A' and 'B' (i.e. where $E' < E$).

For overall section equilibrium the summation of elemental forces (horizontal) across the whole section must equate to zero, i.e. from simple statics there must be zero net horizontal force in the section, as only a pure bending moment is assumed to be applied. Because of this the originally assumed position of the elastic neutral axis will not be the same as the true neutral axis position; thus the latter needs to be determined in an iterative manner which maintains an equilibrium of forces.

If the increment of curvature change has been of only a relatively small value then it may suffice to assume that the individual elemental forces do not change (i.e. that the change in $\bar{Y}$ will be negligible with axis shift) and that only the new position of the instantaneous neutral axis need be determined prior to the next incremental change in the curvature.

The above process is then repeated and a relationship is obtained between the changing curvature and the corresponding value of the bending moment resistance, as given by the summation of the moments of the individual element forces about the appropriate instantaneous neutral axis. Gradually some elements will reach point 'B' in their load-shortening curves and overall beam failure will result when sufficient elements pass this point that overall rapid unrestrained collapse follows. The ultimate bending strength of the box beam section is thus given by the peak value of this resisting moment, as illustrated in Figure 13.8.

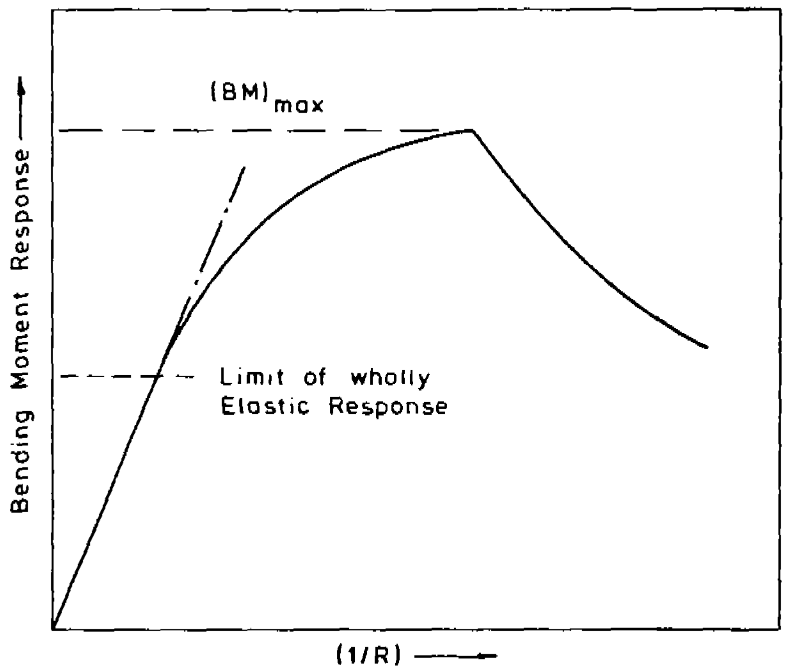


Fig.13.8 Pre- and Post-Ultimate Strength of Box Beams

Other researchers have developed similar methods [13.10, 13.12, 13.14]. There have apparently been only a few limited studies to compare theoretical predictions using such methods with test results, [13.15, 13.16, 13.32]. This is understandable in view of the costs of such test programmes.

13.3 Hull Girder - Structural Redundancy

□ Longitudinal Material

Overall hull girder bending, shear and torsional response and capability is a direct function of the longitudinal material (however the transverse material, in the form of frames and bulkheads, clearly effects the capability in controlling the primary compressive and shear stability of the longitudinal structure).

Redundancy is at both the panel level and the overall general arrangement level:

- at the panel level redundancy appears in the form of the number of discrete longitudinal stiffeners, and

- at the overall general arrangement level redundancy appears in the form of numbers of panels which form decks (both internal and external), side shell, single or double bottoms, internal longitudinal bulkheads (both vertical and inclined) and shelf plates, etc.

The benefits of redundancy are called upon when individual elements or in some cases whole panels, reach their local ultimate load carrying capability and accept no further local loading when the overall hull girder loading is increased. Clearly the gradient of imposed inplane strain along the transverse edge of a stiffened panel is significant in calling into play the redundancy at the individual multi-stiffener panel level. This aspect is clearly catered for in the rigorous methods developed by Smith, Viner, Adamchak and other researchers. However in these fairly recent rigorous analytical methods there is still one implicit and quite major assumption, that is that axial strains are directly proportional to the vertical distance from the instantaneous 'neutral' axis of the overall elastic-inelastic beam section. This assumption leads to the inference that strains imposed upon an horizontal surface, e.g. an internal or external deck, are uniform across the surface. (This is also clearly assumed within the traditional simple theory of elastic bending approach.) However in real ships in actual service there will generally be some variation in strain across the breadth of a horizontal surface - this is due to the complexities of overall loading, the balance between external forces and internal masses and the structural shear lag effects. Whilst exploring this is not the purpose of this study it must be appreciated that this does exist and that fortunately the 'redundancy' of the structure plays an important part in mitigating its effect vis a vis ships in service.

As a general note it is to be appreciated that within the scope of a non-linear static analysis, extended to include the region of panel level post-ultimate strength response, that the analysis may have to be carried out with the assumptions of controlled displacement loading, rather than with dead load, in order to pass and go beyond the peak of the panel's load-deformation curve. In this case the assumption of breadthwise uniform strain would be valid. Additionally uniform strain is also a valid assumption at the midship region of a uniformly loaded (e.g. homogenously and fully loaded) vessel. This latter assumption may be slightly questionable for, say, a bulkcarrier with some holds empty and in other situations where there may be a complex shear lag problem.

13.4 Damage to Hull Structure

In the context of this study damage to hull girder longitudinal structure is the main focus of attention as damage effects both the overall hull girder strength and local panel strength. However clearly damage to transverse structure will also have significant effects on the ability of longitudinal structure to withstand imposed forces and thus cannot be neglected in any in-depth study.

'Damage' to a hull structure can be due to one of many potential causes, e.g.

- ship to ship collisions

- ship to other object collisions (e.g. to berths, shore fixed equipment, etc.)

- grounding

- loading and unloading operations (e.g. grab damage, excessive wheel loads on vehicles, dropped heavy items of cargo or outfit)

- explosions and/or fire.

Such forms of damage would be in the nature of permanent buckles, distorted and ruptured components, etc., and may range from slight/modest in proportions to quite extensive and massive. Depending upon the cause of the damage such damage could occur almost anywhere along the length of the vessel. Clearly collision damage will predominate in the side shell region of the hull girder and towards the forwards end if the vessel involved was the 'striking' rather than the 'struck' ship.

Similarly grounding damage will be limited to the bottom shell and bilge regions with some possible concentration at the forwards end dependent upon the trim condition at the time of the incident.

Other forms of 'damage' would include:

- excessive corrosion and
- fatigue and brittle fracture cracks,

however such would be generally associated with inadequate original design rather than due to 'accidental' causes.

Obviously damage, of whatsoever form, affects the residual strength of the remaining intact hull girder section and depending upon the location of the damage the most significant effects will be related to either vertical bending, vertical shear, horizontal bending, horizontal shear or torsion or combinations thereof.

In the context of simple prismatic box-beam methods of ultimate strength analysis, e.g. of the form proposed by Smith and other researchers, the first and most direct approach would be to delete either individual or simple groups of longitudinal stiffeners and associated hull plating or whole panels, according to the extent and location of the damage being examined or postulated. This will, in general, lead to the remaining hull girder section being unsymmetrical and result in unsymmetrical properties and response capability.

A fundamental problem in analysing the residual strength in damaged hull girders is that both the damage and the hull girder are three-dimensional and assumptions of prismatic box-beam response, even with elements removed to emulate the damage, are not compatible. Prismatic beam theory assumes constant transverse section area properties for some distance both fore and aft of the section being analysed - sufficient for the stresses to redistribute themselves in accordance with the assumptions made in the simple beam theory, either elastic or plastic. Often damage creates a complex three-dimensional stress concentration type of effect and stresses at the boundaries of the damage will differ considerably from the predictions made by simple prismatic beam theory. Thus the prefered, albeit complex and expensive, approach would be to invoke a three-dimensional non-linear finite element method based solution of the form reviewed by Thayamballi [13.19] and Kutt [13.20]. Using an approach of this nature the residual strength of locally deformed structure could be allowed for in the model by suitably representing the shapes of the deformed, but un-ruptured, elements. Similarly cracks of appropriate lengths and locations could be introduced into a structural model.

Returning to the simple prismatic beam model approach. If the 'length' of the damage is assumed to be relatively small (in the context of the prismatic beam model's 'length' direction and other proportions) then the effects of the damage could, in some cases and loading conditions, be allowed for by considering some stress concentration effects superimposed on top of the beam section's response. This will produce higher stresses in the undamaged stiffened structure immediately adjacent to the region of the damage. When such highly stressed elements fail in a ductile manner, either by tensile yielding or compression buckling, then the stress concentration effects will tend to diminish, particularly if the overall ultimate strength of the section is considerably greater than the elastic limit strength.

There are various possible approaches to introducing the effects of cutouts in multi-stiffened structures of the stiffened panel, stiffened cylinder and box beam configurations. Typical conventional stress concentration factor type data generally applies to either solid sections, e.g. bars and rods, or to unstiffened plate-like material. The effects of stiffening arrangements require some form of analysis for each specific situation, geometry and loading condition. Approaches can range from simple internal force redistribution and balance calculations, to give zero internal forces, through the actual cutout or discontinuity, see Fig.13.9, through to the employment of finite element methods. In some ways the internal force redistribution and balance approach and the more simple stress concentration factor approaches are analogous to the energy release approach review in Section 8.6 of this report. Both the stress concentration types of approach and the local energy release approach in continuous structures assume that the effects are of a local perturbation form and that the overall global balance of forces are not changed.

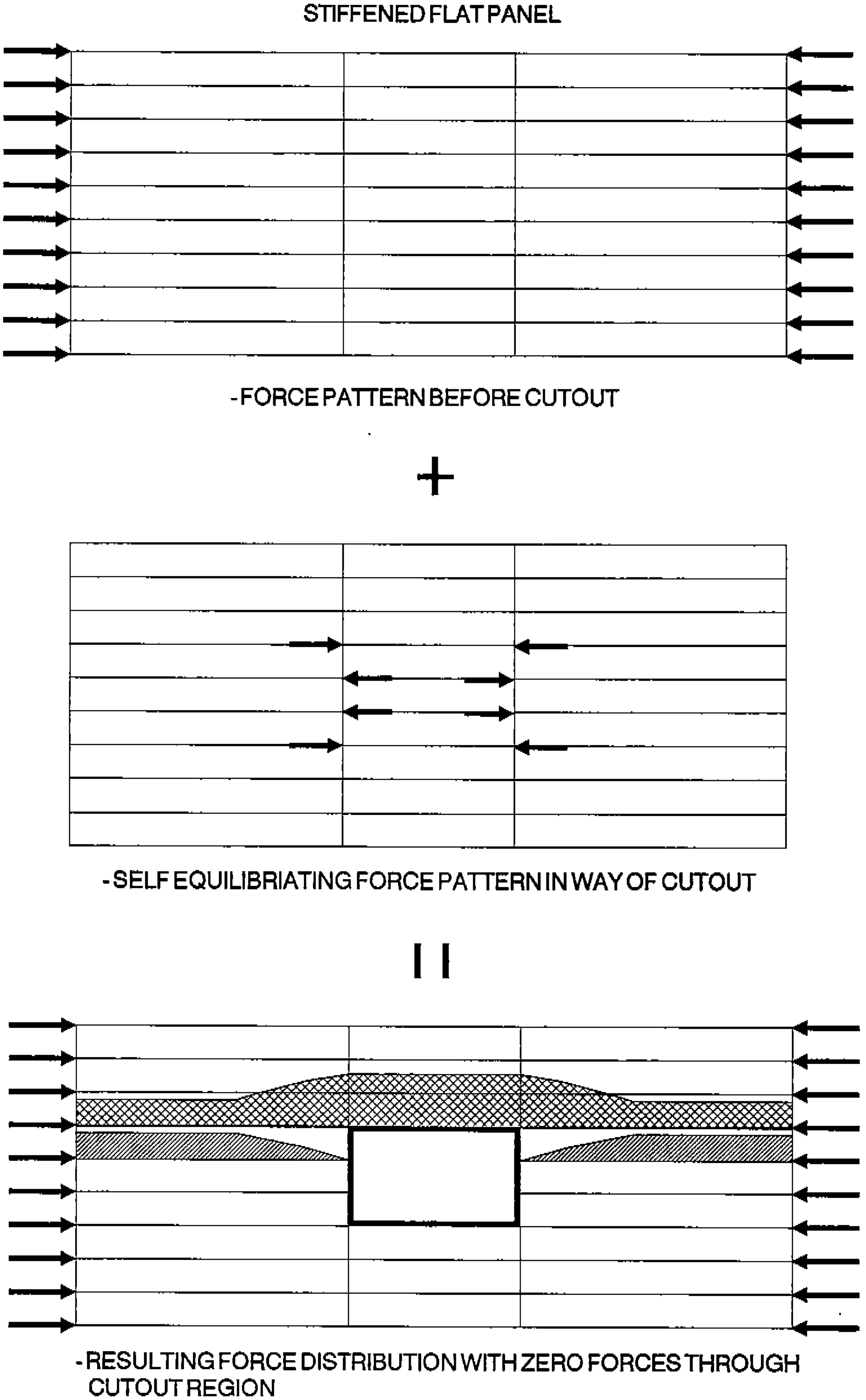


Fig.13.9 Redistribution of Forces in Way of Cutouts or Local Damage

When the 'length' of the damage is increased the response of the overall section (in the middle of the 'length' of the damage) changes to approach that of the remaining structure functioning as a prismatic beam, i.e. the neutral axis moves to that of the remaining structure with the area properties accordingly changing. In general with the damage being unsymmetrical vis a vis the whole section this will produce an unsymmetrical transverse section and, for example, pure vertical bending will need to be resolved into the principal axes of the section as illustrated by Figure 13.10. As noted above, determining the as-damaged structural response is a complex three-dimensional problem and often best approached using an appropriate finite element method based capability, preferably as reviewed for example in [13.19] and [13.20]. In using such a three-dimensional analytical method the effects of damage would be simulated by either removing elements or deforming elements to represent the form, extent and residual capability of the damaged region, in the basic model prior to the application of the loading. A large deflection non-linear response method would be required in order to determine the true residual strength of the damaged hull girder.

(Reference can be made to the studies by Maestro[13.18] and Hegacy[13.17].)

13.5 Analytical Models

The analyses which follow within this section examine the ultimate strength and post-ultimate strength unloading characteristics of several simple box beam models that are subjected to pure vertical bending. Both 'intact' models and ones which contain some symmetrically disposed damage are considered.

Each of the models, both intact and with damage represented, have been analysed using a 'hull girder box beam ultimate strength analysis' computer program that has been developed by Lloyd's Register of Shipping, London and employed by them for inhouse studies. (As this program was still under development and had not been released for general usage when these box beam studies were being undertaken, BMT are grateful that Lloyd's Register freely allowed access to the program, however the preparation of the input data, application of the program and interpretation of the results are of course, clearly BMT's responsiblity.)

□ Brief Description of Method

The computer program employed is similar to a method developed at ARE[13.30] (see also Section 13.2). The method which was originally developed was for vertical bending with allowances for vertical shear only, however this was then extended to allow for both vertical and horizontal bending.

In this approach, the relevant ship transverse section is discretised into elements, generally stiffened panels. The stiffened panel stress-strain curves are obtained from a program[13.33] developed by Lloyd's and based on the theory reported in [13.34].

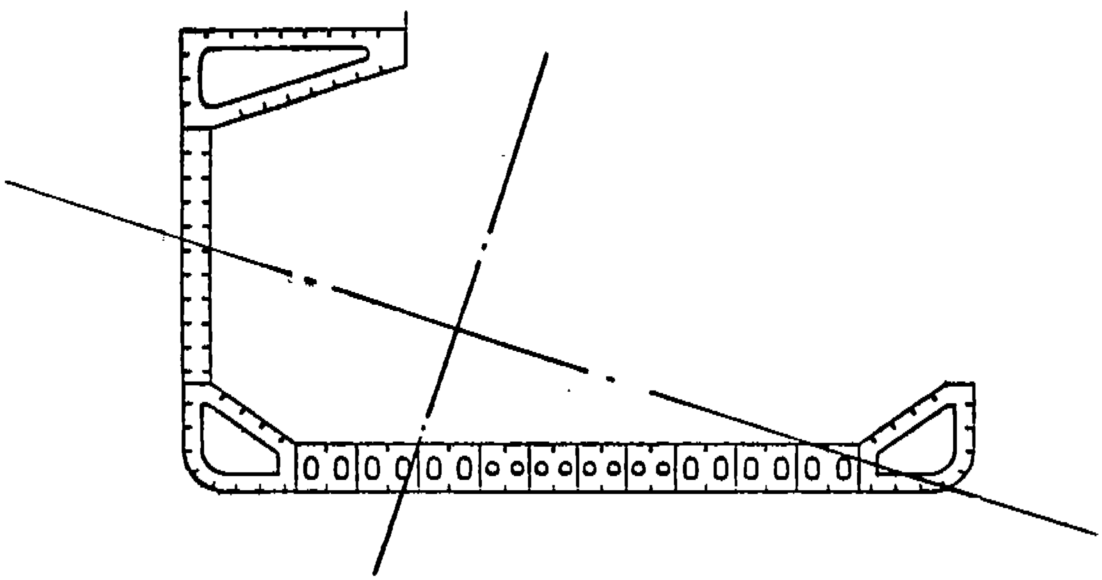


Fig.13.10 Unsymmetrical Section following Side Shell Collision Damage

A typical ship transverse section as shown in Figure 13.11, is subjected to curvatures C_X and C_Y (representing the effects of vertical and horizontal bending) about x and y axes respectively. The combined curvature to which the section will be subjected is given by:

$$C = \pm \sqrt{C_x^2 + C_y^2} \quad \text{and the angle} \quad \cos\theta = C_x/C \; .$$

The strain in an element 'e', positioned at distances x_e and y_e from the neutral axes, is obtained from the following expression:

$$\epsilon_e = C(x_e \sin\theta + y_e \cos\theta)$$

This value of strain is then used to determine the corresponding value of stress σ_e from the specific stiffened panel end load stress-strain curves.

In order to maintain equilibrium, so that the net longitudinal axial force in the whole section is zero, a shift (S) in the neutral axis may be required and is given by:

$$S = \frac{\Sigma\ (A_e \cdot \sigma_e)}{C\ \Sigma\ (E_e \cdot A_e)}$$

This correction is applied iteratively in conjunction with updated stresses and strains until equilibrium is achieved, i.e. the summation of axial forces above the instantaneous neutral axis equals the summation of forces below the axis.

The corresponding vertical bending moment is then calculated from the stresses as follows:

$$M_x = \Sigma\ \sigma_e \cdot A_e \cdot y_e$$

when A_e is the area of the element. In the same way M_y the horizontal moment is also calculated by $M_y = \Sigma\sigma_e \cdot A_e \cdot x_e$.

The curvature is then increased and the corresponding new moments, M_x and M_y, are obtained. Thus by increasing the curvature successively, the moment-curvature diagram is obtained.

As noted earlier, this method assumes a simple prismatic beam model in which the transverse section through the beam is constant for some appreciable distance either side of the specific section being analysed. Thus there is no determination of stress concentration type effects around any representations of damage within the section.

In each of the following studies damage is modelled by simply removing various elements from within the transverse section of the beam and with the damage assumed to be of equal magnitude on both port and starboard sides of the whole section.

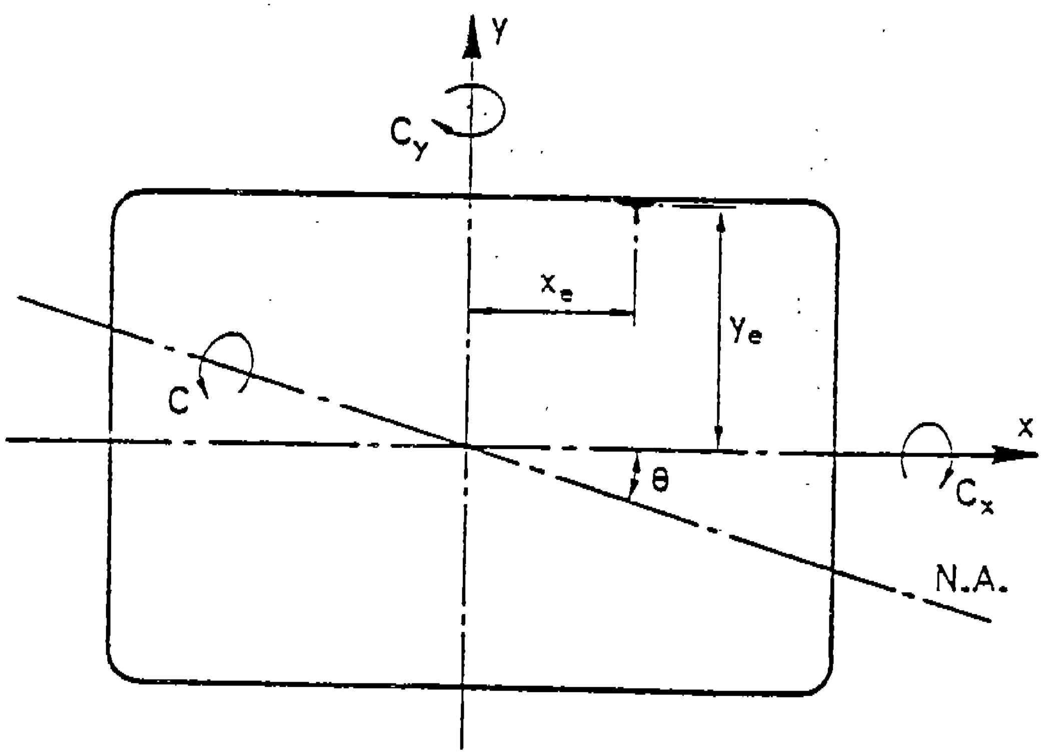


Fig.13.11 Combined Bending of Hull

Within the limits of the method within the program (and which was developed mainly with design studies in mind) the only other possible approach to allowing for the effects of damage would be to employ end load shortening curves derived from analysis of deformed panels or plate elements (e.g. via use of plastic hinge mechanisms or detailed non-linear large deflection finite element studies, etc.). The box beam analysis program will accept general end load shortening curves presented to it in numeric form. For example for a stiffener plus combined attached plate element having some initial large inwards or outwards permanent deformation at some position within its length\ a beam-column analysis could be undertaken to develop such a load shortening curve. Some consideration would have to be given to the scope of the damage, the effects of such extending to more than one stiffener and to the implied locked-in stresses following the initial occurrance of the damage.

Although this modified box beam type of approach has quite considerable limitations compared with a large three-dimensional finite element method based study, for general design and for comparative studies it provides a most cost effective approach.

Three simple models were analysed by this process.

□ Model A - Simple Box Beam

Original work on the developing and testing of this model, shown in Figure 13.12, was undertaken at the Department of Naval Architecture and Ocean Engineering, University of California, Berkeley, as a part of their research programme on an "Experimental Investigation of Ship Hull Ultimate-strength Using Large-scale Models" [13.32]. This model was a simple box-beam structure of 30"×96" overall dimensions in section and with stiffeners of flat bar section of 2.75"×12 gauge, details of which are shown in Figure 13.12.

Figure 13.13 shows the distribution of final experimentally applied bending moment at collapse along the length of the model obtained during the experiment. This result was obtained based on the moment calculation from the final pressure-vacuum records of the loading system, details of which are given in [13.32]. It was noted in [13.32] that the failure of the model might be attributed to lack of continuous welding of the stiffeners, which buckled in torsional and lateral modes (tripping). The experimental collapse moment was reported to be 11,600 kips.in, i.e. 1.319×10^9 Nmm.

In using the Lloyd's Register method, it is necessary to generate the stress-strain curves for various stiffened panels in the overall hull girder section. For this the hull girder was divided into 19 panels, as shown in Figure 13.12. Half the box section is specified and the other half is generated automatically as the program assumes symmetry about the vertical centreline. Basically there are two broad groups of stiffened panels and the response analysis results of these panels were carried out using Lloyd's programme [13.34]. The ultimate strengths of these panels were due to failure of the stiffener in compression.

The stiffened flat panel analysis program released by Lloyd's Register as part of the LRPASS suite of computer programs was employed to determine the end load carrying characteristics of the various panels within the section. This program determines the lowest strength mode of failure based upon three possible conditions:

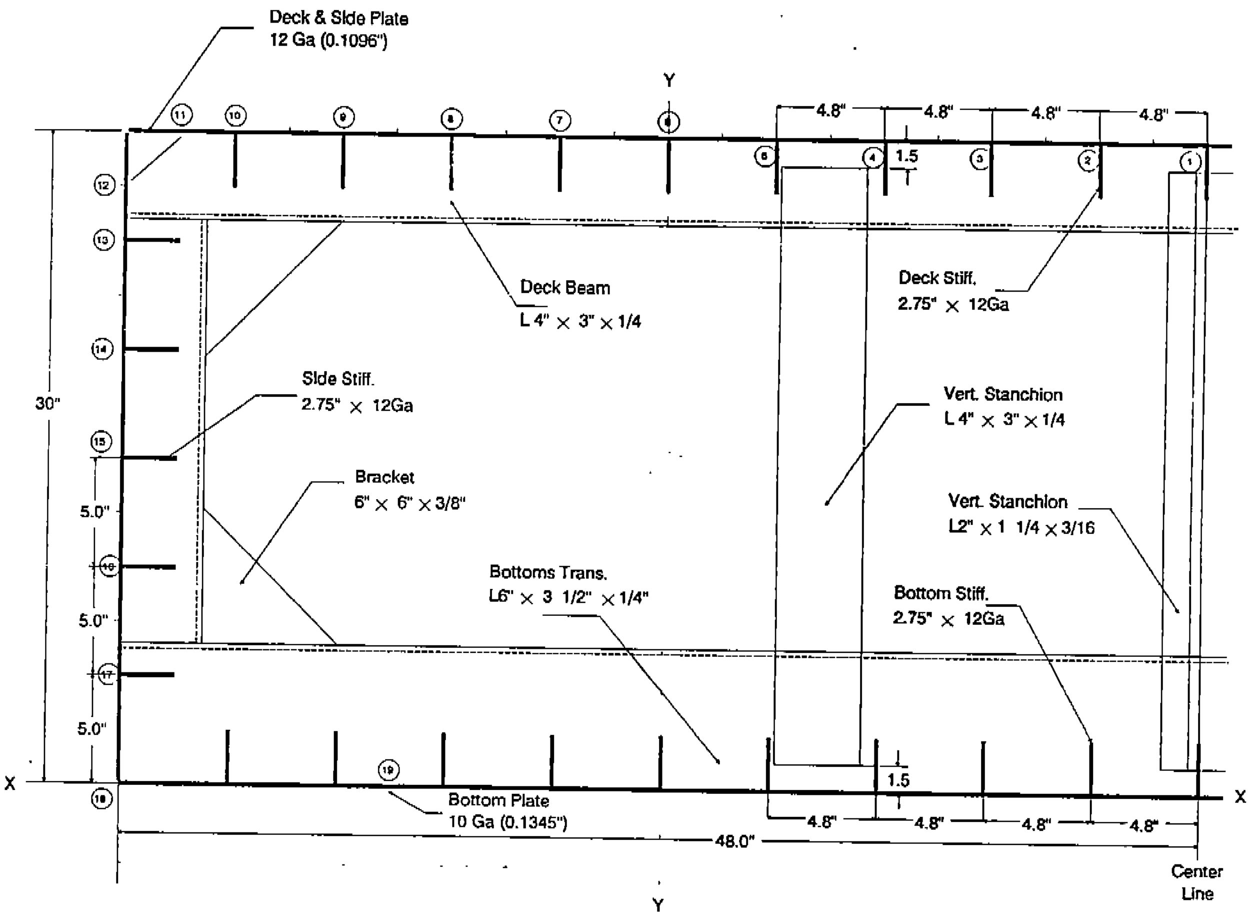


Fig.13.12 Simple Box Beam Model A

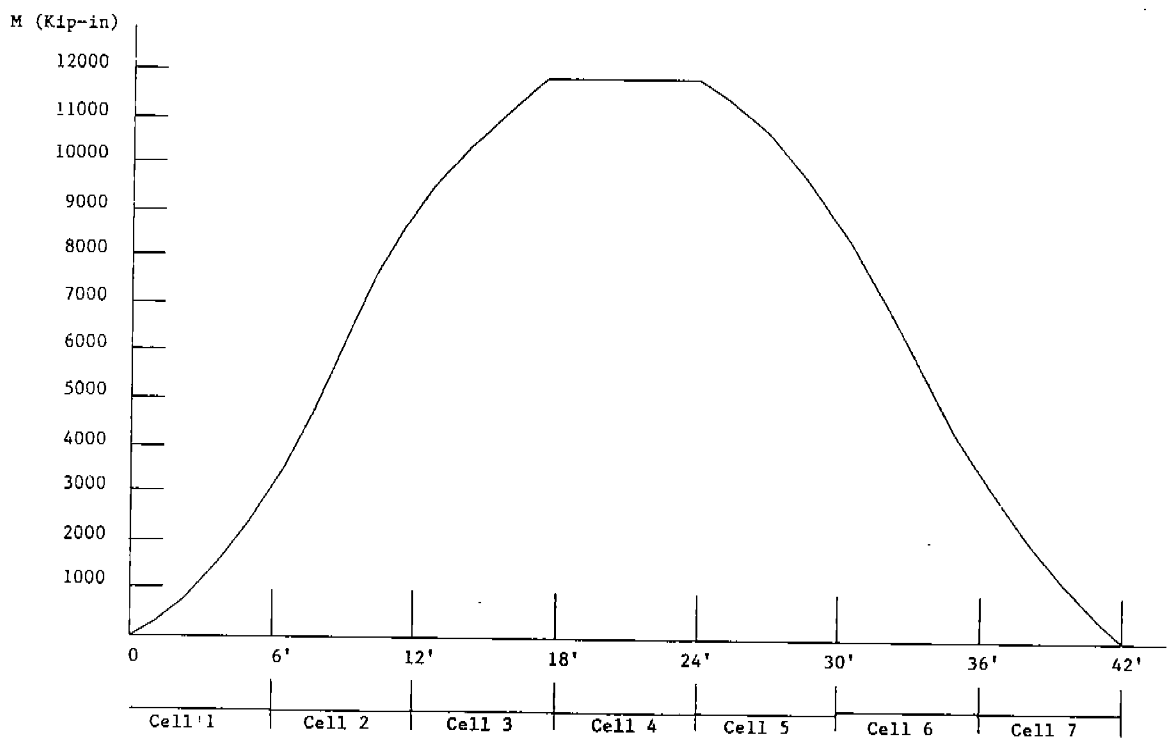


Fig.13.13 Bending Moment Diagram along Midships (Model A)

- failure initiated by plate compression,
- failure initiated by stiffener compression, and
- failure initiated by stiffener tension.

The analysis allows for the effects of initial deformations and residual stresses. In addition to the uniform uniaxial compression applied to the transverse edge of the panel, uniformly distributed lateral pressure forces may also be applied to the panel as long as their effect can be considered to be of secondary importance.

In the case of failure being initiated by stiffener compression (normal column-like instability), torsional buckling checks are also included.

The program allows for stiffener sections of various shapes, e.g. flat bar, tee-sections and angle sections.

The post ultimate strength is then determined by the program depending upon the mode of failure;

- unloading by plate squashing,
- unloading by plate buckling,
- unloading by stiffener squashing, and
- unloading by stiffener buckling.

However, even though the LR stiffened panel analysis program allows for failure by torsional buckling it could be that the effects of the discontinuous stiffener to plate welding in the test model result in as-manufactured distortions larger than employed within the stiffened panel analyses. This may be part of the approximately 28% difference between the experimental and theoretical results. It is also to be noted that whereas the stiffened panel analysis program determines the minimum failure strength mode, as a discrete mode an actual structural failure could include some degree of coupling and interactions between modes, e.g. a coupled euler column-type and torsional lateral instability, and this will generally be at a lower stress level than either of the two discrete modes.

The information obtained from the individual stiffened panel analyses were used as input to the hull girder ultimate strength programme and the moment-curvature relationship diagram from the results obtained were plotted and shown in Figure 13.14. From Figure 13.14, it can be seen that the ultimate vertical bending moment was found to be 1.68×10^9 Nmm compared with an experimental value of 1.313×10^9 Nmm as mentioned earlier. The value of ultimate bending moment of 1.68×10^9 Nmm was for the intact condition of the beam.

Analyses were carried out for the same box beam section assuming damage to stiffened panels Nos.1, 2 and 3, port and starboard, (Figure 13.12). This was accomplished in the analyses by assuming a negligible area of these panels and repeating the moment-curvature analysis. The ultimate moment capacity yielded a value of 1.17×10^9 Nmm, i.e. a reduction of 30% from the original value. Each of these stiffened panels may be thought of as redundant elements and depending on their location with respect to the full section and stiffnesses, they contribute to a reduction of the overall ultimate moment capacity in the case of damage to these panels.

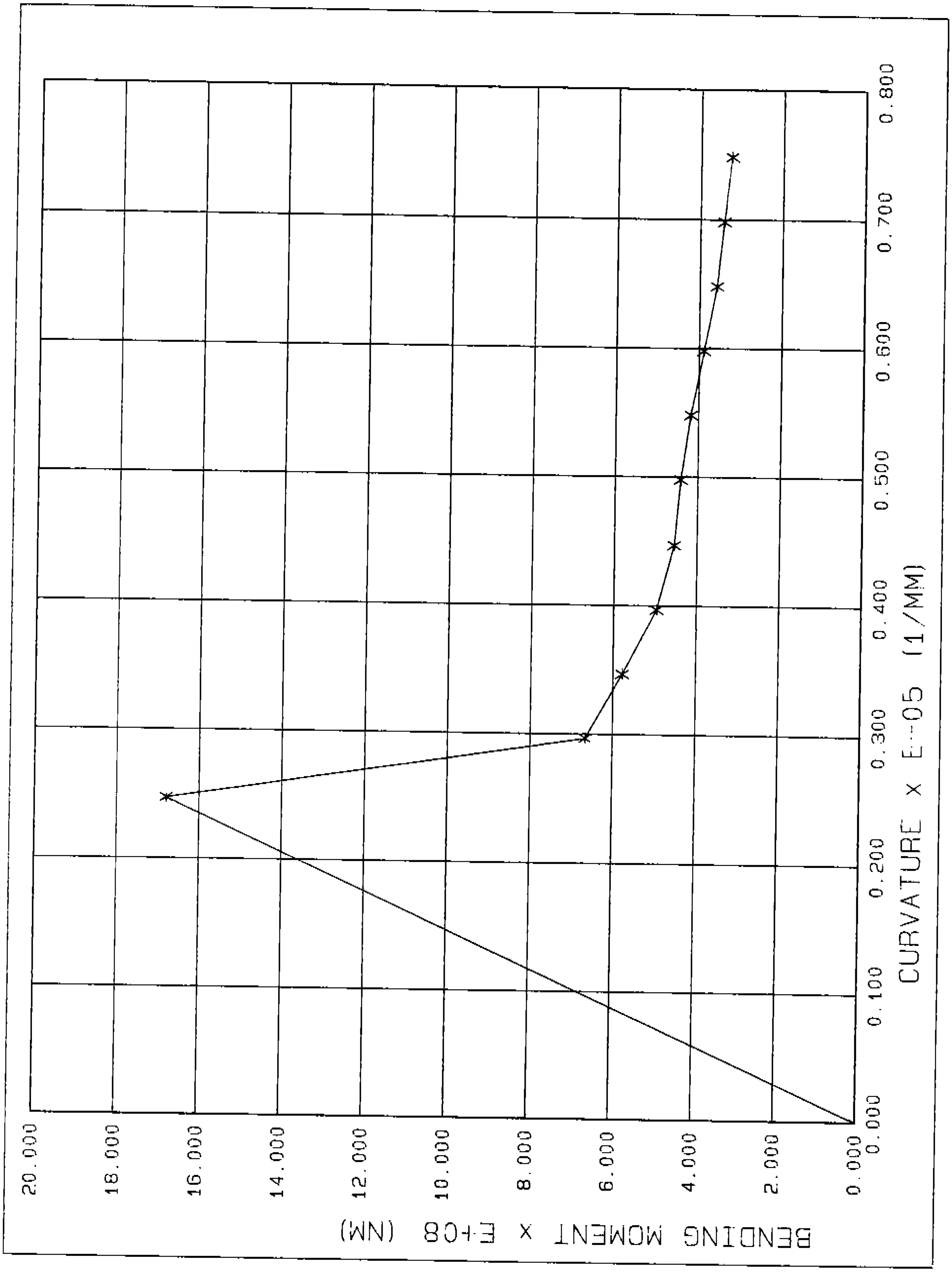


Fig.13.14 Moment Curvature Relationship for Model A

☐ <u>Model B - Simple Box Section</u>

This is a simple rectangular box-girder section of 600mm x 400mm overall dimensions and which was tested by Reckling[13.37] and for which the experimental results are available. (In the test programme, collapse was delayed owing to the strengthening effect of the side walls, although its plastic buckling capacity had been exceeded. Final collapse occurred by nearly simultaneous buckling of the panels between longitudinal stiffeners in the compression flange and of the whole compression flange itself.) The stiffeners used in this model were of both flat bar and angle sections, as shown in Figure 13.15. The model was discretised into 10 panels for the hull girder ultimate strength analysis.

The spacing of the transverses on the compression panel was 500mm. The ultimate bending moment found by application of the Lloyd's Register program to the intact section was found to be 2.36×10^8 Nm. The next run was made assuming damage to two panels, i.e. panels 1 and 2 were removed, (Fig.13.15), and the computed value of the ultimate vertical bending moment was 1.38×10^8 Nm. Thus there was a reduction of 42% in the ultimate strength capability due to the complete damage (removal) of two elements per side, symmetrical on each side of the vertical centreline of the model.

☐ <u>Model C - Typical Tanker Section</u>

The section of the model considered is shown in Figure 13.16. This model represents a typical real ship section of an Oil Tanker type. The details of the various stiffeners are also shown in the same figure and which includes both flat bar and angle section stiffener types. The frame spacing is assumed to be 2.8 metres.

The model is discretised into 48 stiffener plate elements which consisted broadly of three groups of stiffened panels.

The ultimate vertical sagging bending moment for the intact section was found to be 1.73×10^{12} Nmm and the moment curvature plot is shown in Fig.13.17.

Model C is an interim example from a general design study and thus does not represent a final design having scantlings optimised in accordance with a given loading demand. Thus it is not possible to relate to a standard design bending moment, e.g. from a given set of classification society rules. Although this is a fictious design example the overall dimensions and scantlings are of a realistic magnitude, vis a vis oil tankers in service, and it is noted that the residual strength is, usefully, of the order of 50% of the ultimate bending strength capability, in the sagging condition.

☐ <u>Model D</u>

A simple rectangular box-girder section model, shown in Figure 13.18, was studied by Lee [13.37]. This model was analysed by Lee using an independent method approximately similar to that within the Lloyd's Register program. It is to be noted that in this study the whole box section was analysed, rather than assuming a half model symmetrical about the vertical centreline and hence asymmetically disposed damage could be allowed for. The residual strength index (RDI), as defined earlier, was reported [13.37] for this model with two

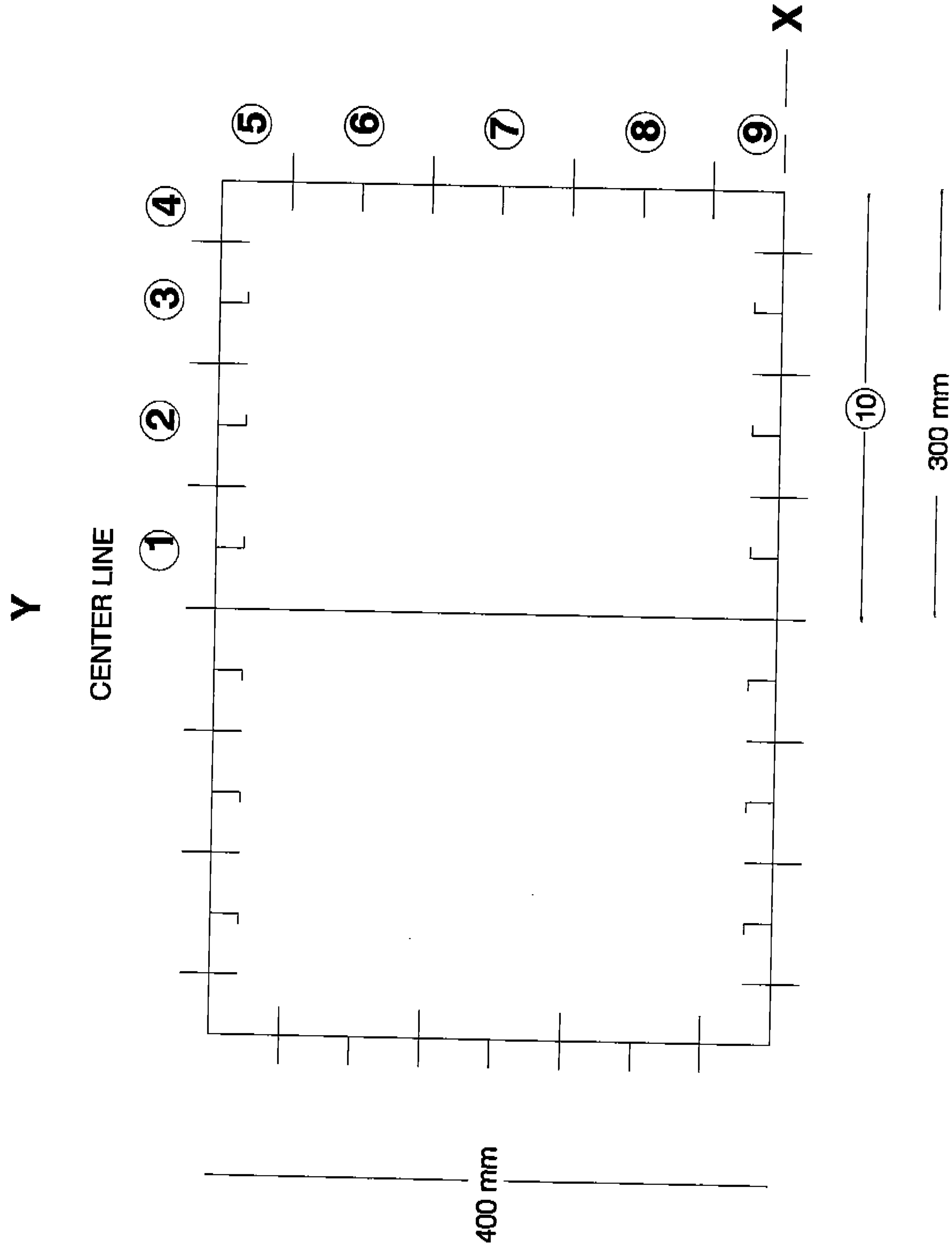


Fig.13.15 Discretisation of Model Tested by RECKLING (Model B)

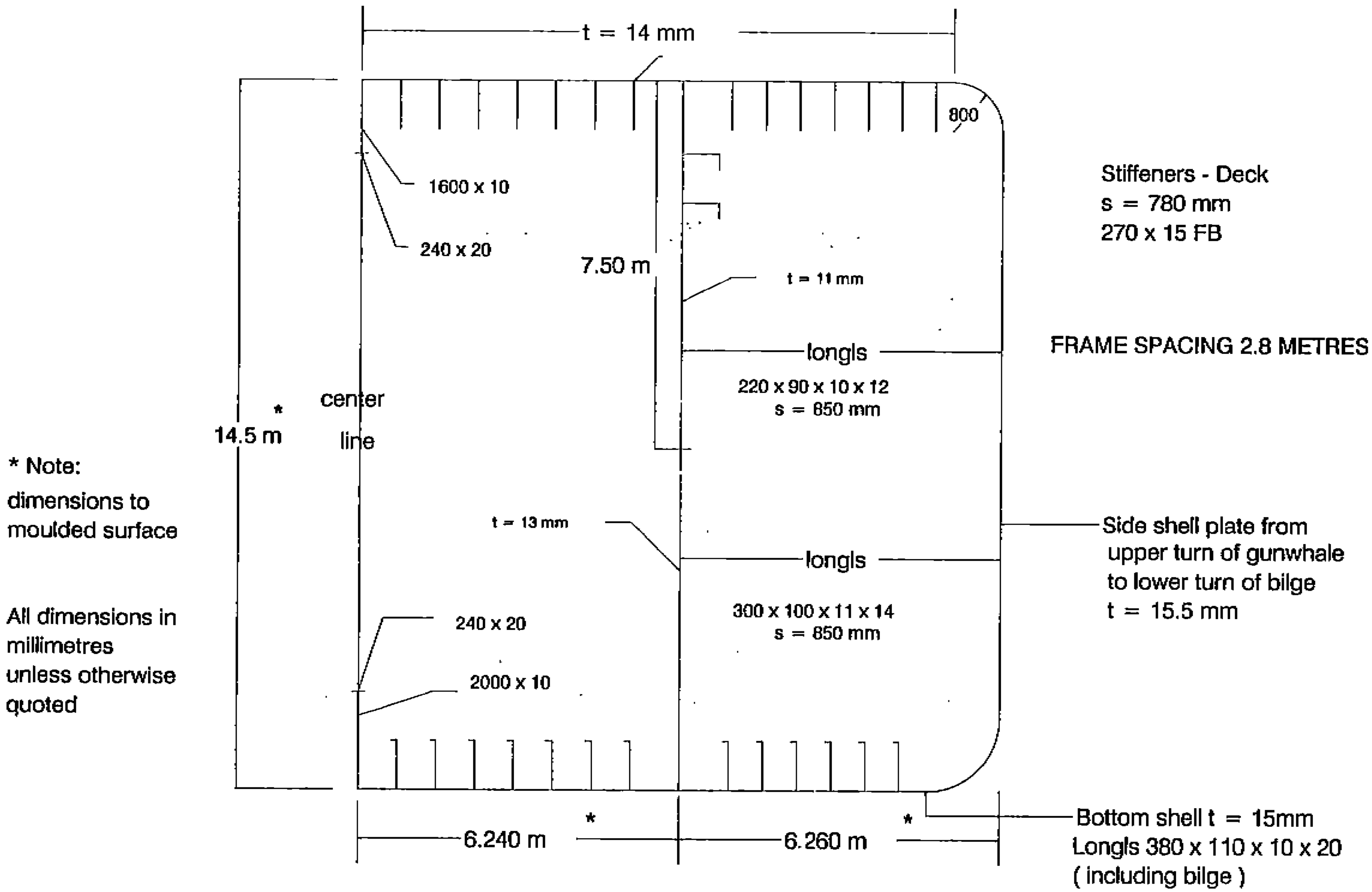


Fig.13.16(a) Model C : Typical Tanker Section - Basic Details

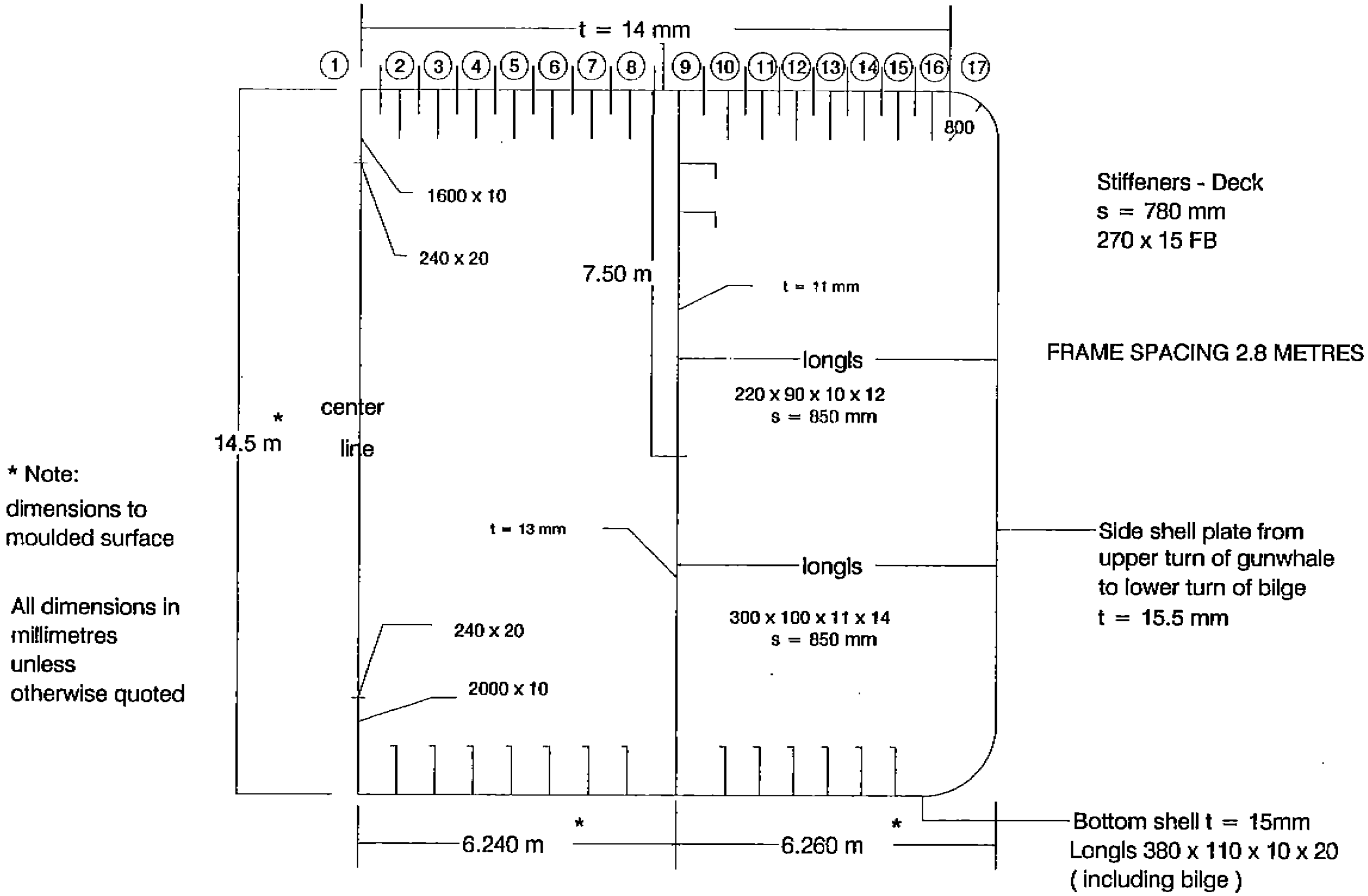
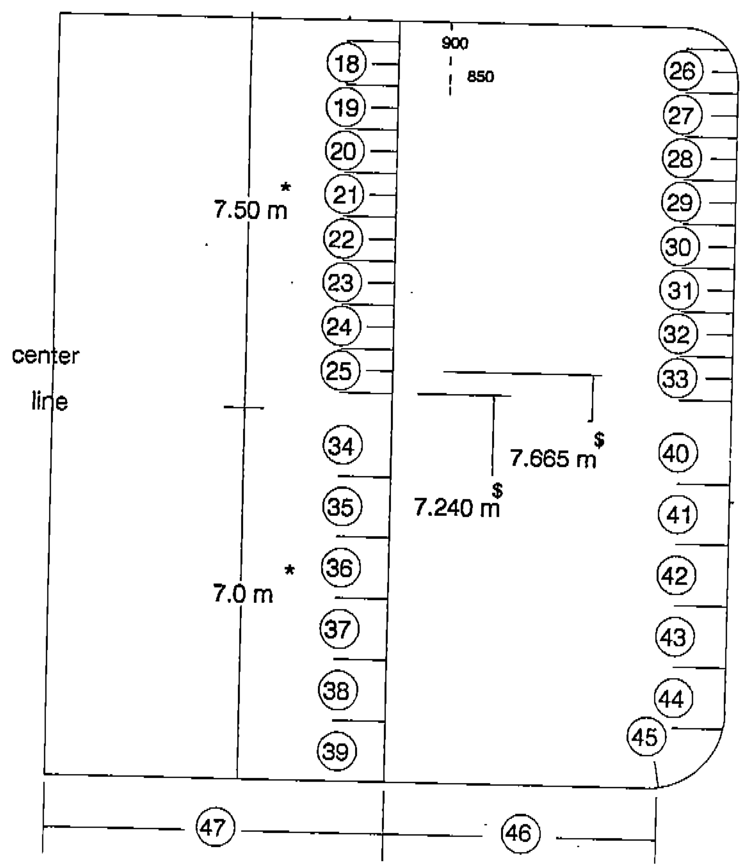


Fig.13.16(b) Model C : Typical Tanker Section - Panel Numbering



Note:
*
dimensions to moulded surface
$
above base of shell

All dimensions in millimetres unless otherwise quoted

Fig.13.16(c) Model C : Typical Tanker Section - Panel Numbering continued

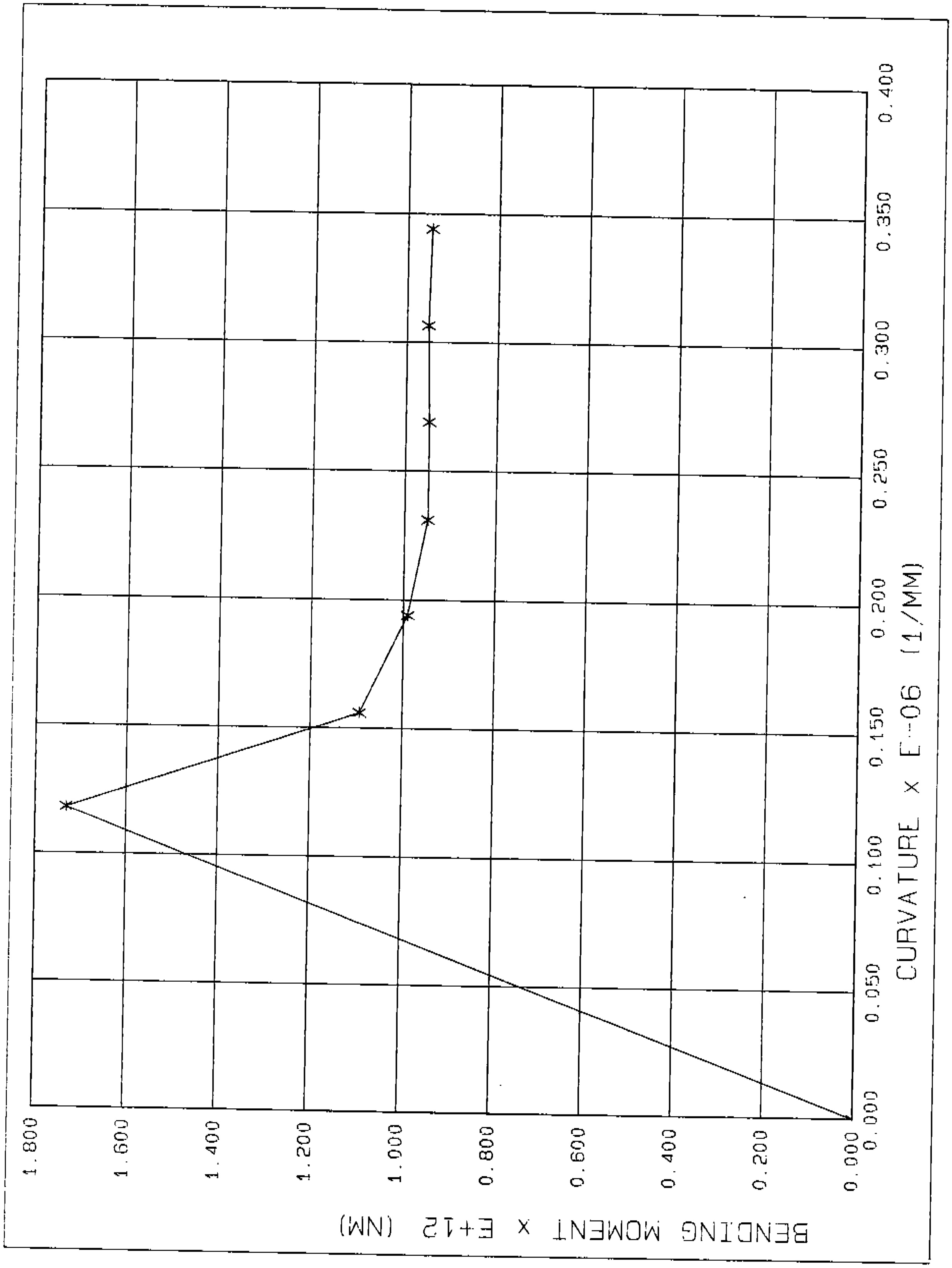


Fig.13.17 Model C : Moment Curvature Results

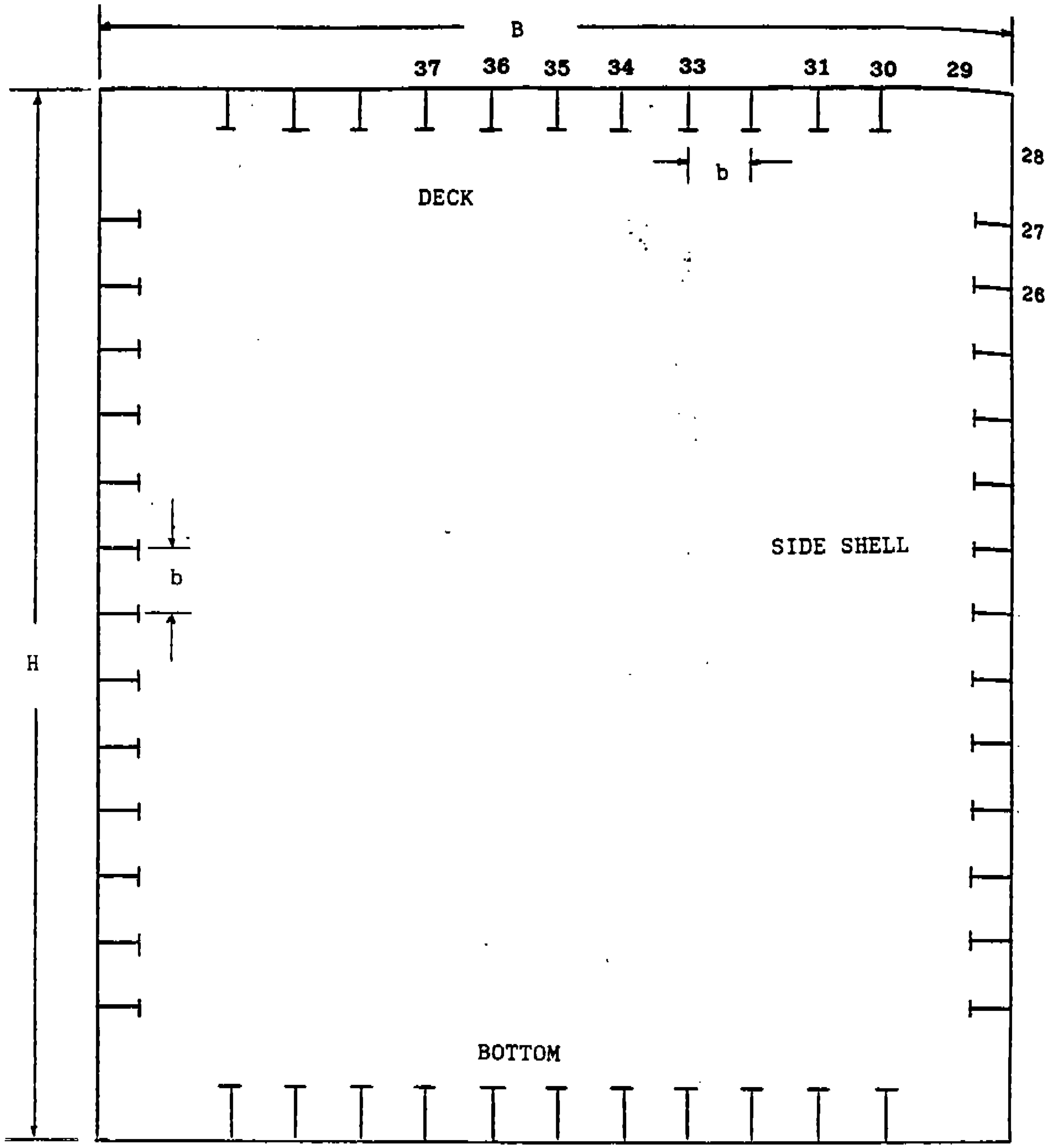


Fig.13.18 Model D : Typical Rectangular Box-Girder Section, Lee[13.37]

damaged conditions (again via simply removing elements) and for several loading cases. One set of analyses being under various combinations of axial compression and bending about the Z-axis and the second a single case for under combined axial compression and bending about both axes. Under these loading conditions, the central parts of the deck were expected to fail. One damaged state was that the stiffened panels, labelled 34, 35 and 36 in Figure 13.18, were assumed to completely fail with no residual strength and the second one was that a further two more stiffened panels, labelled 33 and 37, were assumed to fail.

The results of the RDI calculations for this model, taken from [13.37], are shown in Table 13.1. In this table the loads are non-dimensionalised by their ultimate strength. The ultimate strength in the intact state and the residual strength in the damaged states were calculated by Lee based on his formulations [13.37].

The possible contribution of the residual strengths of the damaged members were not taken into account when calculating the strength of the as-damaged structure. It may be seen from Table 13.1 that residual strength index (RDI) which is a measure of redundancy varied in a manner depending on the types of loads as well as the load levels of the combined loading.

Table 13.1

Example of Calculating Residual Strength Index [13.37]

Case	Assumed State When Critical Elements are Failed				Ultimate State	RDI
	P/P_u	M_z/M_{zu}	M_y/M_{yu}	F_r	F_u	
1A	0.945	0.0	0.0	0.945	1.0	0.945
1B	0.907	0.0	0.0	0.907	1.0	0.907
2A	0.0	0.910	0.0	0.910	1.0	0.910
2B	0.0	0.837	0.0	0.837	1.0	0.837
3A	0.901	0.345	0.0	0.922	0.977	0.944
3B	0.866	0.311	0.0	0.880	0.977	0.901
4A	0.289	0.813	0.0	0.763	0.952	0.802
4B	0.302	0.737	0.0	0.636	0.952	0.668
5A	0.625	0.452	0.395	0.807	0.974	0.829
5B	0.606	0.398	0.358	0.727	0.974	0.746

NOTE:

1A,2A,3A,4A : Stiffened panels at deck labelled 34,35,36 are assumed to fail.

1B,2B,3B,4B : Stiffened panels at deck labelled 33,34,35,36,37 are assumed to fail.

5A : Stiffened panels labelled 27,30 and hard corners labelled 28,29 are assumed to fail.

5B : Stiffened panels labelled 26,27,30,31 and hard corners labelled 28,29 are assumed to fail.

P : Applied axial compression.

P_u : Ultimate axial compression capability

M_z, M_y : Applied bending moments about the z and y axes respectively.

M_{zu}, M_{yu} : Ultimate strength bending moments about the z and y axes respectively.

(NOTE - Unfortunately for this model the design load, P_d, is not known, hence it is not possible to determine the associated RSI values.)

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14.0 GENERAL TRENDS SHOWN BY STUDIES

The objective of this pilot project was to assess the role of Redundancy in marine structures in the context of Reserve and Residual Strength. In order to achieve this a review of the available literature and reported work on this and related subjects was carried out and the various findings have been discussed in various chapters of this report.

It is an accepted fact that in order to carry out a reliability analysis, it is necessary to provide a deterministic physical framework and the general tools upon which the reliability models would be built. In this study an attempt was made to establish and review the role of redundancy applicable to both discrete and continuous structures using deterministic as well as probabilistic models.

In Chapter 11, some deterministic analyses were carried out to illustrate one possible way to quantify the effect of redundancy on the reserve and residual strength of discrete structures. The examples illustrate one possible way of assessing how the failure of a member in a redundant structure affects the overall strength originally designed for and thus showed its importance with reference to the overall structure. Any ranking into order of importance of individual components could possibly evolve from Table 11.1, however in terms of the total system component ranking the two vertical elements must come first, the one between nodes 1-2 having the highest ranking. The next in order of importance must be the horizontal element. Clearly these three members are also the primary members from both strength and function considerations, and from the implications of accidental damage and general degradation.

In terms of ranking the diagonal bracing members the element connecting with the node at which the external force is applied is probably the next in order of importance and the other diagonal elements being of somewhat lesser importance on an individual basis. At this juncture the ranking will probably be effected by the actual loading conditions on the structure and the more complex and three-dimensional the structure in a real multi-direction wave environment the more difficult will be any attempt to rank the many diagonal elements. Although this procedure was applied to a simple plane frame structure, there is no reason why the same concept would not be applicable to the study of a complex jacket structure and to establish the role of various members susceptible to potential early failure or accidental damage with respect to the original strength for which it was designed.

In Chapter 12, both deterministic and probabilistic models were studied for stiffened flat panel structures. The deterministic model studied the effect of failure of some stiffeners on the overall strength of the stiffened panel and thus established their importance with respect to the overall structure.

It should be noted, however, that the calculations were not made with a fixed initial panel geometry and from which individual stiffeners were removed (i.e. taking a panel with N stiffeners and then removing one without respacing the others). Hence the degree and implications of redundancy, particularly with respect to accidental damage, was only examined in a limited manner and not tested in the same way as employed for the two-dimensional framework in Section 11.

For simple longitudinally stiffened flat panels the general modes of ductile failure under axial compression forces applied in the same direction as the stiffeners are:

- stiffener flange or web buckling,
- plate element failure between stiffeners,
- flexural buckling of stiffeners (with the direction of movement being either inwards or outwards), and
- lateral torsional buckling (tripping) of the stiffeners.

Some panels may fail at lower stresses due to combinations and interactions between the various possible individual modes. Panels that are stiffened in two mutually perpendicular directions may fail in an overall grillage type of mode with possible interactions with the above individual component modes. Panel failure can also be influenced by edge support conditions and adjacent bay behaviour, e.g. direction of flexural buckling.

Reliability analyses of stiffened panels were also undertaken using an advanced First Order Second Moment Reliability method on a model structure in which the model uncertainty factor was taken into account, (determined from the results of 21 actual experimental test models compared with a suitable theoretical method). The safety indices inherent in these models for the design loads were determined. Variation of safety indices with reserve strength were also examined.

Deterministic strength models, i.e. via buckling formulations for a typical continuous system specifically ring and ring-stringer stiffened cylinders subjected to different types and combinations of applied loads contained in codes, as well as investigated by various researchers relevant to the design of marine structures, have been reviewed in detail, also in Chapter 12. Significant differences are found between the "knock-down" factors that are introduced to account for the effect of initial as-manufactured geometric imperfections. The API RP2A code appears to be somewhat relatively optimistic for large radius to thickness ratio cylinders when axial load is acting. The effect of plasticity is also handled differently in each of the codes leading to major variations in strength predictions. Some formulations for both ring and ring-stringer stiffened cylinders are identified which give a small bias and coefficient of variation (COV) of the model uncertainty factor when compared with the experimental results obtained by various researchers.

A review of recent work on the Residual strength of ring and ring-stringer stiffened cylinders was carried out. The nature of the damages considered in these studies was mostly in the form of (i) localised denting, (ii) overall bending deformations, and (iii) combination of bending and dents. Comparisons between theory and experiments were shown to be in good agreement. These methods are extremely valuable in estimating the residual strength of damaged tubular beam-columns when assessing the overall integrity of damaged offshore structures. Although most of these formulations are based on numerical step-by-step procedures, there is no reason why they cannot be integrated with a structural reliability procedure to assess the safety of the damaged structure. There are some analytical formulations which can be advantageously used to create the failure surface equation for the structural reliability assessment.

The Residual Strength Index (RDI) was calculated for both ring-stringer stiffened and equivalent ring only stiffened cylinders based on ultimate strength and crushing load formulations. It was shown that the residual strength of the structure was much larger for the ring-stringer stiffened shells than for the ring only stiffened shells.

The Reserve Strength Index (RSI) was also calculated for all the stiffened cylinder models considered in this study, using API RP2A and DnV code formulations to predict the design load capability compatible with each model.

A reliability analysis was then carried out for a selected structural model and the variation of safety index with Reserve Strength Index (RSI) was established. It was shown that for a particular RSI, the ring-stringer stiffened cylinder yielded a higher safety index than for the equivalent ring only stiffened cylinder.

In Chapter 13 simple deterministic analyses to study the role of redundancy in ship hull girders were carried out using a special purpose ship hull girder ultimate strength analysis program. (This computer program, also applicable to general box beams, was one which had been developed by Lloyd's Register of Shipping London and who generously made it available to BMT in the pursuance of this project.) It was shown that the ultimate strength decreases very rapidly with the failure of the top deck part of the structure.

15.0 BASIC ANALYSIS PROCESS TO DETERMINE RESERVE STRENGTH AND RESIDUAL STRENGTH CAPABILITY

☐ Reserve Strength

In the simplest possible terms, this involves calculating the minimum value of the maximum forces (of a specific 'pattern') that a structure can sustain before it collapses. Before the final overall collapse occurs, the structure is likely to have suffered some degree of local distortion, yielding, rupture or failure and may be responding in the large deflection regime. When the pre-collapse forces are removed it is most probable that considerable permanent damage and deformation will have been caused. The reserve strength may be viewed to be dependent of the way in which it fails. The reserve strength will also differ for different applied force pattern systems.

Reserve strength of individual members depends on the type of cross-section to which the section is made of and its mode of failure. For example if the plastic moment (M_o) is considered as the ultimate load (M_u), and the working load (M_w) is given by

$$\text{Working Load } (M_w) = \frac{\text{Yield Load } (M_e)}{\text{Safety Factor } (S.F)}$$

$$\text{then the RSI} = \frac{\text{Ultimate Load}}{\text{Working Load}} = \frac{M_o}{M_w}$$

$$= \frac{M_o \times S.F}{M_e}$$

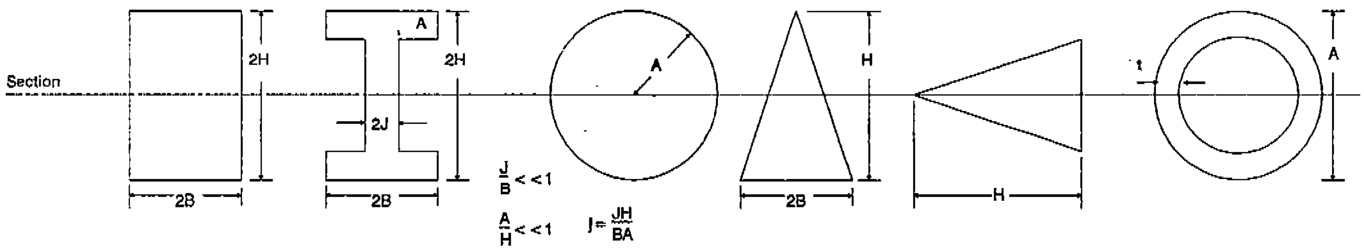
The maximum elastic and fully plastic pure bending moments for some common simple beam sections are shown in Table 15.1.

A safety factor (S.F) of 1.7 is generally adopted in structural steelwork design and if this is taken into account, then the component reserve strength for various simple beam sections are as shown in Table 15.1. The thin-walled tubular section member seems to have less reserve strength compared to most solid sections as seen in Table 15.1 but it has higher reserve strength than the most commonly used structural sections, such as I beams. (The possible effects of crippling stresses in thin-walled sections being lower than yield stress also requires some consideration.)

The results given in Table 15.1 afford a simple comparison assuming that all sections are sufficiently robust to enable the full plastic hinge to develop. However depending on the actual geometry thin walled sections may fail at local stresses that are well below the material yield stress level due to buckling. Additionally hollow circular sections may fail by ovalling (the so-called Brazier effect).

Clearly, as is the basis for pure plastic bending resistance, the form factor, (i.e. M_o/M_e) is highest when there is proportionately more of the cross-sections material towards the neutral axis and conversely the form factor is at its lowest when most of the material is furthest from the bending

Table 15.1 : **Maximum Elastic and Fully Plastic Moments for Simple Beam Sections**



M_e	$\frac{4}{3}\sigma_o BH^2$	$4\sigma_o BHA\left[1+\frac{1}{3}j\right]$	$\frac{\pi}{4}\sigma_o A^3$	$\frac{1}{12}\sigma_o BH^2$	$\frac{1}{6}B^2H\sigma_o$	$\frac{\pi}{4}A^2t\sigma_o$
M_o	$2\sigma_o BH^2$	$4\sigma_o BHA\left[1+\frac{1}{2}j\right]$	$\frac{4}{3}\sigma_o A^3$	$\frac{2-\sqrt{2}}{3}\sigma_o BH^2$	$\frac{1}{3}B^2H\sigma_o$	$A^2t\sigma_o$
$\frac{M_o}{M_e}$	1.50	$\frac{1+\frac{1}{2}j}{1+\frac{1}{3}j}$ †	1.70	2.34	2.00	~1.27*
RSI	2.55	1.92 (for j=1)	2.89	3.98	3.40	2.16

* Depending upon the wall thickness to diameter ratio the M_o/M_e value varies from 1.27 (for thin walled sections) to 1.70 (for a solid section).

† Depending upon the thickness to flange width (or web depth) proportions the M_o/M_e ratio varies from 1.0 (for wide thin flanges) to 1.5 (for solid rectangular section.

neutral axis. A wide flange I-section beam falls in the latter category whilst a thick walled cylinder falls in the former.

Clearly the reserve strength indications given in Table 15.1 are only for a very limited range of simple section examples and for both complex multi-axis loading situations and for elements that may fail in one of several possible modes the individual assessments will need to be made in order to determine the appropriate measure of reserve strength.

Reserve strength also depends for an individual member, on the way it is supported. For example if a beam is supported with restraints, in the axial direction the ultimate load is much higher than if it is supported with zero axial restraint. This, at the overall system level, may require some consideration of the effects of large overall deflections.

In a member which is subjected to a lateral load, the ultimate load carrying capacity decreases with the presence of axial compressive load. Thus the reserve strength is also affected due to the presence of other loads which exacerbate the large deflection and instability phenomena.

To undertake such an analysis on large multi-element redundant structures, other than the most basic arrangements such as fixed ended beams, involves quite a large computer based analytical effort [15.11-15.13]. This undertaking is both expensive and time consuming and needs to be considered for the full spectrum of load conditions for which the structure is intended to withstand. However, in a computer aided environment and with all the current trends in data handling, e.g. automated data generation from a design data base, and in making more power available for less process and mantime costs this should not be an obstacle in the near future.

There are various general purpose analysis programs available which can be used for carrying out such non-linear large deflection analyses and include ADINA[15.11], USAS[15.12], SOLVIA[15.13], FENRIS[15.14], etc., however the complexity of problems of this nature is such that a high degree of user expertise is required. For example a major problem is in allowing for members that may rupture at some point during the overall gradual failure. Common with all finite element and finite difference analyses it is the relevant level of detail within the model which is most important. This relates to both overall topology and fineness of meshes etc.

Fatigue reserve strength can be calculated using Miner's cumulative damage summation as shown in Section 2 of this report and mainly consists of calculating the number of stress cycles in different stress range blocks to which the structure is subjected under the full time-related spectrum of environmental loading conditions and thus will involve dynamic analysis of structure subjected to wave loading. The same finite element programs coupled with wave loading programs can be used for the overall analysis purposes.

Fracture mechanics principles can also be used to determine fatigue reserve strength, crack propagation rates and critical crack lengths.

□ <u>Residual Strength</u>

This parallels the 'reserve strength' process but includes 'modelling' the damage which has been experienced or which it is assumed to have experienced or designed to allow for. The damage may be representative of a simple crack or the whole removal of a component or possibly by the representation of an imposed permanent deformation and attendant residual stresses. The same general purpose finite element or finite difference computer program employed for reserve strength studies can be employed. The concept of residual strength is very important in the sense that in the event of failure of any particular member, it will show its relative importance with respect to the system strength. It also enables one to focus particular attention for appropriate rigorous inspection on the most critical members in the system.

Residual fatigue strength is similar to fatigue reserve strength and involves fatigue analysis of critical members in the system when the 'damage' in the structure is taken into consideration. This would include the possible further degrading effects of damage propagation.

The overall modelling and analysis process, in a considerably simplified form, is as illustrated in Figure 15.1. Clearly a major problem, that would face designers, will be to postulate the many possible locations, forms and physical characteristics of damage and then to include acceptably accurate representations of such within the overall finite element model. This indicates a large number of analyses, particularly if combinations of damage (e.g. accidental plus general fatigue or scantlings diminution) are to be considered.

However for very specific families of structures and in which the most likely forms of damage, particularly accidentally caused, can be identified the process to examine residual strength and associated operating integrity could be codified. Det norske Veritas provide some such guidance for semi-submersibles in [15.15] and for jack-up units in [15.16]. In the DnV guidance for semi-submersibles specific consideration is given to the secondary cross bracing members and to the primary girders in the upper hull/cross deck structure. The maximum design loads are taken to be related to the one year maximum height wave, which in the case of structures operating in the North Sea is taken to be equal to 80% of the 100 year wave. The recommended guidance accepts the occurrence of local yielding and buckling as long as the overall damaged structure can still function - thus re-distribution forces must be accommodated. The guidance for jack-up units relates to overall leg robustness and ductility following local damage due to work/supply boat collision. Again the maximum environmental forces, in the damaged condition relate to the one-year wave return period. It is assumed that one chord element, in any leg, may be damaged but still capable of forming a plastic hinge. Particular emphasis is then placed upon overall response to overall forces and the effects of re-distributions of leg forces. In the damaged condition it is allowed that the structure can work to higher stress levels than in the normal operating condition.

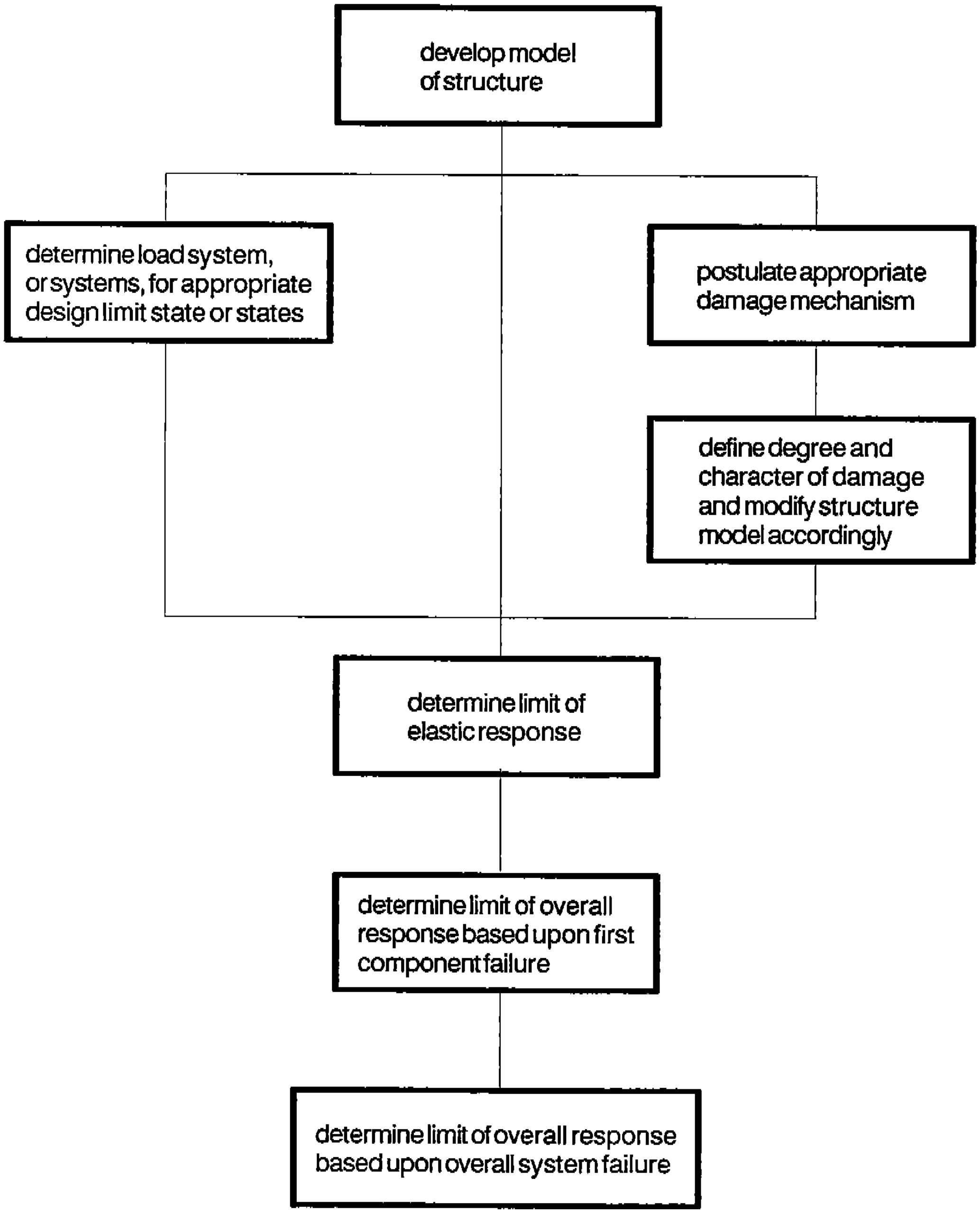


Fig.15.1 Reserve and Residual Strength Analysis Process

The employment of reduced design loads for examining the integrity of damaged structures, that is loads less than the maximum lifetime demands, is similar to the failsafe design evaluation philosophy employed in the aerospace industry. There is an implied assumption that damage will be detected and repaired within a given interval of time from when the damage first occurred and the magnitude of the reduced design load is related to this interval of time. Again this is fundamental within the aerospace failsafe design, inspection and maintenance philosophy.

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16.0 UNCERTAINTIES AND PROBLEM AREAS

16.1 Complexity of Structures

The vast majority of ships and offshore structures, of various types and configurations, are in reality very complex structures and in which the degrees of redundancy are both high and difficult to quantify.

Most steel jacket structures, which are generally for analysis purposes viewed as skeletal three-dimensional frameworks, assemblages of so-called discrete elements, are visibly highly redundant structures and the notional degree of redundancy in the classical (degree of indeterminancy) manner can be numerically assessed if necessary. There may be some form of indeterminate redundancy associated with the response interactions at complex joint regions within the structure. A problem which will exist is in assessing, without the benefits of rigorous numerical studies, the significance/degree of importance of each of the various discrete members within the overall system, allowing for both ductile and brittle local failure modes.

Floating offshore units, whether of fully mobile semi-submersible forms, tethered leg platforms, or of other possible configurations, are analytically quite complex structures. Some simplified models for analysis purposes may be in the form of discrete element three-dimensional frameworks in order to assess the overall global response. At this level of modelling it is possible to quantify, in general terms, the degree of primary/overall redundancy. However most major components of these forms of floating structures are relatively large volume units and which are effectively continuous structures. There is thus another level of redundancy that can be envisaged and that will be particularly important with regard to local damage tolerance and containment.

The spectrum of ship structural forms ranges from simple tankers through to complex multi-deck multi-hold/tank forms. However even the simplest ship is a relatively complex three-dimensional continuous structure possessing a high and possibly numerically indeterminate (in absolute terms) degree of redundancy.

Thus a major problem area, that will require an innovative approach, is recognition of the large range of marine structures and the large variations in levels and forms of redundancy (e.g. primary, secondary and tertiary) possible within each. Clearly an indepth study focussing on a very specific family of structural forms should go a long way to quantifying more closely the inter-relationships between redundancy, reserve and residual strength. The structures used in a study of this nature should be representative examples of ones actually in service.

16.2 Modelling of Structures

The decisions that are made on the most appropriate models for design analysis purposes are generally based upon accuracy and applicability of the results to be obtained, tempered with the costs implicit in undertaking such studies. A common strategy employed for both discrete and continuous structures is to employ relatively coarse mesh models of the overall structure to produce boundary data for subsequent studies on selected areas via fine mesh models. There may even be a progression from coarse mesh through medium mesh to fine mesh very local region models, a typical example being joints in fixed-site steel jacket structures.

Within the overall flow of the design development process the large scale coarse to medium mesh models would be employed in a manner which suffices to enable major decisions to be made re the most suitable overall structural topology, i.e. establishing the prime/basic level of redundancy.

The character of the model, together with the associated analysis process, could influence the mode or modes of failure to be examined. This will particularly apply to the response analysis of discrete structures, e.g. the actual development of failure at steel jacket joints. Clearly the detail level of the model will be very important viz a viz the representation of forms of damage in, say, continuous structures.

16.3 Mixed Modes of Failure

Many, if not most, analytical studies on the ultimate strength and, subsequent to damage, residual strength of complex structures tend to assume that a specific form of failure manifests itself throughout the overall structure. For example studies of skeletal frameworks, representing steel jackets or semi-submersibles, may assume that overall failure occurs when sufficient component plastic hinges have developed for an overall collapse mechanism to form. Thus the implications of local sections or members developing some form of instability before full plastic hinges can develop would not be allowed for.

In studies being undertaken on various forms of continuous structures mixed modes of failure are even more likely to occur, e.g. in a three dimensional structure there will be mixed regions of various forms and modes of buckling and some degrees of inelastic straining (both tension, compression and/or shear related). This potential mixture of modes of both elastic and inelastic behaviour interacting with some forms of secondary levels of redundancy clearly represents a complex problem. In the context of some forms of damage to large three dimensional continuous structures assessments of residual strength will most likely involve, if accurately made, complex buckling and local plastic effects.

16.4 Real versus Theoretical Structures

As is well known 'real' as-manufactured structures contain many geometric imperfections, particularly in various forms of lack of straightness and flatness, and in-built residual stresses due to the effects of welding and fitting actions during assembly, etc. Such imperfections and residual stresses considerably reduce the performance of the structure (for most modes and regimes of response) compared with the theoretical response - except where such theoretical methods have been developed to allow for average imperfections and levels of residual stress and then possibly calibrated against controlled experimental results. One slightly beneficial effect of manufacturing imperfections is that they can have the effect of making failure less precipitative, e.g. by forming a more gradual transition to a ductile plateau at the ultimate strength level.

Another difference between the response of 'real' compared with 'theoretical' structures is in that the latter will generally be (unless using reliability based techniques) based upon approval drawings, and supporting design and manufacturing data which will contain nominal dimensions of various scantlings and geometry. The as-built structure will statistically differ from the approval drawings quite considerably and similarly the mechanical properties of the materials used will differ from the 'rule-minimum' requirements.

The above reinforces the need for the application of reliability based assessment techniques for both component level and system level studies.

16.5 Calibration With Full Scale Performance

There have been many experimental studies made into the performance under loads of various types of structural components. This has enabled comparisons to be made with theory based predictions and the general magnitude of the bias between theoretical and actual performance to be quantified.

However, owing to costs and complexities, there have been in the general ship and offshore structures fields very few large scale tests undertaken. (This is in contrast to the aircraft industry where full scale tests, including ultimate strength and fatigue, are regularly undertaken for each new design). There have been many strain gauge studies undertaken, generally in support of damage investigations, and often considerable differences are found between measured strains and theoretical predictions.

Noting the complexities of most ship and offshore structures, covering the spectrum of discrete and continuous systems, there must thus exist a considerable uncertainty in the ability to theoretically predict ultimate strength and as-damaged residual ultimate strength (even assuming that one can define the full scope of the damage).

16.6 Characteristics and Modelling of Damage

Clearly the form and extent of damage can range from the quite minor (e.g. small fatigue cracks, plate thickness diminution due to corrosion, etc) through to quite massive (e.g. due to a major collision). Within the context of the design-inspection and redundancy triangle and the current interest in damage tolerant design, the range of damage that can result from a major collision are probably outside of the intended scope of this project.

Within the context of discrete structures, the extent of damage can be relatively readily quantified, e.g. a single member could be subject to some form of gross distortion (typically denting and overall bending) or could be partially or completely severed (typically due to a fatigue crack at a joint). The regions within the overall structure most prone to certain forms of damage can be readily identified, typically within the region of the splash zone where collisions with work and supply boats are likely to occur. However member partial or complete failure due to fatigue cracking could occur any where within the structure, particularly in the underwater region where surveillance and inspection is much more difficult.

Within the context of many 'continuous' structures (typically complex stiffened shell and plate type semi-submersible and ship forms) the range of characteristics and positions of various forms of damage and general inservice degradation is clearly quite considerable. Damage can be caused by accidental contact with other marine vehicles or constructions, by dropped objects, by overloaded equipment, etc., and any degradation can be due to combinations of material wastage, fatigue cracking and stress corrosion, etc. This is in addition to damage caused by inadequately designed structure. With regard to ship structures considerable statistics are available concerning collision damage, fatigue cracking and scantlings diminution due to corrosion, etc. Some data is available for semi-submersibles but not with the same statistical basis of a large number of 'ships-years' of service.

Thus a designer, when considering the development of a damage tolerant structure, needs to consider the forms of damage which could statistically occur and for which the structure must be able to accommodate and survive safely.

If a design code is to be, eventually, devised within the DIRT concepts and incorporate interactions between redundancy, reserve strength and residual strength, then it would be important to define sets of 'assumed damage' models that the structure must be capable of surviving. Such proof of capability would need to be demonstrated by approved analytical methods and formal evaluation criteria.

16.7 Phasing Between Damage Loads Inspection and Repair

Clearly, within the concepts of DIRT, the prime function of inspection will be to determine if any form of structural damage and/or degradation has occured. Depending upon various factors, such as times between inspections, probability of detecting damage/degradation, etc., such resulting weakening of the structure will exist, and possibly increase, until repairs are made which either fully or partially restore the integrity of the structure. In the time period between damage initiation and repair, the structure will still be subjected to operational and environmental forces. However, the probability is that these operational and environmental forces will be considerably less than the corresponding maximum design forces - albeit the differences will be a function of the length of the time between damage initiation and repair. This aspect is recognised by some design approval agencies.

16.8 Rigorous Analytical Design Studies

Clearly it would be possible for the designers to undertake comprehensive rigorous analytical studies allowing for various damage scenarios and including combined non-linear response and reliability assessments. Such studies would be quite expensive to undertake, although with the current trends in computer aided design and reductions in computing charges, the studies are becoming more feasible.

If the longer term goal is to evolve design approval codes in which redundancy is specifically allowed for then the main problems will relate to how to quantify the effects of various degrees and forms of redundancy in a manner which is cost effective to both the designers and the approval agencies.

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17.0 RECOMMENDATIONS FOR FUTURE WORK

17.1 Introduction

This avenue of research must have several goals, including:

- to realise the reserves of strength provided by redundancy within structures and which are not, in general, considered within the current design practices (and thus to eventually improve both knowledge of ultimate system strength, overall structural efficiency and overall reliability) and

- to ensure that structures are adequately damage tolerant.

The potential scope of a follow-on research program is clearly quite extensive, noting the considerable range of ship and offshore structure configurations, ranges of feasible failure mechanisms, possible damage scenarios, etc. Thus to identify and select specific aspects that would have some merit and priority for study, within the auspices of the Ship Structure Committee's mandate, is rather difficult.

It is noted that the Ship Structure Committee is able to commission only a small number of new projects each year and which, of necessity, must embrace several diverse subject areas. This chapter reviews a broad range of possible research and development projects without, initially, placing any order of importance or priority against each.

Clearly the Ship Structure Committee will need to consider a direction of focus for future work. There are several possible options that could be taken, for example:

- to aim to provide designers with illustrative worked examples, models, methods, etc.

- to encourage researchers in the development of cost effective ultimate strength analysis tools integrated with reliability assessment capability,

- to provide case study/background data which would be used as input to the future development of design codes,

- to evolve criteria for damage modelling, concepts of survivable damage and appropriate design models and targets, including failsafe design concepts.

The latter, if such eventually became a mandated design requirement, would help to drive the designers to ultimate strength designs, calling upon the effects of redundancy on both reserve and residual strength. Failsafe design concepts should also be reviewed inconjunction with the maximum forces that could occur between damage initiation and subsequent repair, rather than design lifetime maximum forces.

17.2 General Areas Requiring Study

As a general note it is to be appreciated that many of the analytical methods and tools necessary to undertake rigorous assessments of reserve strength (ultimate) and residual strength (ultimate) are already available, albeit sometimes of limited availability and quite expensive to use for some structural families, [15.1, 15.3]. Similarly the techniques and tools for undertaking full statistically based reliability assessment studies are well developed and readily available.

For continuous structures assessments of true reserve and residual strength should really be undertaken using methods that allow for the full three-dimensional nature of such structures. For example in the context of ships Ref.[15.1] illustrates the preferred approach. The use of two-dimensional prismatic box beam models, as typified in [15.9 and 13.24], contain too many assumptions and limitations in order to be able to accurately reflect the effects of local damage.

Thus one general area that would profit from some study and development effort is that for a cost-effective non-linear three-dimensional computer program package that would be capable of allowing for all modes of failure for complex continuous structures, similar to [15.1]. With modern computer aided design systems semi-automated model building should be a reasonably straight-forward task when interfaced with a full design data base. However in recognition that these forms of studies are themselves generally quite complex and requiring appreciable skills from their users (e.g. understanding of strain limits, effects of rupture at large deflections, aspects relevant to 'real' structures and their geometric imperfections, etc.) it is preferable that an 'expert' type of front end to analysis software be provided. This could be expanded to include formal representations of damage.

In focussing on the 'static' implications of the interactions between redundancy, reserve and residual strengths the potential problems associated with dynamic response must not be overlooked. For example, given the energy in a modest to severe wave loading environment the effects of hull girder induced vibrations could lead to considerable damage propagation in a relatively short period of time (this is in addition to the classical low cycle-high stress fatigue problem).

It would be useful to undertake an analysis and evaluation of ship and offshore structure damage and general repair records in order to develop damage/ degradation models and for which future designs would be required to tolerate to a specified level of safety and associated loading demand. This data should be available within the records of the various classification societies.

Clearly 'damage' has to be categorised into both 'cause' and then, subsequently, the 'effects' and in a structural context there will be considerable interactions with redundancy. However the other effects of damage, specifically the effects of flooding and the loss of overall stability, etc., are clearly also of great significance. Thus defining target 'survivable' damage models for use in the design process of various forms of marine structures could be a considerable exercise. When a designer has to produce designs that are survivable in various mandated damage scenarios it is most likely that the design evolution process will involve building-in more redundancy into the design. (This could, in part, stem from the eventual outcome of the Ship Structure Committee's project SR-1332 as described in the Committee on Marine Structures report 'Recommendations for the Interagency Ship Structure Committee's Fiscal 1990 Research Program'. The objectives of project SR-1332, which has the title *'A Structural Life Management Program for Novel Marine Structures'*, are to obtain a procedure for structural life management of novel marine structures, including more efficient inspection, more economical and safer operations, and more effective maintenance, and to recommend research topics for the practical implementation of Marine Structural Integrity Program.)

The target levels of reliability at the overall structural system level need to be established for each of the various relevant classes of structures, allowing for the forms of damage models mandated and the designer should have some means of undertaking cost-benefit studies allowing either explicitly or implicitly for different degrees of redundancy.

As a general point, and related to the longer term goals of the overall program, vis a vis the relationships between redundancy, reserve strength and residual strength, termed the 3R program, some consideration needs to be given to the form of the guidance/formal requirements that may be eventually released to the shipbuilding and offshore industries. For example there are several possibilities ranging from:

- for the designer to be required to undertake complex analytical studies allowing for full system non-linear response, with specific consideration of forms and degree of damage, etc. to

- for the designer to continue with the present methods and criteria but with an additional factor of safety included (or a partial factor) to allow for some measure, or appropriate measure, of overall redundancy.

Other possibilities include the formulation of prescriptive rules on levels of redundancy as promulgated by the American Petroleum Institute for the design of earthquake resistant structures and the specification of strength analysis procedures for damaged structures as given by Det norske Veritas for semi- submersible platforms.

17.3 Specific Recommendations

The following are the outlines of several possible follow-on projects that are offered for consideration, with the aims of:

(a) Making the practicing designers more aware of the methods and implications of the DIRT concepts, including damage tolerant designs.

(b) To provide some quantification of the degree of redundancy in actual typical structures of both discrete and continuous types.

(c) To enable the eventual development of new design codes which reflect structural performance and reliability at the overall systems level.

(d) To define and quantify damage such that life-time cost-effective damage tolerant designs can be evolved.

Clearly this is a broad and complex problem area and many investigative studies could be proposed. However many of the potential study areas are likely to be quite time consuming and expensive to undertake and hence any such proposals must have demonstrable and adequate cost-benefits before they are undertaken. The recommended studies could be in several groups for systematic development

- offshore discrete type structures
- offshore continuous type structures
- conventional ship structures, and
- non-conventional structures.

Within each group examples of progressive complexity could be considered.

17.3.1 The study of a number of actual existing steel jacket type offshore structures in order to quantify the relationships between redundancy, reserve strength and residual strength. In order to reduce the amount of work to a reasonable level the structures should be idealised to an acceptable extent and the applied loading conditions constrained to simple wave environments. Similarly the ultimate capability response of the selected structures could be limited to the large deflection non-linear plastic mode with strain limits.

17.3.2 The study of one or more actual offshore jacket platforms, assuming the availability of full design and response analysis data, considering the overall system reliability, examining the effects and consequences of various damage models and reviewing the sensitivity of the intact and damaged response to the various design and fabrication variables involved.

17.3.3 Again for an actual offshore jacket platform, develop and evaluate the relevant partial safety factors appropriate to that class of structures and examine the effects of various degrees of redundancy. Compare the levels of reliability that result from conventional component level studies and with a system level study.

17.3.4 A comparison between the design of a jacket structure based upon a conventional deterministic process with one designed following a system level reliability based approach and examining the consequences of degrees of redundancy and degrees of damage.

17.3.5 This task would be to take several (two or more) designs of semi-submersibles, preferably with some common overall topology, and to repeat the studies undertaken in tasks 17.3.1 to 17.3.4 inclusive.

(N.B. The above studies represent a progressive evaluation of both fixed site and floating offshore structures. The reports which would result from these studies will provide both detailed descriptions of the terminology, methods and analyses involved, clarification of the most appropriate methods and some data that could be relevant and useful for future code development.)

17.3.6 To examine the damage tolerance of various ship types/structural configurations and to establish the factors and trends which improve reserve and residual strength. This would also include reviewing the implications of damage-caused flooding on both stability and continued operability. This study should include models developed from actual inservice vessels and could also include the model (or models) developed within SSC project SR-1330. However load demands could be based upon simple estimates of maximum wave loading rather than via detailed seakeeping type studies.

17.3.7 To take an existing floating offshore structure, preferably one that has obvious topological as well as detail level redundancy, to review its overall systems level reliability and to illustrate the effects of various damage scenarios on residual system level capabilty.

17.3.8 To take the demonstration example used within SSC project SR-1330, review the structural performance from a system's viewpoint and examine the effects of various damage scenarios on the overall capability.

(N.B. Project SR-1330 is to provide a demonstration of the use of probability-based ship design techniques, comparing the process with the 'traditional' methods and using the example to demonstrate the additional kinds of information that will be required and obtained. In addition to providing a demonstration of probability-based design the project will serve to identify gaps in the present knowledge and thereby help to define goals for follow-on projects in this particular thrust area.)

17.4 Future Work

Several relatively modest peripheral studies would be of value if undertaken.

17.4.1 To review and quantify the time relationships between occurrence of damage, and its subsequent detection and repair and the maximum operational and environmental forces that the structure will have to withstand in this time interval. This study would clearly need to involve the statistical characteristics of all aspects of the problem, e.g. the probability that the inspection process will find and quantify the damage.

17.4.2 Review technology transfer from aerospace to the ship and offshore industry vis a vis damage tolerant and failsafe design concepts.

17.4.3 Undertake a data collection and subsequent analysis of damage records, for both ships and offshore structures, with the data normalised against total 'fleet' years of service in the conventional manner. Assess the damage in the context of vessel survivability and look for correlation with damage tolerant design features and implications of redundancy. Review the condition of the structure before the damage occurred, e.g. degree of general wastage if any.

From this study postulate the details and statistics of damage for which new designs should be made tolerant.

17.4.4 To explore the use of structural reliability theory in the design stage to determine the required level of redundancy for an overall structural system and thereby to establish quantitative trade-offs between the safety level and redundancy.

17.4.5 More research is needed to establish load effects in the reliability approach and thus to improve treatment of the combined effects of the various environmental loading, which, for example, involves joint probability of occurrence of currents and waves.

17.4.6 To assess the quality of existing sources of full-scale and large-scale laboratory data on the environmental loading and response of quasi-static and dynamic fixed structures.

17.4.7 To examine the role of Redundancy in Reliability based fatigue life assessments of marine structure.

17.4.8 To examine the optimisation of offshore structural components based on Reliability and Redundancy.

APPENDIX A

STIFFENED FLAT PANELS

A.1 Background

□ Design Arrangement

The range of stiffened flat panel arrangements which may be encountered in design is quite considerable. Consider in isolation of other structure two basic flat panel arrangements:

1 Quadrilateral panels with basically single-direction stiffeners, the general case is illustrated in Fig.A.1(a) and the more common case in Fig.A.1(b).

2 Quadrilateral panels with intersecting groups of stiffeners. The general case is illustrated in Fig.A.1(c) and the more common case in Fig.A.1(d).

The stiffeners are assumed to be continuously attached to the plating and to be perpendicular to the plane of the plating, as illustrated in Fig.A.2. (In some designs there may be features which result in the stiffeners being not truly perpendicular to the overall panel - however this exception is not considered within this discussion.) They thus possess bending strength in a plane which is perpendicular to that of the flat panel, in addition to having axial load carrying capability.

The panels are assumed to be supported around their external boundaries in a manner which effectively prevents local out-of-plane displacement. However, such boundary supports may permit other forms of displacement, e.g. rotation, depending upon the local detail design and the sub-structure's stiffness.

Both the plating and the stiffeners are normally continuous - the problems of cut-outs and discontinuities and any associated local reinforcement are not considered in this study. There may however be a variation in plate scantlings within a panel.

The stiffeners within a panel may have essentially uniform scantlings, that is, all stiffeners being of the same size and section, or there may be a wide variation in sizes and clearly this will effect the capability of the panel and produce non-uniform response within the panel. These are both illustrated in Fig.A.3.

□ Applied Forces

There is, obviously, a considerable range of balanced force systems that may be applied to stiffened flat panels. Some of these, applied to longitudinally stiffened flat rectangular panels, are illustrated in the following figures. Other force systems could, of course, be combinations of those which are illustrated.

The applied force systems, and their combinations, can be conveniently sub-divided into two general categories:

- inplane, and
- normal to the plane.

(Note - the unusual problems of applied bending moments and torques, in any plane, need to be separately considered.)

These inplane forces may be either:

- uniformly distributed (Fig.A.4),
- non-uniformly distributed (Fig.A.5), or
- point forces.

The inplane forces can also be:

- uniaxial,
- biaxial,
- shear, or
- combinations of the above.

The normal forces can be:

- uniform pressure,
- non-uniform pressure,
- patch-type pressures, or
- point forces.

□ General

Uniformly stiffened flat panel analyses generally assume that the strength of an overall panel is a simple multiple of the strength of a single stiffener plus associated plate element. No special allowance is made for the support provided at the longitudinal edges of the panel, this could be appreciable.

This infers therefore that all elements will fail at the same time and at the same uniform edge load level.

Clearly in a real structure there will be some statistical difference between each of the stiffeners on a panel, even though notionally they are specified as being identical and of uniform spacing. The differences may be in geometry or in as-manufactured imperfections and the results of such differences will be that one stiffener plus attached plate combination will probably reach its ultimate state before the others. The import of this will depend upon the mode of failure of that particular stiffener and its continued behaviour under gradually increasing axial forces. For example if the mode of failure is such that a gradual plateau of local strength is reached then the rest of the panel will be subjected to slight but proportionately higher forces as the overall loading is increased. This can continue until the next stiffener to reach its local capability does so and overall panel failure will occur when the response becomes unbalanced and the rate of loading on the other stiffeners is such that collectively no reserves of overall strength are remaining. The other possibility is that the first stiffener to reach its local load carrying capability then fails in a precipitative manner, i.e. rapidly unloads, and which thus equally rapidly transfers its pre-ultimate strength sustaining forces onto the adjacent stiffeners thus causing them to fail (however, it is unlikely that there will be a significant load carrying difference between adjacent stiffeners).

The above aspects are similar to the situation which will exist where:

(i) the stiffeners are by design of non-uniform size and/or shape and spacing, and

(ii) the edge loading is non-uniform.

The response of a stiffened flat panel to externally caused damage is clearly quite complex and many possibilities exist.

A.2 Stiffened Panel Analysis

To limit the scope of the following analytical study it will be assumed that the panels are simple rectangular uniformly and uniaxially stiffened panels and that uniform uniaxial compressive loading is applied aligned with the stiffener direction.

A number of approaches have been proposed by various researchers and agencies for the ultimate strength analysis of nominally flat plating, uniformly stiffened in one direction and subjected to a longitudinally aligned compressive load. Some of these methods make allowances for manufacturing imperfections and weld induced residual stresses. Comparisons of results obtained from four of these methods[A.1-A.11] with existing test data were made in report[A.12] and a modified approach was suggested for use by Lloyd's Register of Shipping in their direct calculation methods[A.13]. The method adopted is similar to the Imperial College method Ref.[A.12].

All the four methods examined in Ref.[A.12] employ a beam-column idealisation for the analysis of a single bay panel. The theories do not account for stiffener flange buckling, stiffener tripping or for web buckling between plate and flange members however all other failure modes are incorporated. No allowance is made for the effects of the support along the panels' longitudinal edges or the full interactions between adjacent bays, each of which can have an appreciable strengthening effect.

A brief outline of the method adopted in this calculation is as follows.

The analysis method followed assumes a simple eccentrically loaded column, using a Perry-Robertson type of formulation and in which the effective width of plating is allowed for and load line eccentricities owing to both loss of plate effectiveness and manufacturing imperfections are included.

□ Effective Width and Maximum Plate Stress

Formulae for obtaining effective width of plate factors K and maximum plate strengths σ_{pm} are developed in full in Ref.[A.9]. The equations were developed for long panels of width b and thickness t, where an initial as-manufactured deformation in the shape of the critical buckling mode was assumed. The longitudinal edges were constrained to remain straight whilst being free to pull in. This method also allows for the effects of weld induced residual stresses.

The effective width factor K and maximum plate strength σ_{pm} are obtained from the following expressions:

$$K = \sigma/E.\epsilon \tag{A.1}$$

$$\sigma_{pm} = \sigma \tag{A.2}$$

where:

$$\sigma = \sigma_{cr}\left\{1- \frac{1}{m}\right\}+ \frac{3}{8}\ (1-\nu^2)\ \sigma_{cr}\left\{\frac{\delta_o}{t}\right\}^2 (m^2-1) - \sigma_r \tag{A.3}$$

$$\epsilon = \frac{1}{E}\left\{\sigma_{cr}\left\{1 - \frac{1}{m}\right\} + \frac{3}{8}\ (1-\nu^2)\ \sigma_{cr}\left\{\frac{\delta_o}{t}\right\}^2 (2m^2-n^2-1) - \sigma_r\right\} \tag{A.4}$$

$$0.7\left\{\frac{\delta_o}{t}\right\}^2 m^2 - \frac{1}{m} = \frac{\sigma_{yp}}{\sigma_{cr}} + 0.7\left\{\frac{\delta_o}{t}\right\}^2 - 1 \tag{A.5}$$

$$0.7\left\{\frac{\delta_o}{t}\right\}^2 n^2 - \frac{1}{n} = \frac{\sigma_r}{\sigma_{cr}} + 0.7\left\{\frac{\delta_o}{t}\right\}^2 - 1 \tag{A.6}$$

$$\sigma_{cr} = \frac{\pi^2 E}{2.73}\left[\frac{t}{b}\right]^2 \tag{A.7}$$

where m is the plate magnification factor, ν is Poisson's ratio, δ_o is the typical as-manufactured deformation, σ_r is the weld induced residual stress, n is the plate magnification factor appropriate to σ_r, σ_{yp} is the yield stress of the plating and E is the modulus of elasticity.

Using equations (A.1) to (A.7) the values of K and σ_{pm} can be evaluated. Curves (to determine K and σ_{pm}), obtained for values of

$$\sigma_r = 0.1\ \sigma_{yp}$$

and

$$\delta_o = \frac{b}{200}\sqrt{\frac{\sigma_{yp}}{245}}$$

are shown in Fig.A.6.

□ Maximum Stresses

The mean compressive stress across the middle-plane of the plate is given by the following relationship:

$$\sigma_m = \frac{K \cdot P}{Ae} + \frac{K \cdot M \cdot yp}{Ie} \quad (A.8)$$

where Ae is the area of stiffener plus effective area of plate, P is the applied load per stiffener, M is the induced moment at mid-span = $(P \cdot \Delta p \cdot PE)/(PE-P)$, Ie is the second moment of area of effective section, yp is the distance from neutral axis of effective section to the middle plane of the plate. Δp is the eccentricity of the load at mid-span, PE is Euler load = $\pi^2 \cdot E \cdot Ie/Le^2$, Le is the effective span, L is the spacing of transverse frames and K is the plate effective width factor.

P reaches its maximum value when $\sigma_m = \sigma_{pm}$, thus re-arranging equation (A.8) one obtains the following quadratic in terms of the collapse load P:

$$P^2 - P\ (a1 + a2 + a3) + a4 = 0 \quad (A.9)$$

$$a1 = \frac{\sigma_{pm} \cdot Ae}{K}$$

$$a2 = PE$$

$$a3 = \frac{\Delta p \cdot a2 \cdot Ae \cdot yp}{Ie}$$

$$a4 = a1 \cdot a2$$

However failure may also occur when the yield stress is reached in the stiffener outstand. The tensile stress σ in the stiffener outstand is given by the relationship:

$$-\sigma = \frac{P}{Ae} - \frac{M \cdot ys}{Ie} \quad (A.10)$$

where ys is the distance from neutral axis of effective section to the extreme stiffener fibre.

Re-arranging as before with σ made equal to σ_{ys} where σ_{ys} is the yield stress of the stiffener, one obtains:

$$P^2 + P\ (a5 - a2 + a8) - a7 = 0 \quad (A.11)$$

where $a5 = \sigma_{ys} \cdot Ae$

$$a8 = \frac{\Delta p \cdot a2 \cdot Ae \cdot ys}{Ie}$$

$$a7 = a5 \cdot a2$$

The maximum compressive stress in the stiffener outstand can be obtained from the relationship:

$$\sigma_{ys} = \frac{P}{Ae} - \frac{M \cdot ys}{Ie} \qquad (A.12)$$

where $M = (P \cdot \Delta s \cdot PE)/(PE-P)$

Δs = Eccentricity of load at mid-span.

Re-arranging, one obtains the following expression for P:

$$P^2 - P(a5 + a2 - a6) + a7 = 0 \qquad (A.13)$$

where $a6 = \frac{\Delta s \cdot a2 \cdot Ae \cdot ys}{Ie}$

Evaluations of equations (A.9), (A.11) and (A.13) provide the ultimate loads required to cause failure in three possible modes. The lowest of these loads is assumed to represent the limiting strength.

□ Eccentricity of Load at Mid-Span

The eccentricities Δp and Δs consist of two parts, an initial overall curvature defined as ± L/750 in Refs.[A.8–A.10], and a shift of neutral axis resulting from loss of plate effectiveness. The eccentricities are thus defined as follows:

$$\Delta p = \frac{L}{750} + (yp - y) + 1.2ec \qquad (A.14)$$

$$\Delta s = \frac{-L}{750} + (yp - y) + 1.2ec \qquad (A.15)$$

where y and yp are the distances from the middle plane of the plate to the neutral axis of the total and effective sections respectively, and L/750 is the straightness tolerance. The eccentricities, ec, in equations (A.14) and (A.15) and as shown in Fig.A.7 are increased by 20%. This is in line with the recommendations as given in Refs.[A.2, A.3, A.4].

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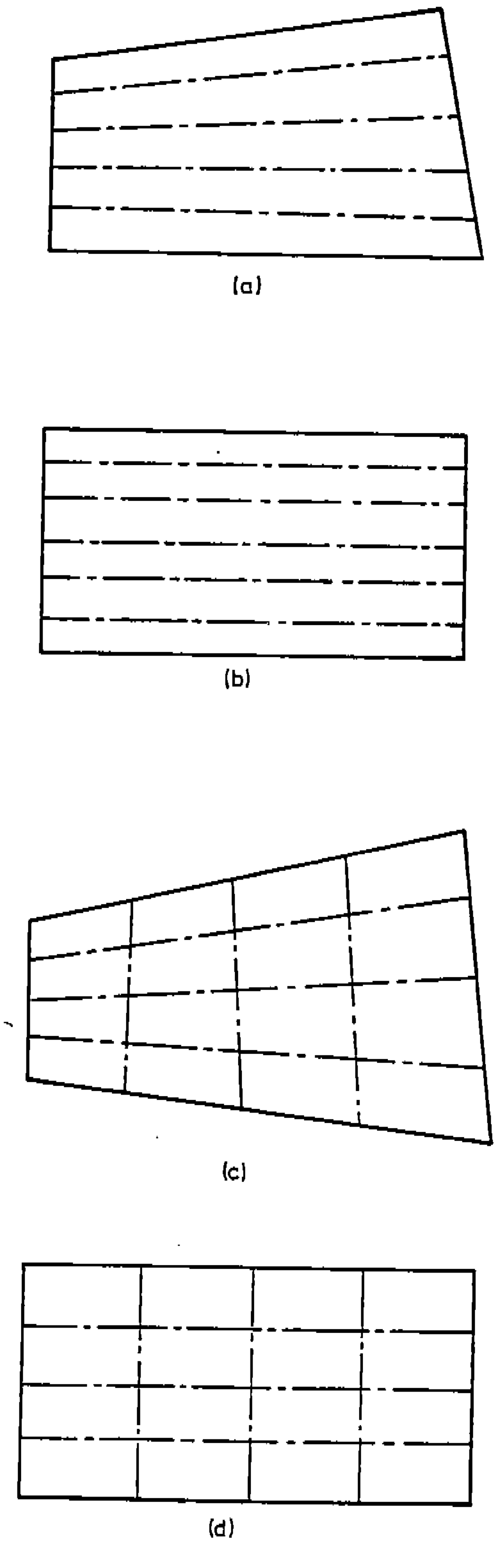

Fig.A.1 Stiffened Flat Panel Configurations

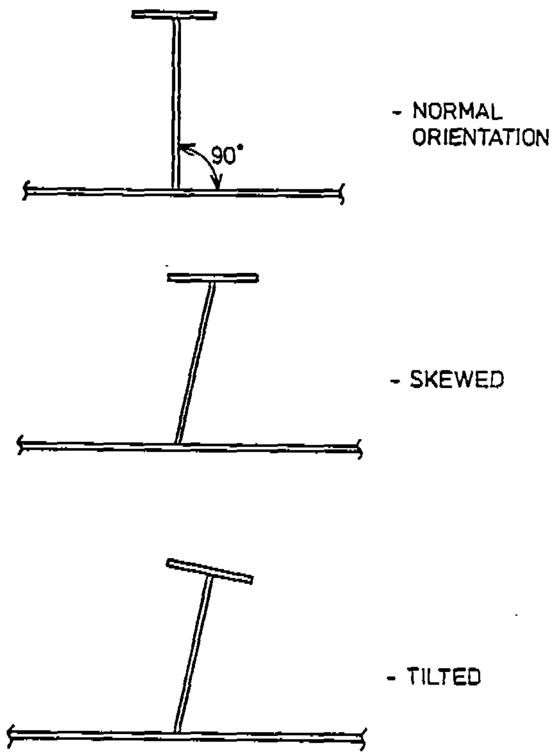

Fig.A.2 Stiffener Orientation

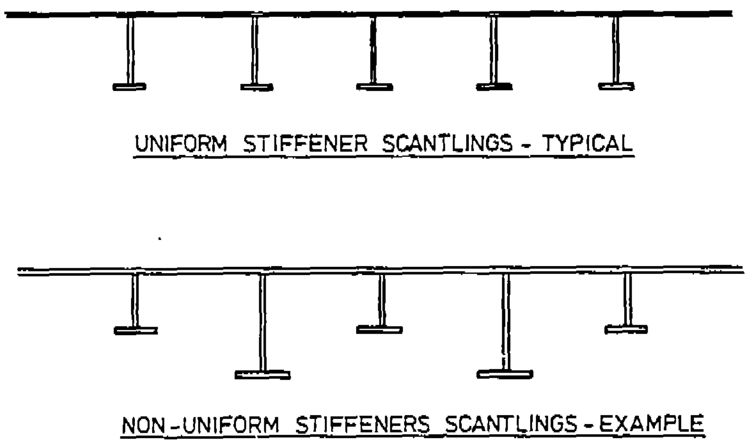

Fig.A.3 Stiffener Scantlings

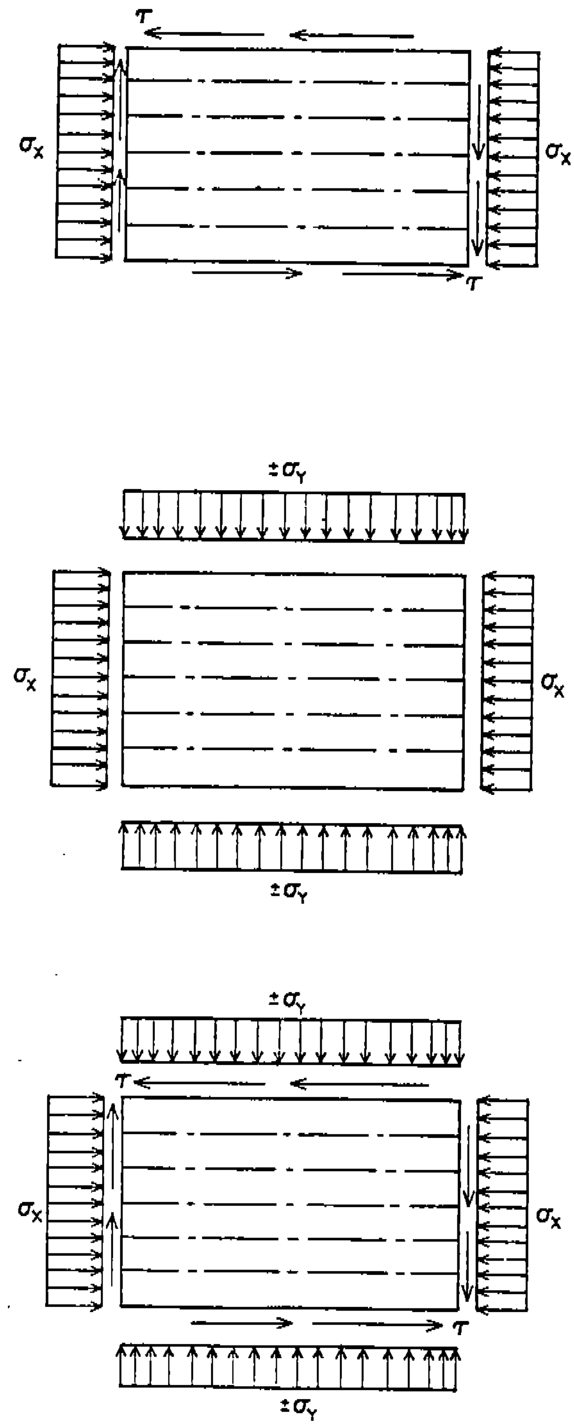

Fig.A.4 Typical Uniform Inplane Loading on Flat Panels

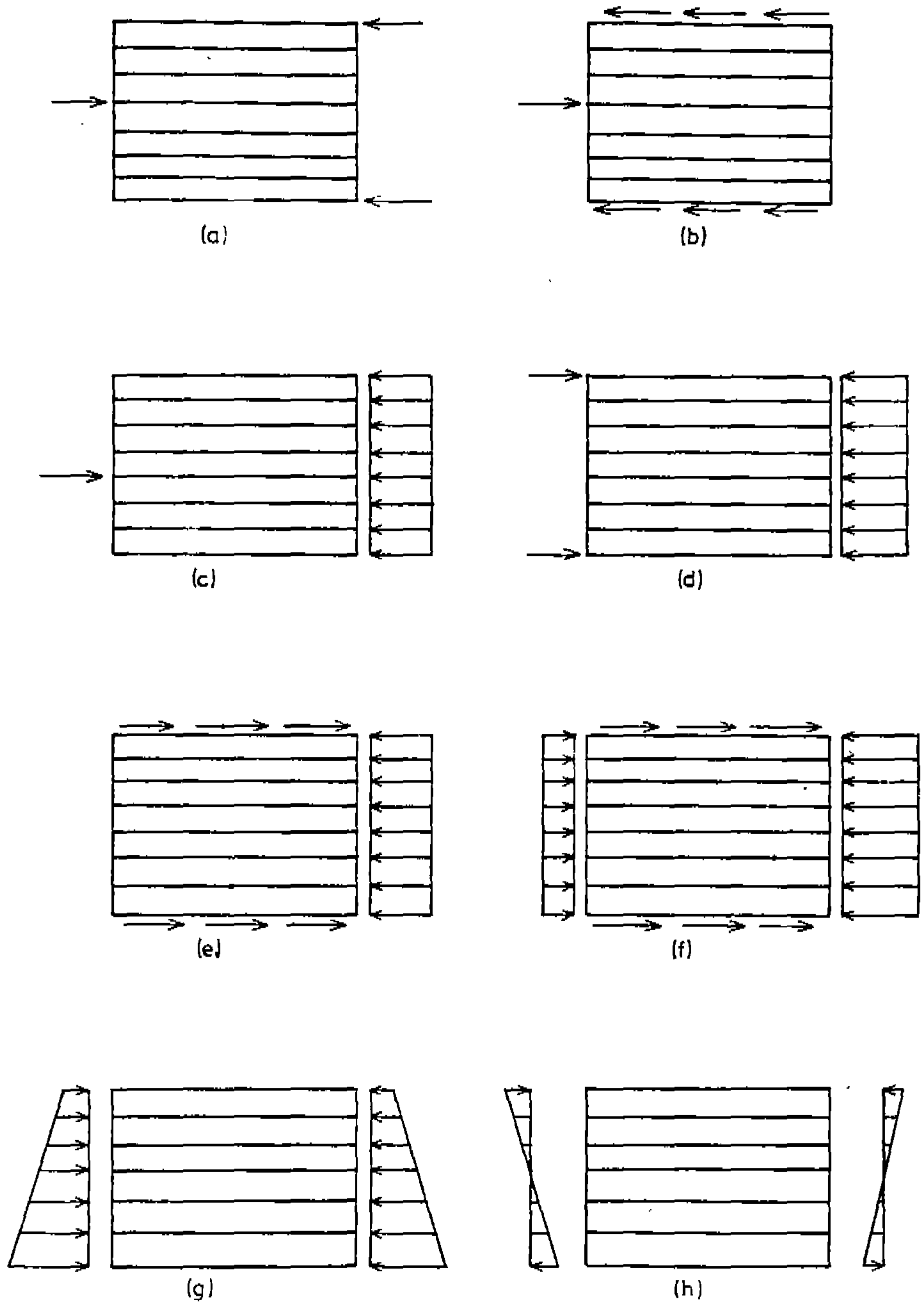

Fig.A.5 Typical Non-uniform Inplane Loading on Flat Panels

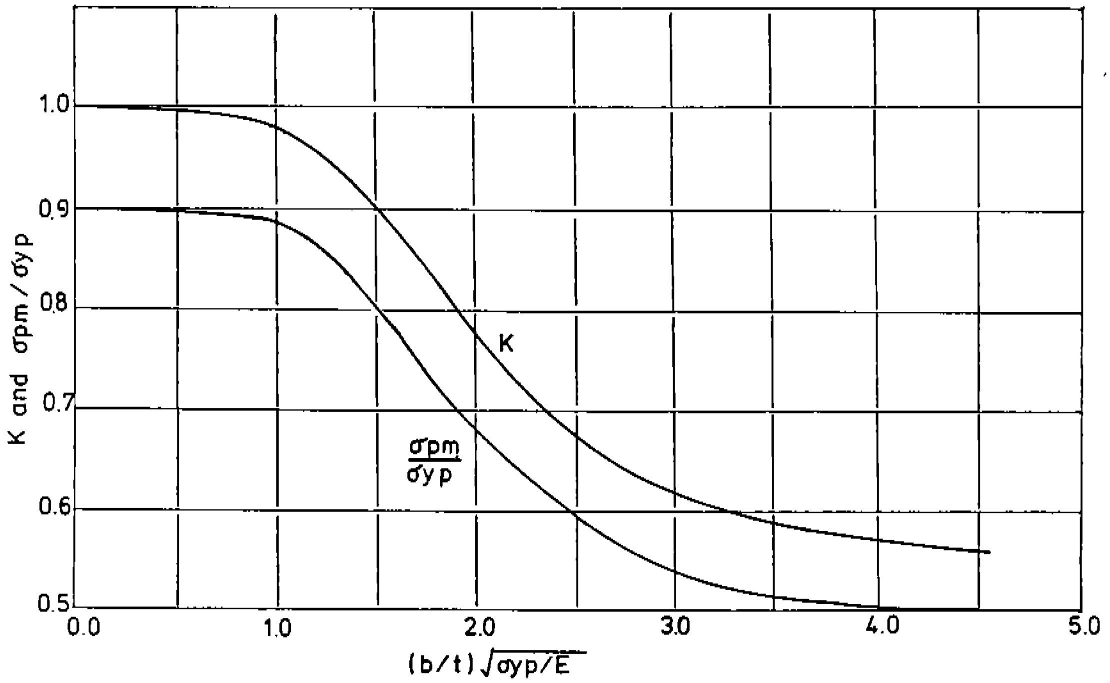

Fig.A.6 Curves for K and σ_{pm} Obtained from the Basic Imperial College Method [A8]

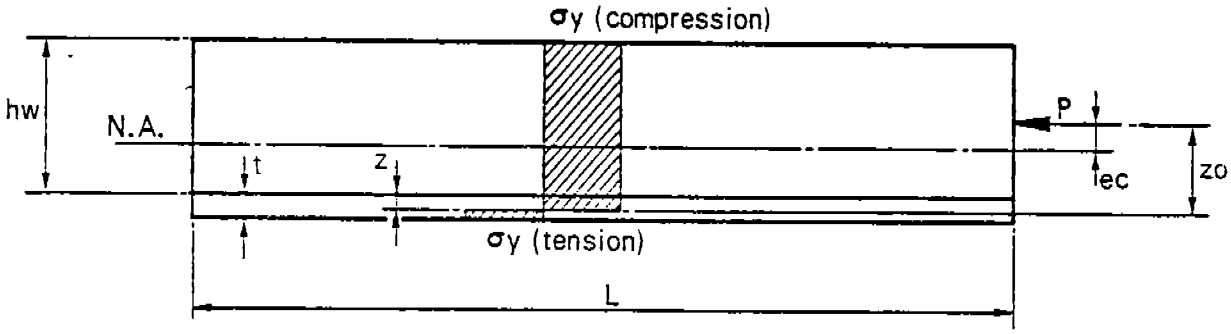

Fig.A.7(a) Squash Mechanism for Failure Towards the Plating

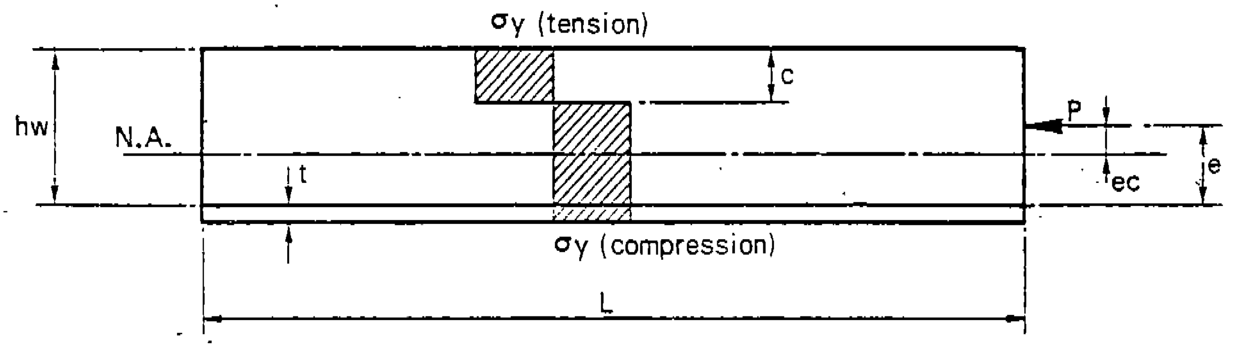

Fig.A.7(b) Squash Mechanism for Failure Towards the Stiffener

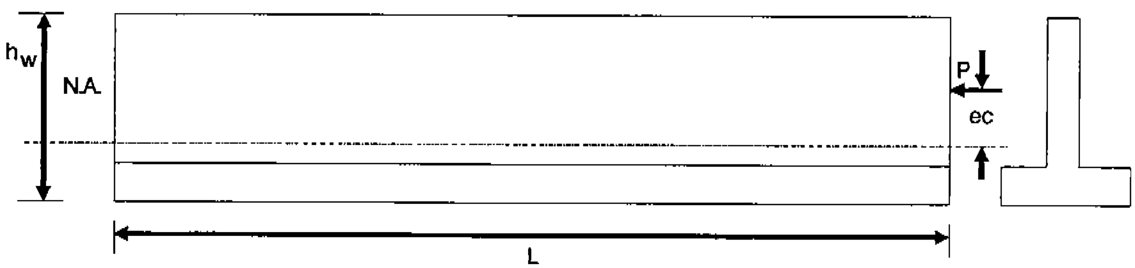

Fig.A.8 Stiffener Subjected to Eccentric Loading (i.e. with eccentricity 'ec' from the neutral axis of the stiffener)

APPENDIX B

STRUCTURAL RELIABILITY ANALYSIS

B.1 General

Structural reliability procedures are used to check safety in a structure within a safety domain after defining limit states which give a detailed description of the structural behaviour. These limit states may be the ultimate limit state, the fatigue limit state or the serviceability limit state. There are various levels of safety checking which are broadly grouped into three levels, namely level III, level II and level I, depending upon the degree of analytical sophistication[B.1]. These differences are mainly mathematical rather than conceptual. A brief description of these methods is as follows.

Level III is an "exact" probabilistic analysis for whole structural systems and involving the convolution integral. It is conceptually straight forward but in practice difficult to formulate and solve. Moreover it cannot be directly used for design, for example for a specified reliability level. However it is the only level which can satisfactorily incorporate all modes of failure when estimating the total reliability. Very clearly, these methods are not suitable for normal design purposes but there is much scope for the limited use of Level III techniques for checking the validity and accuracy of the more simplified Level II and Level I methods, for example by undertaking analyses of specific structures.

Level II methods use mean values and "second moment" properties of load and strength distributions for components and structural assemblies to calculate reliability or the safety index β which corresponds to a notional probability of failure, or level of reliability, for each failure mode or limit state during the life of the structure. Appropriate partial safety factors, PSF's, may then be derived for particular design situations. These safety checks are made only at selected points on the failure boundary (as defined by the appropriate limit state) rather than a continuous process, as at Level III. These methods may be used for analysis and design. Unfortunately, the essential features are conceptually less straightforward than Level III methods which need make no attempt to find the region of basic variable - or state-space, which has the highest probability of failure density. This is central to Level II methods and provides the basis for calculating PSF's at Level I.

Level I approaches provide a workable design method in which appropriate safety margin are provided, usually on a structural element by element basis, by specifying a number of partial safety factors, PSF's, related to some predefined characteristic value of the basic variables. In the strength model these values will usually correspond with the "minimum" values specified in the design, such as minimum yield stress, etc. No explicit reliability calculations are undertaken and the levels of risk in different structures are essentially unknown. Design methods involving a number of PSF's are likely to be of much greater practical value than Level II and III methods.

The Advanced First Order Second Moment (AFOSM) method, i.e. a level II method, has been widely accepted and has been made use of for establishing partial safety factors relevant to various types of offshore structural components and also in calibrating codes such as BS5400 and API RP2A.

3.2 Level-II Analysis

A brief description of the Advanced First Order Second Moment (AFOSM) reliability analysis method is as follows-

If x_1, x_2, ... x_n are the n independent variables involved in a structural design problem, a general expression for any limit state equation for the structure is

$$Z = g(x_1, x_2, \ldots x_n) > 0 \quad \text{(B.1)}$$

where the nature of g depends on the structural type and limit state under consideration. The failure surface is given by

$$Z = 0$$

and a linear approximation to this can be found by using the Taylor series expansion

$$Z \simeq g(x_1^*, x_2^*, \ldots x_n^*) + \sum_1^n (x_i - x_i^*)\ g_i'(x^*) \quad \text{(B.2)}$$

where $g_i'(x^*) = \frac{\partial g}{\partial x_i}$ evaluated at the unknown design point

$$x^* = (x_i^*, x_2^*, \ldots x_n^*).$$

If m_i and σ_i represent the means and standard deviations of the basic variables x_i, the mean value of Z is

$$m \simeq \sum_1^n (m_Z - x_i^*)\ g_i'(x^*) \quad \text{(B.3)}$$

and the standard deviation

$$\sigma_Z \simeq \left[\sum_1^n \{g_i'(x^*)\ \sigma_i\}^2\right]^{\frac{1}{2}} \quad \text{(B.4)}$$

σ_Z may be expressed as a linear combination of σ_i's as follows:

$$\sigma_Z = \sum_1^n \alpha_i\ g_i'(x^*)\ \sigma_i \quad \text{(B.5)}$$

where

$$\alpha_i = \frac{g_i'(x^*)\sigma_i}{\left[\sum_{j=1}^{n} \{g_i'(x^*)\ \sigma_i\}^2\right]^{\frac{1}{2}}} \tag{B.6}$$

and are referred to as sensitivity factors since they reflect the relative influence that each of the design variables has on the strength model.

If the reliability index β of the design is defined as m_Z/σ_Z', then from equations (B.3) and (B.4)

$$\beta = \frac{\sum_1^n (m_i - x_i^*)\ g_i'(x^*)}{\sum_1^n \alpha_i\ g_i'(x^*)\ \sigma_i} \tag{B.7}$$

from which it follows

$$\sum_1^n g_i'(x^*)\ (m_i - x_i^*\ \alpha_i\beta\ \sigma_i) = 0 \tag{B.8}$$

The solution of this equation is

$$x_i^* = m_i - \alpha_i\beta\ \sigma_i \quad \text{for all } i \tag{B.9}$$

and x_i^* is referred to as the 'design point'. This is shown in Fig.B.1. It corresponds to the point of maximum probability of failure density when all the variables are normally distributed. For given values of m_i, σ_i and β, equation (B.9) can be solved in conjunction with equation (B.6).

Finally, the probability of failure for the structure is

$$P_f = \Phi(-\beta) \tag{B.10}$$

where Φ is the normal distribution function.

If any of the design variables have non-normal distributions, the following transformation is adopted

$$m_i^N = x_i^* - \Phi^{-1}\ \{F(x_i^*\}\ \sigma_i^N\ , \tag{B.11}$$

$$\sigma_i^N = \frac{f^N\left[\Phi^{-1}\{F(x_i^*)\}\right]}{f(x^*)} \tag{B.12}$$

where m_i^N, σ_i^N are the mean and standard deviations of the equivalent normal distribution, F is the cumulative distribution function of x_i, f is the probability density function of x_i, and f^N is the normal probability density function which has the effect of equating the cumulative probabilities and the probability densities of the actual and approximate normal distributions at the design point x_i^*.

By replacing σ_i by $m_i \nu_i$, where ν_i is the coefficient of variation, Equation (B.9) can be written as

$$x_i^* = m_i \; (1 - \alpha_i \; \beta \; \nu_i) \tag{B.13}$$

and the term in parenthesis is the central coefficient.

The various advantages of the advanced first order second moment (AFOSM) method are as follows:

(i) It gives results quite accurately to the same degree as that of the Level-III exact method.

(ii) Unlike the Level-III method which can only be used for reliability analysis, i.e. to calculate only reliability from the probability of failure, the Level-II method may be used for both analysis and design, i.e. design for a specified reliability level.

(iii) It does not suffer from lack of invariance in the manner experienced by the mean value first order second moment method.

(iv) It can tackle any type of statistical distribution of the random variables as well as normal distribution.

(v) The sensitivity factors obtained from this method indicate the contribution of each random variable to the total probability of failure.

(vi) The partial factors obtained from this analysis can be used directly for design for a given target reliability level.

Partial safety factors can be made to give a consistent level of safety throughout a structure if these factors are expressed as appropriate continuous functions of the means and the variances of the basic variables and a selected reliability index. As seen in the advanced first order second moment method, the partial factor represents the ratio of the value of a variable at the failure point to its mean value and is expressed as:

$$X_i^* = m_i - \alpha_i \beta_i \sigma_i = m_i (1 - \alpha_i \beta_i V_i)$$

$$= \left[\frac{1 - \beta_i \alpha_i V_i}{1 - k_i V_i} \right] X_{k_i} = \nu_i X_{k_i} \tag{B.14}$$

where m_i is the mean value of X_i, V_i is the coefficient of variation of X_i, σ_i is the standard deviation of X_i, X_{k_i} is the characteristic value of X_i, k_i is a coefficient depending on the probability level p associated with the characteristic value of X and the nature of the probability distribution of X, α_i is a coefficient depending on the form of the failure boundary and ν_i is the desired partial safety factor.

□ References

B.1 Construction Industry Research and Information Association (UK). "The Rationalisation of Safety and Serviceability Factors in Structural Codes", CIRIA Report 63, 1977.

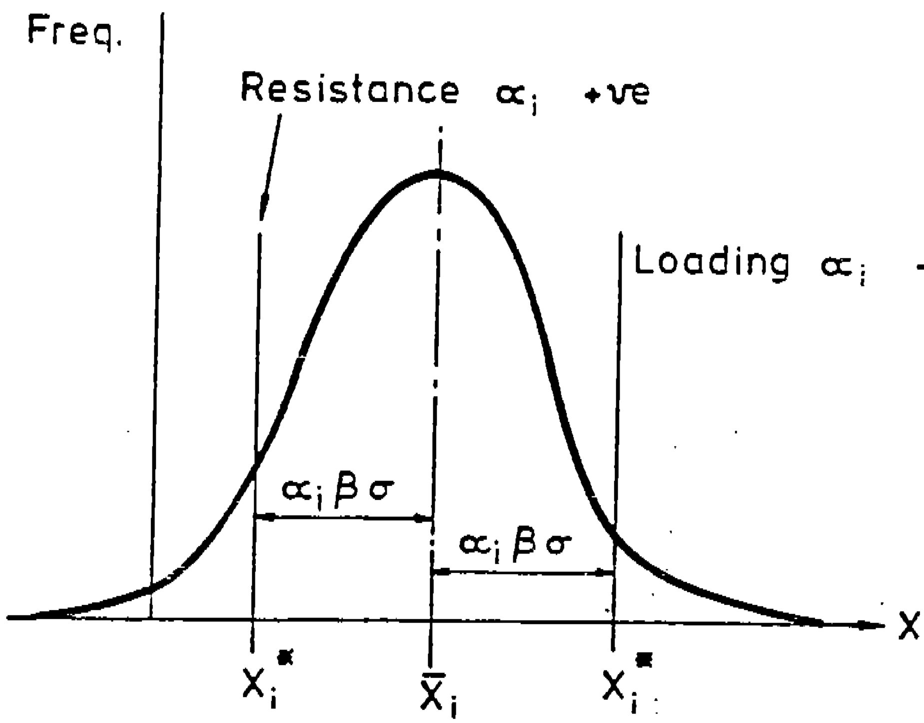

Fig.B.1 Design Points in Design Variable Space

APPENDIX C

STIFFENED CYLINDERS

In order to attempt a reliability analysis of any structure, it is important to have a rational strength model for the structure. In this Appendix the structure considered is a ring-stiffened cylinder without and with stringers. The Appendix is devoted to explaining the background to the rules controlling buckling of this type of structure under axial load, external pressure and bending moment. Particular emphasis is placed on the evolution of different offshore structures codes from the classical solutions for shell buckling and to the empirical fit of experimental data.

C.1 Axial Compression

The Donnel equation [C.1] for the buckling of cylindrical shells in terms of the lateral displacement w is

$$D\nabla^8 w + \frac{Et\partial^4 w}{r^4 \partial x^4} + \nabla^4 \left[N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right] = 0 \qquad (C.1)$$

where E is Young's modulus, t is the shell thickness, r is the shell radius, N_x, N_y, N_{xy} are the inplane stress resultants and x,y are the axial and circumferential directions respectively.

The above equation is strictly valid only for buckling modes in which there are several buckle wave lengths in the circumferential directions.

Under axial compression $N_{xy} = N_y = 0$ and $N_x = f \cdot t$, where f is the axial stress. The governing equilibrium equation reduces to:

$$D\nabla^8 w + f \cdot t \nabla^4 \frac{\partial^2 w}{\partial x^2} + \frac{Et\partial^4 w}{r^2 \partial x^4} = 0 \qquad (C.2)$$

A solution of equation (C.2) that satisfies the boundary conditions of simply supported ends is:

$$w = w_{mm} \sin \frac{m\pi x}{\ell} \sin \frac{ny}{r} \qquad (C.3)$$

where m is the number of axial half waves and n the number of complete circumferential waves. A non-trivial solution of equation (C.2) using equation (C.3) results in an expression for the critical elastic buckling stress f given by equation (C.4).

$$f = k_c \frac{\pi^2 E}{12(1-\mu^2)} (t/\ell)^2 \qquad (C.4)$$

where ℓ is the overall length, μ is Poisson's ratio and k_c is a buckling coefficient:

$$k_c = \frac{4\sqrt{3}}{\pi^2} Z = 0.702Z \qquad (C.5)$$

where $Z = \frac{\ell^2}{rt} \sqrt{1-\mu^2}$ and is called the Batdorf shell parameter.

By substituting equation (C.5) in equation (C.4)

$$f = C\frac{Et}{r} \quad \text{when } C = 1/\sqrt{3(1-\mu^2)} \simeq 0.605 \text{ for } \mu = 0.3$$

$$= 0.605 \frac{Et}{r} \qquad (C.6)$$

The above equation can also be obtained using energy principles, as given in Ref.[C.2].

The range of validity of equation (C.6) is that $Z \geqslant 2.85$ and which corresponds to moderate length cylinders.

For short cylinders, i.e., $Z < 2.85$ it behaves in the same manner as a wide, simply supported column which buckles into one half-wave in the axial direction and none in the unloaded direction. In this case, the buckling coefficient k_c becomes $1 + \frac{12Z^2}{\pi^2}$. If this is substituted into equation (C.4) and re-arranged in the form of equation (C.6), the elastic buckling stress for short cylinders becomes

$$f = \left[\frac{1.425}{Z} + 0.175Z\right] \times 0.605 \frac{Et}{r} \qquad (C.7)$$

For large values of Z, long cylinders buckle in an overall manner as Euler columns with no distortion of the circular cross-section and thus the above approach is no longer appropriate.

The DnV code[C.3] adopts the above formulae (C.6) and (C.7) directly for calculating the elastic buckling stress of cylinders. It thus differentiates between moderate length and short cylinders, whereas other offshore codes, e.g., API RP2A [C.4], ECCS [C.5], ASME [C.6], assume that the cylinder is of moderate length, possibly because for most practical cylinders $Z > 2.85$.

In order to cater for the effect of as-manufactured geometric imperfections, the basic classical equations (C.6) and (C.7) are reduced by factors, called 'knock-down' factors, which aim to reduce the theoretical elastic critical buckling stresses to those more appropriate to practical structures. Thus the elastic buckling stress including imperfection effects is given by:

$$f_e = \rho f \qquad (C.8)$$

where ρ is the so-called knock-down factor, and which is a factor by which the theoretical buckling stress is reduced due to the presence of as-manufactured geometric distortions in the shell, such as ovality, etc.

Various values of ρ can be found in different codes as follows:

DnV

$$\left.\begin{aligned} \rho &= 0.35 - 0.0002r/t && \text{if } Z \geqslant 20 \\ \rho^* &= 0.75 - 0.142(Z-1)^{0.4} + 0.003Z\left[1 - \frac{r}{300}\right] && \text{if } Z < 20 \end{aligned}\right\} \quad (C.9)$$

* An approximate fit to DnV Rules (curves taken from Ref.[C.7]).

ECCS

$$\left.\begin{aligned} \rho &= \frac{0.83}{\sqrt{1+0.01r/t}}, && \text{for } r/t < 212 \\ \rho &= \frac{0.70}{\sqrt{0.1+0.1r/t}}, && \text{for } r/t > 212 \end{aligned}\right\} \quad (C.10)$$

API RP2A

$$\rho = 0.5 \qquad \text{for } r/t > 30 \text{ and } < 150 \quad (C.11)$$

ASME, the larger of the values taken from (a) and (b)

$$\left.\begin{aligned} &(a) && \rho = 0.207 && \text{for } r/t \geqslant 600 \\ & && \left.\begin{aligned} \rho &= 1.52 - 0.473 \log_{10}(r/t) \\ \rho &= 300f_y/E - 0.033 \end{aligned}\right\} && \text{use smaller values for } r/t < 600 \\ &(b) && \rho = 0.826/M^{0.6} && \text{for } 1.73 \leqslant M < 10 \\ & && \rho = 0.207 && \text{for } M \geqslant 10 \qquad \text{where } M = \ell/\sqrt{rt} \end{aligned}\right\} \quad (C.12)$$

When using the ASME code [C.6], equation (C.8) is also multiplied by further slenderness dependent reduction factors.

The knock-down factors calculated in accordance with the above codes are shown in Fig.C.1 for a ring stiffened cylinder which has a radius of 5m, a shell thickness of 50 mm and the ring spacing is 1750mm. The yield stress for the material is assumed to be 350 N/mm². For API RP2A, the constant value adopted is quite obvious. The ECCS formula represents the lower bound of scatter bands of numerous experimental points obtained from tests on plastic and metal cylinders performed over many years. The validity of equation (C.10) has been further examined by Saal [C.8]. Hundreds of experimental results of buckling loads were reviewed and his conclusion is that for shells that meet the requirements of the ECCS Manual, the ECCS curve is a lower bound curve. This factor falls continuously with r/t after initiating at a value of r/t = 0, significantly greater than any of the other adopted values. The ASME factor is determined from four conditions, one of which involves yield stress, although plasticity is intended to be catered for by using a different set of criteria. Only three of these are reflected in the figure as the one involving M (see (b) above) cannot be uniquely represented with the format adopted.

The DnV knock-down factor is also seen to decrease continuously with r/t although at a rate considerably smaller than that demonstrated by the ECCS factor.

The ECCS factor would appear to be most logical in that it approaches unity as $r/t \to 0$. However, the imperfection factor reflects only part of the total combination of criteria which eventually leads to design stresses.

In Ref.[C.7], equation (C.8) has been modified by multiplying it by a bias factor B whilst the knock-down factor remains the same as that in the DnV rules, i.e.,

$$f_e = B\rho f \tag{C.13}$$

It is claimed that with the incorporation of this bias factor, this formulation is within +15% of all steel model test results obtained in the UK [C.9]. It tends to be slightly non-conservative for the stocky (inelastic response) range but conservative for elastic buckling.

C.2 External Pressure

Under external pressure normal to the cylindrical surface $N_x = N_{xy} = 0$ and $N_y = ft = pr$, equation (C1) reduces to:

$$D\nabla^8 w + ft\nabla^4 \frac{\partial^2 w}{\partial y^2} + \frac{Et}{r^2}\frac{\partial^4 w}{\partial x^4} = 0 \tag{C.14}$$

Equation (C.3) is again a solution of this equation provided the ends are simply supported. A non-trivial solution of equation (C.14) using equation (C.3) results in an expression for the elastic buckling stress given by, Ref.[C.1].

$$f = k \frac{\pi^2 E}{12(1-\mu^2)} \left[\frac{t}{\ell}\right]^2 \tag{C.15}$$

where $$k = \frac{\left[1+\beta_1^2\right]^2}{\beta_1^2} + \frac{12Z^2}{\pi^4\beta_1^2\left[1+\beta_1^2\right]^2} \tag{C.16}$$

where $m = 1$ and $\beta_1 = \frac{nl}{\pi r}$.

Thus a cylinder of any length under external pressure buckles into a single half-wave in the axial direction.

For short cylinders, the curvature parameter $Z \rightarrow 0$ and equation (C.16) reduces to

$$k = \left[\frac{1}{\beta_1} + \beta_1\right]^2 \tag{C.17}$$

For moderate length cylinders, i.e., $Z < 100$, an approximation of $1+\beta_1^2 \simeq \beta_1^2$ is reasonable as β_1^2 is much greater than unity so that equation (C.16) reduces to

$$k = \beta_1^2 + \frac{12Z^2}{\pi^4 \beta_1^6} \tag{C.18}$$

Minimising equation (C.18) and substituting the value of β_1 so found back into equation (C.18) gives

$$k = 1.038\ \sqrt{Z} \tag{C.19}$$

For long cylinders, Z increases and the number of half-waves in the circumferential direction decreases until it reaches n=2. The cylinder thus fails due to ovalisation into an elliptical form.

For $n = 2$,

$$\beta_1 = \frac{2\ell}{\pi r}$$

and from equation (C.17)

$$k = \beta_1^2 = \frac{4}{\pi^2 \sqrt{(1-\mu^2)}} \cdot Z \cdot t/r$$

So
$$f = \frac{E}{3(1-\mu^2)} \left[\frac{t}{r}\right]^2 \tag{C.20}$$

Since Donnel's equation is not valid for small numbers of circumferential waves, equation (C.20) is generally corrected to provide more exact solutions [C.2] from which:

$$f = \frac{E}{4(1-\mu^2)} \left[\frac{t}{r}\right]^2 = 0.275E\left[\frac{t}{r}\right]^2 \tag{C.21}$$

As in the case of axial compression, the theoretical 'ideal' elastic buckling pressure is reduced by multiplying it by a knock-down factor to calculate the elastic buckling pressure for the imperfect shell (DnV have adopted equation (C.16)), in conjunction with equations (C.17), (C.19) and (C.21) for the calculation of the elastic critical pressure of cylinders under external pressure. Knock-down factors as a function of Z are given in the DnV code [C.3].

BS5500 [C.10] is based on Von Mises shell buckling criterion amended by Kendrick [C.11] to take into account the buckling into a smaller number of circumferential waves. This relates to interframe shell buckling.

In BS5500, the pressure P_y at which the mean circumferential stress in the shell midway between the stiffeners reaches yield is first calculated, followed by its elastic instability pressure P_m. For the ratio P_m/P_y, P/P_y is then determined graphically where P is the allowable external pressure. This calculation requires prior selection of a minimum shell thickness, and which is not to be less than that required under internal pressure, excluding any corrosion allowance. The implied knock-down factor for elastic buckling collapse is 0.5. This is judged to be somewhat conservative.

In Ref.[C.7] the guaranteed hydrostatic collapse pressure is shown to be approximately given by

$$P_{hc} = \begin{cases} 0.5P_m & \text{if } P_y \geqslant P_m \\ P_y(1-0.5P_y/P_m) & \text{if } P_y < P_m \end{cases} \qquad (C.22)$$

The knock-down factor incorporated in the ASME code [C.6] is 0.8 and is the same in the API RP2A code[C.4].

In ECCS [C.5] for long cylinders, the elastic buckling stress is calculated from equation (C.21) and a knock-down factor of 0.8 is taken for calculating the buckling stress for imperfect shells. For short to intermediate length cylinders, the elastic buckling stress is given by:

$$f = 0.92E \left(\frac{r}{\ell}\right) \left(\frac{t}{r}\right)^{2.5} \qquad (C.23)$$

and a knock-down factor of 0.5 is used.

C.3 <u>Bending Moment</u>

For all practical purposes, instability under pure bending will occur when the compressive bending induced stress reaches the same level as that required to precipitate buckling under pure axial compression, i.e.,

$$f = 0.605Et/r \qquad (C.24)$$

Equation (C.24), which is the same as equation (C.6), is the classical buckling solution in which no allowance is made for geometric and/or material imperfections. However, it may be noted that the buckling stress in bending is generally slightly higher than the axial buckling stress, due to the stress gradient, i.e. stress which is due to local bending is not uniform but gradually reduces from the maximum value at the two extreme fibres, with regard to the cylinder's neutral axis, according to the theory of simple elastic bending. Thus the onset of local instability which must reflect a region or suitable arc of the shell will fail at an average stress which will be slightly less than the maximum stress. However for thin walled sections with typical imperfections the difference between the average stress and the maximum stress is likely to be very small.

DnV have adopted this approach in the same manner as for axial compression except that the knock-down factors[C.6] are different for the calculation of the elastic buckling stress when including geometrical imperfections.

For this case, API RP2A has adopted the above formula and the 0.5 knock-down factor, i.e., the same as that for axial compression.

In the ECCS formula, the knock-down factor is given by:

$$\rho_b = 0.1887 + 0.8113\rho \qquad (C.25)$$

where ρ is the knock-down factor for axial compression as given by equation (C.10).

For relatively thick walled circular cylinders under pure bending failure may take place in an ovalling mode, the so-called Brazier effect.

C.4 Buckling Resistance

In offshore structures, which are mostly constructed of steel, the buckling resistance also depends on plasticity (inelastic) effects including the effects of residual stress e.g. welding induced. A modification of the elastic buckling stress is therefore required. There are basically two methods for doing this which are in general use in practical design.

For inelastic collapse resistance, the so-called η method provides an equation as follows:

$$R_k = \eta f_e \qquad (C.26)$$

where R_k is the collapse resistance, f_e is the imperfect elastic buckling stress, $\eta = E_t/E$, E_t is the tangent modulus and E the elastic modulus. (Unfortunately there is often a sparsity of tangent modulus data).

The Ostenfeld-Bleich [C.12] quadratic parabola for materials having a well-defined yield plateau is:

$$\eta = \frac{f(f_y - f)}{f_p(f - f_p)} \qquad (C.27)$$

where f_y is the yield stress, f_p is the proportional limit stress, and f is the stress under consideration which lies between f_p and f_y.

In the so-called Φ-method, plasticity is accounted for by using an interaction formula such as the Merchant-Rankine equation

$$\frac{1}{R_k^2} = \frac{1}{f_y^2} + \frac{1}{f_e^2} \qquad (C.28))$$

By introducing a reduced slenderness parameter $\lambda = (f_y/f_e)^{\frac{1}{2}}$, equation (C.28) can be re-arranged as:

$$R_k = \Phi f_y$$

where $$\Phi = \frac{1}{(1 + \lambda^4)^{\frac{1}{2}}} \qquad (C.29)$$

The Φ-method has been adopted by DnV for use in their shell buckling resistance calculations.

The well-known Johnson-Ostenfeld interaction formula gives:

$$\left.\begin{aligned} \Phi &= 1 - \lambda^2/4 \quad &\text{for } \lambda \leqslant \sqrt{2} \\ &= 1/\lambda^2 \quad &\text{for } \lambda > \sqrt{2} \end{aligned}\right\} \qquad (C.30)$$

A modification of equation (C.30) has been proposed by ECCS [C.5] as follows:

$$\left.\begin{aligned} \Phi &= 1 - 0.4123\lambda^{1.2} \quad &\text{when } \lambda \leqslant \sqrt{2} \\ &= \frac{0.75}{\lambda^2} \quad &\text{when } \lambda > \sqrt{2} \end{aligned}\right\} \qquad (C.31)$$

Plots of these Φ formulae are shown in Fig.C.2. The DnV and ECCS formulae are seen to predict significantly different resistances. However, the ECCS curve already incorporates a slenderness dependent safety factor, as equation (C.31) indicates, whilst DnV, although overall including a slenderness dependent coefficient, do this via an independent partial safety factor rather than modifying the basic resistance equation. This is discussed later in more detail under combined loading.

In API RP2A the allowable axial compression and bending stress is determined by substituting the local buckling stress f for f_y in the appropriate[C.13] design formulae and f is obtained from

$$f = f_y[1.64 - 0.23(D/t)^{0.25}] \leqslant f_e \qquad (C.32)$$

where f_y is the yield strength in $Kips/in^2$ (Ksi).

In the ASME code the inelastic buckling stress F_{ic} is determined for axial compression by:

$$F_{ic} = \bar{k}_i \eta_i \rho f \qquad (C.33)$$

in which

$\bar{k}_i = 1 \quad \text{for } \lambda_x \leqslant 0.15$

$\bar{k}_i = 1.034 - 0.189\lambda_x - 0.158\lambda_x^2 \qquad \text{for } 0.15 < \lambda_x < \sqrt{2}$

$\bar{k}_i = 0.9/\lambda_x^2 \qquad \text{for } \lambda_x \geqslant \sqrt{2}$

where $\lambda_x = \frac{k_i \ell}{\pi r} \sqrt{\frac{\eta_i \rho f}{E}}$,

ℓ is the length of the cylinder and k_i depends on the end conditions. η_i is the plasticity reduction factor which accounts for non-linearity in material properties and residual stresses. Other parameters have been defined earlier.

C.5 Combined Loading

In offshore structures, a member may be subjected to combinations of various types of loading. As mentioned in Ref.[C.14], the resistance under combined loading can be obtained from

$$\sum S_i^{\gamma i} = 1 \tag{C.34}$$

where $S_i = \frac{\text{applied loading of type i}}{\text{resistance for type i}}$

At DnV, a sum of squares approach is used. Thus when a member is subjected to axial compression, external pressure and bending moments as in the present case, the interaction equation is given by:

$$\left[\frac{\sigma_{ZNd}}{R_{ZNk}} \frac{\kappa_{ZN}}{\psi_{ZN}} + \frac{\sigma_{ZMd}}{R_{ZMd}} \frac{\kappa_{ZM}}{\psi_{ZM}}\right]^2 + \left[\frac{\sigma_{\theta d}}{R_{\theta\kappa}} \frac{\kappa_\theta}{\psi_\theta}\right]^2 \leqslant \frac{1}{\gamma_m^2} \tag{C.35}$$

where σ_{ZNd}, σ_{ZMd} and $\sigma_{\theta d}$ are the stresses due to axial, bending moment and external pressure loading respectively. $R_{ZN\kappa}$, R_{ZMd} and $R_{\theta k}$ are the characteristic strengths (buckling resistances) of the structure under axial, moment and external pressure respectively. γ_m is the material partial factor taken as 1.15, ψ is a factor to reflect post-buckling behaviour and takes values of 1.0 to 0.9 depending on whether or not redistribution is possible, κ is a factor which reflects the uncertainties associated with slenderness in structures prone to instability and assumes values of 1.0 for $\lambda < 0.5$, 1.3 for $\lambda > 1.0$ and $0.7 + 0.6\lambda$ for $0.5 \leqslant \lambda \leqslant 1.0$.

Equation (C.35) may be used in creating the failure surface equation in the reliability analysis, but without the factors associated with stresses and characteristic strengths.

API RP2A [C.4] uses a parabolic interaction approach, the axial load ratio being the linear term. In ASME code case N-284 [C.6], interaction involves a complicated procedure apparently supported by test data.

The DnV Rule for ring-stiffened cylinders has not been substantiated by data relating to tests in the elasto-plastic regime.

A quadratic approach is used in the DnV Rules so that

$$\frac{1}{\sigma_c^2} = \frac{1}{\sigma_y^2} + \frac{1}{\sigma_m^2} \qquad \text{(C.36)}$$

or in its more usual form

$$\sigma_c = \frac{1}{\sqrt{1+\lambda 4}} \cdot \sigma_y \qquad \text{where } \lambda = \sqrt{\sigma_y/\sigma_m}$$

When using equation (C.36) to examine axial compression test data with σ_m derived from the DnV Rules results in a mean value of actual to predicted strengths of 1.44 (i.e. actual > predicted) with a COV of 26.2%. This is shown in Fig.C.3 (Ref.[C.15]). Thus it can be seen that the present DnV formulations from ring stiffened cylinders has a relatively large bias and the coefficient of variation (COV) is also very high.

Over the last few years [C.15] strength modelling for ring-stiffened cylinders has improved significantly, consequently bias from this source is now relatively small (typically with the range of 15%) if these new strength criteria are used and the coefficients of variation (COV's) on the modelling parameters, reflecting the difference between actual and predicted strengths, are in the range of 10 to 13%.

One such formulation is by Frieze etc. reported in Ref.[C.15] and its biaxial stress form, assuming ν = 0.3 throughout, is

$$\left[\frac{\sigma_{xo}}{\varphi_x \sigma_{crx}} + \frac{\sigma_{\theta o}}{\varphi_\theta \sigma_{cr\theta}}\right]^2 + \frac{\sigma_x^2 - \sigma_x\sigma_\theta + \sigma_\theta^2}{\sigma_y^2} = 1 \qquad \text{(C.37)}$$

where

$\sigma_{xo} = -\sigma_x \qquad \sigma_x < 0$

$\phantom{\sigma_{xo}} = 0 \qquad \sigma_x > 0$

$\sigma_{\theta 0} = \sigma_\theta \qquad \sigma_\theta < 0$

$\phantom{\sigma_{\theta 0}} = 0 \qquad \sigma_\theta > 0$

$\varphi_x = 0.3274 + 32.90\ x^{-0.6101}$

$\varphi_\theta = 0.2566 + 431.4\ x^{-0.8532}$

$x = \sqrt{Z}\ E/\sigma_y \qquad$ where $Z = \ell^2 \sqrt{1-\nu r^2}/Rt$

$\sigma_{crx} = 0.605\ Et/R$

$\sigma_{cr\theta}$ is determined using the BS 5500 [C.10] approach for calculating hoop stresses from external radial pressures.

The data on which the above derivations were based relates to 59 compression tests, 67 hydrostatic tests, 30 combined axial compression and radial pressure tests and 28 combined axial tension and radial pressure tests.

There was a very good correlation of the test results with predictions [C.15] using equation (C.37) and the following mean and COV values under various loading conditions, taken from Ref.[C.15], shows the accuracy of equation (C.37).

Loading	Mean	COV
Axial compression	0.994	0.112
Hydrostatic load	0.958	0.089
Axial compression and radial pressure	0.992	0.093
Axial tension and radial pressure	1.104	0.094

A comparison between the actual and predicted axial strengths using equation (C.37) is shown in Fig.C.4, taken from Ref.[C.15].

C.6 Ring and Stringer Stiffened Cylinders

C.6.1 Axial Compression

A typical/generally used formulation is based on the phi-approach, Φ, [C.16, C.17] to allow for the influence of material yield on the critical buckling stress of the stringer stiffened cylinder between ring frames: the latter are designed not to buckle before the longitudinals. The critical buckling stress is determined assuming that a single half-wave forms between rings [C.18]. The critical stress is then given as the simple algebriac summation of the buckling stress for the unstiffened shell between rings and that for the stringer plus attached plate acting as a column between rings, the properties of the column being determined on an effective width basis. The effective width in question is that of the shell panel between stiffeners. It is determined using an approach similar to that derived by Faulkner for stiffened flat panels [C.19] although in this case the imperfect shell buckling stress is used to determine the non-dimensional slenderness parameter instead of the perfect one used in the flat-plate application: this seems necessary because of the more marked sensitivity to lack of shape found in curved members. The formulation is described in detail below.

The curved plate element elastic critical buckling stress is:

$$\left.\begin{aligned} \sigma_e &= 0.605\ Et/r && \text{if } z_s > 11.4 \\ &= \sigma_{OF}(4+3z_s^2/\pi^4) && \text{if } z_s \leqslant 11.4 \end{aligned}\right\} \qquad (C.38)$$

in which $z_s = \frac{s^2}{rt} \sqrt{1-\nu^2}$

and $\sigma_{OF} = \pi^2 E(t/s)^2/12(1-\nu^2)$

where E is Young's Modulus, t the shell thickness, r the shell radius, and s the unsupported width of shell, i.e. stiffener spacing.

The imperfect elastic buckling stress is:

$$\sigma_{cr} = \rho\, \sigma_e \tag{C.39}$$

where $\rho = 1 - 0.019\, z_s^{1.25} + 0.0024 \left[1 - \frac{r}{300t}\right] z_s$

for $1 \leqslant z_s \leqslant 11.4$

$\rho = 0.27 + 1.5/z_s + 27/z_s^2 + 0.008 \left[1 - \frac{r/t}{300}\right] \sqrt{z_s}$

for $11.4 < z_s \leqslant 70$

This knockdown factor ρ was derived as a lower bound to the scatter envelope of test data [C.20].

To generate a mean value for use in reliability analyses, it has been suggested that this is increased by 15%. This implies a coefficient of variation in the range of 7 - 10%.

For the effective width calculation, the slenderness parameter λ is defined by:

$$\lambda = \sqrt{\sigma_y/\sigma_{cr}} \tag{C.40}$$

where σ_y = yield stress.

The reduced effective width s'_{eu} is then given by:

$$\frac{s'_{eu}}{s} = \frac{0.53}{\lambda} \leqslant 1. \tag{C.41}$$

The elastic buckling stress of the column is:

$$\sigma_c = \alpha_e\, \pi^2\, E(r_{ce}/\ell)^2 \tag{C.42}$$

where α_e = effective area ratio = $\frac{A_s + s_e t}{A_s + st}$,

r_{ce} = the effective radius of gyration, of the stiffener plus attached width of effective plate (se)

ℓ = the cylinder length and s_e is determined from

$$\frac{s_e}{s} = \frac{1.05}{\lambda} - \frac{0.28}{\lambda^2} \leqslant 1$$

and A_s is the stringer cross-sectional area.

The unstiffened elastic buckling stress with respect to the total cross-sectional area is:

$$\sigma_s = 0.605\ E(t/r)\ (1+\gamma)^{-1} \qquad (C.43)$$

where γ is the stiffener shell area ratio = A_s/st.

The critical stress for the stiffened cylinder then is:

$$\sigma_{sc} = \sigma_c + \sigma_s$$

and $\Phi = \{1 + (\sigma_y/\sigma_{sc})^2\}^{-\frac{1}{2}}$.

The ultimate stress (on the effective area) = $\Phi\sigma_y$, so that the overall average ultimate failure stress is then given by:

$$\sigma_u = \alpha_e\Phi\sigma_y \qquad (C.44)$$

The above strength model differs from that adopted by the Rule Case Committee (RCC)[C.7] in several ways. These are:

- residual stresses are not specifically allowed for,
- the effective width is determined assuming column collapse occurs at yield stress (and not the simple buckling stress), and
- an alternative method for determining inelastic collapse is permitted.

□ Calibration

Strength models used in design require validation against test data to establish their credibility and the bias between theory based predictions and actual failure (the lower the bias the more accurate the theory, etc.). For use in reliability analysis, however, the result of this validation must be expressed in statistical terms. If only limited data is available, model uncertainty can of course be treated within an analysis as deterministic.

The activity in floating platforms included a number of experimental programmes [C.21, C.22, C.23, C.24]. These studies provided data on the 21 models referred to in Section 12.2 of the main text of this report.

Details of the test models are listed in Table C.1. In the table d_w and t_w are the stringer dimensions, all stringer/longitudinal stiffeners were of simple flat bar section, N is the number of stringers uniformly spaced and σ_{yc} is the material compressive yield stress. The r/t, ℓ/r and z_s values range from 96 to 358, 0.4 to 1.56 and 4.2 to 55 respectively. The models represent both narrow and broad panelled cylinders. The modelling coefficient or bias is quantified by x_m, the ratio of observed test strength to that predicted by theory. The mean and cov of x_m are 1.003 and 10.1% respectively indicating good correlation between theory and experiment. Very similar results were obtained when the test data reported in [C.24] was also included in a similar study [C.7].

C.6.2 External Pressure & Combined Loading

For the determination of critical external pressure, the DnV modified equation[C.18] is used as given below.

The elastic buckling resistance for stringer stiffened cylinders (no end pressure applied, radial pressure only)

$$f_\Theta = \rho_p \psi_1 E \left[\frac{1}{12(1-\nu^2)} \left[\frac{t_s}{r}\right]^2 \frac{[\bar{m}+n^2]^2}{n^2} + \frac{\bar{m}^4}{n^2[\bar{m}^2+n^2]^2} + \frac{I_{ef}}{St_s^3} \frac{m^4}{z^2} \frac{\pi^4(1-\nu^2)}{n^2} \right]_{min} \qquad (C.45)$$

where E is Young's modulus

t_s the shell thickness

r the shell radius

ℓ length of cylinder (distance between rings)

S spacing of stringer (uniform)

ν Poisson's ratio

I_{ef} effective moment of inertia of stiffeners and attached width of effective plate

ρ_p imperfection factor

$z = \ell^2/rt \sqrt{1-\nu^2}$

$\bar{m} = \frac{m\pi r}{\ell}$, m the number of axial half-waves

n circumferential waves

ψ_1 correlation factor (taken unity).

The corresponding buckling mode has to be determined as the combination of (m,n) giving a minimum value for f_θ.

The characteristic circumferential strength resistance is given by

$$R_{\theta\kappa} = \Phi f_y, \qquad f_y = \text{yield stress} \tag{C.46}$$

where

$$\Phi = \frac{1}{\sqrt{1+\lambda^4}}$$

and

$$\lambda^2 = \frac{f_y}{f_\Theta}$$

(The above assumes that the rings have been proportioned to avoid their failure under radial pressure loading, before the stringer/shell failure.)

For the combined action of axial load and external pressure, a parabolic interaction is assumed, i.e.

$$\left[\frac{\sigma_{ZNd}}{R_{ZN\kappa}}\right]^2 + \left[\frac{\sigma_{\Theta d}}{R_{\Theta\kappa}}\right]^2 = 1 \tag{C.47}$$

where $R_{ZN\kappa}$ and $R_{\Theta\kappa}$ are the characteristic strengths for axial load and lateral pressure respectively.

and σ_{ZNd} and $\sigma_{\Theta d}$ are the stresses due to axial load and external pressure respectively.

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Table C1 : Data on Stringer Stiffened Cylinders Tested under Axial Compression [C21→C24]

Model No.	r (mm)	t (mm)	ℓ (mm)	d_w (mm)	t_w (mm)	N	E (kN/mm²)	σ_{yc} (KN/mm²)	σ_u/σ_{yc} Test	σ_u/σ_{yc} Pred.	X_m
UC1	160.7	0.81	64.3	6.48	0.81	20	210	0.324	0.82	0.778	1.054
UC2	160.0	0.81	177.6	6.48	0.81	40	210	0.320	1.03	0.894	1.152
UC3	161.4	0.81	179.1	12.96	0.81	20	210	0.322	0.76	0.787	0.966
UC4	159.8	0.81	177.4	12.96	0.81	30	213	0.320	0.96	0.911	1.053
UC5	159.6	0.81	177.1	12.96	0.81	40	203	0.338	1.04	0.953	1.092
UC6	226.6	0.81	251.6	6.48	0.81	40	211	0.311	0.65	0.744	0.873
UC7	226.5	0.81	251.4	12.96	0.81	40	211	0.311	0.86	0.840	1.023
UC8	289.2	0.81	321.0	12.96	0.81	20	201	0.309	0.51	0.494	1.033
UC9	288.2	0.81	319.8	12.96	0.81	40	211	0.340	0.66	0.662	0.997
B1	226.8	0.81	353.8	12.96	0.81	40	210	0.313	0.82	0.785	1.045
B2	226.8	0.81	353.8	12.96	0.81	20	210	0.324	0.54	0.587	0.919
B3	226.8	0.81	251.7	12.96	0.81	20	210	0.284	0.60	0.660	0.909
B4	226.8	0.81	176.9	12.96	0.81	20	210	0.281	0.61	0.692	0.881
B5	226.8	0.81	353.8	8.67	1.22	40	210	0.318	0.82	0.730	1.123
GU1	571.7	2.0	890.0	32.0	2.0	20	191	0.234	0.57	0.655	0.870
GU2	572.1	6.0	760.6	95.0	6.0	8	197	0.300	0.89	0.901	0.988
GU3	588.5	3.0	650.0	45.0	3.0	30	204	0.420	0.69	0.835	0.827
IC1	160.0	0.84	65.0	6.72	0.84	40	201	0.348	0.96	0.964	0.995
IC2	160.0	0.84	65.0	6.72	0.84	20	201	0.348	0.96	0.766	1.253
IC3	160.0	0.84	180.0	13.44	0.84	40	201	0.348	0.95	0.954	0.995
IC4	599.2	3.53	666.0	48.0	3.53	20	205	0.289	0.87	0.860	1.011

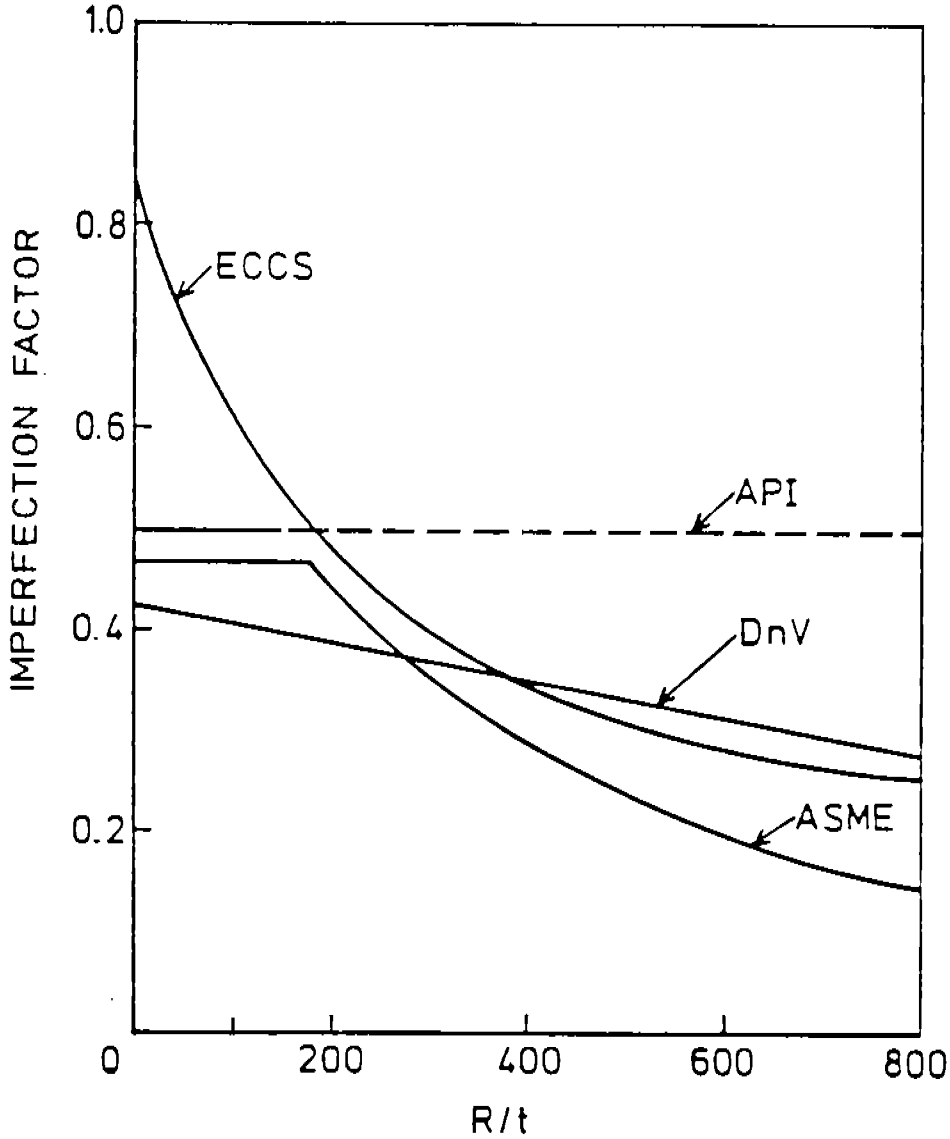

Fig.C.1 Imperfection Factors (Knock-down Factors) for Cylinders Subjected to Axial Compression (σ_y = 350 Nmm^2) [Ref.C.27]

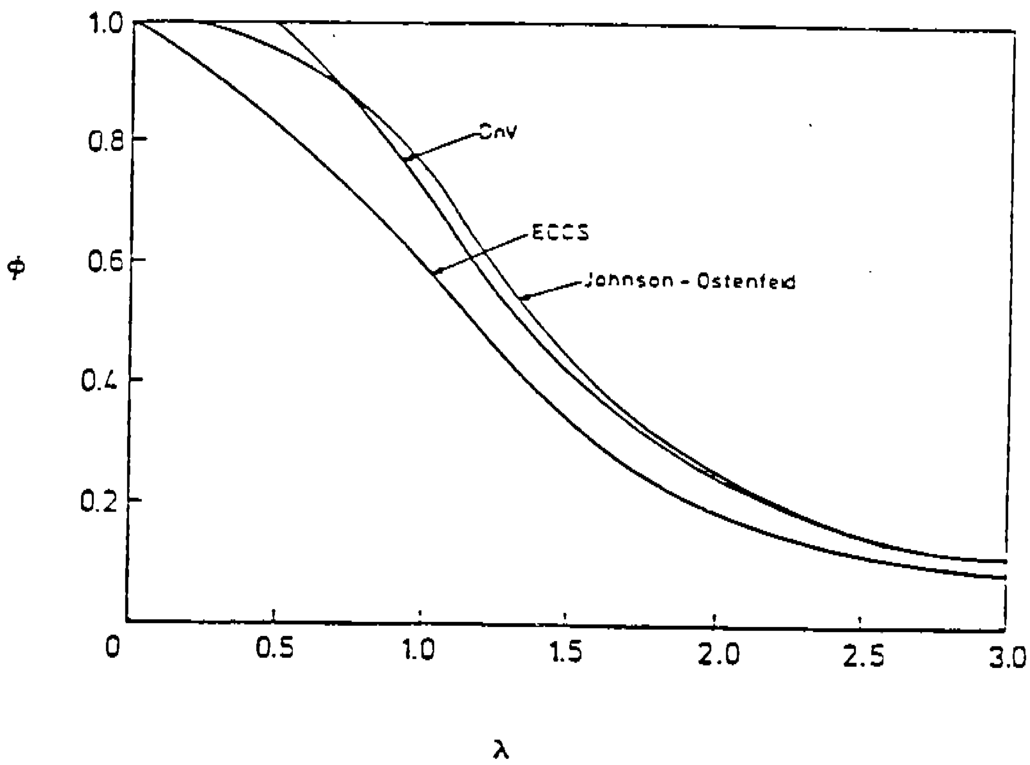

Fig.C.2 Variation of φ with λ [Ref.C.27]

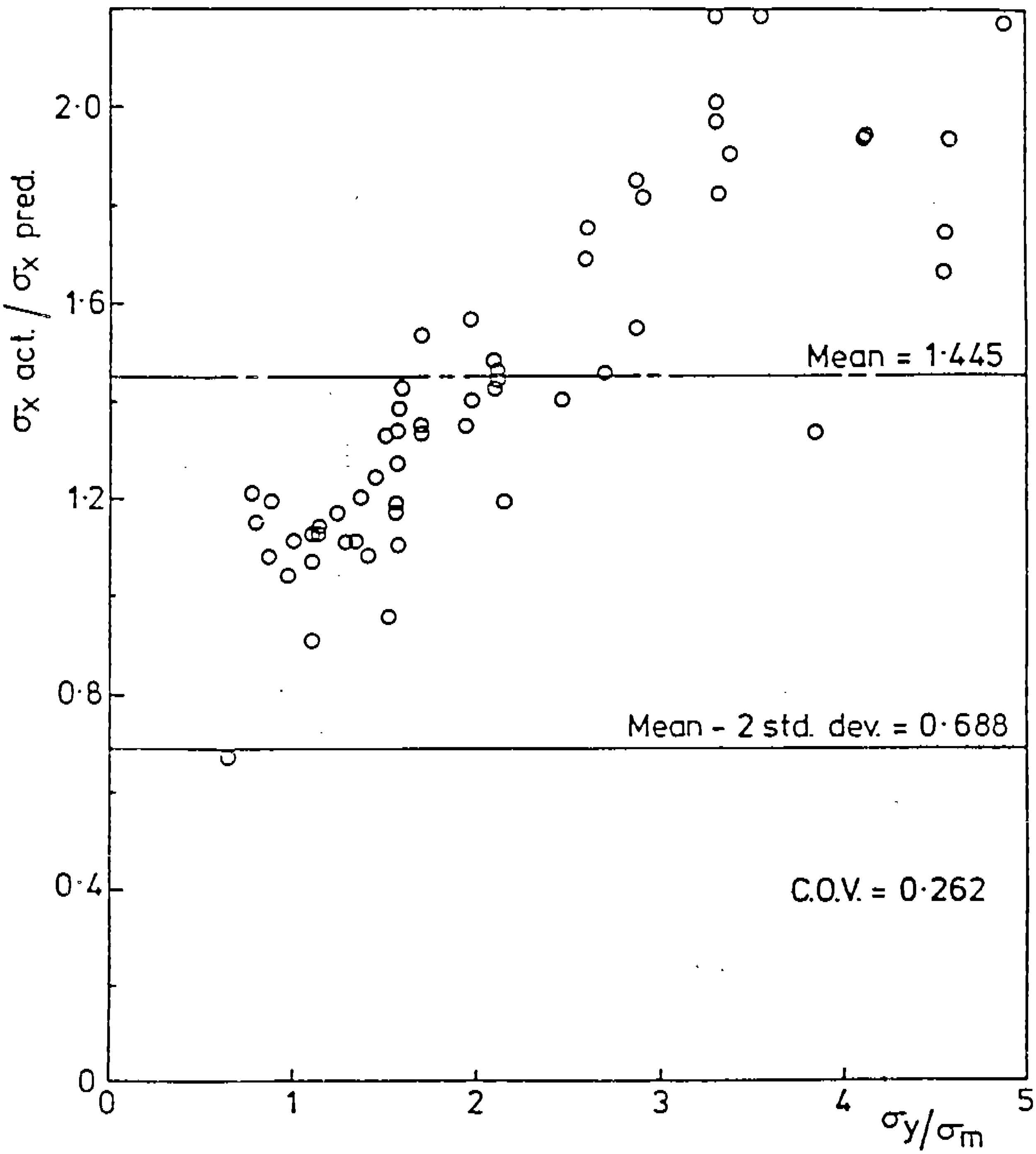

Fig.C.3 Comparison of Actual to Predicted Axial Compressive Strengths Using the DnV Rules [Ref.C.15]

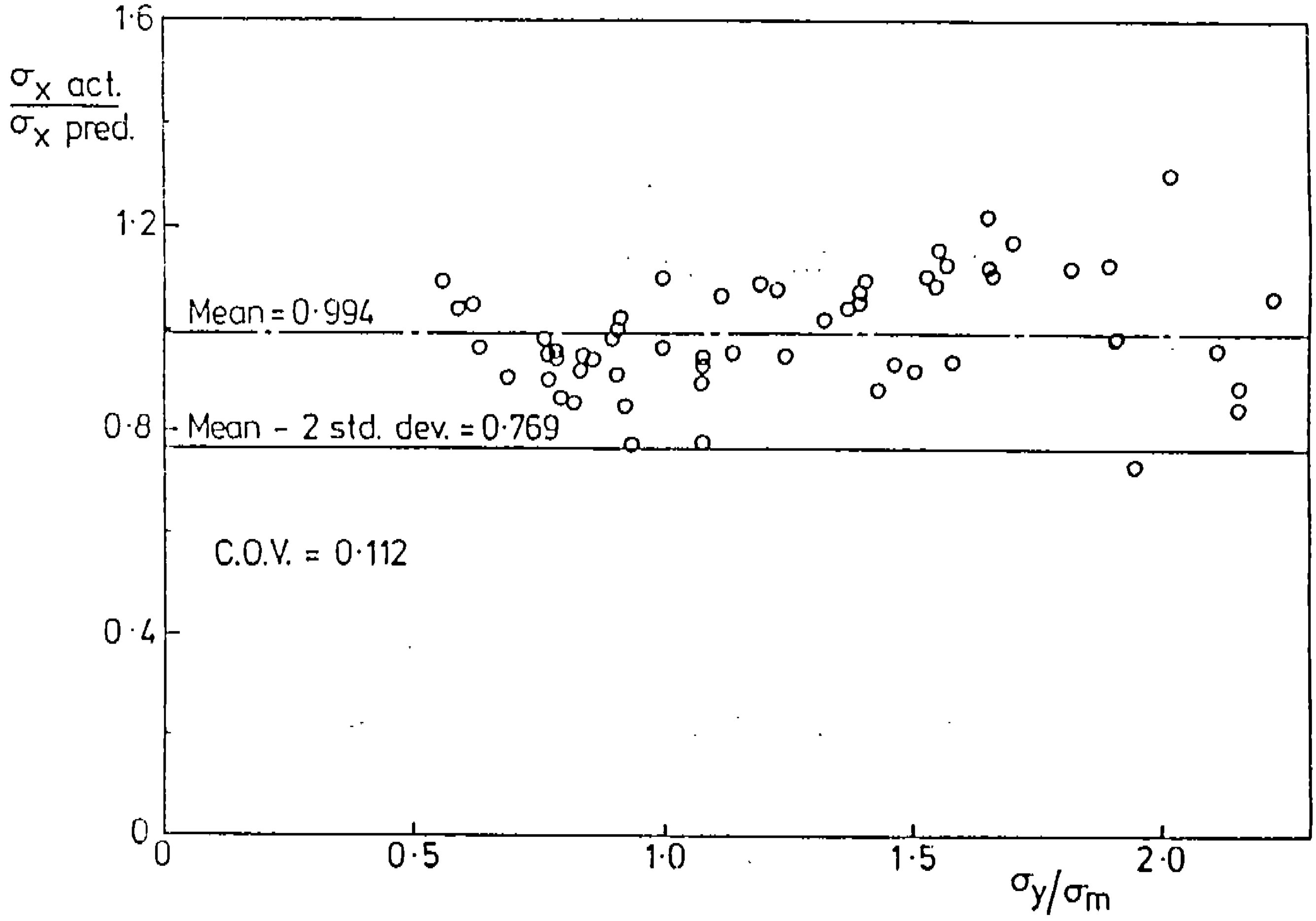

Fig.C.4 Comparison of Actual to Predicted Axial Strengths Using the Proposed Formulation [Ref.C.15]

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