SSC-356

FATIGUE PERFORMANCE UNDER MULTIAXIAL LOADING

SHIP STRUCTURE COMMITTEE

1990

Member Agencies:

United States Coast Guard
Naval Sea Systems Command
Maritime Administration
American Bureau of Shipping
Military Sealift Command

Ship Structure Committee

An Interagency Advisory Committee
Dedicated to the Improvement of Marine Structures

Address Correspondence to:

Secretary, Ship Structure Committee
U.S. Coast Guard (G-MTH)
2100 Second Street S.W.
Washington, D.C. 20593-0001
PH: (202) 267-0003
FAX: (202) 267-0025

December 3, 1990

SSC-356
SR-1323

FATIGUE PERFORMANCE UNDER MULTIAXIAL LOADING

We clearly recognize the need to consider fatigue in the design of marine structures. The analysis of complicated geometries can be difficult where variable multiaxial loads are applied. Fatigue testing of large fabricated details is very costly. This report provides a review and summary of methodologies used to predict the fatigue performance of structural details under multiaxial loading conditions. This investigation should provide a basis for further research leading to increased reliability of marine structures.

J. D. SIPES
Rear Admiral, U.S. Coast Guard
Chairman, Ship Structure Committee

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
SSC- 356		

4. Title and Subtitle Fatigue performance under multiaxial loading in marine structures.	5. Report Date January 1990
	6. Performing Organization Code
	8. Performing Organization Report No. 181-01
7. Author(s) Karl A. Stambaugh, Paul R. Van Mater, Jr., William H. Munse	
9. Performing Organization Name and Address Columbia Research Corporation 703 Giddings Avenue, Suite U-3 Annapolis, MD 21401	10. Work Unit No. (TRAIS) SR-1323
	11. Contract or Grant No. DTCG23-88-C-20021
12. Sponsoring Agency Name and Address U.S. Coast Guard Office of Merchant Technical and Hazardous Materials Washington, D.C. 20593	13. Type of Report and Period Covered 5/88 - 5/89
	14. Sponsoring Agency Code GM

15. Supplementary Notes

The USCG acts as the contracting office for the Ship Structure Committee

16. Abstract

The environmental loading and structural geometry associated with welded marine structures often produces multiaxial stresses. Large scale welded details have been used to characterize multiaxial fatigue response in marine structures; however, cost of these tests is often prohibitive. A review of multiaxial fatigue literature was performed to identify analytical techniques that might be used to predict multiaxial fatigue response. Various approaches are identified and summarized. Supporting literature is referenced. The reliability (bias and scatter) of the multiaxial approaches is presented where available. Various factors influencing multiaxial fatigue response are identified. A welded detail is used as an example to show how multiaxial fatigue life predictions are obtained from uniaxial fatigue test data. Finally, research is recommended to facilitate the technology transfer of multiaxial fatigue research to marine structures.

17. Key Words	18. Distribution Statement
Ship Structures Offshore Structures Fatigue Multiaxial	This document is available to the U.S. Public through the National Technical Information Service, Springfield, VA 22161

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	239	

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

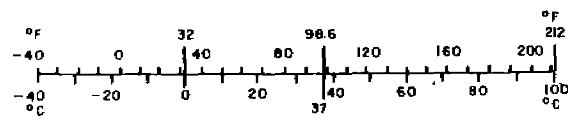


TABLE OF CONTENTS

Page

1.0 INTRODUCTION 1-1

1.1 PROJECT OBJECTIVE 1-1
1.2 SUMMARY 1-1

2.0 FATIGUE IN MARINE STRUCTURES 2-1

2.1 STRESS CHARACTERISTICS AND OTHER FACTORS AFFECTING FATIGUE IN MARINE STRUCTURES 2-1

2.1.1 Loading and Stress Characteristics in Marine Structures 2-1

2.1.1.1 Ship Details 2-1
2.1.1.2 Stresses in Offshore Structures 2-15

2.1.2 Factors Influencing Fatigue Response (General) 2-23

2.2 CUMULATIVE DAMAGE AND CRACK GROWTH APPROACHES FOR PREDICTING FATIGUE RESPONSE 2-29

2.2.1 Stress Concentration Factor (SCF) Approach in Offshore Structures 2-31

2.2.2 Proposed Approach for Fatigue Analysis of Ship Structural Details . . 2-36

2.2.3 Proposed Fracture Mechanics Approaches for Marine Structures . . . 2-37

2.3 IMPETUS FOR A MULTIAXIAL FATIGUE APPROACH 2-38

3.0 REVIEW OF LITERATURE ON MULTIAXIAL FATIGUE RESEARCH. 3-1

3.1 GENERAL MULTIAXIAL FATIGUE RESEARCH 3-1

TABLE OF CONTENTS

Page

3.1.2 Fatigue Crack Propagation Research . . 3-33

3.1.2.1 Observations of Mixed Mode Crack Growth 3-33

3.1.2.2 Prediction of Mixed Mode Crack Growth Rates 3-36

3.2 FATIGUE RESEARCH ON WELDED DETAILS 3-46

3.2.1 Fatigue Life Estimates 3-46

3.2.2 Fatigue Propagation in Welded Structures 3-52

4.0 APPLICATION OF MULTIAXIAL FATIGUE RESEARCH TO MARINE STRUCTURES 4-1

4.1 SHIP STRUCTURE APPLICATIONS 4-1

4.2 OFFSHORE STRUCTURE APPLICATIONS 4-6

4.3 FACTORS INFLUENCING MULTIAXIAL FATIGUE RESPONSE IN MARINE DETAILS 4-9

4.3.1 Multiaxial Stress Fields 4-31

4.3.2 Mean Stress 4-34

4.3.3 Stress Gradient 4-34

4.3.4 Residual Stresses 4-37

4.3.5 Corrosion 4-39

4.3.6 Geometry, Fabrication Treatment, Defects 4-42

4.3.7 Size - Thickness 4-42

4.3.8 Material Strength 4-43

TABLE OF CONTENTS

Page

5.0 EVALUATION OF MULTIAXIAL FATIGUE RELIABILITY 5-1

5.1 RELIABILITY FORMAT FOR EVALUATING MULTIAXIAL FATIGUE IN MARINE STRUCTURES 5-1

5.2 RELIABILITY OF MULTIAXIAL FATIGUE RESEARCH . . . 5-4

5.3 EVALUATION OF MULTIAXIAL FATIGUE RELIABILITY FOR MARINE STRUCTURES 5-7

6.0 CONCLUSIONS AND RECOMMENDATIONS 6-1

6.1 CONCLUSIONS 6-1

6.2 RECOMMENDED MULTIAXIAL FATIGUE RESEARCH 6-3

6.2.1 Define Spacial and Temporal Characteristics of Principal Stresses in Ship and Offshore Welded Details . . 6-3

6.2.2 Conduct Multiaxial Fatigue Tests On Marine Structural Details 6-4

6.2.3 Integrate Multiaxial Fatigue Research Into A Reliability Based Format 6-4

REFERENCES. R-1

APPENDIX A - BIBLIOGRAPHY ON MULTIAXIAL FATIGUE A-1

APPENDIX B - EXAMPLES OF MULTIAXIAL FATIGUE LIFE PREDICTIONS FOR A LATERALLY LOADED PLATE WITH A TRANSVERSE BUTT WELD B-1

LIST OF TABLES

2-1 Various Stresses in Ship Structural Details 2-2

2-2 Influence of Various Stresses on Fatigue in Offshore Structural Details 2-18

2-3 A Summary of Factors and Considerations Related to Fatigue in Welded Joints of Offshore Platforms 2-27

3-1 Summary of Multiaxial Fatigue Approaches for Predicting Fatigue Life Based on Cumulative Damage Approaches 3-31

3-2 Angle of Crack on Surface as Function of Strain State 3-34

3-3 List of the Conditions for Choosing the Proper Terms, Ling and Woo 3-42

3-4 Re-Analysis of Plate Girder Data by Moyer 3-49

3-5 Research on Butt Welds and Crusiform Fillet Welds by Lawrence 3-53

4-1 Summary of Methods for Estimating Fatigue Response to Complex Loading 4-2

4-2 General Configuration of Various Specimen Types . . . 4-10

4-3 Fatigue Strengths of Various Specimen Types 4-14

4-4 HY Steel Test Members Used to Investigate Multiaxial Fatigue Response 4-28

4-5 Effect of Mean Stress on Fatigue Strength of Various Members 4-35

5-1 Potential Improvements in Overall Reliability Using Multiaxial Fatigue Procedures 5-8

B-1 Summary of Fatigue Results B-3

LIST OF FIGURES

2-1 Global Stresses Due to Combined Vertical and Lateral Bending and Torsion 2-3

2-2 Cutout in a Tanker Web Frame 2-5

2-3 Predicted Stress Distribution in a Tanker Web Frame . 2-6

2-4 Constituent Loadings on Structural Web Frame 2-7

2-5 Predicted Stress Distribution in Clearance Cutouts in a Bulk Carrier 2-7

2-6 Stress Distribution in Clearance Cutouts for Fatigue Tests 2-8

2-7 Global Stresses Acting on Hatch Opening 2-9

2-8 Stress Distribution in a Containership Hatch Corner . 2-10

2-9 Fracture of the Forward Hatch Cutout on the High Speed Containership During the Second Winter Season . 2-12

2-10 Biaxial Stress Distribution in a Tanker CVK 2-13

2-11 Biaxial Stresses in Bottom Plate Transverse Butt Weld 2-14

2-12 Illustration of an Offshore Platform with Multibrace Connections 2-16

2-13 Typical Joint Geometry/Loading Configuration 2-17

2-14 Comparison of Stress Concentration Factors with Parametric Equations 2-19

2-15 Stress Concentration Factors for T and Non-Overlapping K, and TK Joints Under Various Types of In-Plane Loading 2-20

2-16 Stress Distribution Around Circumference of a T-Joint Under Various Loads 2-21

LIST OF FIGURES (continued)

2-20 Schematic Representation of Crack Growth in Steels 2-32

2-21 Illustration of Stress Concentration Factors and Hot-Spot Stress for Offshore Structures 2-35

3-1 Schematic Representation of High Cycle and Low Cycle Fatigue as a Function of Strain State and Strain Amplitude 3-3

3-2 Comparison of Predicted Combined Bending-Torsion Fatigue Limit with Tests Data for a Circumferentially Notched Round-Bar Specimen 3-5

3-3 Comparison of Life Predictions Based on the Tresca Equivalent Strain Criterion with SAE Test Data 3-10

3-4 Comparison of Life Predictions Based on the von Mises Equivalent Strain Criterion with In-Phase SAE Test Data, Taking Circumferential Notch Strain as Poisson's Ratio times Notch Bending Strain 3-10

3-5 Comparison of Life Predictions Based on the von Mises Equivalent Strain Criterion with In-Phase SAE Test Data, Using a Variable Poisson's Ratio 3-11

3-6 Actual versus Predicted Lives: Maximum Principal Strain Theory 3-11

3-7 Actual versus Predicted Lives: Effective Strain Theory 3-14

3-8 Actual versus Predicted Lives: Maximum Shear Strain Theory 3-14

3-9 Actual versus Predicted Lives: Lohr and Ellison Theory 3-15

3-10 Actual versus Predicted Lives: Kandil, Brown and Miller Theory 3-15

LIST OF FIGURES (continued)

3-13 Correlation of Test Data Including Mean Stress Effects (a) Lohr and Ellison Parameter and (b) Kandil, Brown and Miller Parameter 3-21

3-14 Stage I and Stage II Crack Growth Systems Under General Multiaxial Cyclic Strains 3-22

3-15 Uniaxial Stress-Strain Approximation for 1 Percent CR-Mo-V Steel 3-24

3-16 Plastic Work Per Cycle (Calculated) versus Life to Failure (Observed) 3-25

3-17 Plastic Work Per Cycle (Calculated with a Weighing Factor on Shear Work) versus Life to Failure (Observed) 3-22

3-18 In-Phase vs. Out-of-Phase Straining (Comparison) . . . 3-28

3-19 Elastic and Plastic Strain Energy Densities for a Uniaxial Cyclic Loading Case 3-29

3-20 The Predicted Strain Energy Density W^t versus the Number of Cycles to Failure, N , for Various Strain Ratios 3-30

3-21 Crack Propagation Plotted as Crack Length versus Crack Growth Rate to Compare Mode I, Mixed-Mode, and Mode II Growth 3-35

3-22 Initial Fracture Angle versus the Ratio of Stress Intensity Ranges K_{II} and K_I 3-37

3-23 Comparison Between the Proposed Method and Ling and Woo Method for $\alpha = -1.0$ 3-44

3-24 θ - β Relationship for Biaxiality Ratio, $\alpha = -1.0$ . . 3-45

3-25 θ - β Relationship for Biaxiality Ratio, $\alpha = -0.45$ . . 3-45

3-26 Details of Various Types of Stiffener 3-47

3-27 S/N Diagram for Maximum Bending Stress at Failure Section 3-47

3-28 S/N Diagram for Maximum Principal Tensile Stress at Failure Section 3-47

LIST OF FIGURES (continued)

3-29 Specimen Geometry of Tube-to-Plate Welds 3-51

3-30 Finite Element Mesh of Tube-to-Plate Welds 3-54

3-31 The Maximum "Worst Case" Local Principal Stress Range versus Cycles to Failure 3-55

3-32 The "Worst Case" Local von Mises' Effective Stress Range versus Cycles to Failure 3-56

3-33 Findley's Equivalent Nominal Shear Stress Range versus Cycles to Failure 3-57

3-34 Illustration of Estimating Stress Intensity by Finite Element Analysis 3-59

3-35 Predicted Crack Path a a Welded Tubular Intersection 3-61

3-36 Stress Intensity Factors of Weld Toe Surface Flaw (In-Plane Bending) 3-63

3-37 Stress Intensity Factors of Weld Toe Surface Flaw (Torsion) 3-63

3-38 Stress Intensity Factors of Weld Toe Surface Flaw (Axial Tension) 3-64

3-39 Stress Intensity Factors of Weld Toe Surface Flaw (Out-of-Plane Bending) 3-64

4-1 Tension Member and Connector with High Pre-Load . . . 4-8

4-2 S-N Curves for Type I Specimens of HY-80 Steel 4-16

4-3 S-N Curves for Type I Specimens of HY-80, HY-100 and HY-130/150 Steels 4-17

4-4 S-N Curves for Type II Specimens of HY-80, HY-100 and HY-130/150 Steels 4-16

4-5 S-N Curves for Type IVB Specimens of HY-80 Steel Plate and Type VIC of HY-80 Cast Steel at Stress Ratio R=0 4-18

LIST OF FIGURES (continued)

4-6 S-N Curves for Type IVB Specimens of 1-1/2" HY-130/150 Steel and Type XIVA Specimens of 3/4" A302B and 70B Steels at Stress Ratio R=0 4-18

4-7 S-N Curves for Type VA and VB Specimens of HY-80 and HY-130/150 Steels at Stress Ratio R=0 (Life at 10% increase in deflection) 4-19

4-8 S-N Curves for Type VA and VB Specimens of HY-80 Steel at Stress Ratio R=0 (Life at 100% increase in deflection) 4-19

4-9 S-N Curves for Type VA and VB Specimens of HY-80 Steel at Stress Ratio R=0 (Life at 100% increase in deflection) 4-20

4-10 S-N Curves for HY-80 Type VI Specimens Under Various Treatments at Stress Ratio R=0 (Life at 10% increase in deflection) 4-21

4-11 S-N Curves for HY-80 Type VII Specimens Under Various Treatments at Stress Ratio R=0 (Life at 100% increase in deflection) 4-21

4-12 S-N Curve for Type VII Spimens of HY-80 Steel at Stress Ratio R=-1 4-22

4-13 S-N Curves for Type VII Specimens of HY-80 Steel at Stress Ratios of R=0 and R=-1 4-23

4-14 S-N Curve for Type VIII Specimens of M. HY-80, HY-100 and HY-140 Steels at Stress Ratio R=0 4-24

4-15 S-N Curves for Type IX and X Specimens of HY-80 Steel at Stress Ratio R=0 4-25

4-16 S-N Curves for Type XI Specimens of HY-80 and HY-100 Steels 4-17

4-17 S-N Curves for Type XII and XIII Specimens of HY-80 and HY-100 Steels at Stress Ratio R=-1 4-25

4-18a S-N Curve for Type XIVB Notched Specimens of A302B and 70B Specimens at Stress Ratio R=0 4-26

4-18b Comparison of Plain and Notched Type XIV Specimens . 4-26

LIST OF FIGURES (continued)

4-19 Comparison of Axial Fatigue of Type III Specimens and Bending Fatigue of Type VI and VII Specimens at Stress Ratios R=0 and R=-1 4-27

4-20 Modified Goodman Diagram for HY-80 4-36

4-21 Comparison of Axial Fatigue of Type II Specimens and Bending Fatigue of Type VB Specimens of HY-80 Steel 4-38

4-22 S-N Curves for As Welded and Mechanically Peened Type VIB Specimens of HY-100 Steel at Stress Ratio R=0 4-40

4-23 Ratio of Fatigue Strength to Tensile Strength for Plain Plate Specimens of HY-80, HY-100 and HY-130/150 at Stress Ratio R=0 4-41

B-1 Typical 56 by 28 by 1-1/2" HY-80 Steel Butt Welded Plate Elements B-3

B-2 S-N Diagrams for HY-80 . . . . B-4

B-3 Strain-Life Curves Showing Total Elastic and Plastic Strain Components B-6

1.0 INTRODUCTION

Ships and offshore platforms are designed and built to endure a rugged environment typified by extreme events [1-1, 1-2] that are highly unpredictable. The random seaway is often characterized by amassed probabilities [1-2, 1-3]. This random seaway acts on marine structures that are designed to resist the random loads by welded structural geometries with intersecting structural members. More often than not the loadings are complex; that is, they produce two or three principal stresses that may be nonproportional or whose directions may change during a cycle of loading. Structural details such as intersections in longitudinal and transverse framing and adjacent butt welds in ships and nodal joints (K, T) in offshore structures are a few examples. Complex stress systems are also common at notches or geometric discontinuities. The variable loading of these details produces a fatigue response [1-4, 1-5] under such stress systems. The multiaxial fatigue response is generally unaccounted for on a detail level. Computational techniques for predicting the state of stress in structural elements have improved over recent decades because of the increased availability and capability of finite element computer programs. They also require substantial effort in predicting stresses on a detail level. The fatigue prediction methods used in practice for designing ship and offshore structures do not generally provide a comparable level of detail because they are based on the nominal stress field applied to boundaries of welded configurations.

1.1 PROJECT OBJECTIVE

The objective of this project is to survey and review methodologies for predicting multiaxial fatigue performance of structural details pertinent to marine structures. The research should ultimately lead to increased levels of reliability in designs and performance evaluations of existing structures and potentially minimize the need to conduct full-scale fatigue tests of ship and offshore structural elements.

1.2 SUMMARY

To accomplish the project objective a detailed literature survey was performed identifying over 600 references on structural fatigue under complex loading of various structural configurations. The database is presented in Appendix A. The references include multiaxial fatigue approaches used in structural systems such as nuclear reactors, aircraft, gas turbines, automobiles and heavy moving equipment.

The factors affecting fatigue in marine structures were reviewed including the stress characteristics in marine structural details. Also, existing fatigue design procedures were reviewed as a baseline to judge procedures associated with multiaxial fatigue approaches and to identify the extent multiaxial fatigue response is considered in the existing techniques. Essentially, the existing design approaches are based on structural components tests where complex stress distributions are internal to the applied nominal stress field boundary.

Several basic multiaxial fatigue approaches were identified including stress and strain-based approaches where an equivalent stress or strain is correlated to simple uniaxial test data, critical plane approaches where crack initiation is dependent on a critical stress or strain plane and strain energy approaches for both crack initiation and crack propagation. These approaches were reviewed and the engineering significance discussed in Section 3.0. The multiaxial fatigue approaches were compared to test data for typical structural details found in marine structures including a web frame cutout, center vertical keel (CVK), hatch corner and a butt weld for ship structures and K and T joints in offshore structures. These comparisons indicate that there are candidate approaches for predicting multiaxial fatigue response in marine structures. The amount of experimental verification has been extremely limited and there are a large number of variables that have not been quantified for marine environments.

To support the evaluation of multiaxial fatigue data, statistical characteristics (bias and scatter) were identified and compared to existing fatigue design approaches by Munse [1-6], the American Petroleum Institute (API) [1-7] and the United Kingdom Department of Energy (UK DOE) [1-8]. There are indications from the data that there are possible gains in reliability to be realized by applying multiaxial fatigue approaches. However, these gains must be evaluated in context of the overall system reliability and associated level of uncertainties and the additional effort required to apply multiaxial fatigue procedures as discussed in Section 5.0.

Finally, recommendations are provided to incorporate the multiaxial fatigue design approaches into a fatigue design procedure for marine structures including experimental work.

2.0 FATIGUE IN MARINE STRUCTURES

Fatigue in marine structures is a function of the loading characteristics and a given material's capability to perform without developing fatigue cracks in the operational environment. These factors will be examined and engineering techniques used to estimate the load and response, and to provide a proper perspective for reviewing and evaluating the applicability of multiaxial fatigue research.

2.1 STRESS CHARACTERISTICS AND OTHER FACTORS AFFECTING FATIGUE IN MARINE STRUCTURES

2.1.1 Loading and Stress Characteristics in Marine Structures

There are a number of multiaxial fatigue analysis procedures for predicting crack initiation and growth. Each method is applicable to a specific set of stress, strain, and strain energy loading characteristics. Each method must be evaluated by comparing the loading assumed in the multiaxial fatigue criteria with the actual loading on the structure of interest. Therefore, it is beneficial to review the state of stress in typical marine structural details, many of which are where fatigue cracks are known to initiate and propagate. The actual stress distribution in the structural details varies depending on the operational environment encountered; however, design generalizations are usually made to characterize basic stresses for ships and offshore structures.

2.1.1.1 Ship Details

Sources of fatigue loads on ships are summarized in Table 2-1. Global loads are distributed through the structure via plates, girders and panel stiffeners at welded structural details.

In the steel structure of a ship, the stress or strain cycles are generally caused by the seaway and by changing still water bending moments. These loads produce bending stress and shear stress in the ship's hull girder. These global stresses are illustrated in Figure 2-1 for a typical tanker where vertical bending, lateral bending and torsional bending stresses combine in the primary structural members. Superimposed on the hull girder loads are local stresses caused by changes in hydrostatic pressure and local loading from ships cargo or ballast. As shown in Figure 2-1, the stresses are plane stresses within a thin walled plate structural member. In a transverse plane, bending and shear stresses are caused by

TABLE 2-1
VARIOUS STRESSES IN SHIP STRUCTURAL DETAILS

Residual Stress - This includes the locked-in stresses in a structural element which occur during fabrication and assembly as well as the stresses induced by the support of the ship's own structure. The local stress is then the state of stress that exists in the light ship condition.

Initial Mean Stress - The still water bending stress (SWBS) may be induced by the addition of the deadweight which includes cargo, fuel and lube oil, potable water, stores, crew and effects, ballast and light ship bending stress.

Varying Mean Stress - This refers to stress changes due to fuel burn-off, consumption of consumables, and change in ballasting that affect the total displacement and attitude of the ship and, consequently, the stresses a structural element may experience.

Stress Due to Ship's Own Wave - This stress is induced by the pressure of the ship's own wave system. Methods are available to estimate the speed dependent bending moment contribution and thus the stress contribution from the ship's own wave system.

Diurnal Thermal Stresses - These stresses arise from the thermal expansion of the topside in the day and contraction during the night. The thermal stresses are also affected by the amount and location of sun exposure occurring during daylight hours.

Low Frequency Wave-Induced Stresses - These stresses are caused by the wave forces on the hull and the ship motions due to these forces. These cyclic stresses occur at the frequency of encounter of the ship with the wave system. The level of stress experienced is directly related to (although not directly proportional to) the significant wave height of the encountered seaway.

High-Frequency Wave-Induced Stresses - These stresses are induced by dynamic wave loads which act on the ship's structure. The most common are bottom slamming, shipping of water on deck, and flare impact. Dynamic loads produce whipping and springing elastic motions of the hull, typically at higher frequencies than the frequency of wave encounter. The impact-induced stresses will produce an initial spike in the stress records followed by high-frequency vibrations.

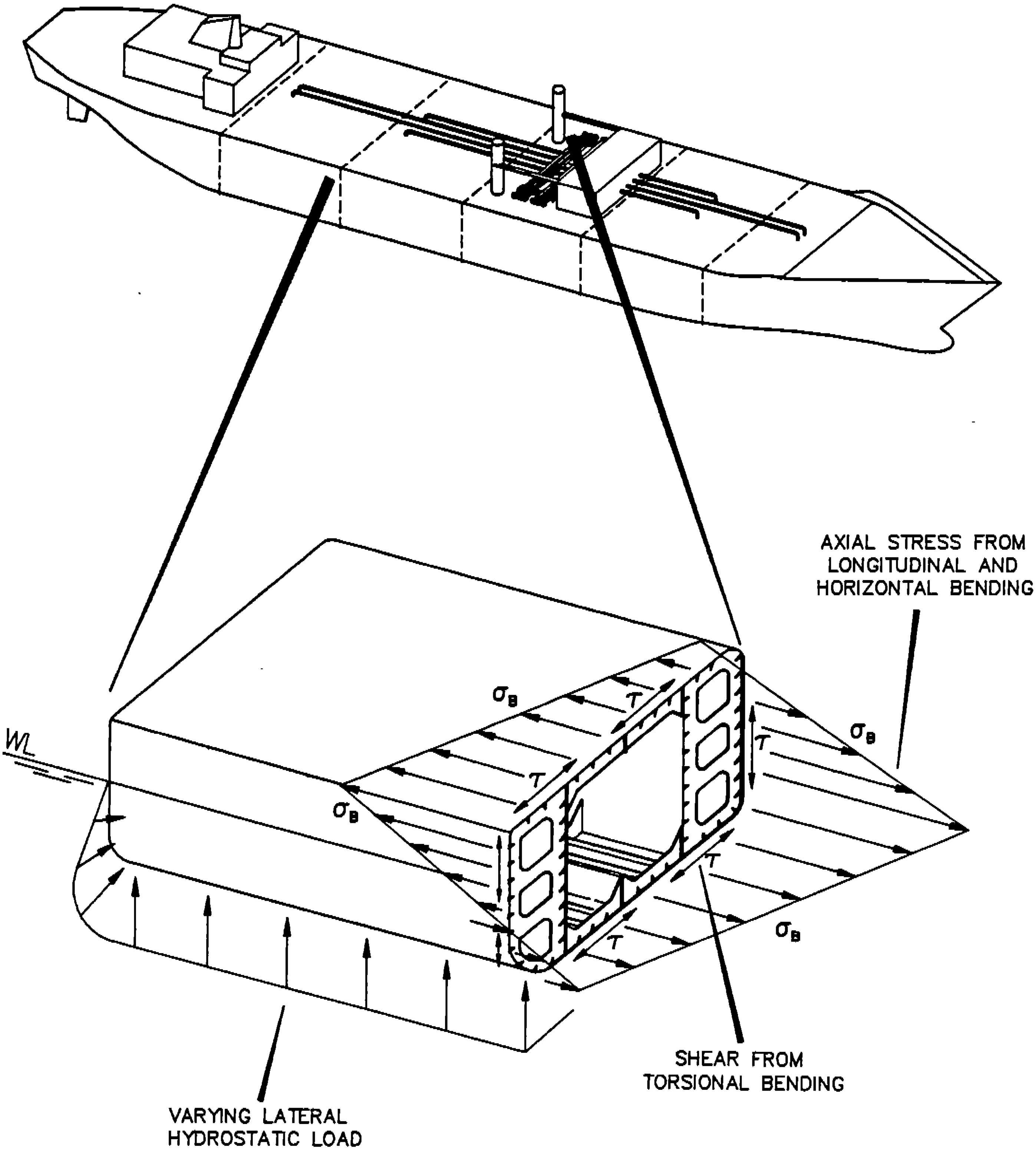


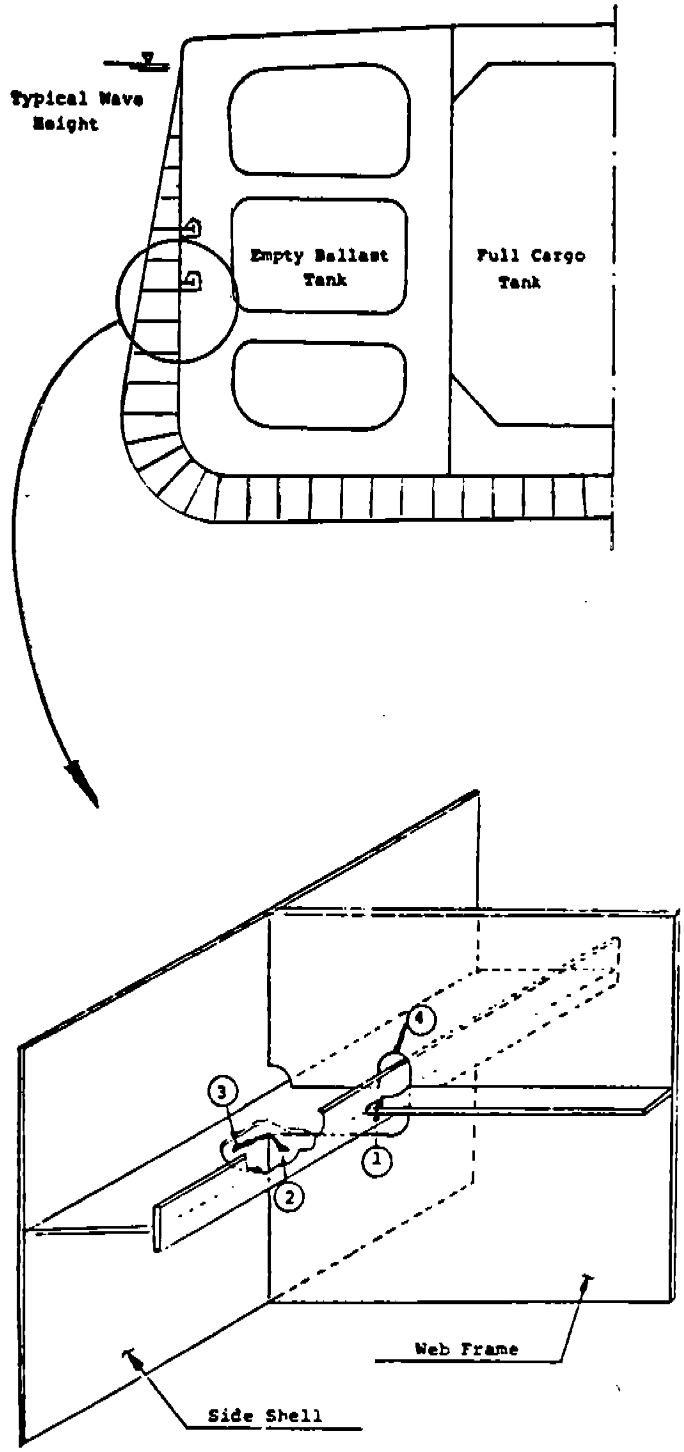
Figure 2-1

GLOBAL STRESSES DUE TO COMBINED VERTICAL AND LATERAL BENDING AND TORSION differences in hydrostatic pressure and internal cargo loads or ballast.

These stress patterns are transmitted to structural details. The stress patterns in elemental details vary; however, the planer character of stress remains because the geometry of a typical structure. These stress patterns will be investigated further for specific details where fatigue cracks are known to exist for a web frame cutout, a hatch cover detail, a CVK and a bottom plate butt weld.

The web frame cutout detail was chosen as an example detail because of frequent cracking found in the web plating. Figure 2-2 shows the location of the cutout under consideration. This detail is commonly found in tankers. The state of stresses was investigated by ABS [2-1] and is illustrated in Figure 2-3. The stress patterns in the cutout were investigated by Fricke [2-2] for a similar cutout in a bulk carrier. The constituent loadings are illustrated in Figure 2-4 and the resulting stress patterns are shown in Figure 2-5. This cutout detail was also fatigue tested by Munse [2-3]. The stress distribution was measured by Munse with strain gauges as shown in Figure 2-6. The load was applied as a concentrated load between cutouts, however, the stress patterns are consistent with those shown in the previous figures. The constituent stress characteristics in the vicinity of the cutout are biaxial. However, the stress is uniaxial on the extreme fiber of the cutout. This is because a free surface cannot support stress normal to the surface. Shear strains can exist near the free surface. Another important characteristic is the stress concentration that exists around the cutout. This concentration produces local stresses that can exceed yield while adjacent biaxial field stresses are well below yield. The behavior has been confirmed by stress predictions for this cutout conducted by ABS [2-1] and can result in strain-controlled fatigue cracking. Fricke [2-2] also describes this strain-controlled phenomenon for a similar structural detail.

Hatch corners often experience excessive stresses that lead to fatigue cracking. Compressive and tensile stresses result from longitudinal and lateral bending due to the hogging or sagging condition. Also, torsionally induced hull stresses are high and common in large hatch ships. Figure 2-7 illustrates a hatch opening through a deck with associated stresses present. These stresses concentrate around the corners of the hatch opening. Figure 2-8 illustrates stress



1 Initiation of Crack in Flat Bar Stiffener
2 Crack at Free Edge of Cut-out
3 Crack in Side Shell Plating
4 Crack at Radius of Cut-out

Figure 2-2. Cutout in a Tanker Web Frame

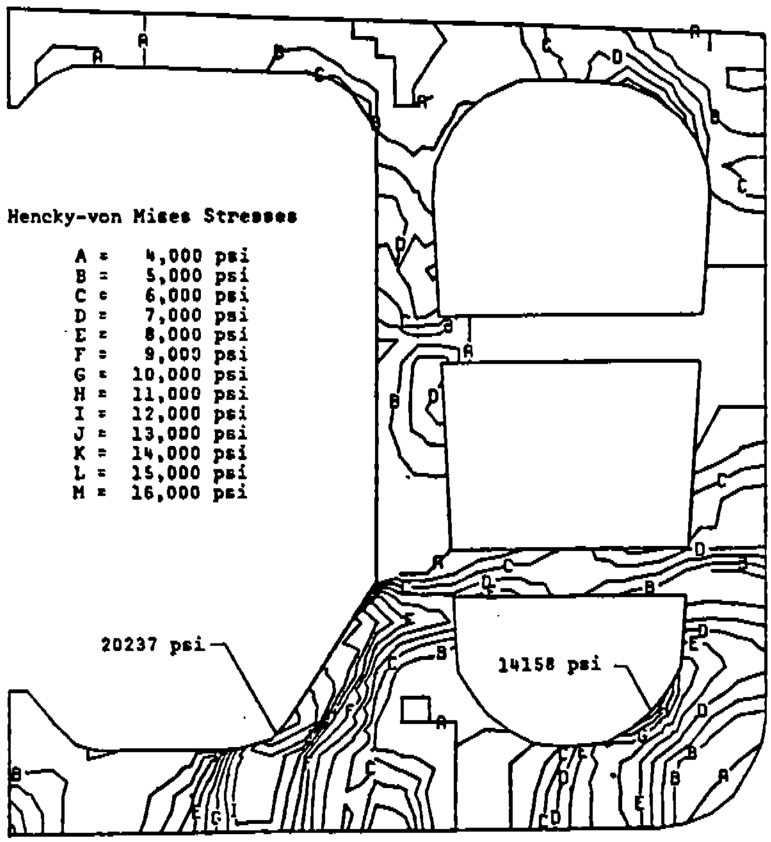


Figure 2-3. Predicted Stress Distribution in a Tanker Web Frame

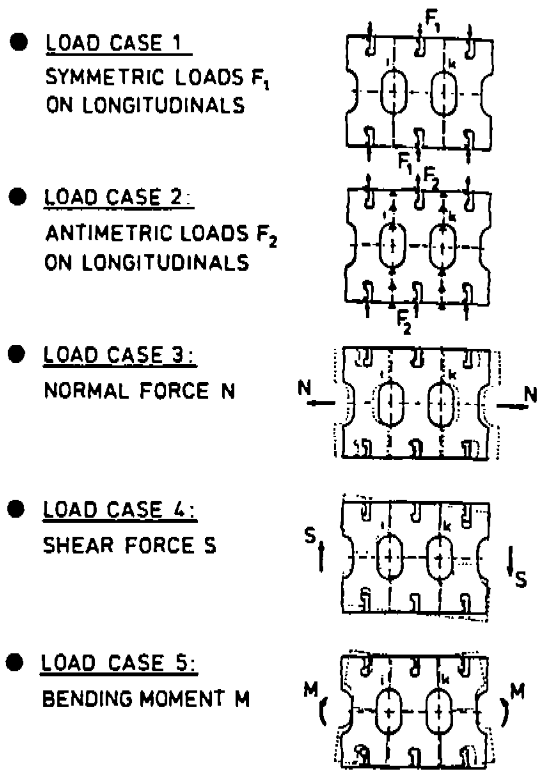


Figure 2-4. Constituent Loadings on Structural Web Frame

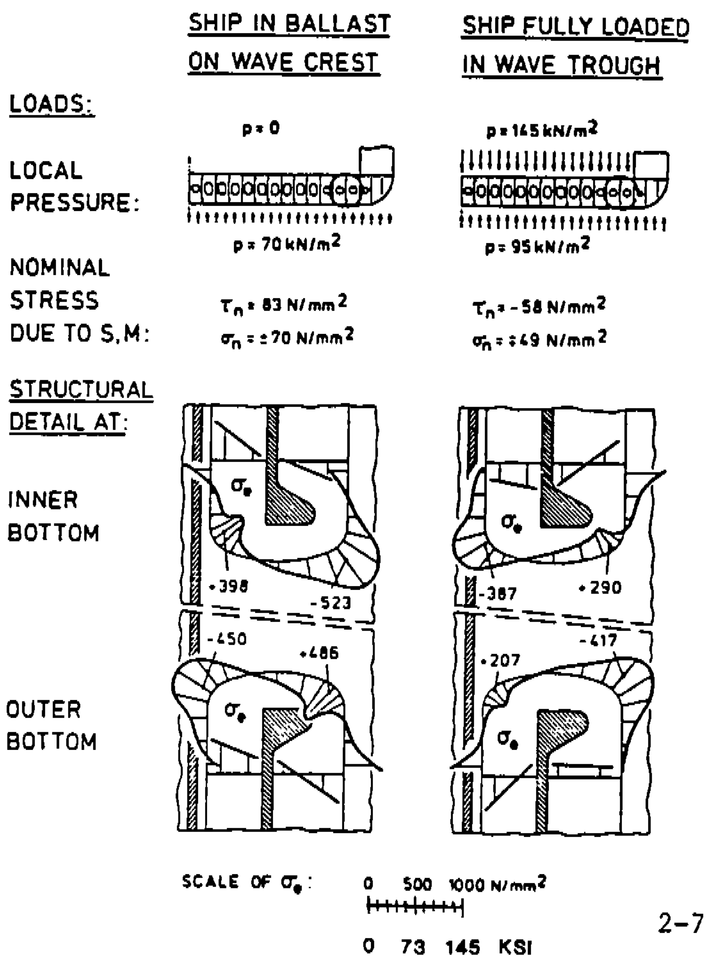
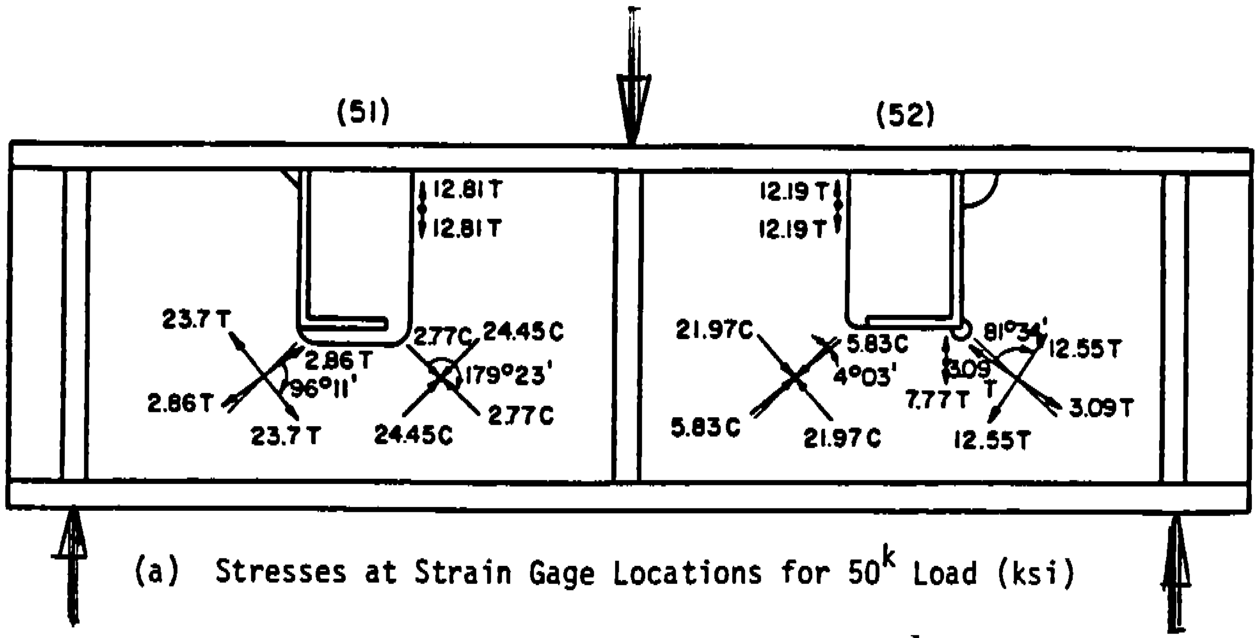
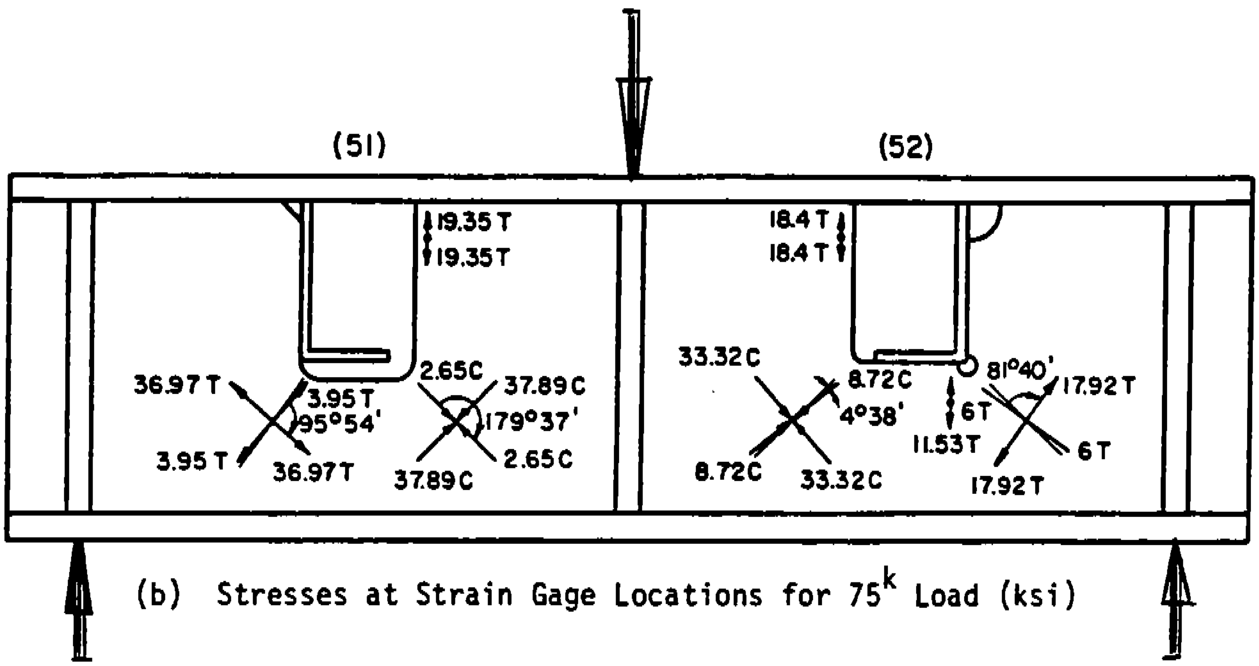


Figure 2-5. Predicted Stress Distribution in Clearance Cutouts in a Bulk Carrier



(a) Stresses at Strain Gage Locations for 50^k Load (ksi)



(b) Stresses at Strain Gage Locations for 75^k Load (ksi)

Figure 2-6. Stress Distribution in Clearance Cutouts for Fatigue Tests

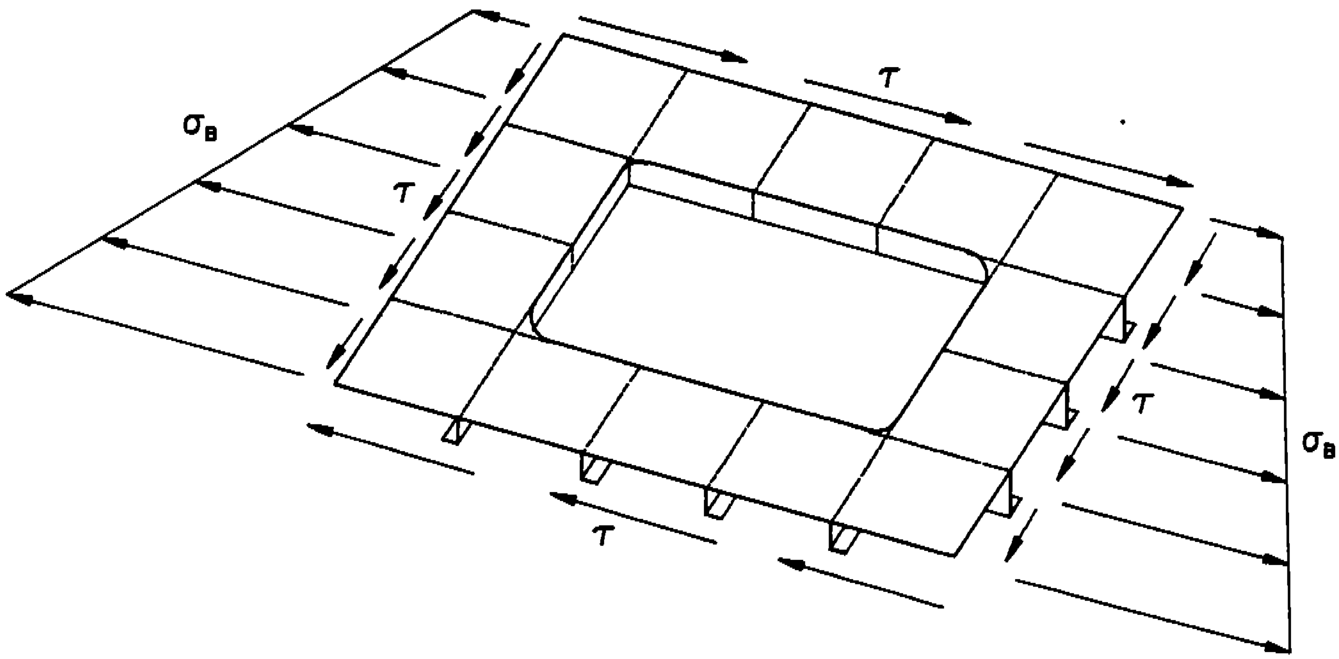
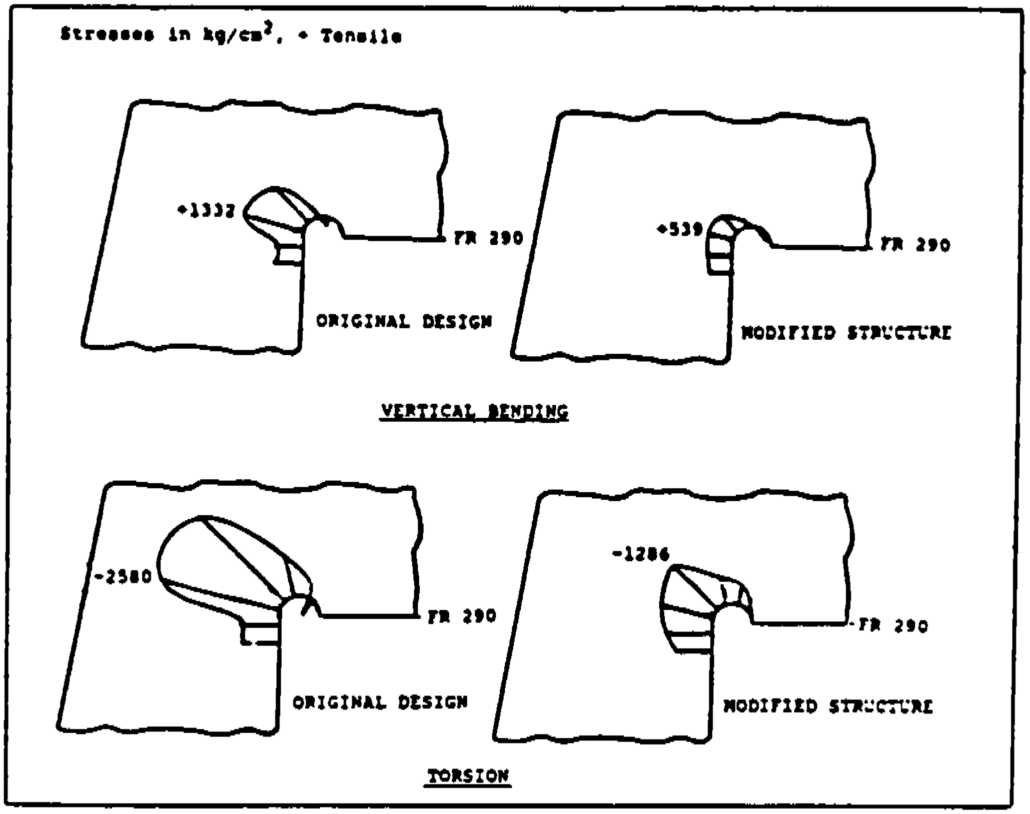
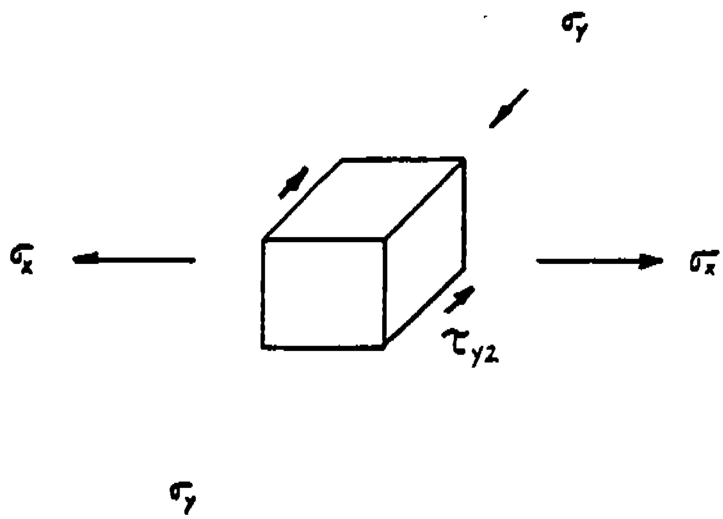


Figure 2-7

GLOBAL STRESSES ACTING ON HATCH OPENING



Hatch corner stresses from finite-element analysis



HATCH CORNER ELEMENT STRESSES

Figure 2-8. Stress Distribution in a Containership Hatch Corner patterns that were determined for a class of containerships from finite element analysis.

The stresses that act on an element of the deck plate surrounding a hatch corner are tensile, compression, and shear. The proportionality of principal stress is variable, depending on the status of mean stresses and encountered waves. Stresses in hatch corners were measured as part of the well known and well documented SL-7 instrumentation program [2-4]. Stresses in excess of yield were measured in the vicinity of the hatch corner. Fatigue cracking occurred early in the ships service life as documented by Stambaugh [2-5] and others [2-6] and shown in Figure 2-9. The fatigue crack growth is along a path perpendicular to the maximum principal stress field indicative of Mode I crack extension. Again, a local strain controlled condition exists where local stresses are in excess of yield and nominal stresses are below yield in the vicinity of a stress concentration.

Fatigue cracking has been reported in longitudinal CVK details where they bracket to a transverse bulkhead. Figure 2-10 illustrates the global stresses acting on the CVK due to bending and lateral hydrostatic pressure. As shown, axial bending stresses and vertical shear are present in the CVK. Local shearing is present due to the longitudinal deflection of the transverse bulkhead which acts on the CVK through the bracket. Figure 2-10 shows stresses on the detail level, and stresses present on an elemental level. Bending and shear act along the two principal axes forming a biaxial stress field in the CVK web.

Stambaugh [2-5] reported on fatigue cracking in transverse butt welds located in the bottom shell of a ship. The fatigue stresses in the butt weld result from primary, secondary and tertiary loading all with mean stress and residual stresses present as shown in Figure 2-11. The net response is a biaxial field with varying longitudinal principal stresses.

In summary, stresses in ship structure are uniaxial on the edge of plates and cutouts and biaxial in plate details. Principal stresses generally align with the major longitudinal axis of the hull or in the direction of major transverse frames. Secondary and tertiary stresses are produced by lateral hydrostatic loadings and induce biaxial components. However, the direction of principal stresses is highly variable. Phasing of the stresses is also variable. All variations are random at wave encounter frequencies. Long-term and short-term load distributions have been investigated

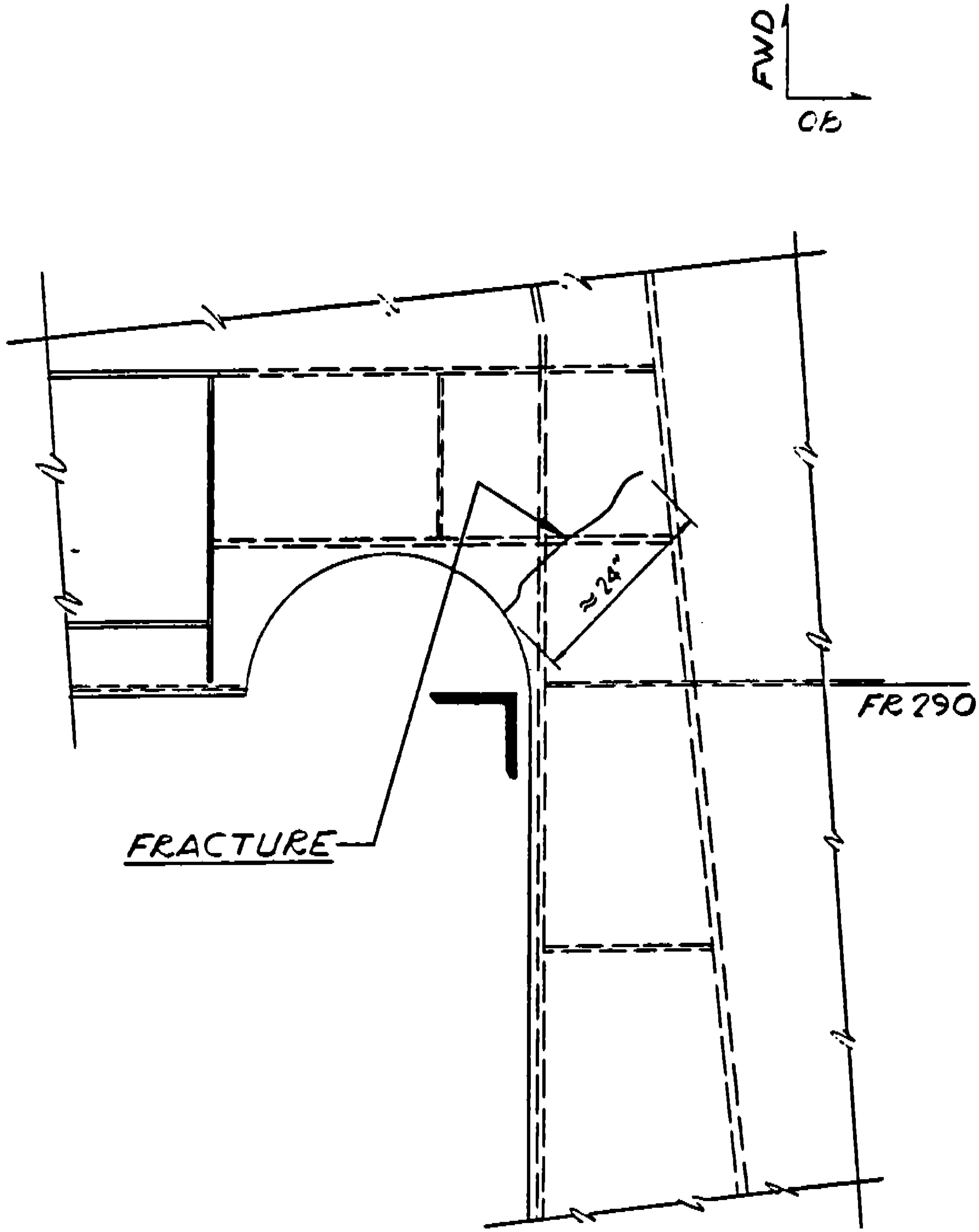


Figure 2-9. Fracture of the Forward Hatch Cutout on the High Speed Containership During the Second Winter Season

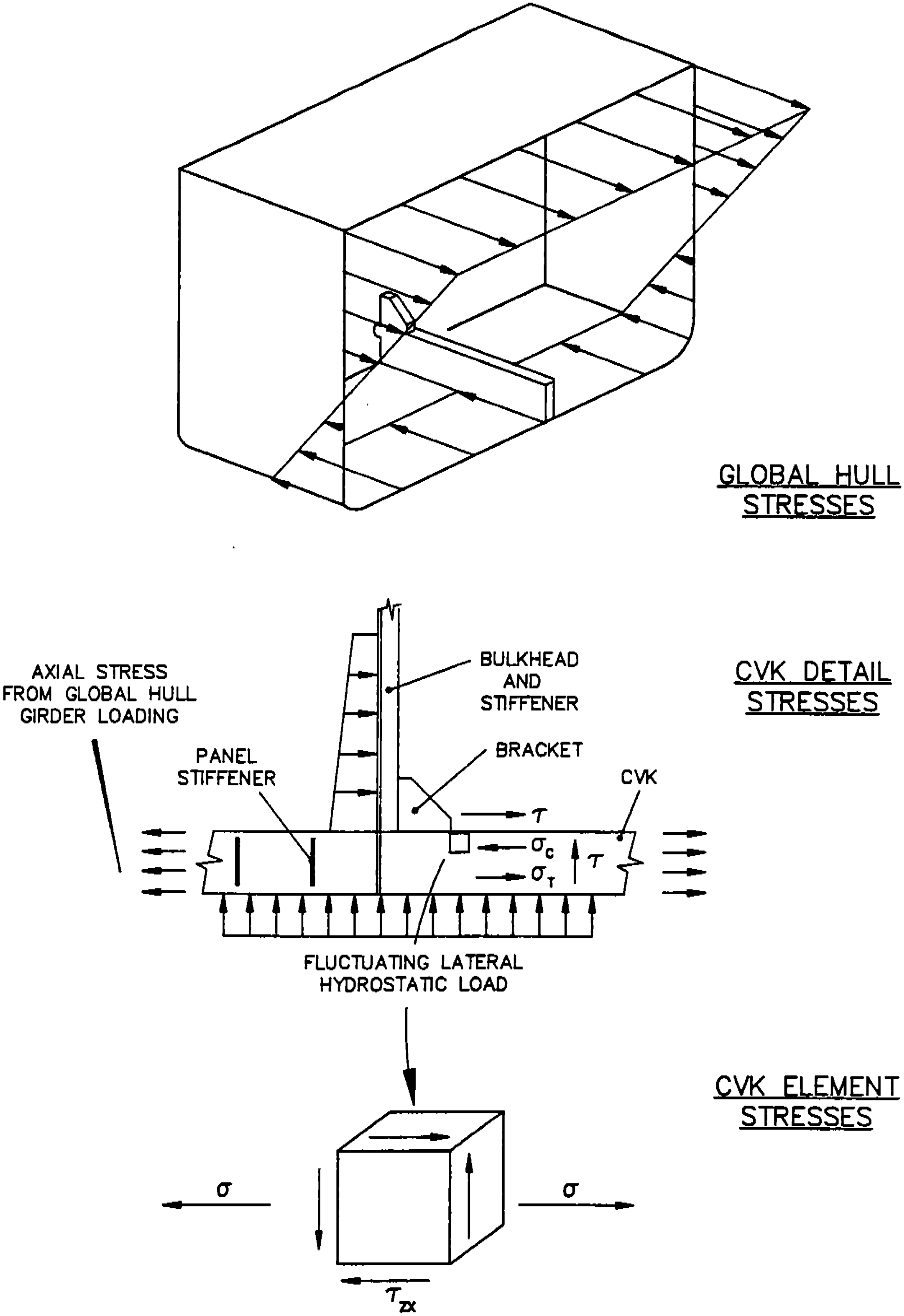


Figure 2-10

BI-AXIAL STRESS DISTRIBUTION IN A TANKER CVK

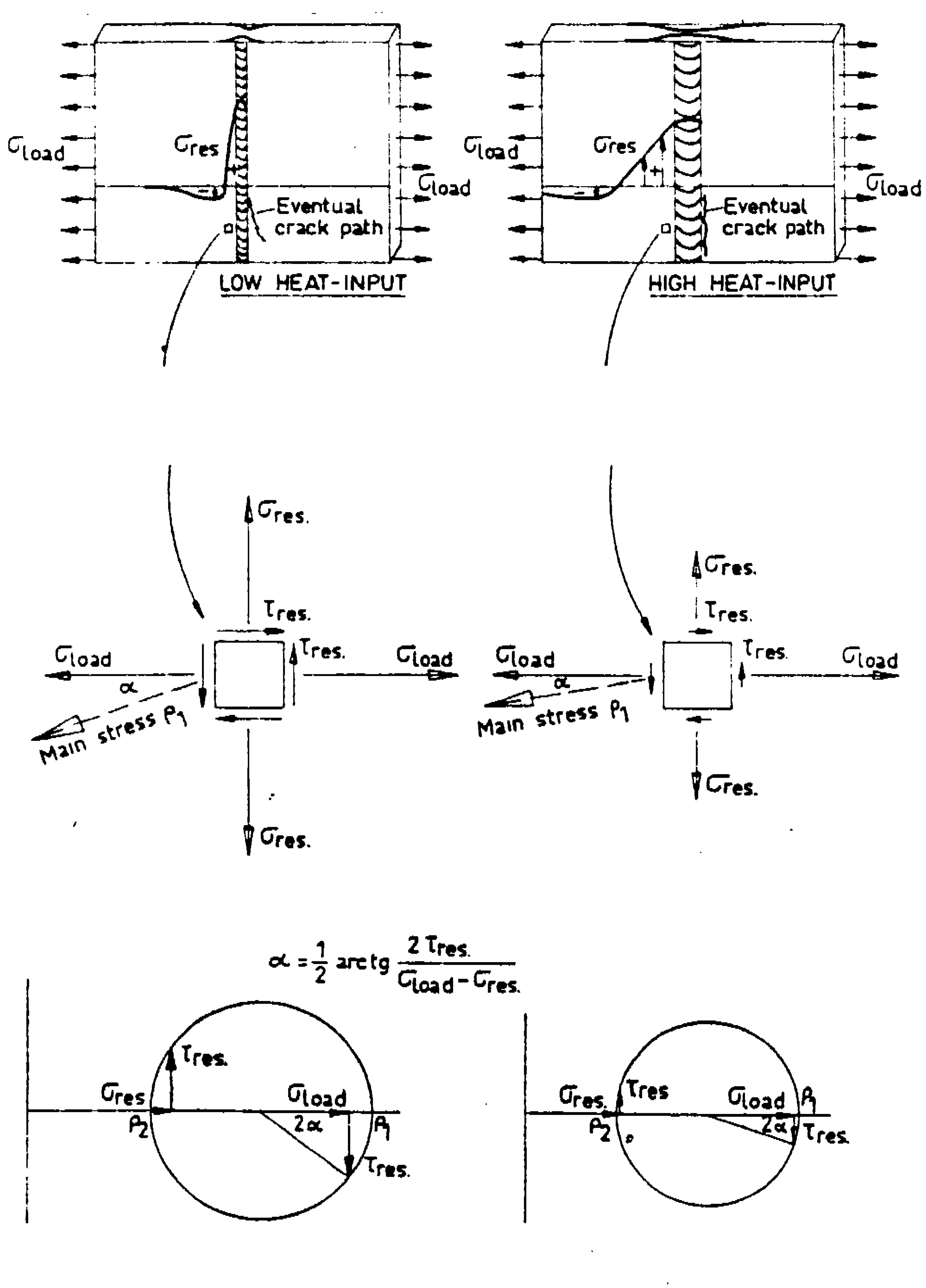


Figure 2-11. Biaxial Stresses in Bottom Plate Transverse Butt Weld

extensively with little agreement between researchers as to the exact type of distribution that will account for random load effects. Mean stresses should also be taken into account. Also, large residual stresses are present from the weld process, in many instances on the order of magnitude of yield, and effect the mean stress level.

2.1.1.2 Stresses in Offshore Structures

Stress analyses of tubular offshore structures (see Figure 2-12) begin with a global analysis of the jacket as shown in Figure 2-13 and are generally made through space frame analyses. The nature of member end loads changes with direction of wave forces, as shown. Other loads on offshore structures are summarized in Table 2-2. The common joint stress analysis procedure utilizes parametric equations [2-7, 2-8, 2-9] to estimate the hot spot stresses at selected locations. The parametric equations are based on either finite element analysis or on laboratory testing. Currently, parametric formulations are available for X, T, K, and TK joints subjected to simple loadings of axial, in-plane bending and out of plane bending applied separately to the joints as shown in Figures 2-14 and 2-15. The principal stress distribution of a simple T joint is illustrated in Figure 2-16. Stress distributions around the weld toe of a Y joint are shown in Figure 2-17. The stress state is biaxial on the surface and triaxial through the thickness with significant shear stresses developing across the thickness of the chord and the brace at the intersection. Stress "hot spots" occur at the saddle point (±90° to chord axis) of the intersection where stress concentrations of greater than six times the nominal axial stress in the brace may occur depending on the geometry of the joint. Peak stresses which are higher than those at the intersection occur at the toe of the weld. Principal stresses at the saddle points lie within 16° to 30° of the normal to the weld line. At crown points (0°, 180° to chord axis) the angle to the normal is about 8°. Stress decays rapidly away from the weld toe.

The state of stress in tubular joints is biaxial on the surface. Stresses will generally be compressive on one side of the chord brace intersection and tensile on the other depending on direction of applied in-plane bending moment. Stress hot spots will be located near or at crown points of the intersection. Stress concentrations vary from four to seven times the nominal bending stress in the brace. Maximum principal stresses at the hot spots generally lie within 8° from normal to the weld line; however, the magnitude and

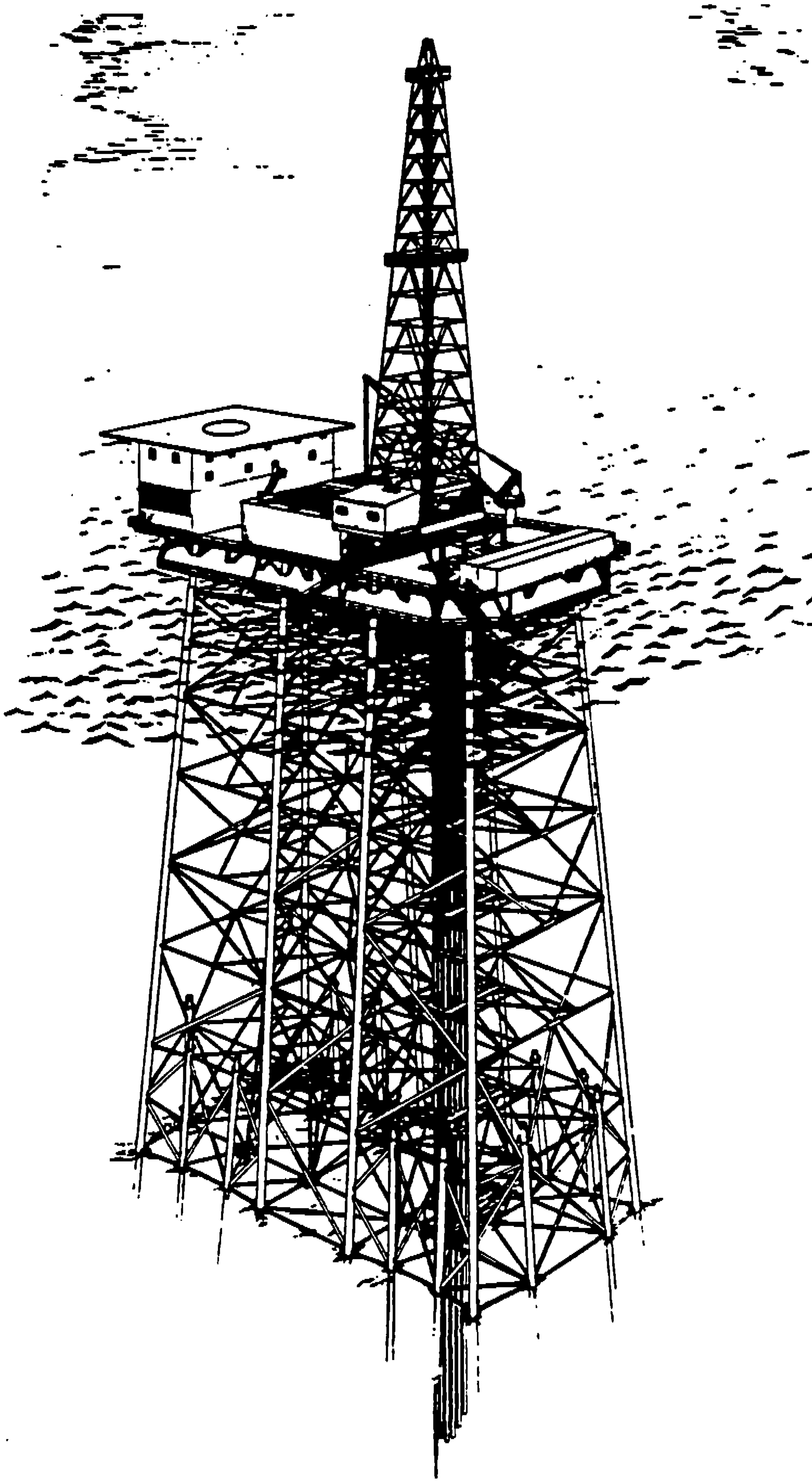


Figure 2-12. Illustration of an Offshore Platform
with Multibrace Connections

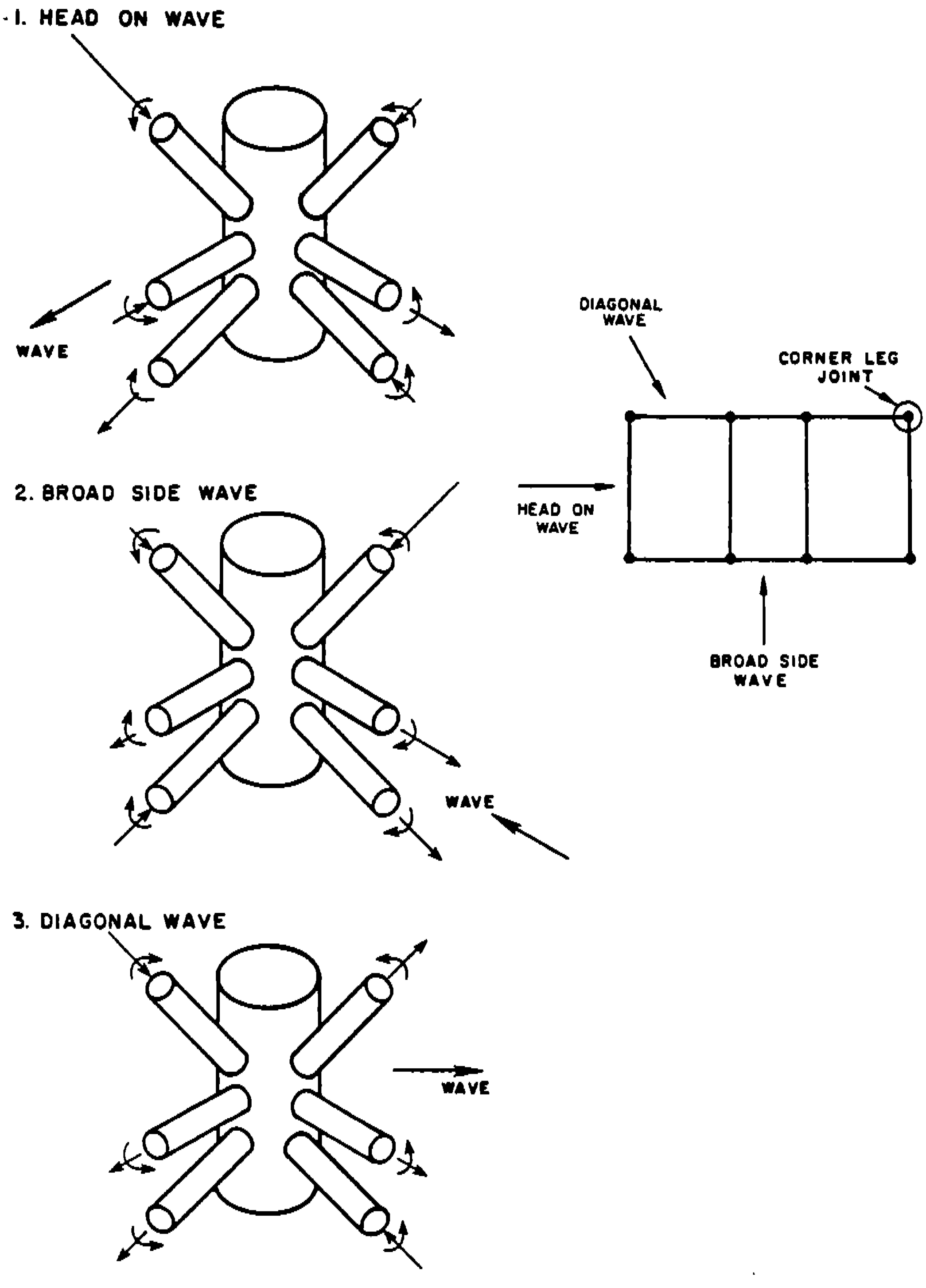


Figure 2-13. Typical Joint Geometry/Loading Configuration

TABLE 2-2

INFLUENCE OF VARIOUS STRESSES ON FATIGUE IN OFFSHORE STRUCTURAL DETAILS

TYPE OF LOADING	CONTRIBUTION
Wave	Fatigue Crack Initiation and Propagation
Current and Wind (Static and Quasi-Static)	Mean Stress (R ratio) in Crack Initiation and Propagation
Current and Wind (Dynamic)	Fatigue Crack Initiation and Propagation
Deadweight	Mean Stress (R ratio) in Crack Initiation and Propagation
Fabrication	Mean Stress (R ratio) in Crack Initiation and Propagation Fatigue Crack Initiation Initial Flaw Size in Crack Propagation
Launching/Installation	Crack Initiation
Live Loads	Fatigue Crack Initiation and Propagation
Transportation	Low-Cycle Fatigue Crack Initiation and Propagation
Buoyant and Hydrostatic	Mean Stress (R ratio) in Crack Initiation and Propagation Crack Initiation
Foundation Movement/ Earthquake	Low-Cycle Crack Initiation
Floating Ice Impacts	Crack Initiation

JOINT AND LOADING CONFIGURATION	BRANCH ID	FINITE ELEMENT ANALYSIS CHORD (OUTSIDE SURFACE)					PARAMETRI... KUANG	
		0°	45°	90°	135°	180°	CHORD	WORDSWORTH CHORD
	A	-2.01	-3.91	-6.01	-3.91	-2.01	4.67	6.16^s 2.91^c
	A	-1.88	-1.58	0	1.58	1.88	1.58	2.03^c
	B	1.12	2.45	2.78	2.73	1.64	2.80	2.36^s 2.50^c
	C	-1.12	-2.45	-2.78	-2.73	-1.64	-2.80	2.36^s 2.50^c
	B	-2.23	-1.83	0.22	1.60	2.61	2.53	2.68
	C	0.07	0.17	0.30	0.27	1.03		
	A	-1.48	-0.23	0	0.23	1.48		
	B	1.36	2.84	3.28	2.74	3.16	3.51	3.52
	C	-1.36	-2.84	-3.28	-2.74	-3.16	3.51	3.52

NOTES: s = SADDLE POINT (90° POSITION)
c = CROWN POINT (0° or 180° POSITION)

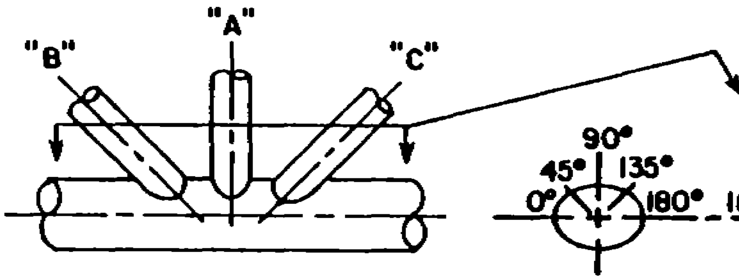


Figure 2-14. Comparison of Stress Concentration Factors with Parametric Eq

JOINT AND LOADING CONFIGURATION (CASE NO.)	BRANCH ID	HOT SPOT STRESS ON CHORD(KSI) 0°	45°	90°	135°	180°
(T-1)	A	-2.01	-3.91	-6.01	-3.91	-2.01
(K-2)	B	-2.88	-3.58	-4.01	-4.89	-5.16
	C	-1.75	-1.13	-1.28	-2.33	-3.54
(K-3)	B	-1.75	-1.13	-1.28	-2.23	-3.54
	C	-2.88	-3.58	-4.01	-4.89	-5.16
(K-11)	B	+1.12	+2.45	+2.73	+2.65	+1.64
	C	-1.12	-2.45	-2.73	-2.65	-1.64
(K-12)	B	-4.63	-4.71	-5.29	-7.12	-8.70
	C	-4.63	-4.71	-5.29	-7.12	-8.70
(T-4)	A	-1.88	-1.56	0	1.56	1.88
(K-5)	B	2.23	1.83	-0.22	-1.60	-2.61
	C	-0.07	-0.17	-0.30	-0.27	-0.03
(K-6)	B	+0.07	+0.17	+0.30	+0.27	+1.03
	C	-2.23	-1.83	+0.22	+1.60	+2.61
(K-13)	B	-2.16	-1.66	+0.52	+1.87	+3.64
	C	-2.16	-1.66	+0.52	+1.87	+3.64
(K-14)	B	+2.30	+2.00	+0.08	-1.33	-1.58
	C	-2.30	-2.00	-0.08	+1.33	+1.58
(K-15)	B	+3.42	+4.45	+2.81	+1.32	+0.06
	C	-3.42	-4.45	-2.81	-1.32	-0.06
(K-16)	B	-2.33	-2.71	-5.21	-8.45	-10.28
	C	-6.93	-6.71	-5.37	-5.79	-7.12

JOINT AND LOADING CONFIGURATION (CASE NO.)	BRANCH ID	HOT SPOT STRESS ON CHORD (KSI) 0°	45°	90°	135°	180°
(TK-1)	A	-2.92	-3.51	-4.51	-3.51	-2.92
	B	-0.76	-0.48	-1.18	-1.61	-1.50
	C	-0.76	-0.48	-1.18	-1.61	-1.50
(TK-2)	A	-2.16	-2.02	-1.32	-2.26	-3.63
	B	-3.07	-3.72	-3.98	-4.73	-5.57
	C	-1.71	-0.89	-0.69	-2.00	-2.40
(TK-3)	A	-3.63	-2.26	-1.32	-2.02	-2.16
	B	-1.71	-0.89	-0.69	-2.00	-2.40
	C	-3.07	-3.72	-3.98	-4.73	-5.57
(TK-11)	A	-1.48	-0.23	0	0.23	1.48
	B	1.36	2.84	3.28	2.74	3.16
	C	-1.36	-2.84	-3.28	-2.74	-3.16
(TK-12)	A	-5.79	-4.28	-2.64	-4.28	-5.79
	B	-4.78	-4.61	-4.67	-6.73	-7.97
	C	-4.78	-4.61	-4.67	-6.73	-7.97
(TK-4)	A	-1.97	-1.30	0	1.30	1.97
	B	-0.05	-0.01	0.13	0.29	0.15
	C	0.05	0.01	-0.13	-0.29	-0.15
(TK-5)	A	-0.14	-0.30	-0.56	-0.53	-1.03
	B	2.28	1.85	-0.65	-1.50	-2.59
	C	-0.06	-0.12	-0.28	-0.24	-0.20
(TK-6)	A	+1.03	+0.53	+0.56	+0.30	+0.14
	B	+0.06	+0.12	+0.28	+0.24	+0.20
	C	-2.28	-1.85	+0.65	+1.50	+2.59
(TK-13)	A	1.17	0.83	1.12	0.83	1.17
	B	-2.22	-1.73	0.93	1.74	2.79
	B	-2.22	-1.73	0.93	1.74	2.79
(TK-14)	A	0.89	+0.23	0	-0.23	-0.89
	B	2.34	1.97	-0.37	-1.26	-2.39
	C	-2.34	-1.97	0.37	1.26	2.39
(TK-15)	A	-0.59	0	0	0	0.59
	B	3.70	4.81	2.91	1.48	0.77
	C	-3.70	-4.81	-2.91	-1.48	-0.77
(TK-16)	A	-4.90	-4.05	-2.64	-4.51	-6.68
	B	-2.44	-2.64	-5.04	-7.99	-10.36
	C	-7.12	-6.58	-4.30	-5.47	-5.58

Figure 2-15. Stress Concentration Factors for T and Non-Overlapping K, and TK Joints Under Various Types of In-Plane Loading

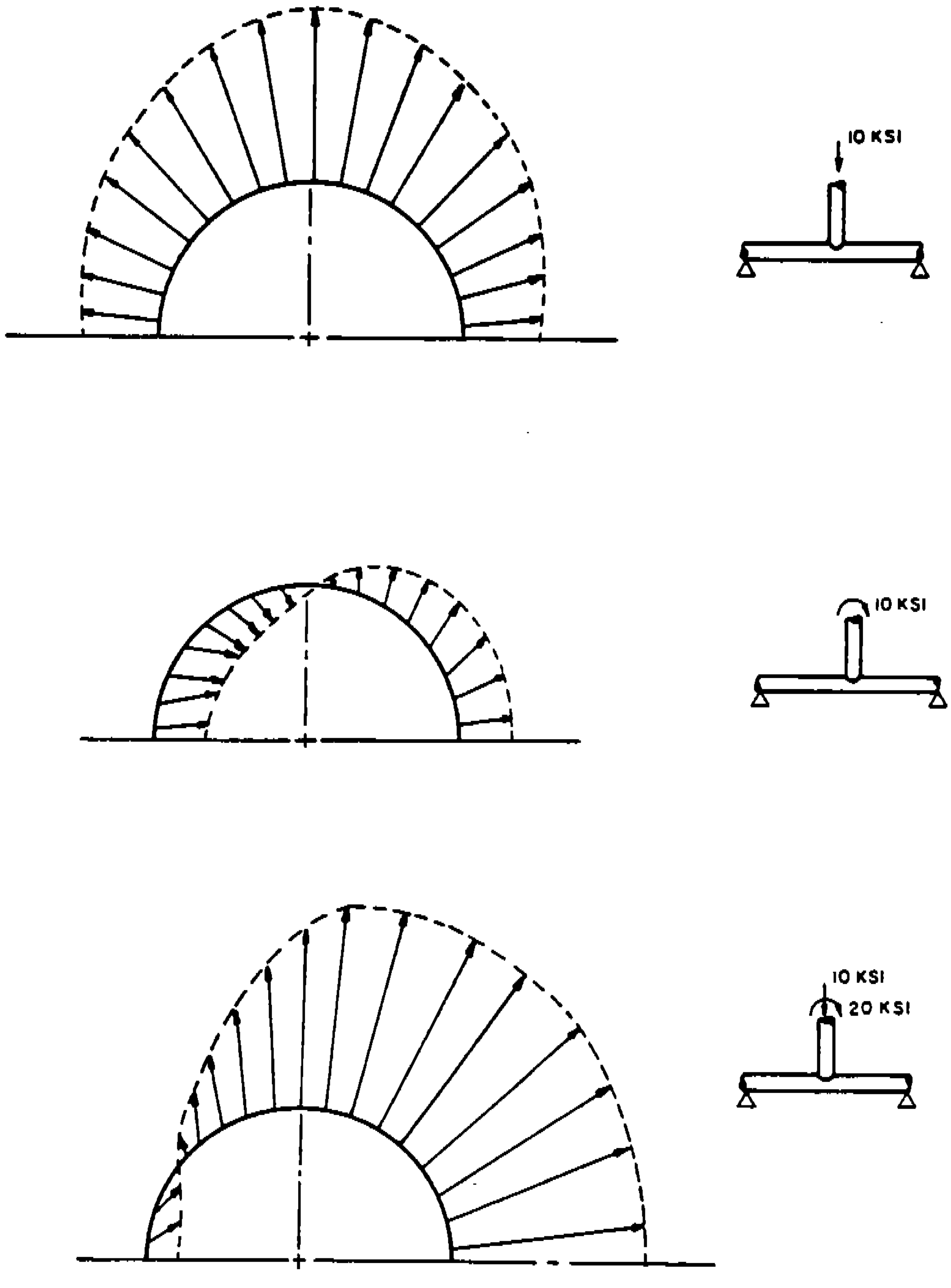


Figure 2-16. Stress Distribution Around Circumference of a T-Joint Under Various Loads

STRESS DISTRIBUTION AT 12H FULL PENETRATION, TWO-SIDE WELD

NUMBERS DENOTE SCF VALUES

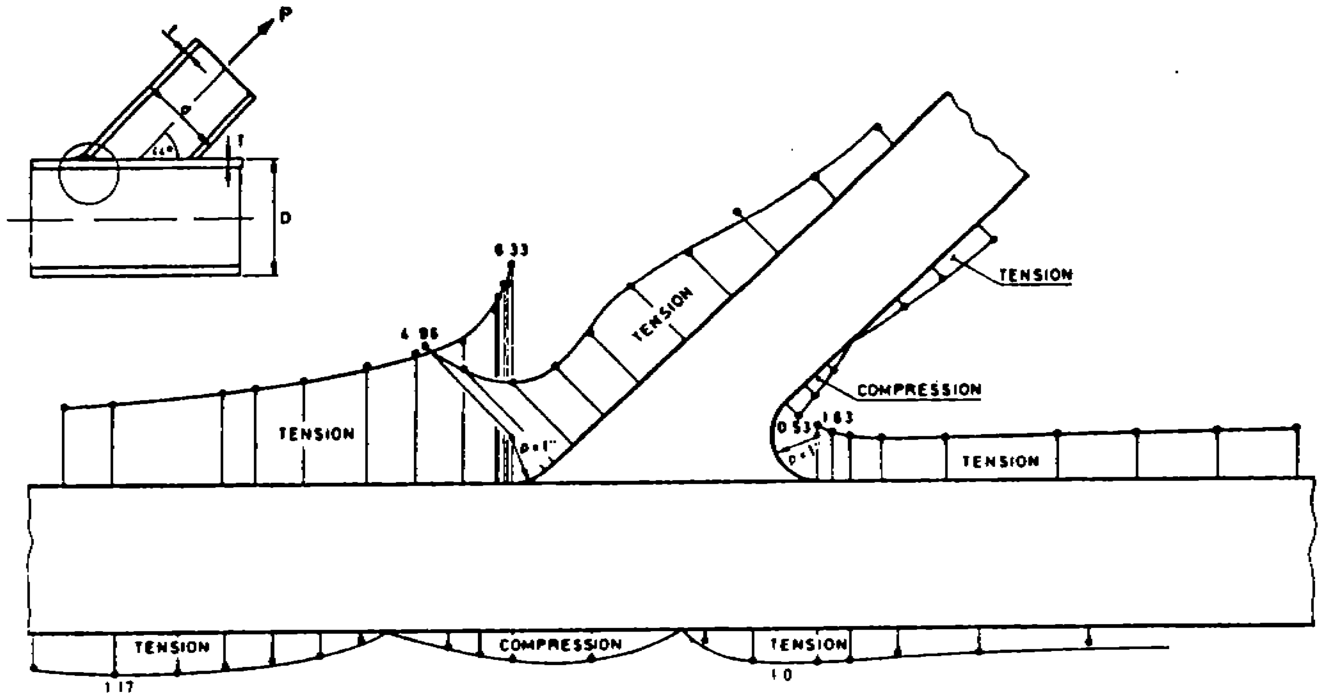


Figure 2-17. Stress Distribution at a Welded Y-Joint Connection with an Axial Load

direction vary during the complete cycle. Out-of-plane bending loads tend to produce higher stress than in-plane loads; however the location and magnitude of stress hotspot and the direction of the principal stress at that location vary with loading and joint geometry parameters.

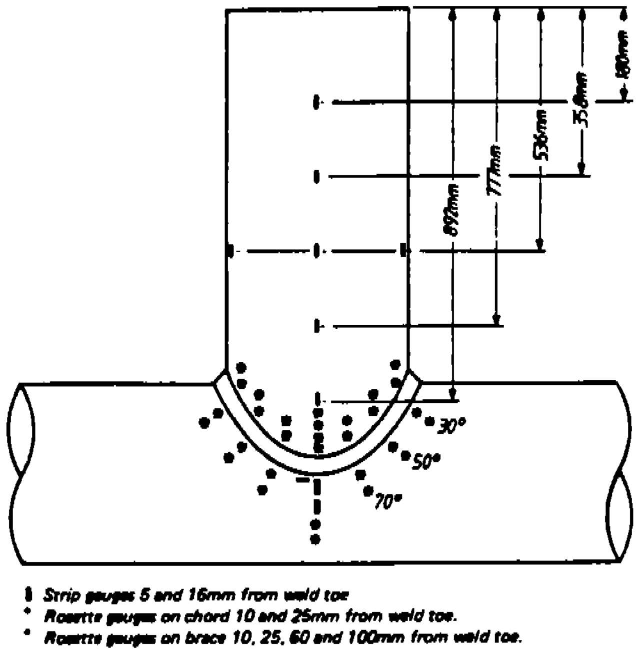
The interaction between members of in-plane joints has been investigated by Gulati [2-7] where stress concentration factors are provided for stresses normal to the weld toe in complex KT joints typical of jacket structures; however, little is presented in the literature on the multiaxial stress distributions between multi planar joints. Rather, loading interaction effects are accounted for by superimposing moments. Stress concentration factors are then determined. Typical stress concentration factors are presented by Gulati [2-7], Marshall [2-8, 2-9] and Rodabaugh [2-10]. Gulati also discusses the fact that principal stress in joints subjected to combined loading is not orthogonal to the weld lineal direction. This condition is shown in Figure 2-18 for a simple T joint. Furthermore, the direction of the principal stress changes with the changing magnitudes of constituent loadings. The effect of the non-perpendicularity to the weld lineal direction of the principal stress (at the hotspot) on the crack trajectory and fatigue life of an as welded joint is not known at this time.

In summary, stress characteristics in offshore structures are triaxial within the tube wall thickness. Biaxial stress fields exist at the tube free surface at the weld toe in chord and brace intersections with large shear stresses in the trough thickness of the brace wall. Principal stresses vary in proportion with the random nature of wave encounter. The location of maximum stresses changes with the encounter and passing of waves. Mean stresses are present in terms of dead loads, current loads and residual stresses.

2.1.2 Factors Influencing Fatigue Response (General)

In addition to the state of stress, there are other factors that influence fatigue response that must be considered in the evaluation of multiaxial fatigue analysis and design procedures. But first a brief description of the fatigue process will be extremely helpful in understanding and evaluating multiaxial fatigue mechanisms.

Fatigue cracks initiate in local slip planes or in the plane of a discontinuity in welded details depending on the size and shape of the defect and local stress conditions. In the



Strain Gauge Locations for Tested T Joint

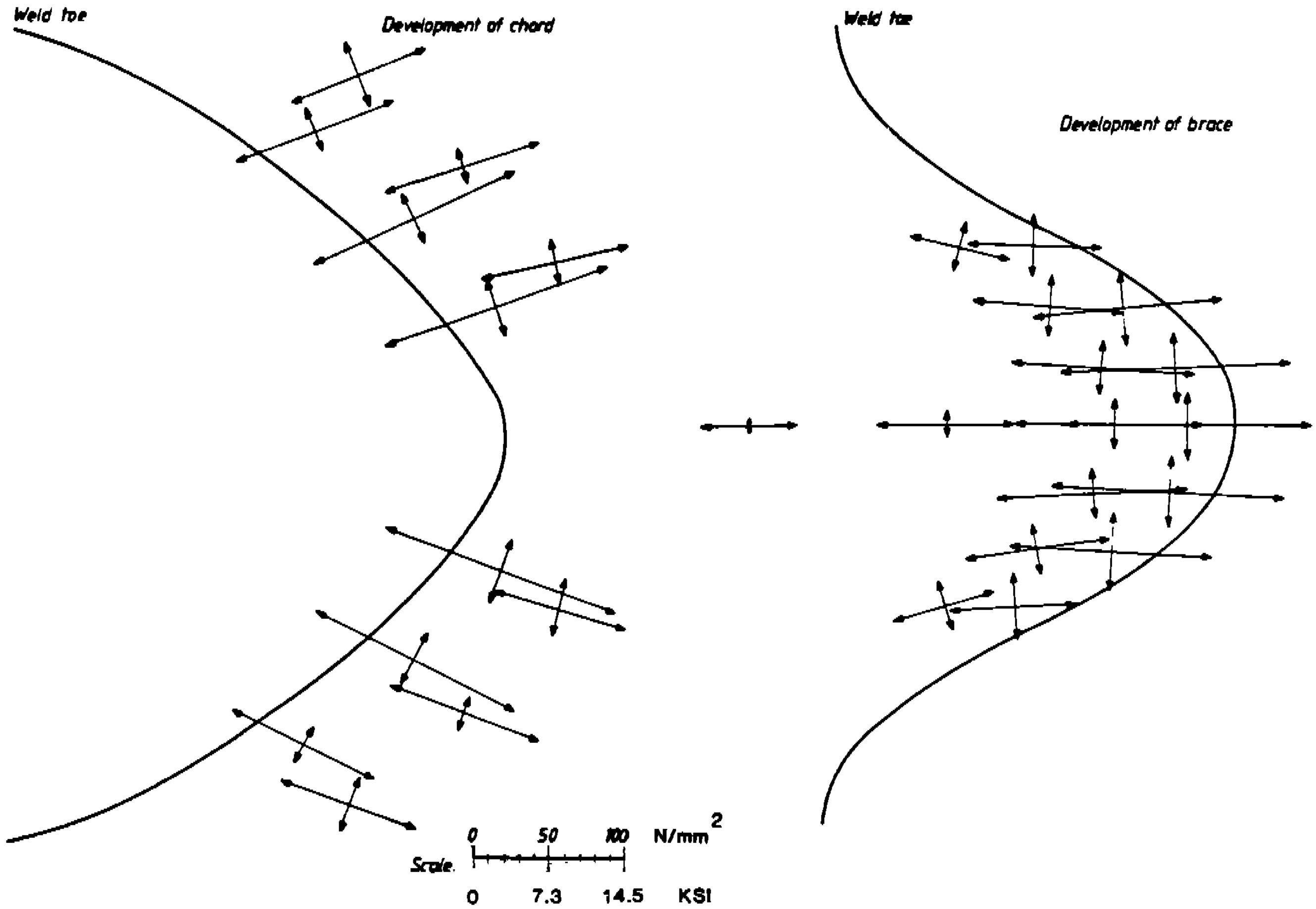


Figure 2-18. Principal Stress Distribution in a T-Joint. Note Stress Direction Relative to Weld Toe

absence of defects, fatigue cracks tend to grow in a plane of maximum shear stress range. This growth is quite small, usually on the order of several grains. As cycling continues, fatigue cracks tend to coalesce and grow along planes of maximum tensile stress range. When defects are present the mode of crack growth is more complex. The two stages of fatigue crack growth are called Stage I and Stage II as shown in Figure 2-19. The stages are important because multiaxial fatigue prediction techniques are generally applicable to one stage or the other and some attempt to account for both.

Fatigue life is influenced by numerous factors. Bea [2-11] has presented Table 2-3 listing the factors influencing fatigue in offshore structures indicating the level of complexity and amount of effort required to predict fatigue response in offshore structures. Of a total of almost 40 general topics, multiaxial fatigue is included in 1 or 2 areas. Other factors affecting fatigue response in welded marine structures are considered as subcategories to Table 2-3. They are as follows:

1. Material properties
    a. Base metal
    b. Heat affected zone
    c. Weld metal
2. Stress characteristics
    a. Stress gradients in weld geometry
    b. Stress proportionality
    c. Elastic and plastic strain relationships
    d. Mean and residual stresses
    e. Stress phasing
    f. Random loading
    g. Stress relieving
3. Corrosion
4. Thickness effects
5. Flaws
6. Fabrication procedures
7. Surface finish

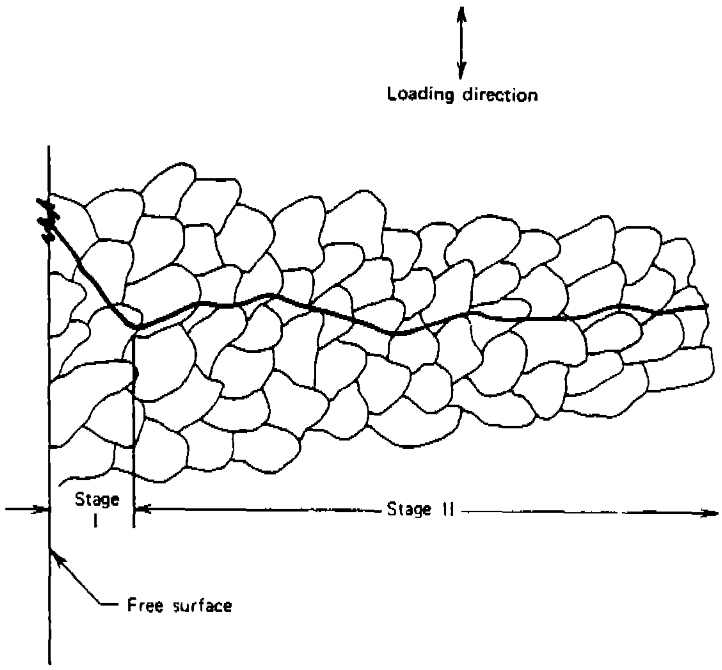


Figure 2-19. Schematic of Stages I and II Transcrystalline Microscopic Fatigue Crack Growth

TABLE 2-3

A SUMMARY OF FACTORS AND CONSIDERATIONS RELATED TO FATIGUE IN WELDED JOINTS OF OFFSHORE PLATFORMS

I. FATIGUE BEHAVIOR OF WELDED JOINTS
1. Definition of fatigue failure in S-N data
2. Size effect in S-N data
3. Effect of weld profile
4. Effect of corrosion and cathodic protection
5. Assumption of a linear model and lognormal distribution for N
6. Classification of joint on the basis of geometry rather than load pattern
7. Relationship between stress at joint and stress used to obtain S-N curve
8. Ignoring possible stress endurance in S-N curve
9. Compatibility of determination of hot spot stress with S-N curve

II. MANUFACTURING CONSIDERATIONS
1. Fabrication uncertainties
2. Requirements on weld contours not met

III. DEFINITION OF THE ENVIRONMENT
1. Use of full scatter diagram of wave height and wave period
2. Variations in wave period
3. % occurrence estimates
4. Wave directionality
5. Interaction of waves and currents
6. Theoretical model used for ocean waves

IV. HYDRODYNAMIC LOADS ON STRUCTURE
1. Inertia and drag coefficient
2. Directional wave spectra which accounts for wave spreading
3. Marine growth
4. Sheltering effects

V. STRUCTURAL RESPONSE TO HYDRODYNAMIC LOADS

1. Assumptions made in spectral analysis

   a. linear response during transfer function development
   b. linearization of drag term
   c. at joints
      i. no flexibility
      ii. effect of can
      iii. center to center coordinates

TABLE 2-3 (continued)

A SUMMARY OF FACTORS AND CONSIDERATIONS RELATED TO FATIGUE IN WELDED JOINTS OF OFFSHORE PLATFORMS

d. soil stiffness in dynamic model
e. damping effects in structural response
f. dynamic response not accounted for in analysis

VI. FATIGUE STRESSES AT JOINT

1. Method of analysis to evaluate stress concentration factors (SCF)
2. Parametric equations used for SCF
3. Point at intersection where failure occurs

VII. FATIGUE DAMAGE EQUATIONS

1. Assumption of Miner's Rule
2. Assumption of narrow band damage equation in spectral approach
3. Assumption of Weibull distribution for stress ranges in stress distribution approach

VIII. OTHER CONSIDERATIONS

1. Errors by designers
2. Bad judgment during towing and installation
3. In service loads

These factors should be kept in mind in reading the review of multiaxial fatigue prediction approaches. Few have been addressed adequately if the intent is to incorporate the multiaxial fatigue approaches in design of welded marine structures.

As we will present later, there are gains to be made in overall structural reliability by considering multiaxial effects if the designer has the proper design tools, information and financial resources to do so.

2.2 CUMULATIVE DAMAGE AND CRACK GROWTH APPROACHES FOR PREDICTING FATIGUE RESPONSE

As with stress distributions, it is important to understand existing fatigue design procedures before reviewing multiaxial fatigue research. Most multiaxial fatigue approaches are extensions of fatigue life and fracture mechanics approaches developed for uniaxial loading.

The fatigue life of a structural detail is determined by the sum of the elapsed cycles required to initiate a fatigue crack and propagate the crack from subcritical dimensions to a critical size. (Note that the critical crack size and criteria for failure often differs from one set of data to another.) The two basic fatigue prediction approaches that are most widely used in structural design include the Miner's Linear Rule for fatigue life (crack initiation and growth of short cracks) and the fracture mechanics theory for crack growth.

Miner's [2-12] approach is based on knowledge of the structural loading and the resistance of the structure in terms of stress range and number of cycles to failure. This method is developed from test data (S-N curves) together with the hypothesis, that fatigue damage accumulates linearly. According to this hypothesis, the total fatigue life under a variety of stress ranges is the weighted sum of the individual lives at the various stress ranges, S, as given by the S-N curves, with each being weighted according to the fractional exposure to that level of stress range. To apply this hypothesis, the long-term distribution of stress range is replaced by a stress histogram, consisting of a convenient number of constant amplitude stress range blocks, S_i, and a corresponding number of stress cycles, η_i. The constraint against fatigue fracture is then expressed in terms of a nondimensional damage ratio, η:

$$\sum_{i=1}^{\beta} \frac{\eta_i}{N_i} \leq \eta_L$$

where β = number of stress blocks

η_i = number of stress cycles in stress block i

N_i = number of cycles to failure at a constant stress range, S_i

η_L = limit damage ratio

The limit damage ratio η_L depends on the number of cycles at each stress level.

Theories of crack propagation associated with most crack growth methods are based on linear elastic or elastic-plastic fracture mechanics methods.

In linear elastic fracture mechanics, the stress field at the tip of a crack is described in terms of stress intensity with a function of the following form:

$$K = F \cdot \sigma \sqrt{a} \ (\text{Ksi}\sqrt{\text{in}})$$

where σ = gross stress (ksi)

a = crack length (in)

F = geometry correcting factor, dependent upon crack and part geometry, and stress gradient

K = stress intensity factor

In the literature there exists stress-intensity solutions for a wide variety of crack shapes and loading cases.

The stress intensity is used in all known schemes for calculating crack-growth rates. For constant-amplitude crack growth, it has been shown that crack growth is primarily a function of ΔK where $\Delta K = K_{max} - K_{min}$. That is, $da/dn = f(\Delta K)$.

When constant-amplitude crack-growth rate (da/dn) is plotted against ΔK on log/log paper, an S-shaped curve results as shown in Figure 2-20. Traditionally, this curve is broken into three regions called Stage I, II and III crack growth. Stage I crack growth, sometimes called threshold crack growth, applies when crack-growth rates are very low. Small changes in ΔK substantially alter the crack-growth rates. Stage II crack growth, sometimes referred to as steady-state crack growth, is characterized by a nearly linear relation between log da/dn and log ΔK, while Stage III crack growth is characterized by a rapidly increasing crack-growth rate and results from crack instability.

Several models have been presented which attempt to describe the relation between da/dn and ΔK. The simplest, and limited strictly to Stage II crack growth, is the Paris equation:

$$\frac{da}{dn} = C(\Delta K)^m$$

where C and m are material-dependent constants and are determined by experiment. Other relations account for stress ratio effects or include Stage I or Stage III crack growth but are modifications of this approach.

Both the fatigue life and fracture mechanics approaches have either been applied or proposed for use in marine structures as described next.

2.2.1 Stress Concentration Factor (SCF) Approach in Offshore Structures

The American Petroleum Institute (API) [2-13] and the American Welding Society (AWS) [2-14] provide a method of evaluating the fatigue life of offshore platform tubular joints using either an experimental or theoretically determined "hot-spot stress" range or "hot-spot strain" range. The hot-spot stress/strain is the maximum stress/strain in a given tubular joint due to a specified load range, but does not include the "peak" stress/strain which arises due to the geometrical discontinuity at the toe of the fillet welds. The hot-spot stress/strain with this definition can be measured by strain gauges (the peak stress/strain cannot be so measured) or it can be calculated by finite-element analysis; the model used in the finite-element analysis does not include the fillet weld contour or the extremely fine element mesh needed to determine peak stresses.

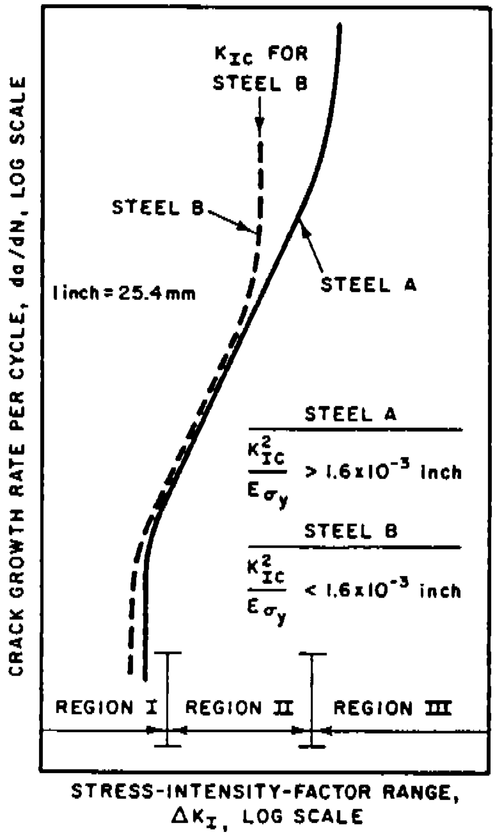


Figure 2-20. Schematic Representation of Crack Growth in Steels

The AWS Code states that the X-curve is to be used with "greatest total range of worst hot-spot stress or strain on the outside surface on intersecting members at the toe of the weld joining them -- measured after shakedown in model or prototype connection or calculated with best available theory".

The British have also developed an approach for predicting fatigue life of welded details and offshore structural joints [2-15]. The non-nodal joints are classified according to weld configuration and geometry. The nodal joints are all represented by one S-N curve. Unlike the API code, this approach recognizes combined stresses and presents data in terms of principal stresses at the weld toe. This approach is based on nominal field stresses and tests of actual joint configurations.

During the past twenty years, a substantial amount of work has been conducted and results published on hot-spot stresses in simple tubular joints. This work includes stresses as experimentally determined by strain gauges on tubular joint models; stresses as experimentally determined by photo-elastic tubular joint models, and by finite-element analysis of tubular joint models. These results have been used to develop correlation methods for estimating elastic hot-spot stresses in a wide variety of types and joints and parameters.

Comparisons between measured hot-spot stresses and finite-element analysis involve considerations of exactly where the hot-spot stresses are located and which type of data is more appropriate for use with the AWS X-curve. Potvin [2-16] discussed this aspect using the illustration in Figure 2-21.

Potvin et al [2-16] noted that there is a ±20% difference between their analytical hot-spot stresses and measured hot-spot stresses and they attribute this to the difference in the analytical and experimental hot-spot locations. This variation in hot-spot location is a result of two factors: the absence of the weld fillet in their finite element model and the difficulties involved with obtaining consistent experimental data. Their finite-element model predicts that the hot-spot occurs at the intersection of the midsurface of the brace and chord. The inconsistency in the experimental data involves the actual location, with respect to the weld toe, at which the experimental database is obtained. The experimental uncertainty is compounded by the size and number of strain gauges used (i.e., is the gauge small enough to be placed close to the toe of the weld and/or are there a sufficient number of gauges so that an accurate extrapolation of stresses to the toe of the weld is possible as indicated in Figure 2-21).

The test-data available and correlation methods, in addition to being restricted to the elastic regime, are also restricted in the sense that they are based on "isolated" joints (i.e., joints in a chord where any other joint is sufficiently far away so that the two joints are not influenced by each other). In offshore platforms, quite often two or more joints are placed at the same axial location on a chord. There are no available data on such joints. There are "rule of thumb" approaches for joint spacing such that data on isolated joints would be reasonably applicable.

Correlation methods for joint hot-spot stresses based on joint configuration parameters have been developed by a large number of researchers. These correlation methods include:

1. Marshall's [2-17] presentation of the Kellogg [2-18] equation for hot-spot stresses in the chord and his equations for hot-spot stresses in the brace;
2. Bijlaard's [2-19] method for hot-spot stresses in the chord;
3. Kuang et al [2-20] method for hot-spot stresses;
4. Wordsworth [2-21] for chords;
5. Gibstein [2-22] for chords.

Other authors [2-23 through 2-26] have developed correlation methods. These correlation methods are developed for in-plane joints as shown by Kuang in Figures 2-14 and 2-15. Gulati [2-7] reviewed several methods mentioned above and compared them to finite element analyses, the results of which are shown in the same figures.

The interaction of stress fields of neighboring connections is accounted for by direct super position of stresses. However, results have been proven suspect in magnitude and location of hot-spot stress. Additionally, local interaction formulations are based on parametric formulations and rules of thumb and these limitations will be discussed in Section 2.3.

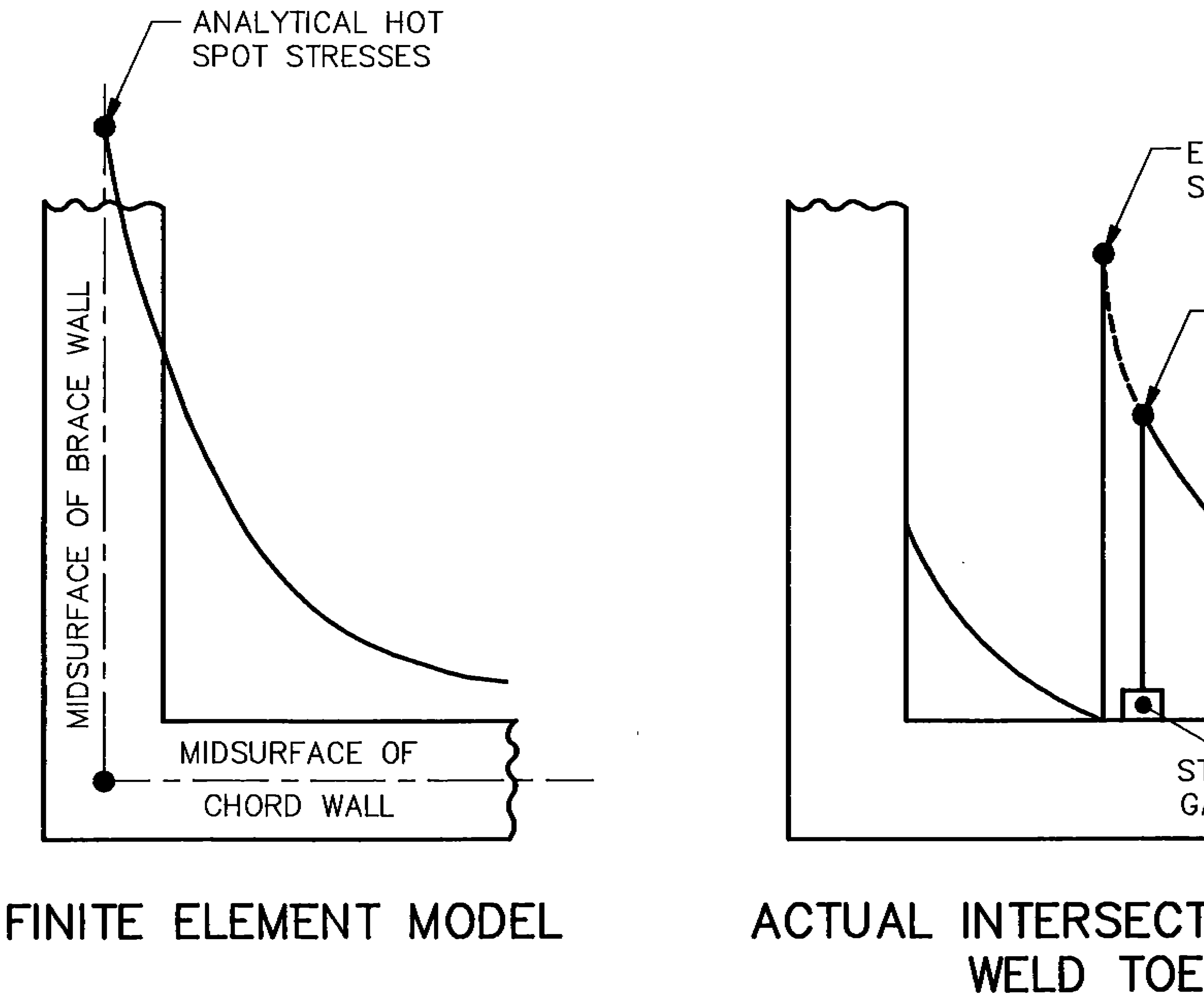


Figure 2-21. Illustration of Stress Concentration Facto
Hot-Spot Stress for Offshore Structures

2.2.2 Proposed Approach for Fatigue Analysis of Ship Structural Details

While there is no approach specifically written into codes or rules for ship structures (other than empirical stress allowables), there have been approaches proposed to account for fatigue life in ship structural details [2-3, 2-27 through 2-29]. The most notable approach is that developed by Munse et al [2-3] and presented in SSC 318. This approach is based on calculating a "design allowable" stress range, S_{rd} for fatigue. This stress range is the maximum peak-to-trough stress range expected at the point in question once under the most severe sea state and during the entire life of the structure. Comparing that stress range to the allowable stress for other failure modes indicates the controlling mode of failure. In any case, the maximum stress computed from the fatigue design stress range, S_{rd}, must, generally speaking, be less than the nominal permissible stress permitted once by the basic design rules. According to the Munse approach, the design stress range, S_{rd}, is found using the following equation:

$$S_{rd} = S_n \cdot R_f \cdot \epsilon$$

where S_n = mean value of the constant amplitude stress range at the design life, N_d

R_f = reliability factor

ϵ = random load factor.

The mean value of the constant amplitude stress range, S_n, is found by entering the S-N curve of the structural detail of interest at the number of cycles expected in the design life, N_d. The probabilistic nature of the design method is introduced by the other two factors in the equation.

The reliability factor, R_f, is meant to account for uncertainties in the fatigue data, workmanship, fabrication, use of the equivalent stress range concept, errors in the prediction of load history and errors in the associated stress analysis. The factor comes from the assumption that fatigue life is a random variable with a Weibull distribution, and the use of a relationship for the probability of survival through N loading cycles. The effect of the reliability factor is to reduce a mean constant stress range to an equivalent stress range which corresponds to a designated probability of survival greater than the 50% level of the mean stress range. The random load factor, ϵ, is introduced in the design procedure to make possible the use of existing constant-cycle fatigue data in designing for variable loading service conditions.

The work of Munse et al represents a significant step forward in the design of ship structures. It presents the first truly probabilistic approach to fatigue design for ship structural details. This approach integrates well with the ship design process where a nominal level of stress is developed and no detail stress calculations are required. Multiaxial stress distributions are not accounted for within the detail, however, reliability factors are presented which are to account for these effects such as scatter about the mean line of the S-N curve. When detailed stress analysis is warranted, other approaches are required for correlation to the basic S-N curve for the materials.

2.2.3 Proposed Fracture Mechanics Approaches for Marine Structures

Thayamballi et al [2-30] proposed a fracture mechanics approach for ship structures based on the Paris equation presented above. This approach takes into account various factors influencing fatigue response and reliability considerations. There are approaches presented to account for multiaxial loading sources and complex stress fields. However, the authors emphasize Mode I stress intensity factors being typical of ship structural loading and fatigue crack growth.

Chen used this approach for the fatigue crack growth analysis of the SL-7 hatch corner cracking [2-6]. An integration of the Paris equation and use of the equivalent stress concept yield the cycles to failure. As described earlier, Thayamballi found that initial crack length is proven a major consideration and source of uncertainty.

Fracture mechanics analysis for offshore structures has been used as a fitness for purpose tool for some time and has been proposed for fatigue life analysis by Rhee [2-31] and Haung [2-32] among others. Again, the Paris equation for linear elastic fracture mechanics is utilized. The important steps associated with the procedure are summarized as follows:

1. calculate storm member forces through frame analysis for a jacket under a given environment;

2. determine the crack location for fatigue crack growth simulation analyses based on the stress analysis results and defect distribution status;

3. calculate the stress intensity factors for the fatigue crack growth simulation analyses, and other fracture parameters required for crack instability analyses. Calibrate the material fatigue and fracture properties required consistently with the fracture parameters;

4. perform fatigue crack growth simulation analyses;

5. perform crack instability analyses to determine the critical crack size;

6. correlate the results of the fatigue crack growth and crack instability analyses to determine the component fatigue life.

2.3 IMPETUS FOR A MULTIAXIAL FATIGUE APPROACH

Over the past decade, the application of finite element analysis to structural details has proliferated, especially with the advent of personal computers with memory capacity for finite element computer programs. Increasing use of this tool has made it possible to analyze the state of stress in a structural detail to a level of detail beyond that required by existing nominal stress fatigue design procedures. This tool also makes it quite possible to predict combined stresses from multiple loads and from geometric concentrations. The designer now has a choice: 1) to use techniques that he has to predict nominal stresses in joint boundaries and accept limitations on conservatism associated with the correlation equations or test data from similar welded components; or 2) cross the nominal stress boundary and predict the state of stress within the detail. More structural designers for both ships (ABS [2-1] and Columbia Research Corporation (CRC) [2-34]) and offshore platforms (Gulati [2-7] and Rhee [2-31]) are choosing the latter option. However, the new knowledge has also raised additional questions.

Existing S-N curves and crack growth data is typically presented from uniaxial tests. Finite element techniques estimate principal stresses in multiple directions resulting from various load and geometric effects. The designer now requires additional knowledge on how to characterize the state of stress to correlate with uniaxial fatigue life predictions and improve design reliability.

This situation is evident as a result of a finite element study conducted by Gulati [2-7] for offshore structures. As shown in Figure 2-18, the principal stress in the neighborhood of the concentration point at the chord-brace junction of a T-joint subjected to combined loading is not orthogonal to the weld lineal direction assumed by the current fatigue life prediction methods. Furthermore, the direction of the principal stress changes with the changing magnitudes of the constituent loadings. The effect of the non-perpendicularity to the weld lineal direction of the principal stress (at the "hot spot") on the crack trajectory and fatigue life of an as welded joint is not known.

Because designers are asking increasingly difficult questions on multiaxial fatigue response in real structures, it became necessary to review the general multiaxial fatigue research and those approaches that have been applied to welded structural details and evaluate to their applicability to marine structures.

3.0 <u>REVIEW OF LITERATURE ON MULTIAXIAL FATIGUE RESEARCH</u>

Interest in problems of metal fatigue and fracture in engineering has a long history dating back well into the last century. There was certainly an early awareness of the problem of dealing with the effect of combined loads, but the major thrust of early fatigue development was in the field of uniaxial fatigue testing and the development of uniaxial data for material fatigue characteristics in the form of the well-known S-N diagrams. Interest was centered in the area of long material lives, that is, High Cycle Fatigue (HCF). In this regime the nominal stresses in the material are in the elastic range, only a fraction of the yield stress for the material, and plasticity is confined to the region in the immediate vicinity of the fatigue crack tip. The discipline of fracture mechanics was not born until after World War I, but it remained until World War II for this field to experience really active development [3-1 through 3-8]. As understanding of the mechanisms of crack initiation and growth improved, interest increased in the area of higher loadings and shorter lives -- that is, the Low Cycle Fatigue (LCF) regime. As the demands of technology made clear the need for a more sophisticated understanding of the entire fatigue problem, testing machines were developed which had the capability of testing in two modes, most often in torsion and axial push-pull, sometimes in torsion and bending, and less frequently in in-plane linear orthogonal loads. Multiaxial fatigue research has been active, lively and growing for the past twenty-five years, but it is not by any means a mature discipline. There are a multitude of proposed multiaxial fatigue criteria, the most prominent of which will be discussed in subsequent sections of this chapter, and there are codes and formulations (e.g., American Society of Mechanical Engineers (ASME) Pressure Vessel Code) which are in place and may be used on a provisional basis, but there does not exist a single unified and validated criterion, accepted by the research community, which can be used for engineering design applications.

In the following sections, the general multiaxial fatigue research and candidate approaches to marine structure design and evaluation will be reviewed.

3.1 GENERAL MULTIAXIAL FATIGUE RESEARCH

3.1.1 <u>Fatigue Life Estimates for Crack Initiation</u>

The first general category of multiaxial fatigue research is for those approaches based on the cumulative damage approach for estimating fatigue life. An equivalent stress is used to correlate multiaxial stresses or strains to uniaxial test data. Each approach is applicable to a specific type of fatigue response. Generally, shear stress based criteria are applicable to fatigue crack initiation in the high cycle regime. Principal stress criteria are applicable to fatigue life approaches that include crack growth. Combinations of approaches have been developed. Approaches have also been developed for estimating fatigue life under low cycle conditions where plastic work dominates. Traditional interest had been in long-lived behavior in which nominal stresses in the material remained in the elastic range well below the tensile yield stress. Figure 3-1 taken from Reference [3-8] illustrates schematically the partition between low cycle fatigue and high cycle fatigue regimes. The differentiation between high cycle and low cycle is important in further categorizing multiaxial fatigue approaches.

In the following sections the general multiaxial fatigue research developed for applications in other industries such as pressure vessels, aircraft, moving vehicles and heavy equipment is reviewed. Much of this research is based on extension of existing uniaxial research with newer approaches based on plastic work estimates.

3.1.1.1 Stress Based Criteria

Early research in multiaxial fatigue was, quite naturally, an extension of uniaxial work and dealt with combined stresses in the HCF regime. The work of Mason [3-9] and Mason and Delaney [3-10] is representative.

Two well-known and widely used yield criteria, those attributed to Tresca and to Von Mises were adapted early to cyclic stresses.

Tresca: $$\tau_{max} = (\sigma_1 - \sigma_3)/2 = \text{constant}$$

Von Mises: $$\tau_{octahedral} = 1/3 \left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \right]^{1/2} = \text{constant}$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the principal stress and the value of the constant is specified for a given fatigue life.

A significant contribution was made by Gough and colleagues in the mid-30's through the early-50's [3-11 through 3-14]. Gough found that the arc of an ellipse provided a good fit to his experimental data from in-phase bending and torsion tests of brittle materials and ductile materials. Guest

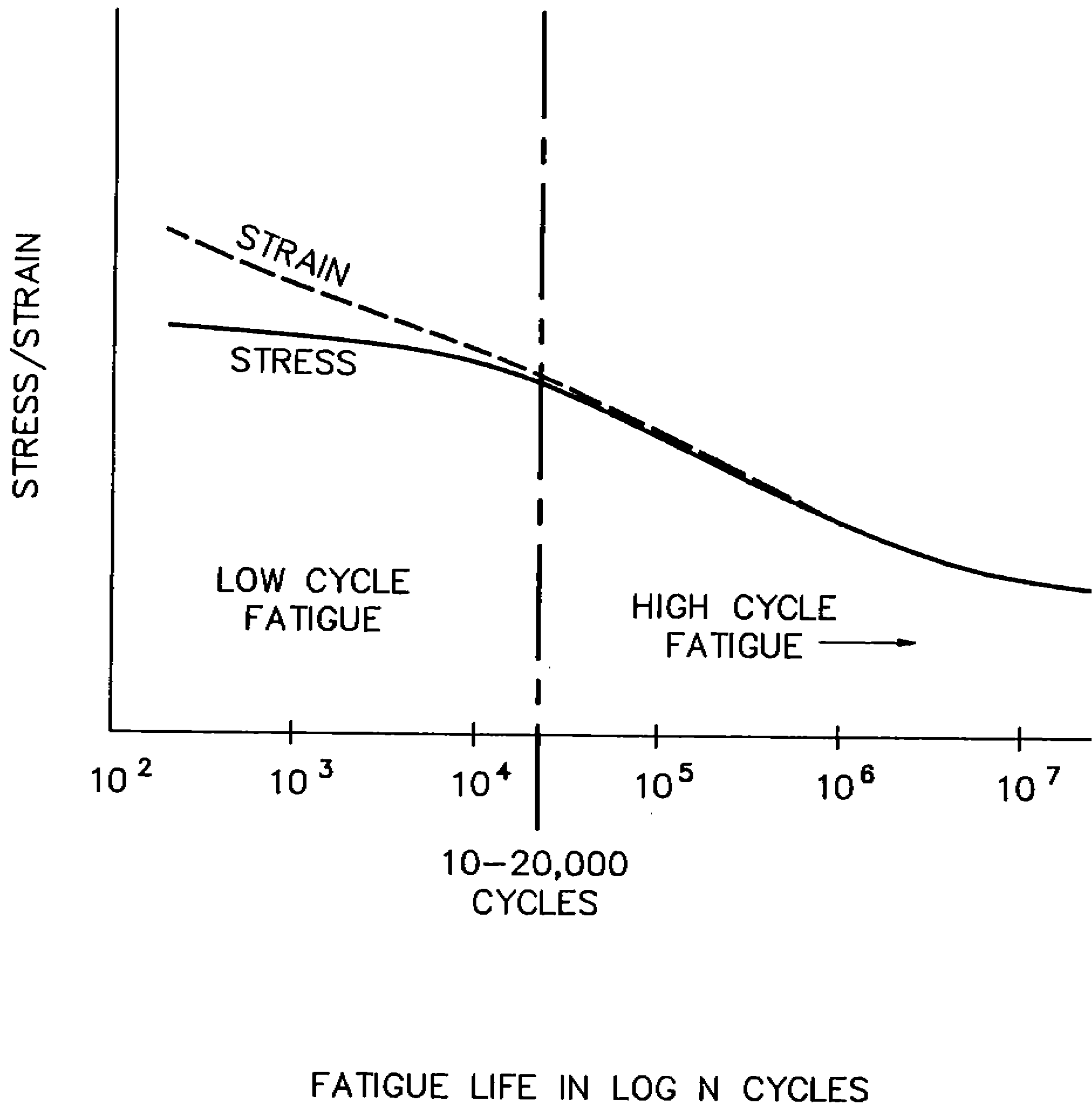


Figure 3-1. Schematic Representation of High Cycle and Low Cycle Fatigu
as a Function of Strain State and Strain Amplitude

[3-15, 3-16] noted that Gough's empirical formulations could be reduced to a single expression:

$$\frac{\sigma_1-\sigma_2}{2} + k\,\frac{\sigma_1+\sigma_3}{2} = \text{constant} \qquad \textbf{(3-1)}$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are principal stresses and k is a measure of the relative ductility or brittleness of the material. Guest's relationship is a statement that constant fatigue lives are a function of the maximum shear stress, $(\sigma_1-\sigma_2)/2$, modified by a fraction, k, of the normal stress on the plane of maximum shear stress, $(\sigma_1+\sigma_3)/2$. Variations of this theme in terms of both stress and strain will be seen to occur repeatedly in the following years. Tipton and Nelson [3-17] have expressed the Guest criteria for combined in-phase bending and torsion as:

$$t_n = \left[\frac{1}{2}\sqrt{\sigma^2+4\tau^2} + \left(2\,\frac{t_n}{b_n} - 1\right)\sigma\right] \qquad \textbf{(3-2)}$$

where σ = amplitude of cyclic bending stress

τ = amplitude of cyclic torsion shear stress

b_n = fatigue limit for <u>notched</u> specimens in bending (determined experimentally)

t_n = fatigue limit for <u>notched</u> specimens in torsion (determined experimentally)

Figure 3-2, taken from Reference [3-17], shows an excellent correlation of data taken from multiaxial notched specimen tests in torsion and bending with the torsion-bending fatigue limit predicted by (3-2) above. Also shown in Figure 3-2 are fatigue life predictions using the well-known Von Mises distortion energy yield criteria applied to constant-amplitude bending and torsional stresses. Stresses in the notch of the specimen have been calculated in two ways: (1) using elastic stress concentration factors, K_t; and (2) experimentally determined stress concentration factors, K_τ. In this case, using the elastic stress concentration factors gives a very poor correlation. Other cases displayed in [3-17] using K_t are in better agreement. K_τ predictions are generally better, but in most cases unconservative. Further confirmation of the form of the Guest Law was provided by Findley [3-18]. McDiarmid [3-20] also proposed a criterion for fatigue failure based on the critical <u>range</u> of shear stress modified by the normal stress acting on the plane of maximum shear stress. Sines [3-19] proposed a tensorial method of combining biaxial alternating

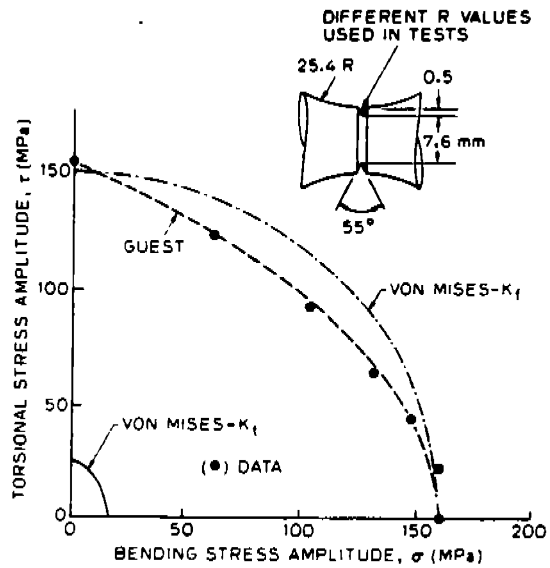


Figure 3-2. Comparison of Predicted Combined Bending-Torsion Fatigue Limit with Test Data for a Circumferentially Notched Round-Bar Specimen [3-17]

(in-phase) stresses with a static mean stress. The method is general in nature and could be extended to triaxial stress systems with a superimposed static stress.

Langer [3-21] has discussed the application of Tresca's criterion to the design of pressure vessels involving multiaxial fatigue. Tresca's criterion was favored over Von Mises' "criterion" because of its simplicity, conservatism and adaptability to more situations. This particular criterion was adopted as the basis for evaluating multiaxial fatigue in the ASME Boiler and Pressure Vessel Code procedures [3-22]. The approach is as follows:

S_{alt} = the value of (1/2)(S1D-S3D) maximized with respect to the least and greatest principal stress values of the stress cycle where S1D and S3D are the maximum and minimum stresses respectively.

The parameter S_{alt} is called "stress-intensity" which is used as the parameter correlating fatigue life (to crack initiation). It is the maximum range (over the complete "cycle" of loading) of shear stress acting on a particular pair of planes and in a particular direction; its absolute value is important and not the planes and directions along which it acts. This is a stress based criterion. This approach has been refuted under out-of-phase multiaxial loading because of the path independence assumption.

Reviews of the ASME Boiler and Pressure Vessel Code exist [3-23, 3-24] indicating mixed results for out-of-phase loading. In the case of biaxial in-phase constant amplitude loadings, the principal stresses will act in a constant direction, but with oscillating amplitudes. The out-of-phase monofrequency constant amplitude problem is much more realistic for many rotating machinery and pressure vessel applications. Note that most of the following equivalent stress approaches are presented in a format in which sinusoidal loading functions are known, for example, in applications such as rotating machinery.

More recently, ASME adopted the following formulation in the 1974 ASME Boiler and Pressure Vessel Code [3-25] for an equivalent stress, SALT (S_{alt}) which for fully-reversed bending and torsion can be expressed as:

$$\mathrm{SALT} = \frac{B}{\sqrt{2}} \left[1 + K^2 + \sqrt{1 + 2K^2\cos 2\phi + K^4} \right]^{1/2} \tag{3-3}$$

where

	B	=	amplitude of cyclic bending stress
	T	=	amplitude of cyclic torsion stress
	K	=	2T/B
	ϕ	=	phase angle between bending and torsion.

In a modification of the above method, an equivalent stress parameter, SEQA ($S_{equiv.}$), was presented as an ASME Code case [3-26]. For fully reversed out-of-phase bending and torsion the parameter is given by:

$$\mathrm{SEQA} = \frac{B}{\sqrt{2}} \left[1 + \frac{3}{4} K^2 + \sqrt{1 + \frac{3}{2} K^2\cos 2\phi + \frac{9}{16} K^4} \right]^{1/2} \tag{3-4}$$

It can be shown that the SEQA formulation reduces to the Von Mises criteria for in-phase loadings. Lee [3-27] developed a criterion in the spirit of Gough's ellipse quadrant applicable to constant amplitude, monofrequency, fully reversed, out-of-phase torsion and bending loads.

$$\mathrm{SLEE} = b \left[(B/b)^{\alpha} + (T/t)^{\alpha} \right]^{1/\alpha} \tag{3-5}$$

where

	b	=	bending fatigue strength for a given life N
	t	=	torsional fatigue strength for the same life N
	α	=	variable power dependent on phase difference, ϕ and material
	α	=	$2(1 + \beta\sin\phi)$
	β	=	material constant

Lee, in Reference 3-27, reports a more favorable correlation with test data using Equation 3-5 than with Equations 3-3 or 3-4.

The ASME Codes are widely considered to be quite conservative. Tipton and Nelson [3-17] confirm this but note that out-of-phase loadings are more damaging than in-phase loadings, a feature not well predicted by the SEQA criterion. Lee [3-27] also found out-of-phase loadings to be more damaging. For 90° out-of-phase loadings and for certain ranges of the parameter K, Lee found that both SALT and SEQA underestimated the danger of fatigue failure.

The central theme in the development of stress-based criteria has been the reduction of a complex stress state to a single scalar parameter which can be related to the cyclic material properties and lives determined from uniaxial fatigue tests and used to predict life under the more complex loading conditions. While cyclic applications of Tresca and Von Mises yield criteria are the most prevalent, there are a number of others, each with their proponents. Stress-based criteria find their greatest success in the HCF (long-lives) regime where nominal stresses remain in the elastic range and plastic straining effects remain very localized.

Brown and Miller [3-28] note that a common feature of all biaxial HCF criteria is that they contain two constants. For example, in the case of torsional and bending loading systems the constants, t and b, determined experimentally from separate tests, are present. But for bending tests the value of b depends on the physical size of the specimen and will vary from one test situation to another. Standard push-pull fatigue tests do not exhibit this size dependent behavior. Brown and Miller's comment does not bode well for the eventual development of a completely general criteria.

3.1.1.2 Strain Based Criteria

Strain-based life prediction methods gained momentum in the 1960's with increased interest in the LCF regime. Again, the use of an equivalent amplitude, in this case a strain amplitude, is determined from measured or calculated strain histories as an argument to enter an ϵ-N curve to determine fatigue crack initiation life. On one hand, strain-based methods are inherently better suited for applications where loading is such that relatively large regions of plasticity are involved. On the other hand, this requires a knowledge of the cyclic plastic behavior of the material and component including variation in Poisson's Ratio, υ , with strain level. Thus while strain-based methods may be more realistic, strain-based life predictions are not as tractable for engineering purposes as stress-based methods.

Application of Tresca and Von Mises criteria, written in terms of strains rather than stresses, is an obvious starting point. The Tresca method assumes that the maximum shearing strain amplitude under multiaxial conditions will correlate fatigue life with that determined from the maximum shearing strain in uniaxial tests. For bending and torsion this is given by Tipton and Nelson [3-17] as:

$$\gamma_{max} = \left[\epsilon_{xx}^{2} \; (1+ \;)^{2} + \;{}_{xy}^{2} \right]^{1/2} \quad \text{(in-phase loading)}$$

$$_{max} = \text{greater of} \left\{ \begin{matrix} \gamma_{xy} \\ \epsilon_{xx} \; (1+\upsilon) \end{matrix} \right\} \quad \text{(90° out-of-phase loadings)}$$

where γ_{xy} = torsional shear strain amplitude

ϵ_{xx} = bending strain amplitude

υ = Poisson's Ratio

The equivalent strain amplitude is given by:

$$\epsilon_{a} = \;{}_{max}/(1+\upsilon)$$

In the case of a standard SAE uniaxial bending-torsion fatigue specimen, the fatigue strain life for a normalized 1045 steel is given by Tipton and Nelson as:

$$\epsilon_{n} = 0.00481(2N_{f})^{-0.102} + 0.182(2N_{f})^{-0.433}$$

where ϵ_{a} = strain amplitude

$2N_{f}$ = cyclic reversals to failure (two reversals per cycle)

Results are shown in Figures 3-3 through 3-5.

Note that the coefficients and exponents in the above equation, known as the "strain-life" equation, are material-specific and at this time are available for only a limited number of materials.

Tipton and Nelson have also evaluated a Von Mises strain-based approach under assumptions of $\epsilon_{yy} = -\upsilon\epsilon_{xx}$, Figure 3-4, and $\upsilon = \overline{\upsilon}$, Figure 3-5, where $\overline{\upsilon}$ is a variable Poisson's Ratio calculated by Gonyea's method [3-29]. The effective strain used for entering the uniaxial ϵ-N curve is given by:

$$\epsilon = \frac{1}{\sqrt{2(1+\upsilon)}} \left[(\epsilon_{1}-\epsilon_{2})^{2} + (\epsilon_{2}-\epsilon_{3})^{2} + (\epsilon_{3}-\epsilon_{1})^{2} \right]^{1/2}$$

where ϵ_{1}, ϵ_{2}, ϵ_{3} are principal strain amplitudes and υ is Poisson's Ratio = .29 or .50.

Tipton and Nelson comment on several problems that are characteristic of fatigue testing. First, there is not a

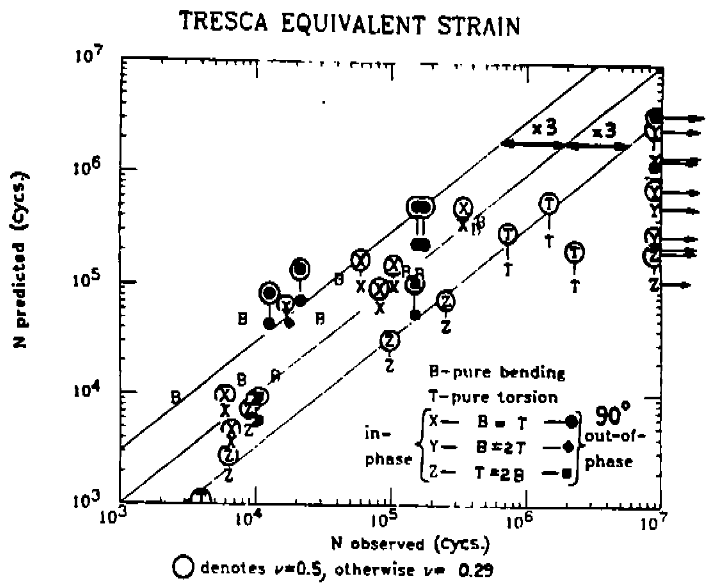


Figure 3-3. Comparison of Life Predictions Based on the Tresca Equivalent Strain Criterion with SAE Test Data [3-17]

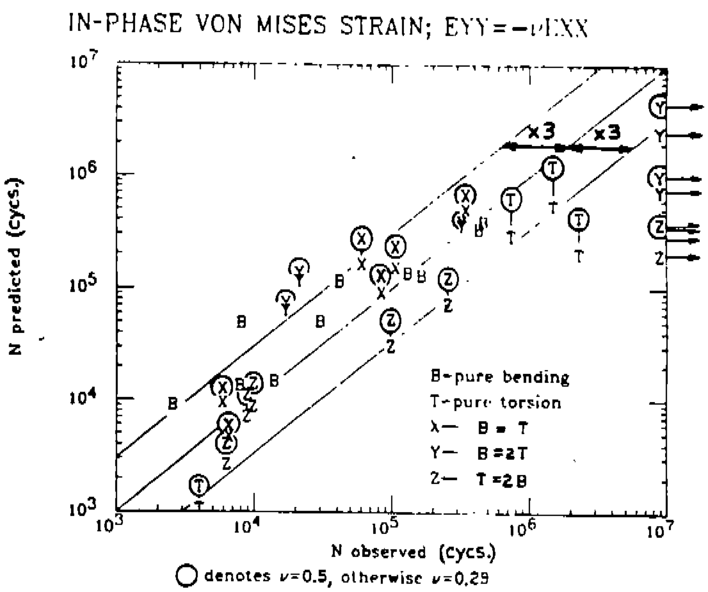


Figure 3-4. Comparison of Life Predictions Based on the Von Mises Equivalent Strain Criterion with In-Phase SAE Test Data, Taking Circumferential Notch Strain as Poisson's Ratio times Notch Bending Strain [3-17]

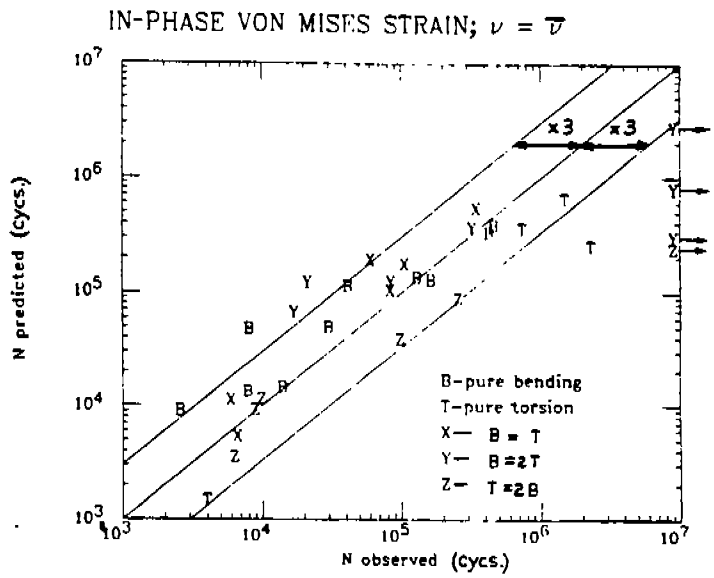


Figure 3-5. Comparison of Life Predictions Based on the von Mises Equivalent Strain Criterion with In-Phase SAE Test Data, Using a Variable Poisson's Ratio [3-17]

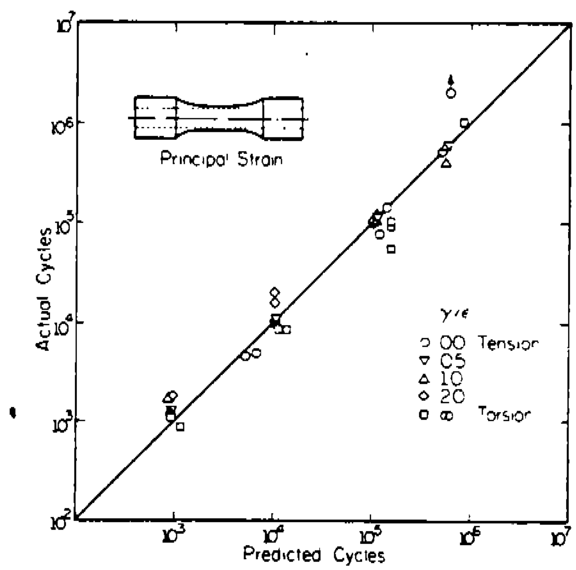


Figure 3-6. Actual versus Predicted Lives: Maximum Principal Strain Theory [3-8]

commonly accepted standard for the size of a crack that constitutes initiation. A crack detectable by visual inspection is one definition. In other cases, optical microscopes are used or ultrasonic acoustic transducers may be employed. 0.1mm and 1mm crack lengths are other definitions which are used. "Life" may be life to crack initiation, or life to failure which may be defined as a percentage drop in load-carrying capability. Ten percent, twenty percent and one hundred percent drop-offs are often used. Tipton and Nelson use a fifty percent drop-off in Reference [3-17].

A further difficulty in using uniaxial data is that, depending on the loading modes, their relative amplitudes and the multiaxial strain state, the type of cracking experienced by a multiaxial specimen may be quite different than that experienced by the uniaxial specimen under "equivalent" conditions.

It may well be that application of equivalent uniaxial data to predict multiaxial fatigue phenomena will be, at best, an interim procedure which will be useful until more general multiaxial criteria are developed together with specifically applicable supporting material data.

The use of a constant value for Poisson's Ratio is also a problem which must be dealt with when large plasticity or high temperatures are present. Gonyea [3-29] has proposed an iterative method for determining a variable value for Poisson's Ratio.

Brown and Miller [3-30 through 3-32] have presented an important strain-based multiaxial fatigue life criterion which is a natural extension of Guest's earlier work. The approach is one of several known as "critical plane" methods. It assumes that initiation is dominated by the amplitude of the maximum shear strain, γ_{max}, and that propagation is strongly influenced by the strain, ϵ_n, normal to the plane on which γ_{max} is acting, a plane which they called the T-plane. The general form of the criterion they recommended formed a family of ellipses with life as a parameter.

$$\left(\frac{\gamma_{max}}{g}\right)^j + \left(\frac{\epsilon_n}{h}\right)^j = 1$$

The specific application is to strain-controlled in-phase combined tension and torsion tests, with experimentally determined values of g, h and j which were found to vary with life. Figure 3-6 shows these results.

Fash et al [3-8] conducted experiments on a thin-walled tube specimen in tension-torsion and on a solid notched shaft in bending-torsion and compared results with predictions based on five different theories. They concluded that:

1. maximum principal multiaxial strain correlates life with maximum principal uniaxial strain (Figure 3-6);

2. the effective strain (Von Mises) may be used to obtain life from uniaxial data (Figure 3-7);

3. maximum multiaxial shear strain may be correlated by converting uniaxial data into shear strains (Tresca method) (Figure 3-8);

4. a theory proposed by Lohr and Ellison [3-33] is based on the maximum shear strain, γ , which occurs on a "critical plane" which in the case of tension-torsion loading of thin-walled tubes occurs on planes intersection the surface at 45°. The maximum shear strain range, γ , is modified by a fraction of the strain normal to the critical planes, ϵ_n (Figure 3-9);

5. a theory proposed by Kandil, Brown and Miller [3-34], essentially an extension of the earlier Brown and Miller work [3-31, 3-32], is similar in appearance to that of Lohr-Ellison but the methods differ in the definition of the shear strain and normal strain components (Figure 3-10). Lohr and Ellison and Kandil, Brown and Miller approaches yield essentially equivalent correlations of thin-walled tube specimen fatigue life.

The relationship between fatigue life and <u>applied strain amplitude</u> for uniaxial tests is given by the <u>strain-life equation</u>:

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_f'}{E} (2N_f)^b + \epsilon_f' \ (2N_f)^c$$

where $\Delta\epsilon$ = axial strain range

σ_f' = fatigue strength coefficient

ϵ_f' = fatigue ductility coefficient

b = fatigue strength exponent

c = fatigue ductility exponent

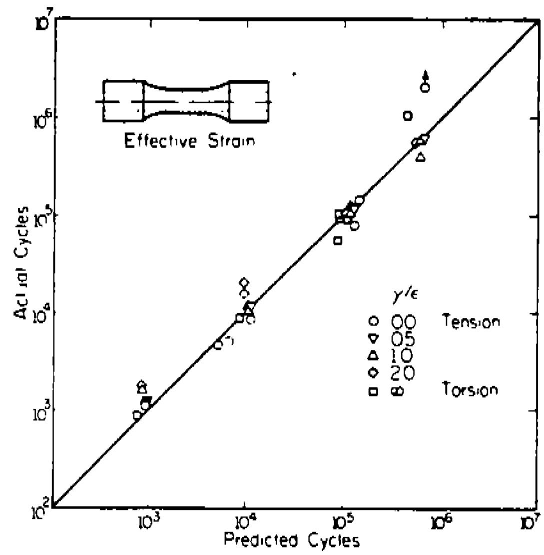


Figure 3-7. Actual versus Predicted Lives:
Effective Strain Theory [3-8]

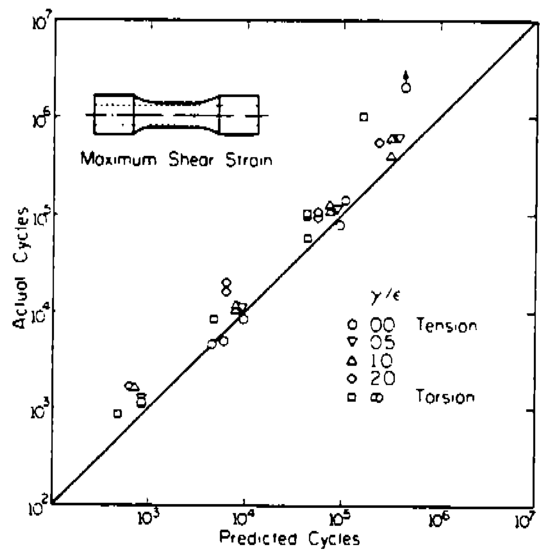


Figure 3-8. Actual versus Predicted Lives:
Maximum Shear Strain Theory [3-8]

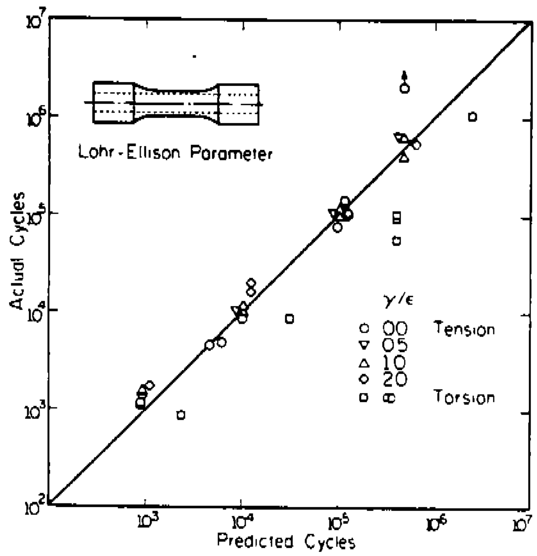


Figure 3-9. Actual versus Predicted Lives:
Lohr and Ellison Theory [3-8]

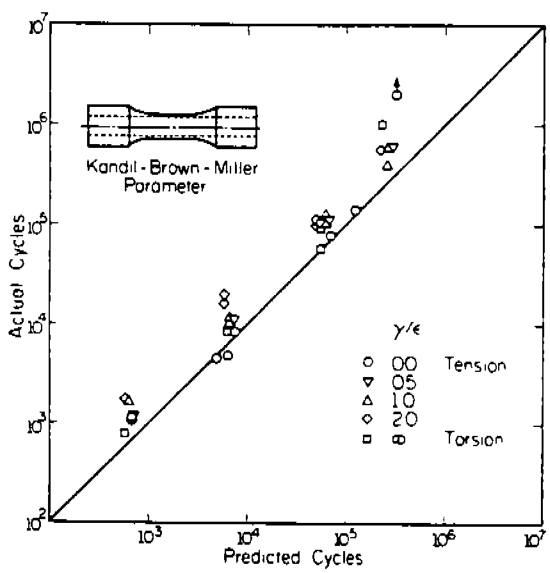


Figure 3-10. Actual versus Predicted Lives:
Kandil, Brown and Miller Theory [3-8]

E = Young's Modulus

N_f = cycles to failure

$2N_f$ = reversals to failure.

Multiaxial strains as determined in the above theories are equated to the strain life equation above with coefficients and exponents appropriate to the material. For the work of Fash et al, failure was defined as a 10% load drop-off for the thin-walled tube tests and the occurrence of a 1 mm crack for the notched shaft specimen tests.

In the uniaxial test the maximum principal strain and amplitude is equal to the applied axial strain amplitude. Life estimates based on the <u>range of maximum principal strain</u> are based on a similar form of the strain-life equation:

$$\frac{\Delta\epsilon'}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c$$

where $\Delta\epsilon'$ is the range of maximum strain.

Effective (Von Mises) strains are given by:

$$\bar{\epsilon} = \frac{2}{3}\left[(\epsilon_1-\epsilon_2)^2 + (\epsilon_2-\epsilon_3)^2 + (\epsilon_1-\epsilon_3)^2\right]^{1/2} \qquad (\upsilon = .5)$$

then $$\frac{\Delta\bar{\epsilon}}{2} = \frac{\sigma_f}{E}(2N_f)^b + \epsilon_f(2N_f)^c$$

For uniaxial data:

$$\frac{\Delta\gamma_{max}}{2} = (1+\upsilon)\frac{\Delta\epsilon}{2}$$

For multiaxial loadings, the maximum shear strain from Mohr's circle is:

$$\frac{\Delta\gamma_{max}}{2} = \Delta(\epsilon_1-\epsilon_3) \qquad (\epsilon_1 \geq \epsilon_2 \geq \epsilon_3)$$

Substitution of υ=.3 and υ=.5 for elastic and plastic values of Poisson's Ratio gives the <u>maximum shearing strain (Tresca) criterion</u>:

$$\frac{\Delta\gamma_{max}}{2} = 1.30 \frac{\sigma_f'}{E} (2N_f)^b + 1.50\ \epsilon_f'(2N_f)^c$$

For the Lohr and Ellison criterion:

$$\frac{\Delta\gamma^*}{2} = \Delta\ (\epsilon_1-\epsilon_2)$$

and $$\epsilon_n^* = \frac{\Delta\ (\epsilon_1+\epsilon_2)}{2}$$

where the asterisk is used to designate the Lohr and Ellison parameters, substituting appropriate values of Poisson's Ratio gives the Lohr and Ellison form of the strain-life equation.

$$\frac{\Delta\gamma^*}{2} + .4\ \epsilon_n = 1.44\ (2N_f)^b + 1.60\ \epsilon_f\ (2N_f)^c$$

The Kandil, Brown and Miller criterion differs in the definition of ϵ_n and adds a term modifying the maximum shear strain range, $\Delta\gamma$ max.

$$\epsilon_n = \Delta \frac{(\epsilon_1-\epsilon_3)}{2}$$

Substituting appropriate values of Poisson's Ratio yields the Kandil, Brown and Miller variant of the strain-life equation.

$$\frac{\Delta\gamma_{max}}{2} + k\epsilon_n = 1.65 \frac{\sigma_f'}{E} (2N_f)^b + 1.75\ \epsilon_f'\ (2N_f)^c$$

A comparison of actual lives versus the predicted lives by the five theories is shown for the smooth thin-walled tube tests in Figures 3-6 through 3-10. Correlation for all five methods falls within a scatter band of three of the perfect correlation line, except for the torsion data which is attributed by the authors to anistropy measured in the 1045 steel specimens. The Lohr and Ellison theory appears to give the slightly better results but not enough to justify any strong preferences.

The results for the notched specimen tests are shown in the Fash et al paper but are not included here. Correlations based on all five theories were rather poor and unconservative showing that there is still much to understand about the effect of stress concentrations in notches.

At this point it appears that the Kandil, Brown and Miller approach and the Lohr and Ellison approach are emerging at the head of the pack in multiaxial fatigue theories, thus the emphasis placed in the preceding paragraphs. Leese [3-35] in a recent state-of-the-art review, shows the difference in the critical planes assumed in the two theories on a Mohr's circle representation of the three-dimensional strain state, Figure 3-11. Qualitatively, the difference between the two theories is that Lohr and Ellison reflect tendencies for the crack to be driven through the specimen whereas the Kandil, Brown and Miller approach reflects the strains which drive surface cracking.

Socie, Waill and Dittmer [3-36] conducted standard uniaxial tests using Inconel 718 specimens and biaxial strain-controlled tests under: (a) axial loading only; (b) torsional loading only; and (c) combined axial and torsional loading. Results of the uniaxial tests are given in Figure 3-12 for three different criteria for failure: occurrence of a crack 0.1mm long, occurrence of a crack 1.0mm long, and separation of the specimen. For this material, strain at the .1mm crack level was 15% of that at separation, while strain at the 1.0mm crack was 90% of that at separation. Socie et al also examined modifications to the Lohr and Ellison parameter and to the Kandil, Brown and Miller parameter for the effect of mean stress, an important consideration in ship fatigue. Both modified theories gave reasonably good correlations, but Lohr and Ellison was slightly the better. The comparison is shown in Figure 3-13. The constants in the Lohr and Ellison equation may be obtained from uniaxial tests.

Brown and Miller [3-37] have identified two distinct cracking systems which they define as Case A and Case B, each of which gives rise to different fatigue lives. The two cases are depicted schematically in Figure 3-14. In Case A, strains ϵ_1 and ϵ_3 are parallel to the surface (as in the case of combined tension and torsion) and tend to generate shallow surface cracks. Under other loadings ϵ_3 is normal to the surface tending to drive deep cracks. This is indicated in Case B. Different functions were proposed for each case.

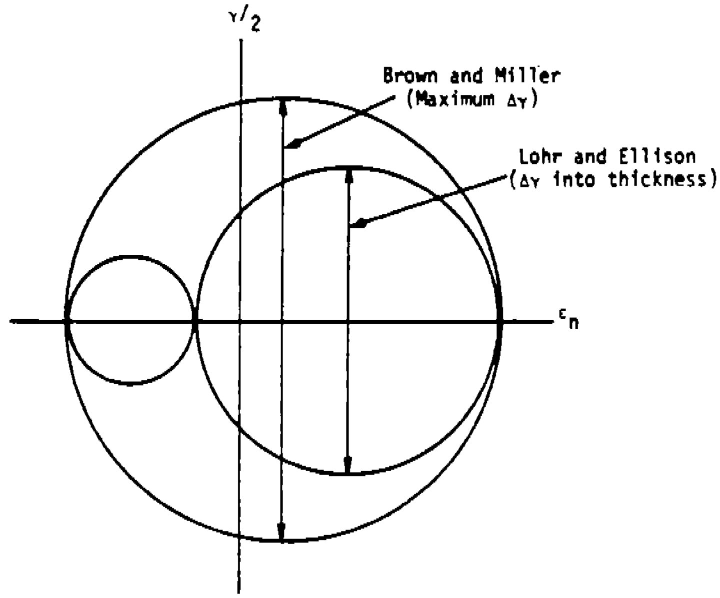


Figure 3-11. Difference in Critical Planes of Brown and Miller and of Lohr and Ellison for Tension/Torsion Loading on Mohr's Circles of Strain (from Ref. [3-35])

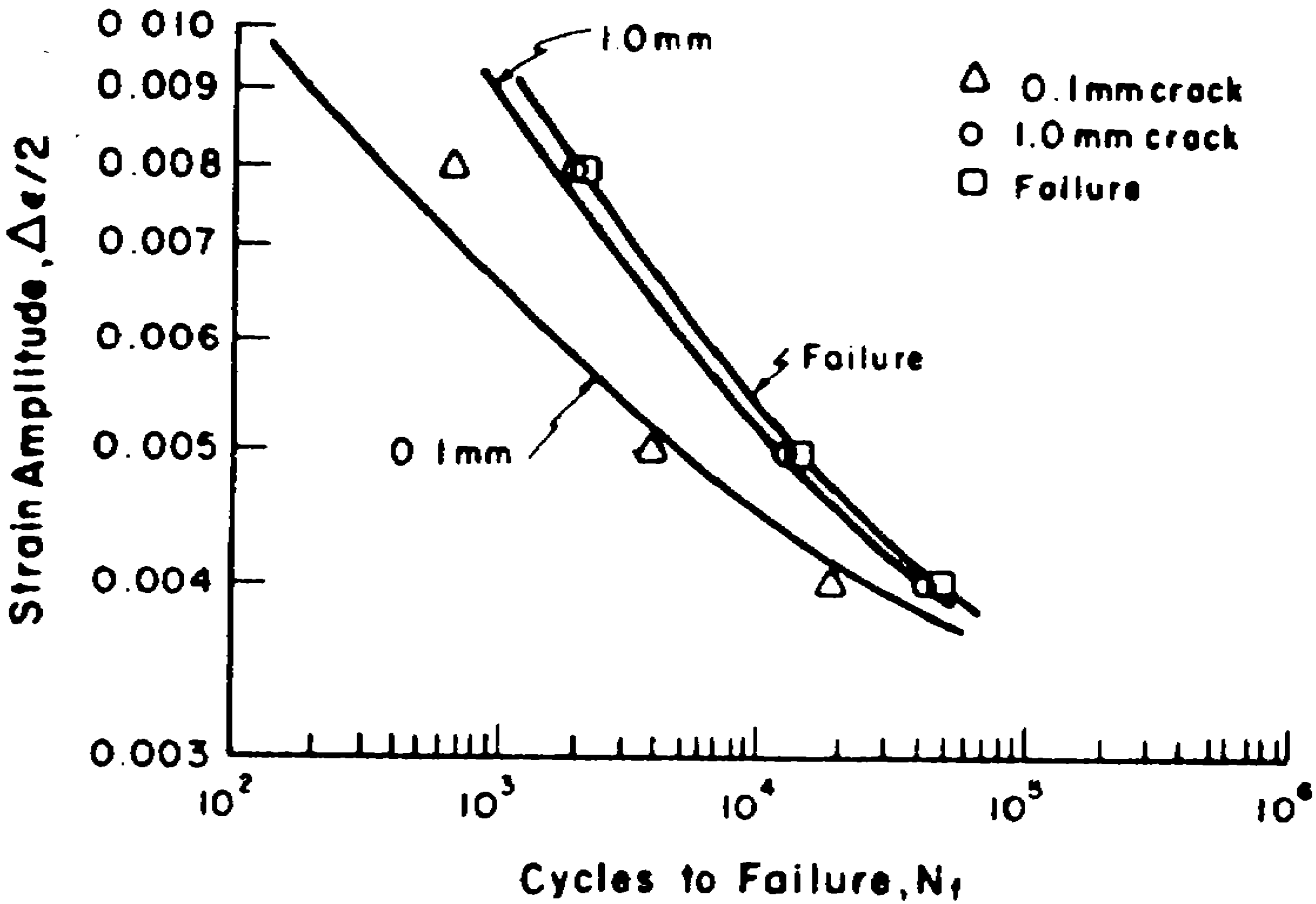


Figure 3-12. Uniaxial Test Results from Solid Specimens

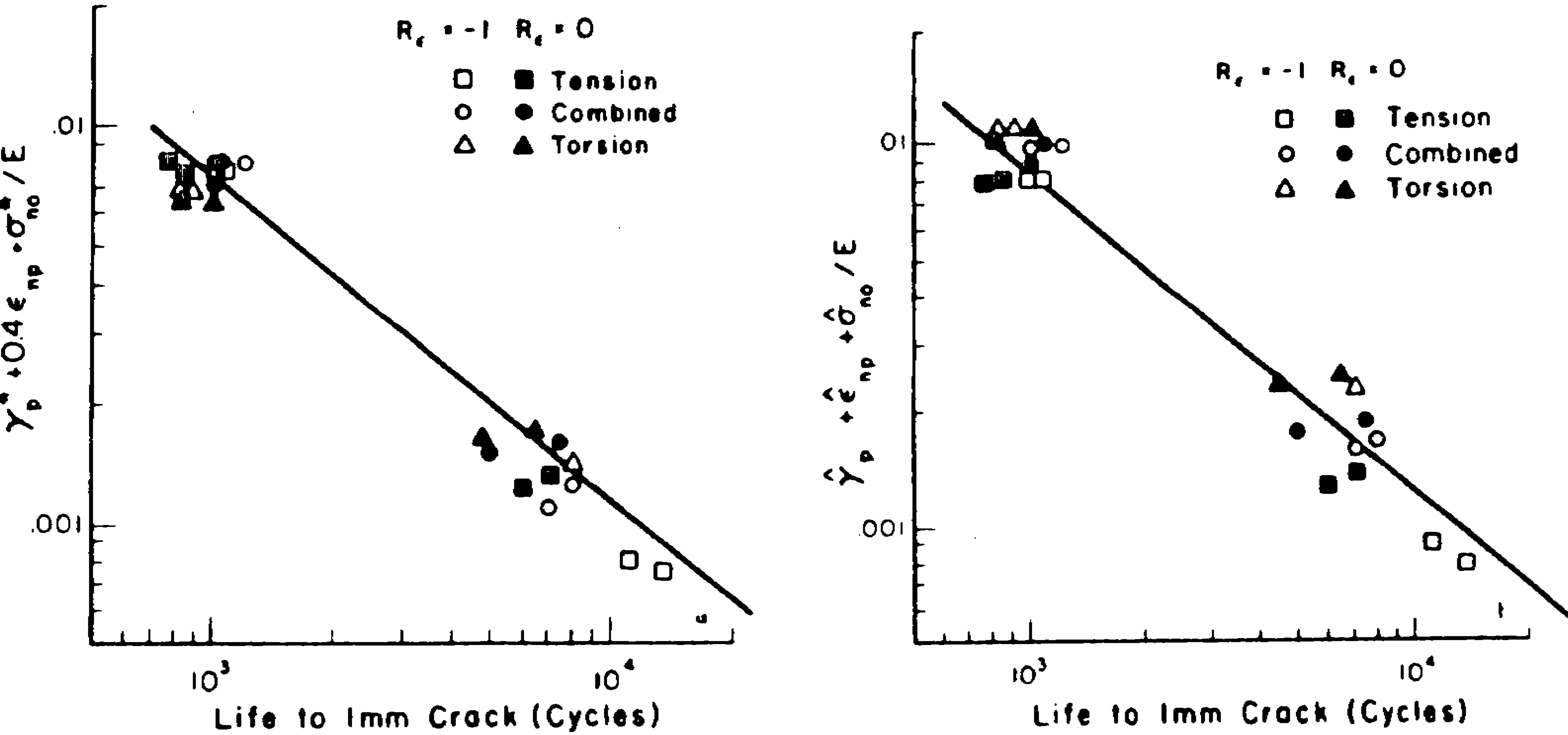


Figure 3-13. Correlation of Test Data Including Mean Stress Effects (a) Lohr and Ellison Parameter and (b) Kandil, Brown and Miller Parameter

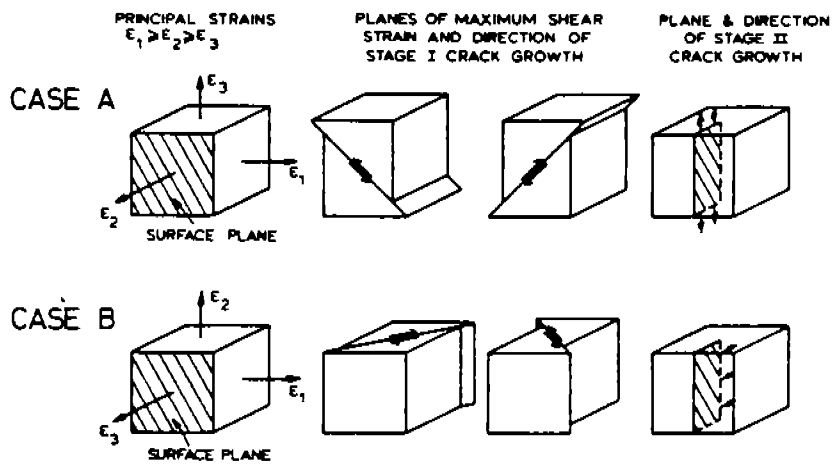


Figure 3-14. Stage I and Stage II Crack Growth Systems Under General Multiaxial Cyclic Strains

Other authors also have made contributions in the area of multiaxial strain-based methods. Konter, Janssen and Husslage [3-38] have presented a quadratic method for equivalent strain range using plastic strain components:

$$\epsilon_{eq}^2 = (16-9\beta) \quad \epsilon_n^2 + \beta\ (\gamma_{max}/2)^2$$

Different values of the constant, β, apply to Cases A and B. Zamrik and Frishmuth [3-39] proposed use of maximum total strain to correlate out-of-phase fully reversed bending-torsion tests:

$$\epsilon_T = \sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2}$$

Grubsic and Simburger [3-24] proposed examining all planes in a body and selecting the most unfavorable combination of mean and alternating shear stress on each plane as the basis for a life criterion.

3.1.1.3 Plastic Work Methods

The difficulties associated with extending stress-based or strain-based criteria to the more general cases of cumulative damage under random loadings of mixed frequency and phase has led a number of investigators to consider methods in which the material's resistance to fatigue under multiaxial loading conditions can be related to the work expended in each stress-strain cycle. Early foundations for energy approaches were laid by Feltner and Morrow [3-40], Morrow [3-41] and Halford [3-42] during the 1960's. In 1977, Leis [3-43] proposed a damage parameter utilizing strain energy per cycle which was intended to account for multiaxiality of stress and stain, mean stress, and viscous deformation response. The method assumed isothermal stress-strain cycles of constant strain amplitude obtained from uniaxial tests. Using existing published data, Leis obtained an excellent data collapse for uniaxial data using a log energy versus a log life plot, but the collapse of biaxial data was much less impressive. Leis' definition of failure was the termination of Stage I crack growth. In 1981, Garud [3-44] proposed an energy method, again based on evaluation of the plastic work per strain cycle, but now introducing a modified strain hardening method into the stress-strain constitutive relations for the material. Garud used the axial-torsional data of Kanazawa, Miller and Brown [3-45] to correlate his results. Figure 3-15 shows the uniaxial stress-strain curve used to determine the plastic work per cycle. Figure 3-16 shows log plastic energy versus log life for the various tests in the Kanazawa data. Note that the axial test data falls on the lower bound and torsional data falls on the upper bound of a factor-of-three scatter band with the phase angle biaxial

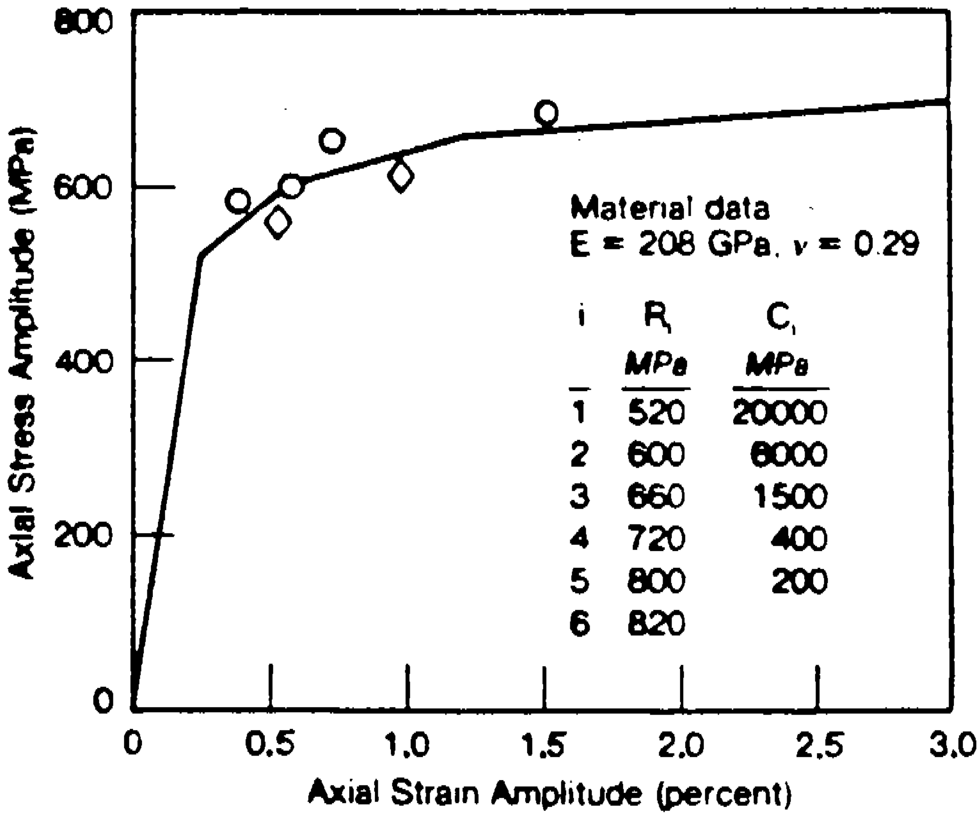


Figure 3-15. Uniaxial Stress-Strain Approximation for 1 Percent CR-Mo-V Steel (segments 4, 5 and 6 extrapolated; segments 5 and 6 not shown)

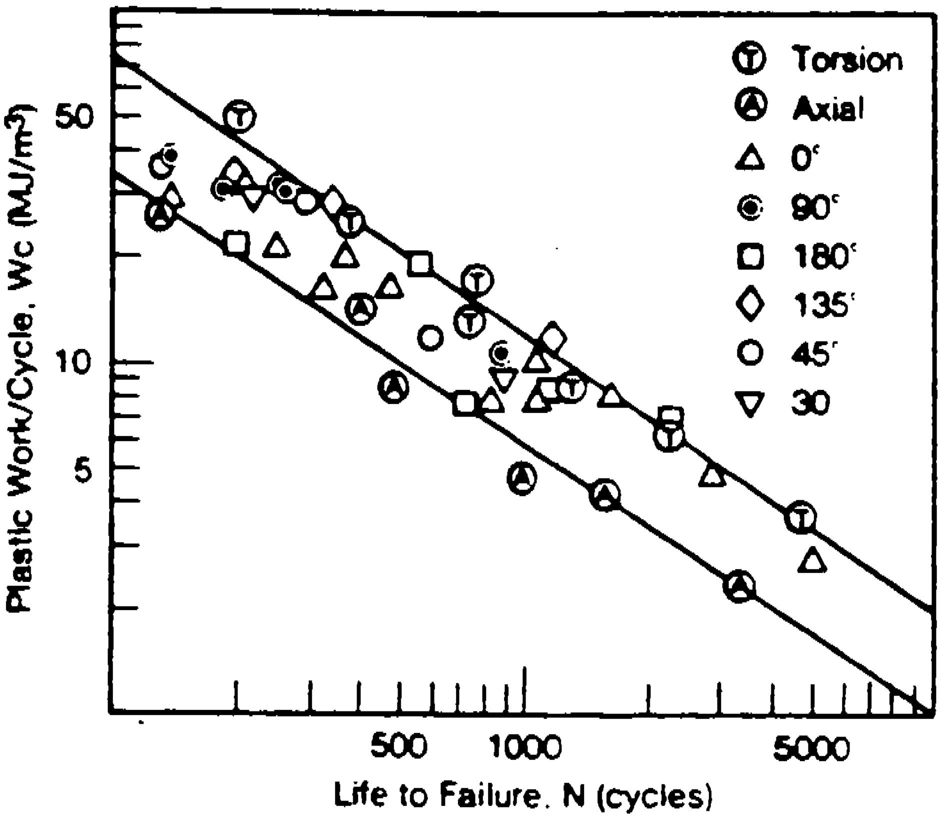


Figure 3-16. Plastic Work Per Cycle (Calculated) Versus Life to Failure (Observed)

tests generally falling in between. When a weighing factor of ½ was applied to the plastic work done by the shear straining, the correlation improved as shown in Figure 3-17. The effect of phase angle is shown in Figure 3-18. At the higher axial strains (above the point C), the 90° out-of-phase loading absorbs the most plastic work, confirming the findings of other investigators that this condition is the most damaging for this type of loading. Garud states that hydrostatic stress and mean stress effects may be included in the method; however, strain rate and creep effects are excluded.

Ellyin and his colleagues [3-46 through 3-49] have also worked on the development of energy based multiaxial fatigue criteria throughout the 1980's. The criterion proposed in Ellyin's recent paper, Reference 3-49, is based on the theory that the damage caused as a result of cyclic loading is a function of the mechanical work input to the material. The total work per cycle, ΔW^t, is partitioned into elastic, W^e, and plastic, ΔW^p, components with separate formulations for each. The elastic and plastic components are depicted graphically for a uniaxial cyclic loading case in Figure 3-19. The Ellyin criterion includes the effect of hydrostatic pressure and mean stress but in its present form is limited to proportional or nearly proportional loadings in the biaxial mode. This implies that the principal stresses are nearly constant in direction through the cycle. Experimental results for two strain ratios are required to determine the constants in Ellyin's equation; however, these could be determined from relatively simple tests such as a standard uniaxial push-pull fatigue test and a simple torsion fatigue test. Results of such tests are generally available for a wide range of materials. Ellyin and Golos have applied the criterion to correlate test data obtained from thin-walled tube tests with alternating internal pressure. Results for four different strain ratios are shown in Figure 3-20.

Plastic strain energy methods have great appeal because of their intrinsic flexibility. However, there are practical considerations. They are basically better suited for the LCF regime where high straining is present. For the HCF regime where straining levels are much lower, the amount of plastic work is low and much more difficult to quantify. An assumption present in plastic work methods is that, after the first few cycles of straining, the constitutive relationships remain constant. For materials or under loading conditions for which this assumption is less realistic, the method would be much more complex. A summary of the multiaxial fatigue life approaches is presented in Table 3-1.

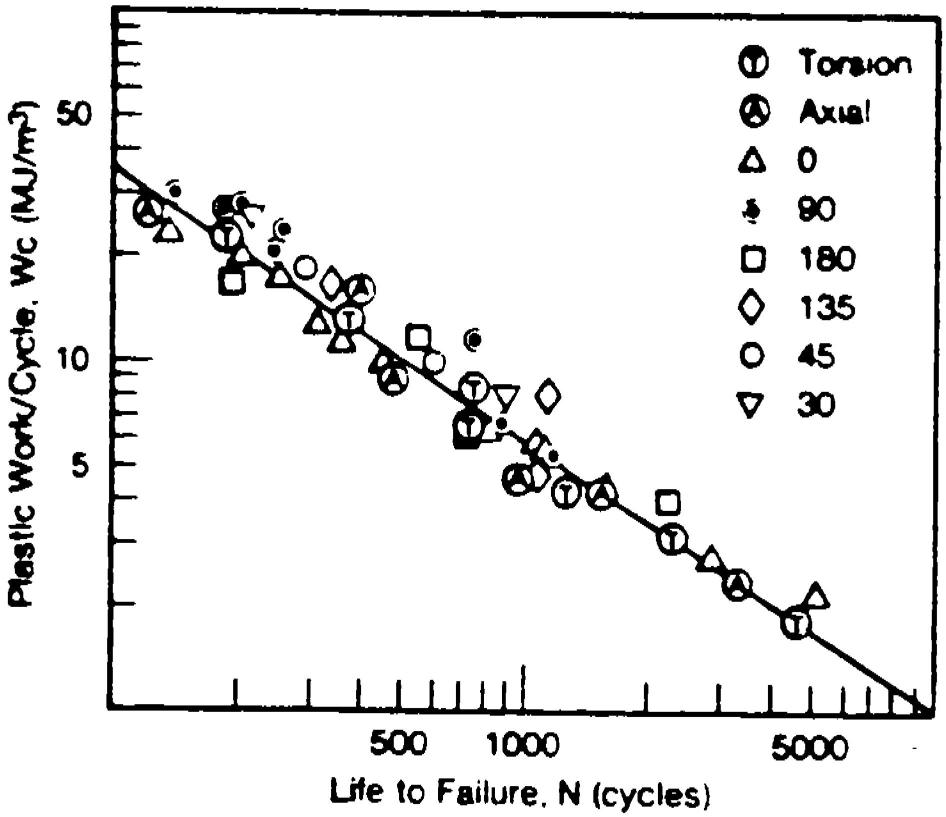


Figure 3-17. Plastic Work Per Cycle (Calculated with a Weighing Factor on Shear Work) versus Life to Failure (Observed)

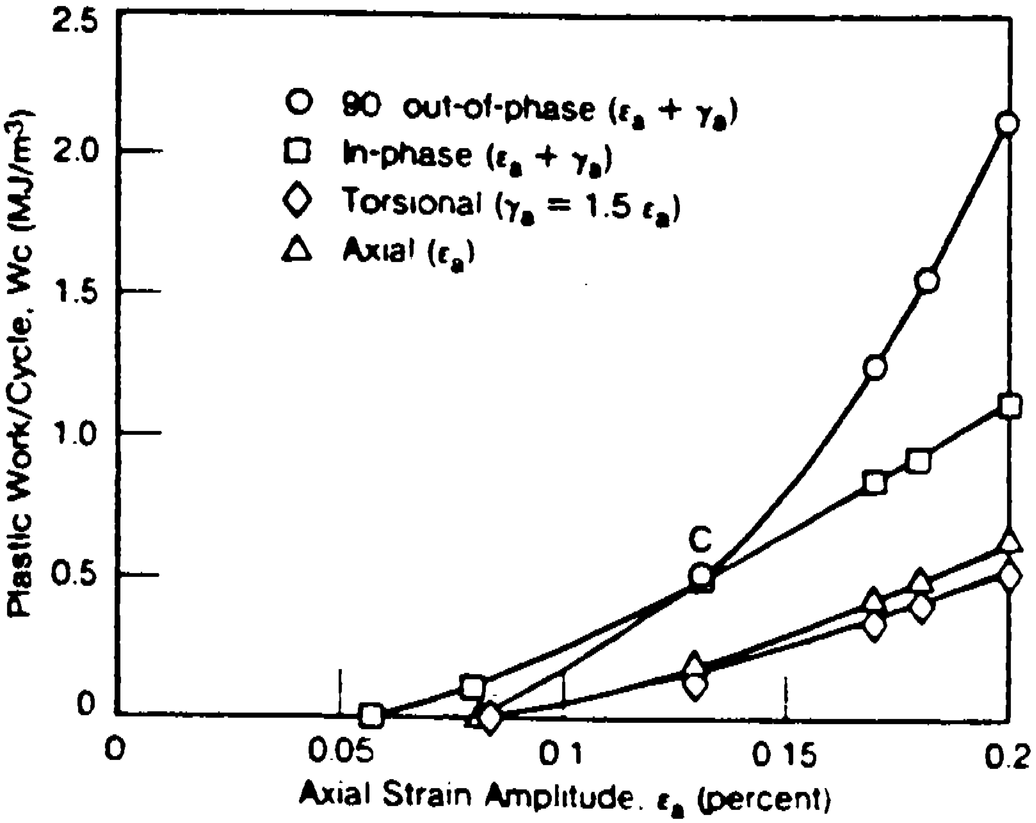


Figure 3-18. In-Phase vs. Out-of-Phase Straining (Comparison)

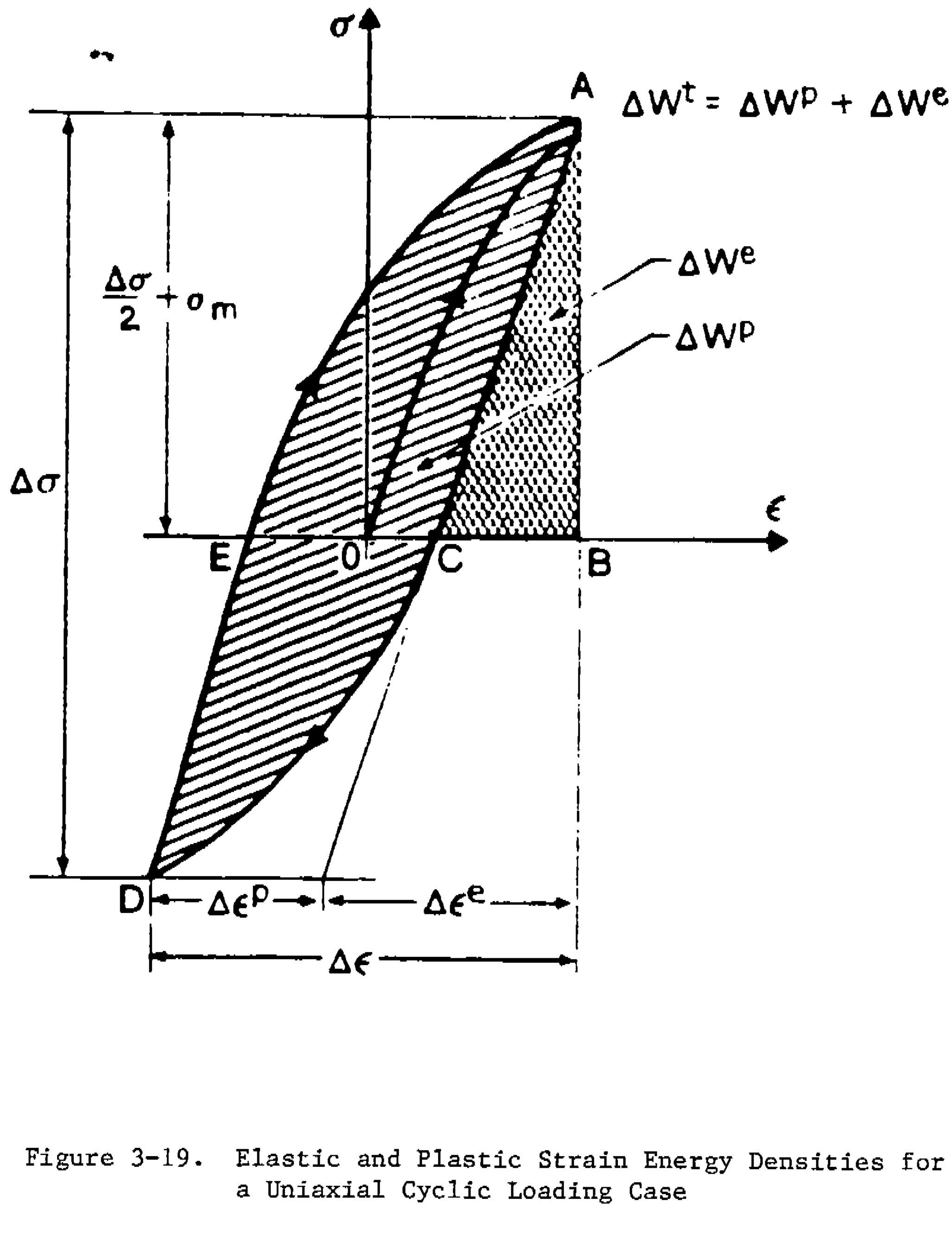


Figure 3-19. Elastic and Plastic Strain Energy Densities for a Uniaxial Cyclic Loading Case

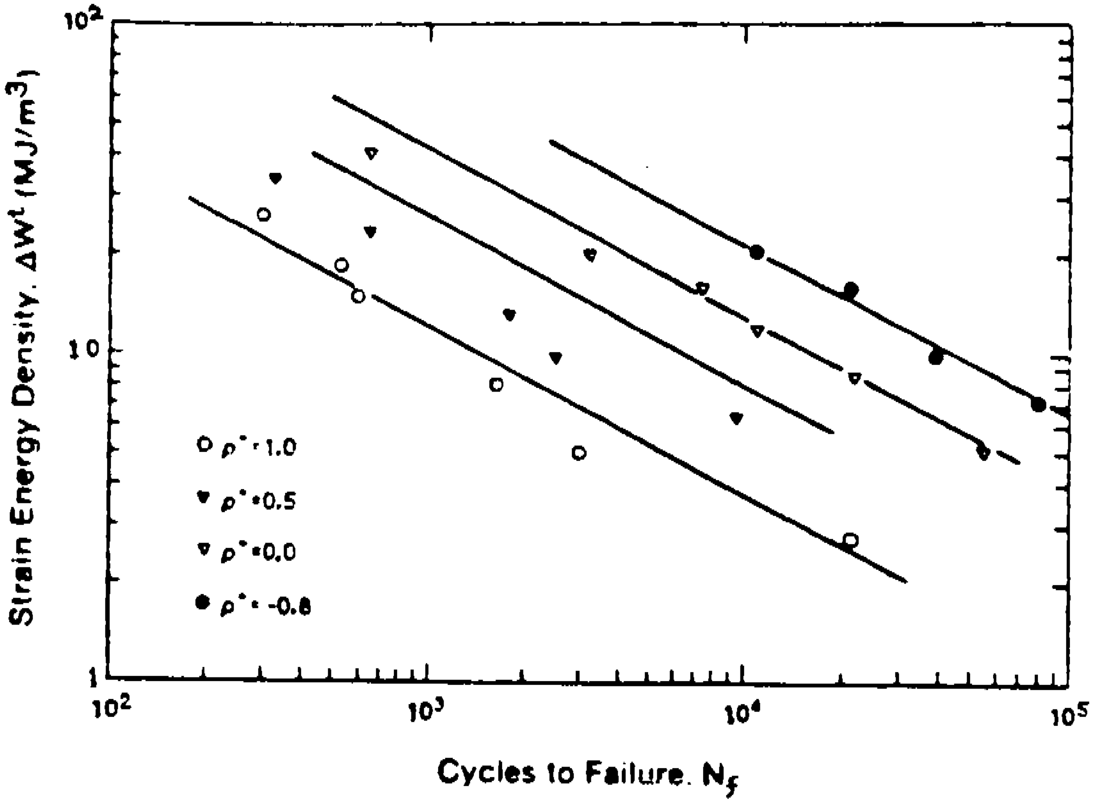


Figure 3-20. The Predicted Strain Energy Density W^t versus the Number of Cycles to Failure, N_f, for Various Strain Ratios

TABLE 3-1

SUMMARY OF MULTIAXIAL FATIGUE APPROACHES FOR PREDICTING FATIGUE LIFE BASED ON CUMULATIVE DAMAGE APPROACHES

APPLICABLE MAF APPROACHES		REMARKS	REF
Equivalent Stress Methods			
I. Von Mises methods	$\tau \text{ octahedral} = 1/3\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right)^{1/2} = \text{constant}$	Biaxial von Mises stress correlates with uniaxial von Mises stress at failure. Applicable to HCF regime.	3-1
II. Tresca methods	$\tau_{max} = \sigma_1-\sigma_3/2 = \text{constant}$	Biaxial Tresca stress correlates with uniaxial Tresca stress at failure. Applicable to HCF regime.	3-1
III. Guest method	$\frac{\sigma_1-\sigma_2}{2} k \frac{\sigma_1+\sigma_2}{2} = \text{constant}$	Maximum shear stress modified by a fraction of normal stress amplitude predicted using uniaxial fatigue limits. Applicable to HCF regime.	3-1
Strain-based methods			
IV. Maximum shear strain method	$\Delta_\gamma \max = \left[\epsilon_{xx}^2 (1+\upsilon)^2 + \gamma_{xy}\right]^{1/2}$	Maximum biaxial shear strain is correlated with maximum shear strain in uniaxial tests. Applicable to LCF and HCF regimes.	3-8
V. Effective strain method	$\epsilon a = \Delta \max/(1+\upsilon)$	von Mises strain in biaxial tests is correlated with von Mises strain in uniaxial test. Applicable to LCF and HCF regimes.	3-8
VI. Maximum principal strain	$\epsilon = \frac{1}{\sqrt{2}\,(1+\upsilon)} \left[(\epsilon_1-\epsilon_2)^2 + (\epsilon_2-\epsilon_3)^2 + (\epsilon_3-\epsilon_1)^2\right]^{1/2}$	Maximum principal strain under biaxial conditions correlates with maximum principal strain in uniaxial test. Applicable to LCF and HCF regimes.	3-8

TABLE 3-1 (continued)

SUMMARY OF MULTIAXIAL FATIGUE APPROACHES FOR PREDICTING FATIGUE LIFE BASED ON CUMULATIVE DAMAGE APPROACHES

APPLICABLE MAF APPROACHES		REMARKS	REF
<u>Critical plane methods</u>			
VII. Lohr-Ellison method	$\frac{\Delta\gamma}{2} = \Delta(\epsilon_1-\epsilon_2)$ $\epsilon_n = \Delta\frac{(\epsilon_1-\epsilon_2)}{2}$	Fatigue life is predicted using maximum shear strain range on a plane 45° to the free surface and a fraction of the normal strain to that surface. Exponents in relationship are determined from uniaxial tests.	3-8
VIII. Kandil Brown and Miller method	$\frac{\Delta\gamma}{2} = \Delta(\epsilon_1-\epsilon_3)$ $\epsilon_n = \Delta\frac{(\epsilon_1-\epsilon_3)}{2}$	Fatigue life is predicted using maximum shear strain range on the plane of maximum shear strain and a fraction of the normal strain on that plane. Exponents determined from uniaxial tests.	3-8
<u>Energy methods</u>			
IX. Plastic work method, Ellyan	$\Delta\bar{\sigma}\,\Delta\bar{\epsilon} = KN_f^{\alpha}$	Failure is predicted by estimating the amount of work done on the biaxial specimen to cause failure as determined from integration of the stress-strain hysterisis loops. Material properties are determined from uniaxial tests. Applicable primarily to LCF regime.	3-4

3.1.2 Fatigue Crack Propagation Research

General research on fatigue crack propagation using fracture mechanics has been reviewed and is presented here. While few approaches have been proven for incorporation into design procedures, they do provide insight for future directions in crack growth studies.

3.1.2.1 Observations of Mixed Mode Crack Growth

Smith and Pascoe [3-50] show that in HY100 steel, cracks can either initiate and grow in shear (Stage I-Mode II) or change to grow in the opening mode (Stage II-Mode I) under different stress status. They tested thick plate specimens under the applied strain conditions in Table 3-2.

Under Mode II dominated conditions, crack bifurcation was observed which had a significant effect on crack growth rates as shown in Figure 3-21. Under these conditions the Paris equation is invalid. They also noted that crack tip plasticity had a large effect on crack growth rates for HY100. The most significant finding of their study is the stable crack growth under Mode II loading. This finding is attributed to crack tip plasticity of this ductile material. Most literature indicates a lack of full understanding of crack propagation in mixed mode stress/strain fields.

Brown and Miller [3-50] show the importance of crack tip plasticity in controlling crack speed. Crack growth rates are always faster than linear elastic fracture mechanics (LEFM) predictions suggest. Their rudimentary approach to predicting propagation rates requires further verification.

The work of Kitagawa et al [3-51] confirms that LEFM is only applicable below one-third of yield stress and that at higher stress levels there is a need to account for biaxial effects. They also confirm the observation that long cracks under mixed mode loading branch to follow the direction where K_{II} is zero or, in other words, in a direction normal to the principal stress.

In contrast, Gao [3-52] observed stable mixed mode growth at stress levels close to threshold in four materials. Smith and Pascoe [3-50] show that stable Mode II growth is possible for long cracks and high strain amplitudes. This shows that at least one question still to be resolved is why shear mode cracks should change to Mode I. Again, large-scale yielding (or lack of) in shear planes is the suspected cause.
Mode III crack growth studies were conducted by Ritchie et al [3-53], Hourlier et al [3-54] and Pook [3-55]. The researchers found it difficult to maintain Mode III growth

TABLE 3-2

ANGLE OF CRACK ON SURFACE AS FUNCTION OF STRAIN STATE

APPLIED STRAIN STATE $\xi = \frac{\Delta\varepsilon_2}{\Delta\varepsilon_1}$	CRACK ANGLES FROM PRINCIPAL AXIS
-1 (shear)	45° and 135°
-0.5	55° and 125°
0 (plane strain)	90°
+0.5 (uniaxial strain)	Between 45° and 135°
+1 (equibiaxial)	Any angle

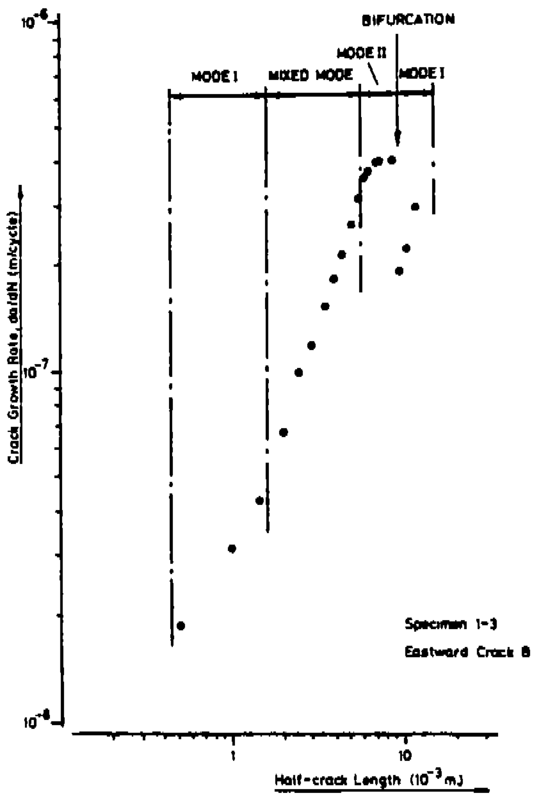


Figure 3-21. Crack Propagation Plotted as Crack Length versus Crack Growth Rate to Compare Mode I, Mixed-Mode, and Mode II Growth

for long cracks except at the highest stress levels where presumably large crack-tip plastic zones permitted Mode III extension. The fracture surface morphology, frequently observed in torsional loading situations, indicates a preference for Mode I cracking. However, the crack path appears to be mean stress and material dependent. Hourlier et al tested structural steel and other materials and suggested that cracks will always seek the mode that can generate the greatest propagation rate, thus adding additional observational information on the direction of crack propagation under mixed mode loading.

3.1.2.2 Prediction of Mixed Mode Crack Growth Rates

The inclined crack, shown in Figure 3-22, has served as a useful configuration for biaxial fatigue testing. Research on inclined and branched cracks is extensive. Kfouri [3-56] presents a review of this research. Nearly all the studies have been in the context of LEFM. Briefly, the main criteria are: (a) the maximum tangential stress criterion; (b) the criterion of local symmetry; (c) the minimum strain energy criterion; and (d) the maximum energy release rate criterion. A criterion based on the maximum normal strain has been proposed and another uses the crack-tip opening displacement (CTOD) to predict the direction of the onset of crack extension.

The work of Erdogan and Sih [3-57] can be considered among the initial efforts in the area of mixed mode cracking. They treated the problem of a plate under uniform tension σ with an angled central crack of length 2a to determine the crack direction. They assumed that the crack grows in a direction θ_o for which the hoop stress, σ_θ, at the crack tip is maximum. Williams and Ewing [3-58] modified this theory by including the non-singular terms in the series expansion for better correlation. Later, Finnie and Saith [3-59] pointed out that the proposed modification neglected the contribution of the normal stress to the crack.

Among the models that were proposed to deal with mixed mode cracking, Swedlow's Model [3-60] was considered as a basic model for the biaxial stress investigation. Swedlow proposed a compression model to account for the crack closing and frictional effect in cracks under compression. He used Williams analysis [3-61] to derive general expressions for the stress distribution near the crack tip. These expressions are:

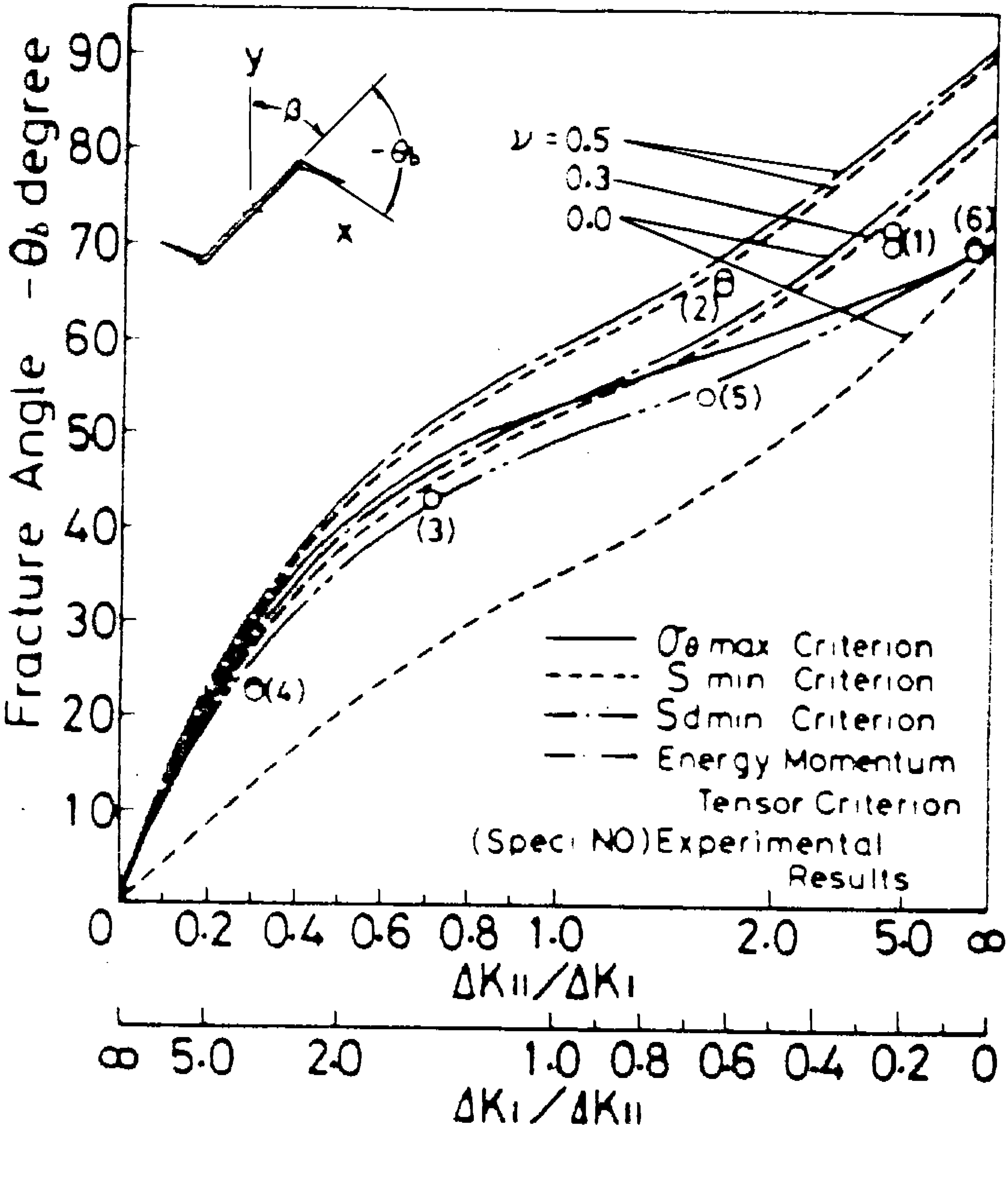


Figure 3-22. Initial Fracture Angle versus the Ratio of Stress Intensity Ranges K_{II} and K_I

$$\sigma_r = \frac{K_I}{\sqrt{2\pi r}} \left[\frac{5}{4} \cos \frac{\theta}{2} - \frac{1}{4} \cos 3 \frac{\theta}{2} \right]$$

$$+ \frac{K_{II}}{\sqrt{2\pi r}} \left[- \frac{5}{4} \sin \frac{\theta}{2} + \frac{3}{4} \sin 3 \frac{\theta}{2} \right] + \sigma \cos^2\theta$$

$$\sigma_\theta = \frac{K_I}{\sqrt{2\pi r}} \left[\frac{3}{4} \cos \frac{\theta}{2} + \frac{1}{4} \cos 3 \frac{\theta}{2} \right]$$

$$- \frac{3K_{II}}{4\sqrt{2\pi r}} \left[\sin \frac{\theta}{2} + \sin 3 \frac{\theta}{2} \right] + \sigma \sin^2\theta$$

$$\tau_{r\theta} = \frac{K_I}{4\sqrt{2\pi r}} \left[\sin \frac{\theta}{2} + \sin 3 \frac{\theta}{2} \right]$$

$$+ \frac{K_{II}}{\sqrt{2\pi r}} \left[\frac{1}{4} \cos \frac{\theta}{2} + \frac{3}{4} \cos 3 \frac{\theta}{2} \right] - \sigma \sin\theta\cos\theta$$

where:

$$K_I = \sigma\sqrt{\pi a}(\sin^2\beta)$$

$$K_{II} = \sigma\sqrt{\pi a}(\sin\beta\cos\beta)$$

$$\sigma = \bar{\sigma}(\cos^2\beta - \sin^2\beta)$$

where a is the semi-elliptical crack length and σ is the applied stress.

To take into account the crack closing and the frictional effect, Swedlow suggested that σ should not be replaced by $-\sigma$ to express a compressive model. He postulated that when K_I has negative values, it means that the crack is closing on itself and by such action, it causes friction on the surface of the crack. Thus, in compression, K_I can be neglected and the equations can be expressed as:

$$\sigma_r = \frac{K_{II}}{\sqrt{(2\pi r)}} \left[\frac{5}{4} - \sin \frac{\theta}{2} + \frac{3}{4} \sin 3 \frac{\theta}{2} \right]$$

$$+ \sigma(\sin^2\theta - \mu_f \sin 2\theta) \sin^2\beta + \sigma_t \cos^{2\theta}$$

$$\sigma_\theta = \frac{3K_{II}}{4\sqrt{(2\pi r)}} \left[\sin \frac{\theta}{2} + \sin 3 \frac{\theta}{2} \right]$$

$$+ \sigma(\cos^2\theta - \mu_f \sin 2\theta) \sin^2\beta + \sigma_t \sin^2\theta$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{(2\pi r)}} \left[\frac{1}{4} \cos \frac{\theta}{2} + \frac{3}{4} \cos \frac{\theta}{2} \right]$$

$$+ \bar{\sigma}(\cos\theta \sin \theta - \mu_f \cos 2\theta) \sin^2\beta + \mu_t \sin\theta\cos\theta$$

where $\sigma = -1$, μ_f is the coefficient of friction, and

$$K_{II} = \bar{\sigma}\sqrt{\pi\alpha}(\sin\beta\cos\beta - \mu_f \sin^2\beta)$$

and

$$\sigma_t = \bar{\sigma}(\cos^2\beta - \sin^2\beta)$$

These equations are applicable along the crack faces. The coefficient of friction may take a static or dynamic value depending on the motion of the crack faces. Using the maximum hoop stress criterion of Erdogan and Sih and taking into account the crack closing and the friction effect, Swedlow was able through his model to predict the initial fracture angle, θ_o, for an inclined crack under uniaxial compression.

Another model related to this problem is one proposed by Woo and Ling [3-62]. They reviewed a number of fracture criteria dealing with angled crack initiation θ_o and its relation to crack inclination angle β. They examined the biaxial loading effect on the parameter used in each criterion for the prediction of the fracture behavior under mixed mode. In their experimental work, they used cruciform type specimens of 1/8 in. thickness with an initial crack length of 1-1/4 in. The material used in their investigation was polymethyl-methacrylate (PMMA). Their analysis was an extension of Swedlow's approach to the case of biaxial tension-compression stressed models. This was accomplished by modifying Swedlow's near-tip stress equations to the following forms:

$$\sigma_r = \frac{K_I}{\sqrt{(2\pi r)}} \left[\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos 3\frac{\theta}{2}\right]$$

$$+ \frac{K_{II}}{\sqrt{(2\pi r)}} \left[-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin 3\frac{\theta}{2}\right] + \sigma_t\cos^2\theta$$

$$+ \sigma_n\sin^2\theta + \sigma_f\sin 2\theta$$

$$\sigma_\theta = \frac{K_I}{\sqrt{(2\pi r)}} \left[\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos 3\frac{\theta}{2}\right]$$

$$- \frac{3K_{II}}{4\sqrt{(2\pi r)}} \left[\sin\frac{\theta}{2} + \sin 3\frac{\theta}{2}\right] + \sigma_t\sin^2\theta$$

$$+ \sigma_n\cos^2\theta - \sigma_f\sin 2\theta$$

$$\tau_{r\theta} = \frac{K_I}{4\sqrt{(2\pi r)}} \left[\sin\frac{\theta}{2} + \sin 3\frac{\theta}{2}\right]$$

$$+ \frac{K_{II}}{\sqrt{(2\pi r)}} \left[\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos 3\frac{\theta}{2}\right] - \sigma_t \sin\theta\cos\theta$$

$$+ \sigma_n \sin\theta\cos\theta + \sigma_f \cos 2\theta$$

where

σ_n = normal compression stress on the crack faces

σ_f = frictional resistance due to the compressive stress $(=\mu_f\sigma_n)$

μ_f = friction coefficient

σ_t = $\sigma(1-\alpha)\cos 2\beta$

They stated that the terms in the above equations will not appear at the same time and the selection of the proper terms for a given problem was listed in a tabulated form as shown in Table 3-3. On the basis of their experimental work, they concluded that the maximum hoop stress criterion was the most appropriate one to use for both the open and closed crack conditions.

Zamrick [3-63] developed a biaxial stress model to predict crack initiation angle, θ_o, with respect to an inclined crack angle β:

$$(\alpha + \tan^2\beta) - (1-\alpha)\left[\frac{1-3\cos\theta}{\sin\theta}\right]\tan\beta$$

Loading conditions \ Selection of terms	K_I	K_{II}	σ_t	σ_n	σ_f
$\alpha<0$	$\sigma\sqrt{(\pi\alpha)}(\sin^2\beta+\alpha\cos^2\beta)$	$\sigma\sqrt{(\pi\alpha)}(1-\alpha)\sin\beta\cos\beta$	$\sigma(1-\alpha)\cos 2\beta$		
$\alpha<0$: $\sigma\sin^2\beta>\lvert\alpha\rvert\sigma\cos^2\beta$	$\sigma\sqrt{(\pi\alpha)}(\sin^2\beta+\alpha\cos^2\beta)$	$\sigma\sqrt{(\pi\alpha)}(1-\alpha)\sin\beta\cos\beta$	$\sigma(1-\alpha)\cos 2\beta$		
$\alpha<0$: $\lvert\alpha\rvert\sigma\cos^2\beta\geq\sigma\sin^2\beta$: $\sigma\sin\beta\cos\beta(1-\alpha)>$ $\mu\sigma(\lvert\alpha\rvert\cos^2\beta-\sin^2\beta)$		$\sigma\sqrt{(\pi\alpha)}\left[(1-\alpha)\sin\beta\cos\beta+\mu\sigma(\alpha\cos^2\beta+\sin^2\beta)\right]$	$\sigma(1-\alpha)\cos 2\beta$	$\sigma(\alpha\cos^2\beta+\sin^2\beta)$	
Ditto (In Swedlow's form)		$\sigma\sqrt{(\pi\alpha)}\left[(1-\alpha)\sin\beta\cos\beta+\mu\sigma(\alpha\cos^2\beta+\sin^2\beta)\right]$	$\sigma(1-\alpha)\cos 2\beta$	$\sigma(\alpha\cos^2\beta+\sin^2\beta)$	$\mu\sigma(\alpha\cos^2\beta+\sin^2\beta)$

Table 3-3. List of the Conditions for Choosing the Proper Terms, Ling and Woo

$$- \frac{16}{3}\lambda \left[\frac{\sin\frac{\theta}{2}}{\tan\theta} \right] (1 + \alpha\tan^2\beta) = 0$$

This equation is the proposed model for relating θ to β and α in a biaxial stress state. The roots of the equation are denoted by $\theta_\circ$.

A comparison of Zamrick's model to the Woo and Ling approach is shown in Figure 3-23. The two approaches are in close agreement. However, comparisons to experimental data shown in Figure 3-23 are not as convincing.

Griffith's energy release rate, $G(\alpha)$, which is equal to the crack separation energy rate, $G\Delta(\alpha)$, when the material is elastic depends on the direction of the crack extension given by the angle θ. When the extension is coplanar (θ = 0) and the material is elastic we have:

$$J = G(o) = G\Delta(0)$$

where

$$J = (K_I^2 + K_{II}^2)/E'$$

and

$$E' \;.\; E/(1-\upsilon 2)$$

The energy rates, $G(\alpha)$ and $G(\alpha)$, can be expressed as the sum of the Mode I and Mode II components:

$$G(\alpha) = G_I(\alpha) + G_{II}(\alpha); \text{ and}$$

$$G\Delta(\alpha) = G_I\Delta(\alpha) + G_{II}\Delta(\alpha)$$

Estimates for $G(\alpha)$ for non-planar crack extension may be obtained from finite element analyses on cracks of different

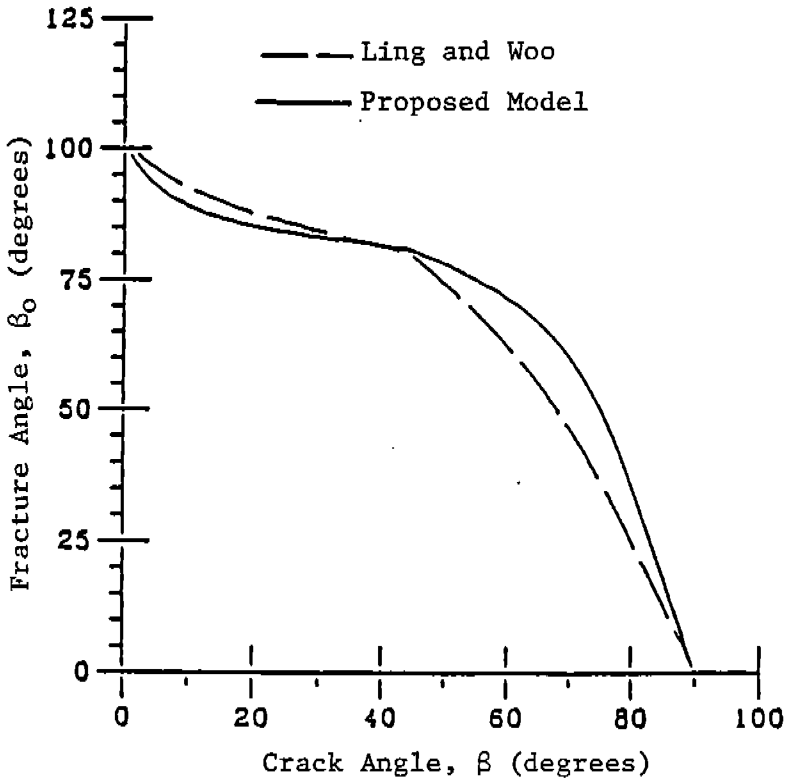


Figure 3-23. Comparison Between the Proposed Method and Ling and Woo Method for $\alpha = -1.0$

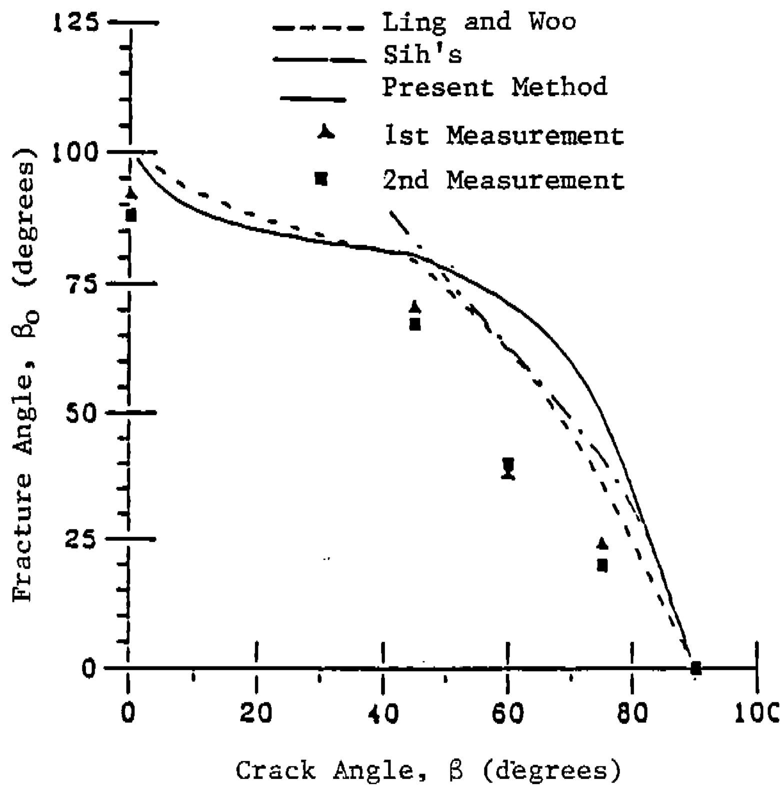


Figure 3-24. - Relationship for Biaxility Ratio, α = -1.0

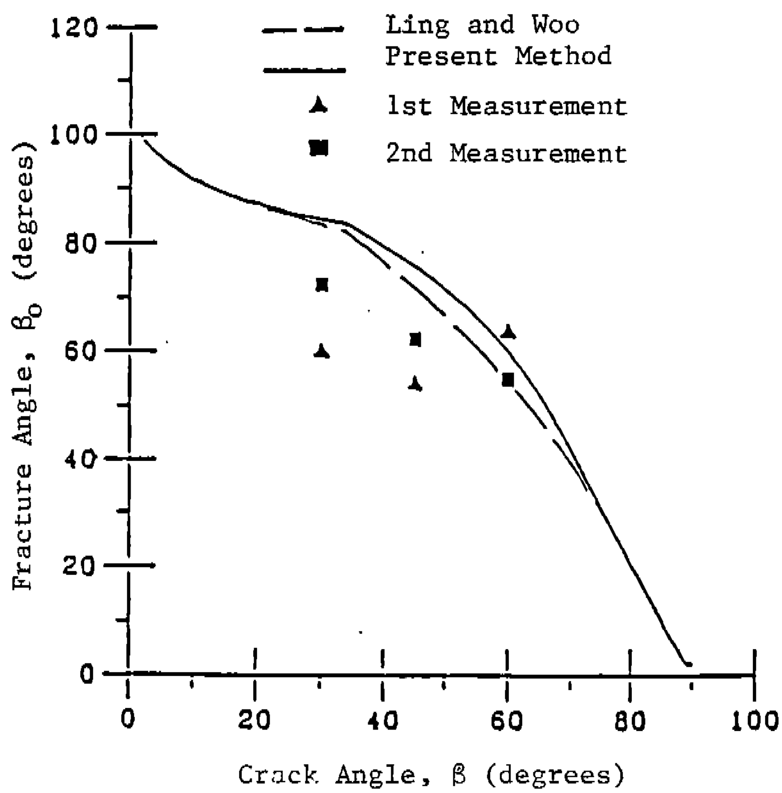


Figure 3-25. - Relationship for Biaxiality Ratio, α = -0.45

The agreement of any one approach is encouraging; however, no one approach is significantly highlighted by these correlations. Also, few data points are presented for correlation. Therefore, further development of these approaches will be required before design applications can be considered.

3.2 FATIGUE RESEARCH ON WELDED DETAILS

Few multiaxial fatigue approaches have been developed for welded details due to the number of variables associated with the weld process and geometry. Nonetheless, a few researchers have attempted correlations for relatively simple weld configurations and loadings and have had some success. Review of this important research is presented next.

3.2.1 Fatigue Life Estimates

Munse and Stallmeyer [3-65] investigated the fatigue of welded plate girders with panel stiffeners shown in Figure 3-26. Various stress formulations were tried to correlate S-N data. The most important finding is that most of the fractures occurred at stiffeners that were not in the region of pure moment. In fact, most fractures occurred at stiffener's ends where the flexural stresses were considerably lower than the maximum flexural stresses.

An S-N curve is presented in which the maximum bending stress on the extreme fiber at the location of failure is used as the ordinate. These data are shown in Figure 3-27. It is apparent from this plot that the maximum bending stress at the fracture section does not provide a consistent relationship in these tests either. A much better correlation is obtained when the data are analyzed on the basis of the maximum principal tensile stress (including the effect of shear) at the point of failure. Data plotted on the latter basis are presented in Figure 3-28. Although there is still scatter in the test results, there is a well-defined scatter band for the entire range of fatigue lives. It appeared to the investigators that the shear and flexural stress in the web of the welded members and the resulting principal tensile stresses, along with the stress concentrations at the stiffener welds, produced the most critical fatigue conditions in the weldments with stiffeners. Maximum principal tensile stress predicted fatigue life including initiation and propagation.

Later, Moyar and Garg [3-66] re-analyzed some of the Munse data using other fatigue criteria. They calculated the stresses in the vicinity of fatigue cracking and correlated the data based on maximum principal stress, modified range

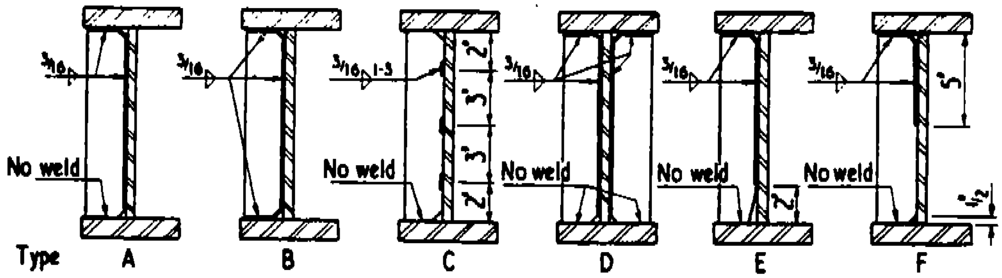
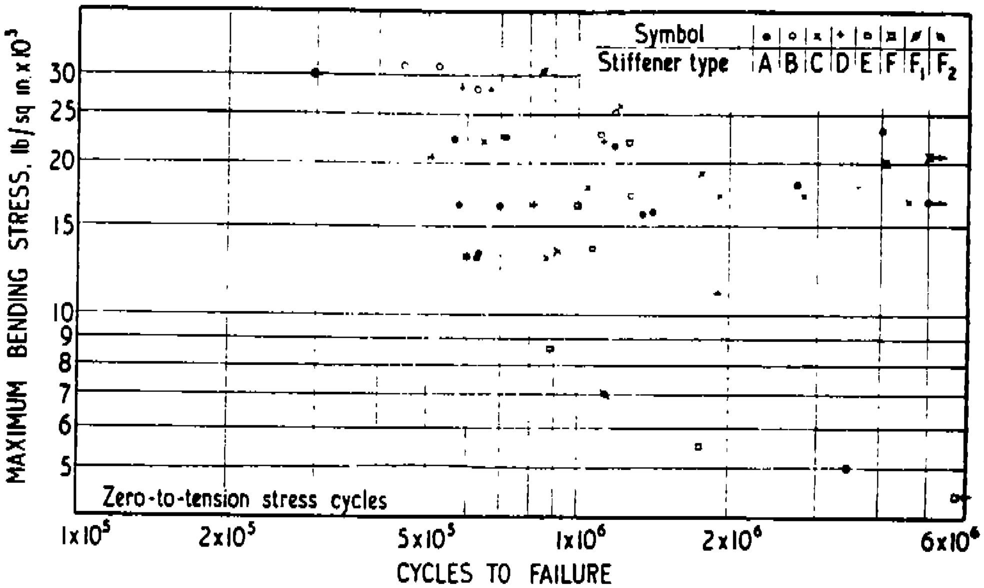


Fig. 3-26 —*Details of various types of stiffener*

Fig. 3-27

—*S/N diagram for maximum bending stress at failure section*



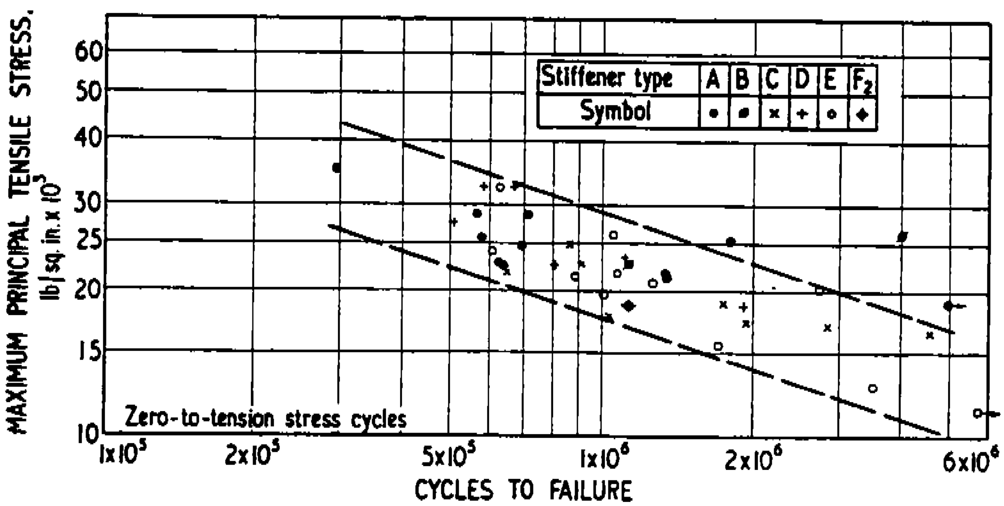


Fig. 3-28

—*S N diagram for maximum principal tensile stress at failure section* of shear stress, and maximum octahedral shear stress approaches. The results of these tests are summarized in Table 3-4. Both the modified range of shear stress and maximum octahedral shear stress criterion provided better correlation with observed data for stiffened welded plate beams. However, this analysis is based on a few tests and correlations are not very convincing in the low cycle region.

Lawrence [3-67] recently proposed an approach for predicting crack initiation and growth in simple weldments under combined loading (tensile and flexural moment). This approach is based on the ability to determine a stress fatigue notch factor for axial and bending load conditions in the vicinity of the weld toe and predicting the state of stress at the notch by superimposing tensile stress. This approach also includes the effects of mean stress and residual stress.

Lawrence proposed that for long fatigue lives and constant amplitude loading conditions, the notch root stresses are mostly elastic and the residual stresses can be considered not to relax. Under these conditions, the Basquin equation can be used to estimate fatigue life:

$$S_a K_f = (\sigma'_f - K_f S_m - \sigma_r)(2N_I)^b$$

where σ_f is the fatigue strength coefficient ($\sigma'_f = S_u + 50$ (ksi units)), S_m is the remotely applied mean stress, σ_r is the notch-root residual stress and K_f is the appropriate fatigue notch factor. Expanding the mean stress (S_m) to include both (applied or induced) axial and bending mean stresses (S_m^A and S_m^B), and considering both applied and induced cyclic axial and bending stresses through the following equation:

$$K_{fmax}^{eff} = (1-x)K_{fmax}^{A} + xK_{fmax}^{B}$$

$$x = S_a^B / S_a^T$$

where K_{fmax}^{A} and K_{fmax}^{B} are the worst-case-notch fatigue notch factor for axial and bending load conditions, respectively;

Table 3-4

Re-Analysis of Plate Girder Data by Moyer

SPEC. # AA-+SC	OBS. CYCLES TO FAIL	MAX.PRIN. STRESS THEORY		MAX.SHEAR STRESS THEORY		MAX.OCT. STRESS THEORY	
		PREDICT	DIFF.	PREDICT	DIFF.	PREDICT	DIFF.
29(0)B	0.6278	0.1426	-0.4852	0.3868	-0.2410	0.3456	-0.2822
30(0)B	0.4467	0.0731	-0.3736	0.2836	-0.1631	0.2355	-0.2112
31(0)B	0.5236	0.0744	-0.4492	0.2884	-0.2352	0.2399	-0.2837
41(0)B	1.1103	0.4438	-0.6665	1.0261	-0.0842	0.9188	-0.1915
43(0)B	1.2719	0.8111	-0.4608	1.2522	-0.0197	1.1994	-0.0725
	MEAN		-0.4871		-0.1486		-0.2082
	STD.DEV.		0.1086		0.0962		0.0864

COMPARISON OF PREDICTED CYCLES TO FAILURE (IN MILLIONS) OF TYPE B BEAMS USING PROPERTIES FROM TYPE C BEAM DATA AND BIAXIAL FATIGUE THEORIES

S_a^B and S_a^T are the bending stress and the total stress amplitude, respectively. From the above, one can derive an expression for the fatigue strength of a weldment subjected to axial and bending mean and constant-amplitude cyclic stresses:

$$S_a^T = \frac{(1/K_{fmax})(\sigma_f' - \sigma_r) - S_m - XS_m^B}{1 - x(1-x)} (2N_I)^b = S_F(2N_I)^b$$

where X is the ratio of K_{fmax}^B to K_{fmax}^A.

If the assumptions of the approach are valid, then this expression should predict the constant amplitude fatigue strength at long lives ($N_T > 2x10^6$ cycles) for which initiation is thought to dominate the total fatigue life.

The variables in the equation can be divided into either constants of known quantities (S_m^A, S_m^B, $2N_I$) and random variables (σf, σ_r, K_{fmax}^A, x). Of the random variables K_{fmax}^A and x are considered as determining the variation in fatigue strength. Neither material property, the fatigue strength coefficient (σ_f') which is proportional to the UTS or the residual stress (σ_r) which is equal to the base metal yield strength (S_y), vary greatly for one material and welding process.

To show the improvement associated with the notch stress approach, Lawrence calculated mean and standard deviations for experimental data and compared it to Munse's approach considering nominal field stresses. This comparison is discussed in Section 5.0.

Yung and Lawrence [3-68] conducted a study of fatigue analysis of plate and tube weldments under torsion and bending as shown in Figure 3-29. The combined bending and torsion loadings studied gave lives about a factor of three shorter than pure bending stresses, showing that multiaxial stress states do influence the fatigue resistance of weldments despite the two-dimensional nature of the weld toe notch and its relative insensitivity to pure torsional loading. Residual stresses were found to play an important role in determining the portion of the fatigue life period devoted to crack initiation and early growth. Lawrence

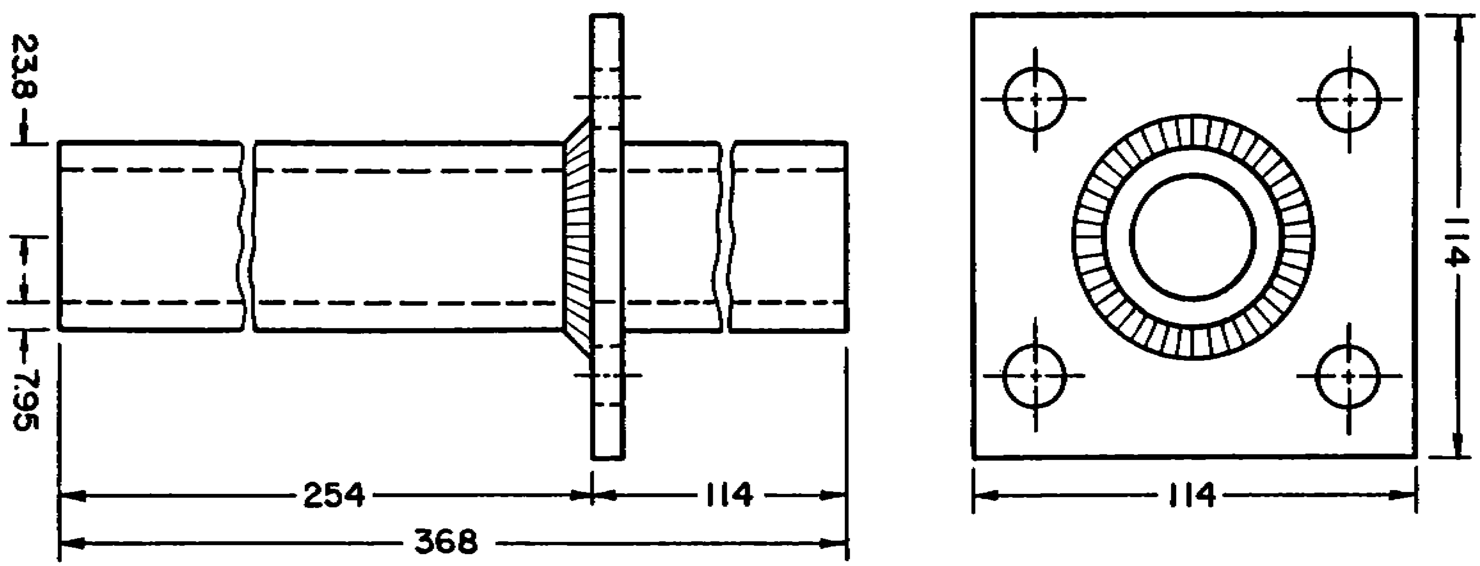


Figure 3-29. Specimen Geometry of Tube-to-Plate Welds (units in mm)

found that the multiaxial fatigue theories of Lohr and Ellison and Kandil, Brown and Miller gave best estimates of fatigue crack initiation life, as shown in Table 3-5, when the notch root strains could be determined by finite element analysis as shown in Figure 3-30. Estimates of initiation life based on Basquin's equation modified for multiaxial stresses (principal stress and an equivalent stress) gave predictions for long-life high-cycle fatigue which were within a factor of three of the observed total lives, as shown in Table 3-5.

Siljander et al [3-69] recently investigated the proportional and non-proportional multiaxial fatigue of tube-to-plate weldments. This work was conducted to support the development of a multiaxial fatigue design procedure for built-up plate bridge girders. Bridges experience non-proportional out-of-phase stresses as vehicles move across the bridge. One of the most interesting aspects of the study is the analysis of the principal stresses in girders with moving loads to determine representative out-of-phase loadings. They correlated their data using various damage criteria. As in many structural codes, principal stress range used von Vises effective shear stress range and a critical shear plane approach originally developed by Findley. Of these approaches the critical shear plane approach correlated the proportional and non-proportional data quite well as seen in Figures 3-31 through 3-33. Several of their findings in achieving this degree of correlation are noteworthy. First, the test specimens were stress relieved to eliminate residual weld stresses. This produced a significant reduction in data scatter. Second, the investigators used stress concentration factors at the weld toe that are developed in part by a complex 3-D finite element analysis with the added advantage of having knowledge of weld profiles, information that is not available with certainty to structural designers.

3.2.2 Fatigue Propagation in Welded Structures

Multiaxial fatigue procedure for crack propagation have been examined because weld flaws are often present at crack initiation sites as discussed by Thayamballi [3-70] and many others.

Procedures have been identified for estimating the stress intensity factor for Mode I, K_I, propagation in complex stress fields. The first approach identified is the influence function technique presented by Bueckner [3-71]:

$$K = \int_0^a h(x)\sigma(x)dx$$

Table 3-5

Research on Butt Welds & Cruciform Fillet Welds by Lawrence

Statistical Summary of the Departures of Predicted Lives from Fatigue Test Data

	Munse's	Miner's	Gurney's	RMS Method	I-P Model
No. of Cases	29	29	29	13	29
$\bar{F}_P$	1.061	1.015	0.894	0.906	1.016
Ω_{F_P}	0.124	0.093	0.081	0.052	0.067

$\bar{F}_P$: Mean value of F_P; $F_P = \dfrac{\log_{10}(N_{prediction})}{\log_{10}(N_{Test})}$, a unity of F_P value represents the perfect agreement between the prediction and fatigue data.

Ω_{F_P} : Coefficient of Variation of F_P.

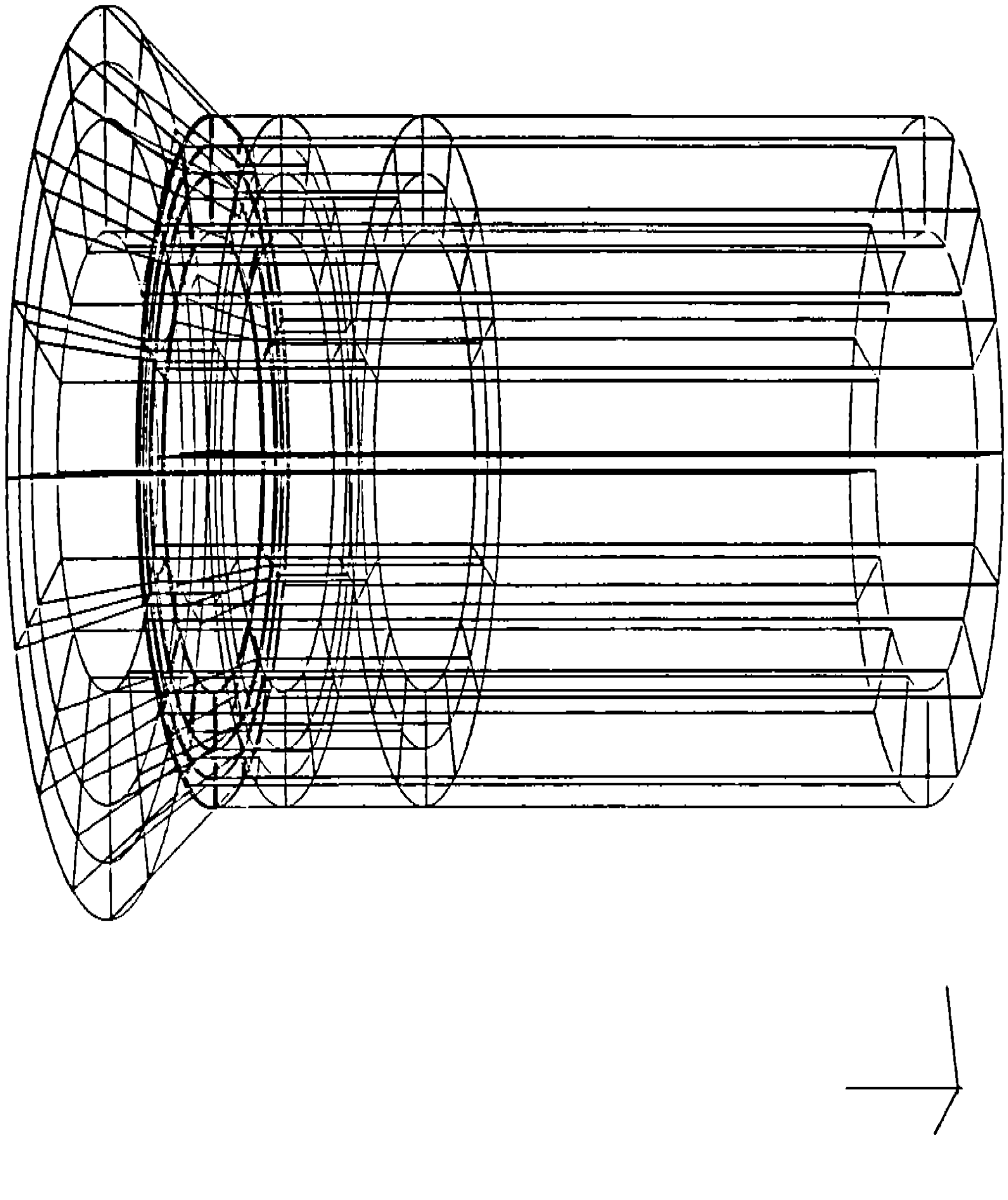


Figure 3-30. Finite Element Mesh of Tube-to-Plate Welds (Ratio of Notch-Root Element Size to Notch Radius = 1:4)

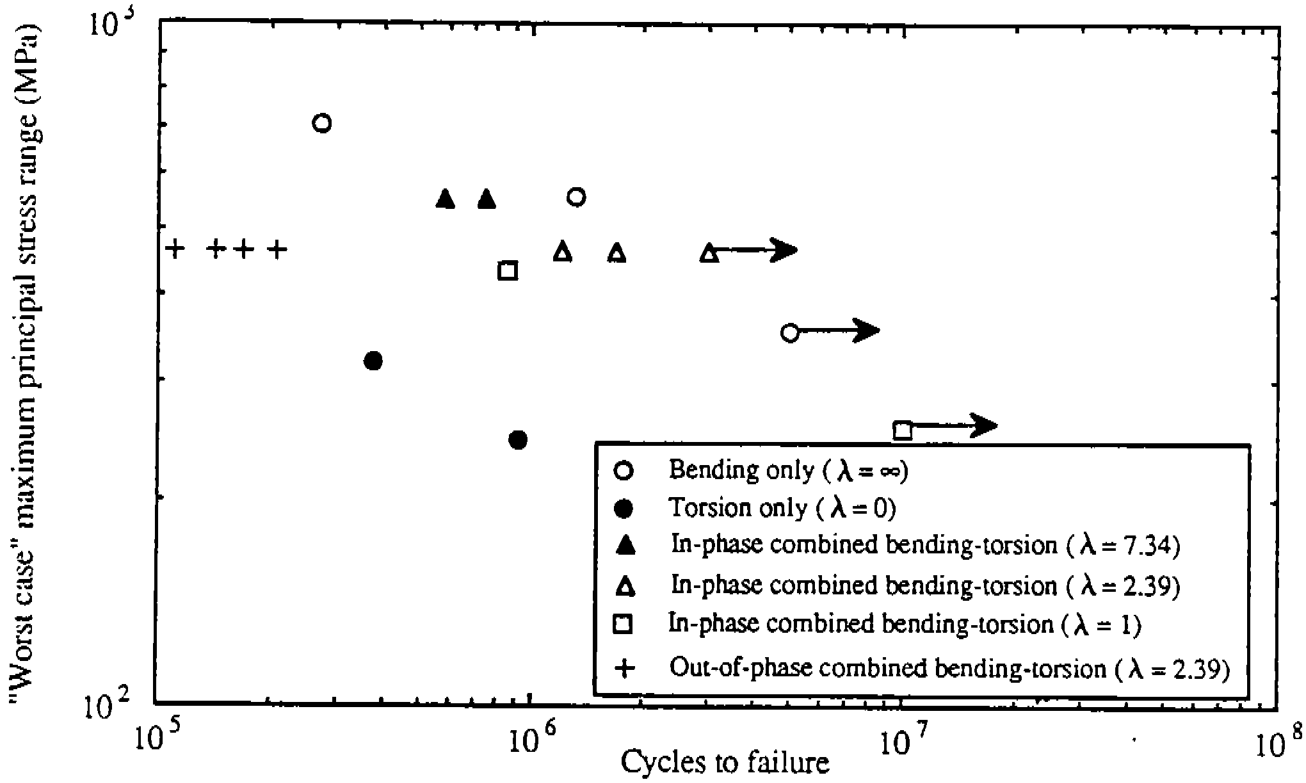


Figure 3-31. The Maximum "Worst Case" Local Principal Stress Range versus Cycles to Failure

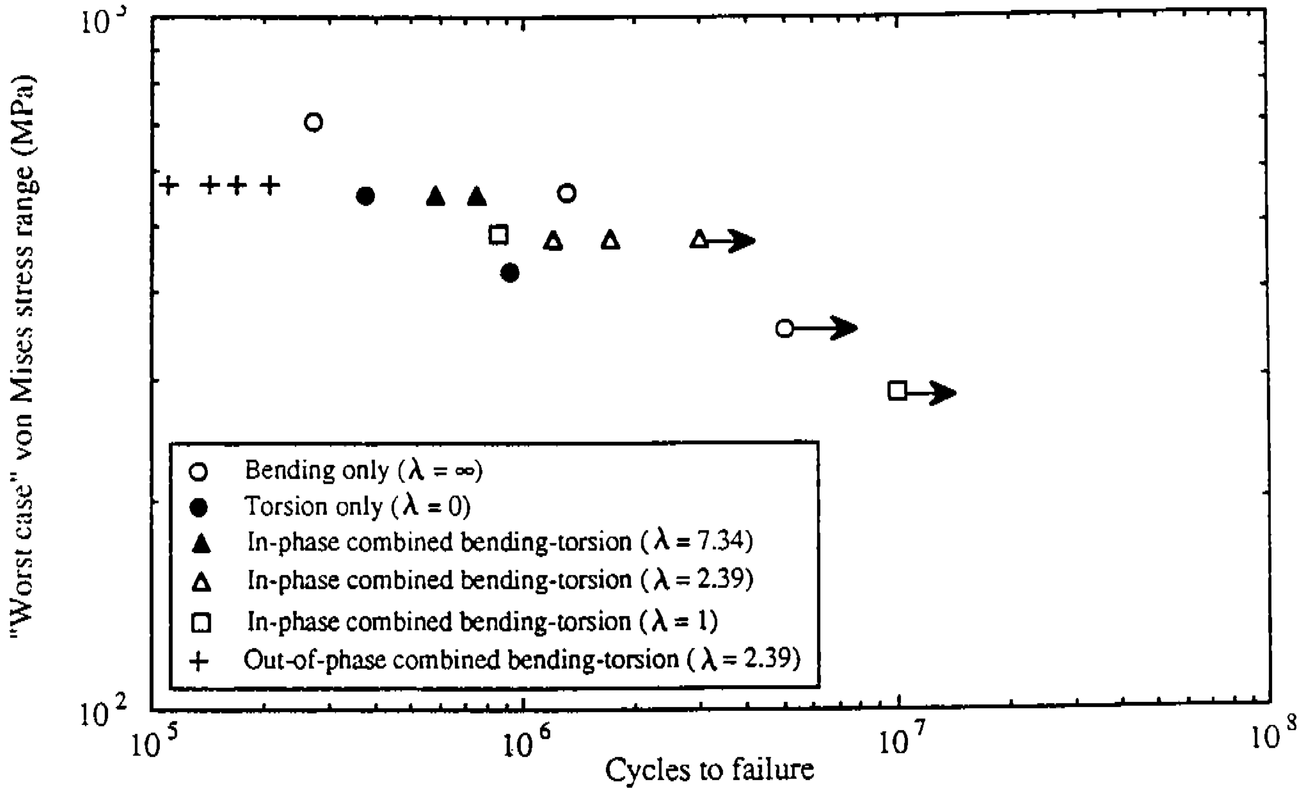


Figure 3-32. The "Worst Case" Local von Mises' Effective Stress Range versus Cycles to Failure

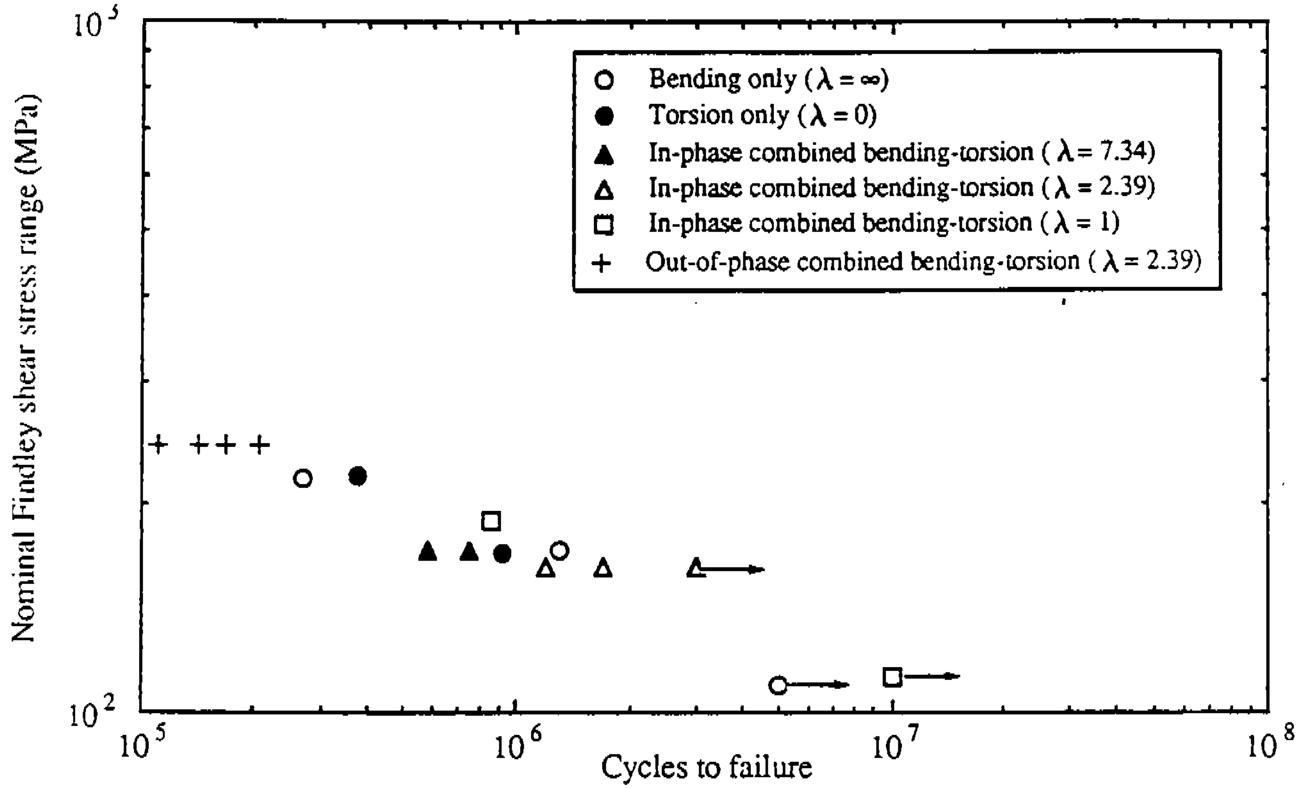


Figure 3-33. Findley's Equivalent Nominal Shear Stress Range versus Cycles to Failure

where

$h(x)$ = intensity arising from a unit load at location x on the crack face

$\sigma(x)$ = uncracked stress distribution

This approach has the advantage of being relatively simple to develop. It's disadvantages include the assumption that the load is symmetric about the crack face.

The second approach for estimating stress intensities in complex stress patterns from geometric configurations is given by Gurney [3-72]:

$$K = \frac{M_k M_f M_b}{\phi} \sigma \sqrt{\pi a}$$

where

K = $\sigma \sqrt{\pi a}$ is baseline stress intensity for the through thickness crack

M_k = stress concentration factor at welded toe

M_f = front face correction for secondary bending at crack tip

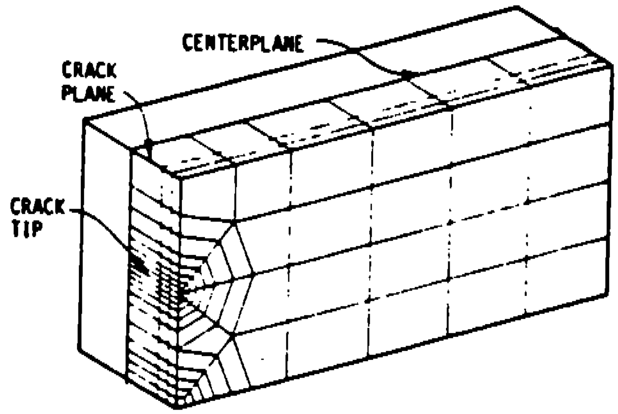
M_b = back face correction for finite thickness plate

ϕ = geometric factor for semi-elliptical surface flaw

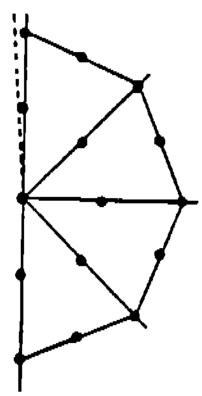
The advantages of this approach are, again, its relative ease of application. However, factors must be developed for specific crack geometries and stress conditions.

The third approach for estimating stress intensity factors is by finite element techniques with a crack element in the model. This approach is illustrated in Figure 3-34. The advantage of this approach is that it accounts for complex stress patterns and crack geometries and elastic and plastic conditions around the crack tip. Disadvantages of this technique are that it is relatively time consuming and expensive at this time. This situation is expected to change in the future as more computational techniques are made available and computer capabilities increase.

Of the three approaches, the later has been proposed most recently for applications. Most importantly, crack growth under multiaxial loading has been shown to occur along a



Finite Element Model of Structure Adjacent to the Crack Tip



Crack Tip Element

Figure 3-34. Illustration of Estimating Stress Intensity by Finite Element Analysis path that maximizes Mode One component K_I (Hellen [3-73]) and strain release rate G (McDonald [3-74], Haung [3-75]). This is a controversial finding with conflicting experimental results. However, if validated for use in marine structures, it would facilitate development of an approach for predicting multiaxial fatigue response in welded structures. This factor alone makes prediction of fatigue crack growth in welded structures worth further consideration.

There is evidence that multiaxial stress affects the fatigue life of welded joints as indicated by Sih [3-76]. The crack trajectory emanating from a flaw at an angle to the principal stress is neither exclusively a function of principal stress nor the stress component perpendicular to the flaw. This is also true for welded tubular joints by the experimental observations of McDonald et al [3-74]. Furthermore, there is evidence [3-76] to indicate that cracks grow along a curved path through thickness under chord-brace intersections. The direction of a growing fatigue crack in a complex stress field has been the subject of several hypotheses. It is argued by Hellen [3-73] that the crack develops in such a way as to maximize the Mode I component K_I, or that the path maximizes the strain energy release rate G [3-77, 3-78]. Insight into the direction of crack growth can be obtained by considering the effect of a small angle kink on the crack tip. The local Mode I and Mode II stress intensity factors on the kink are denoted K_I and K_{II} can be expressed in terms of the Mode I and Mode II components K_I and K_{II} on the main crack following the work of Bilby [3-79]; Hussain et al [3-80]; and Masahiro et al [3-81]. Although the analyses differ on detail they give broadly similar results. The angle of the crack for a tubular joint weld is shown in Figure 3-35. The angle of the kink which maximizes K_I and makes K_{II} zero is close to 15 degrees for crack length to thickness ratio a/T=.2, 20 degrees for a/T=.6, and approximately 55 degrees for a/T=.9. The local strain energy release rate G for the kink can be determined by combining the local K_I and K_{II}, or be obtained by aft axis virtual crack extension. For the case shown in Figure 3-35 crack paths which maximize K_I and G are similar. These are plotted as three piece wise linear segments and compared with the path observed experimentally by Rice as presented by Haung et al [3-82].

Rhee [3-76] has presented two techniques for computing stress intensity factors for welded structural details. The first method is based on development of an effective stress intensity factor (K_e) for mixed mode crack propagation and the second is by using a special finite element program for estimating stress intensity factors.

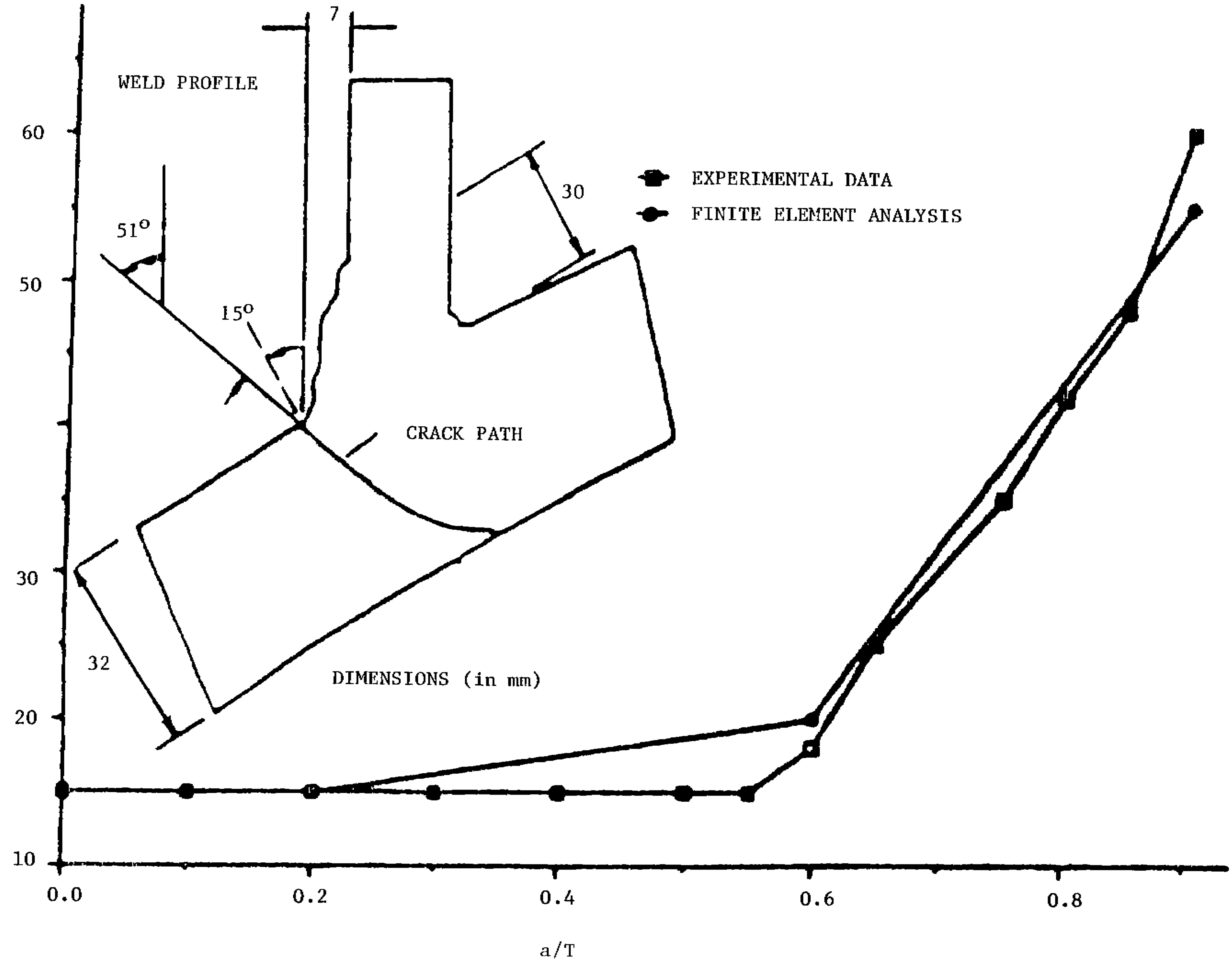


Figure 3-35. Predicted Crack Path at a Welded Tubular Intersection

The equivalent stress intensity factor, K_e, is converted from the appropriate energy release rate, G:

$$\frac{K_e^2}{E} = G = \frac{1-\upsilon^2}{E} K_I^2 + K_{II}^2 + \frac{K_{III}^2}{1-\upsilon}$$

This equivalent stress intensity factor represents the crack driving force which incorporates all the K_I, K_{II} and K_{III} components of a mixed mode problem. For a mixed mode problem, any single component of the stress intensity factors cannot represent the crack driving force properly since crack propagation is a combined result of the contributions of all three stress intensity factors of the problem. A comparison between K_e and any individual component of K_I, K_{II} and K_{III} can indicate the contribution of such an individual component to crack propagation. For the brace tension cases, it is apparent from Figures 3-36 through 3-39 that, throughout the crack front, the fatigue crack propagation will dominate the Mode I stress intensity factor K_I. However, for the in-plane bending cases, the K_I contribution to K_e is negligible except for a small length near the surface. Therefore, if the considered flaw geometry is practical for a fatigue load system which consists of a significant contribution of in-plane bending, a fatigue crack growth analysis should consider all three -- K_I, K_{II} and K_{III}.

A crack driving force parameter such as K_e has yet to be established as a parameter through which the fatigue crack growth behavior of a mixed mode flaw can be directly calculated in a manner similar to the K_I of a Mode I problem. Expressions similar to the present K_e have been studied for limited mixed mode problems [3-83, 3-84] such as problems with K_I and K_{II} mixed and with K_I and K_{III} mixed. However, few fully mixed mode problems, such as those present, have been studied.

Rhee has also presented a method to calculate the stress intensity of an elliptical flaw using a specialized finite element computer approach. This method, called the Finite Element Alternating method, can solve the problem of an elliptical flaw under arbitrary normal and shear stresses which can be expressed in higher-order polynomials [3-85 through 3-87]. The Finite Element Alternating method iterates through the following steps.

1. Using the solution of an elliptical crack in an infinite solid and subjected to a constant pressure, the surface traction σ_{yy}, τ_{yx} and τ_{yz} are computed on y=-h plane.

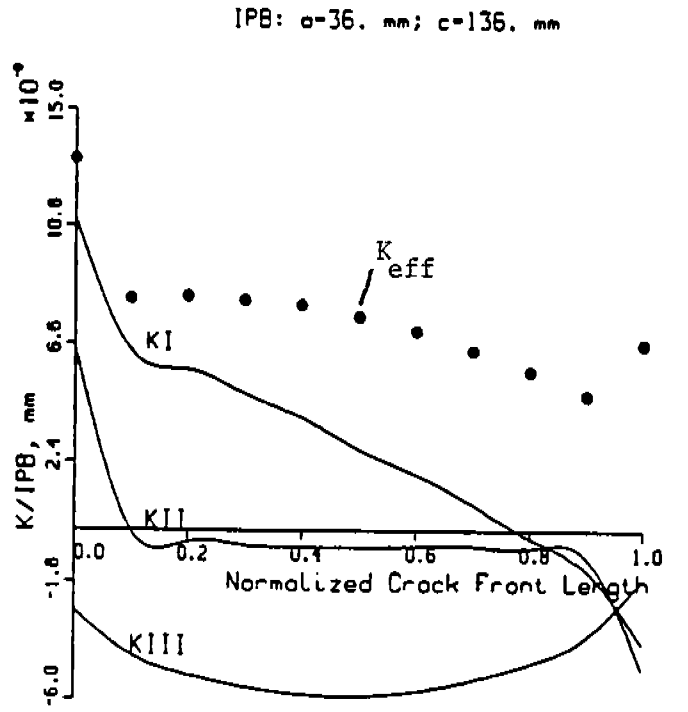


Figure 3-36. Stress Intensity Factors of Weld Toe Surface Flaw (In-Plane Bending)

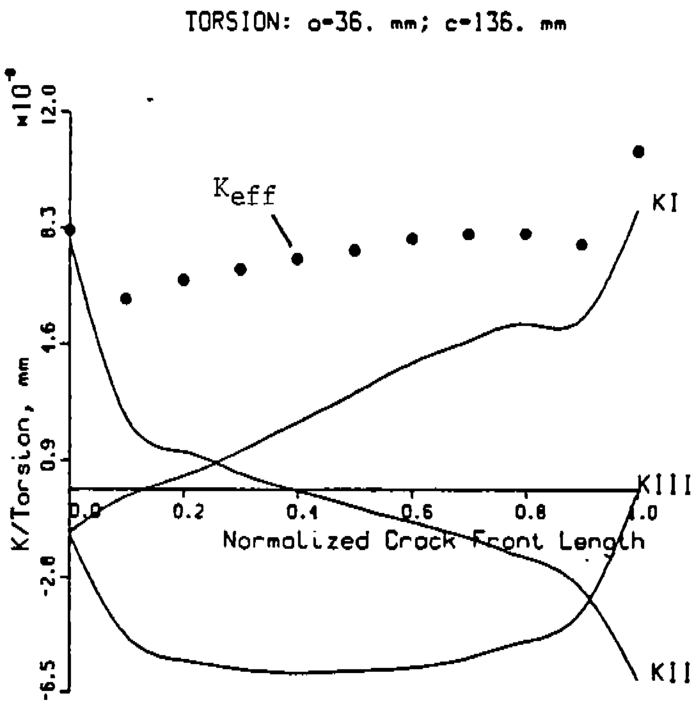


Figure 3-37. Stress Intensity Factors of Weld Toe Surface Flaw (Torsion)

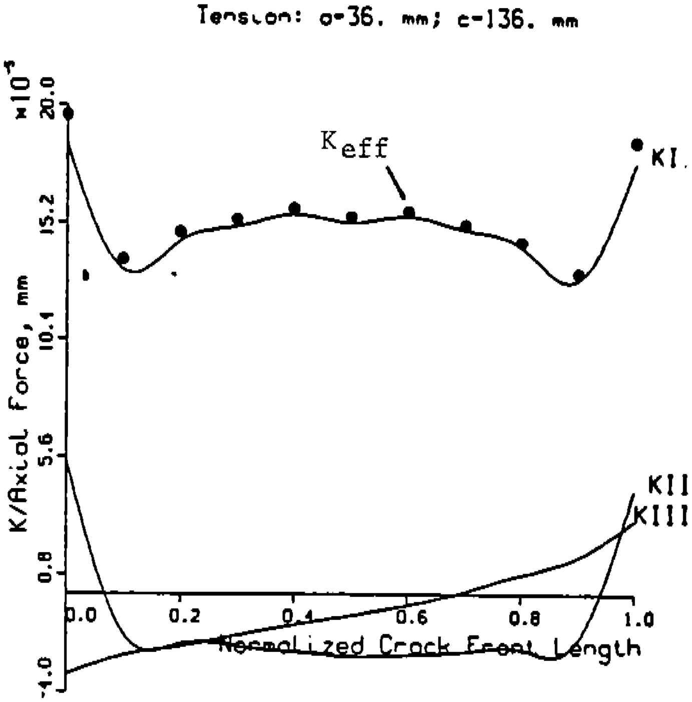


Figure 3-38. Stress Intensity Factors of Weld Toe Surface Flaw (Axial Tension)

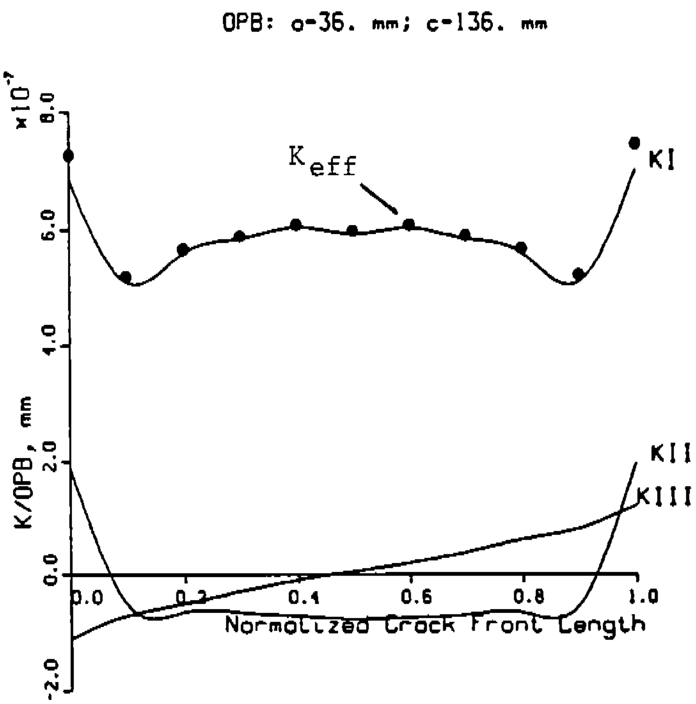


Figure 3-39. Stress Intensity Factors of Weld Toe Surface Flaw (Out-of-Plane Bending)

2. The residual tractions acting on y=-h plane are then freed by applying opposing surface tractions on the plane of an uncracked, semi-infinite solid. The resulting residual normal stress, σ_{zz}, on the crack surface location is calculated.

3. The residual normal stress, σ_{zz}, on the crack surface is removed by an opposing stress. This opposing stress results in the stress intensity factors along the crack front and the surface tractions on y=-h plane, which can be calculated using Segedin's potential function [3-88].

4. Steps 2 and 3 are repeated until the resulting residual stresses on the crack tip surface become negligible when compared to the applied stress of τ. The result of all the iterations are superimposed to obtain the stress intensity factors for the elliptical crack of the original problem.

This approach has been successfully applied to obtain accurate stress intensity factors in finite thickness plate [3-87], in pressure vessels [3-89] and for quarter-elliptical corner crack emanating from a pinhole in plates and in aircraft attachment lugs [3-90].

In the finite element alternating method, the solutions of an elliptical flaw in an infinite body are obtained through the Trefftz's potential function method [3-91]. In this procedure, extensive numerical calculations are involved to evaluate the various forms of elliptical integrals required for the solutions. The finite element solution method is used to obtain the solutions of the uncracked body under arbitrary boundary tractions. The boundary tractions, which result from the above elliptical flaw problem, are first converted into the boundary nodal forces of the finite element model without a crack and then applied as the external loads to calculate the stresses on the crack location. These solutions are used following the sequences discussed earlier to obtain the stress intensity factors.

Another finite element based approach for calculating crack propagation in welded details was presented by Haung [3-92]. His approach is based on the work of Rice and Levy [3-93] where the crack is represented by a series of generalized line springs which act across a discontinuity in a thin shell. The approach was originally developed for Mode I loading but has been further generalized by Parks [3-94] and Desvaux [3-95] to incorporate both Mode II and Mode III loadings and is implemented in the finite element code ABAQUS [3-96].

Haung [3-92] presented results comparing the line spring model to three-dimensional finite element analysis and to experimental crack growth rates for a chord and brace intersection.

A fracture mechanics fatigue analysis method explicitly considers the flaw geometry, which is one of many critical characteristic of the problem. The necessity to consider the flaw geometry in a fatigue analysis, in turn, can make a rigorous crack growth analysis prohibitively expensive. The effort required for the evaluation of the stress intensity factors can be significant, even for a simple flaw geometry. Especially for a surface flaw, which is a common form of weld toe defects in a marine structure and whose stress intensity factors can only be practically evaluated through a numerical method such as the finite element method, the investment required for the evaluation of the stress intensity factor solution is enormous.

To perform a rigorous fatigue crack growth analysis of a surface flaw, which changes its size and shape continuously under fatigue loading, the stress intensity factors of a group of flaws with different dimensions are required to cover a certain range of flaw shapes and sizes. For the stress intensity factor calculation using the finite element method, which is the most popular at present, the majority of the analysis effort is devoted to modeling the finite element meshes.

4.0 APPLICATION OF MULTIAXIAL FATIGUE RESEARCH TO MARINE STRUCTURES

Marine structures are subjected to various loads and local effects that are combined in structural details. These loadings contribute to fatigue in the details. In this section, the stress characteristics described in Section 2.1.1 and multiaxial fatigue approaches presented in Section 3.0 are compared to identify candidate multiaxial fatigue approaches for marine structures and to help identify areas where additional research is required. Table 4-1 has been prepared to summarize the proposed approaches identified during the course of this project and to highlight, for quick reference, areas in which gaps in technology exist. Further discussion of the applicability of multiaxial fatigue research to marine structures follows.

4.1 SHIP STRUCTURE APPLICATIONS

Returning to our examples of representative structural details in ship structures, we will present relevant methods that have been proposed for predicting multiaxial fatigue response.

The first detail considered is the cutout in a transverse web frame. The cutout is to allow longitudinal panel stiffeners to pass through continuously in order to carry longitudinal primary hull bending stresses. There have been studies conducted on this detail [4-1] to determine the state of stress. As discussed in Section 2.0, there are biaxial nominal stresses adjacent to the cutout caused by distributed lateral load on the girder resulting in flexural bending and shear stresses. However, at the edge of the cutout, where cracks are known to propagate, the stress is axial tension or compression. The principal stress field has mean stress components from hydrostatic loads and varying principal stresses from variations in hydrostatic loads as waves are encountered. Therefore, on a detailed level, the stresses are not at all multiaxial, but on a nominal stress level, the stresses are biaxial as indicated by Munse and others. Munse suggested shear stress as the correlating nominal stress for this flexural member with cutouts.

The second example detail is a hatch corner cutout. The hatch corner is a source of stress concentration. The state of stress in the vicinity of this cutout is biaxial resulting from combined axial torsional hull loading. However, as with the previous cutout, the stress state on the face of the cutout is axial. Research by Chen [4-2] and Munse [4-3]

TABLE 4-1

SUMMARY OF METHODS FOR ESTIMATING FATIGUE RESPONSE TO COMPLEX LOADING

	LOADING[1]		RESPONSE	MULTIAXIAL APPROACHES[4]		RELI (INI APPR O
STRUCTURAL DETAIL	GLOBAL	DETAIL[2]	LOCAL STRESS/STRAIN[3]	INITIATION	PROPAGATION	BIAS
<u>Ships</u>						
Longitudinal Cutout in Web	Lateral loading from hydrostatic wave variations. Cargo loading internally (tankers)	Flexural bending and shear from lateral load	Axial stress at edge of cutout, biaxial opposing principal stresses adjacent to cutout	Maximum shear stress (Munse) Maximum stress* (Fricke)	-	-
Hatch Corner	Longitudinal wave bending, torsional hull flexing	Axial loading from hull bending and shear from flexing	Axial stress at edge of cutout, opposing principal stresses adjacent to cutout	Maximum stress* (Chen)	Mode I, max principal in plane stress (Chen, Wirsching)	-
CVK	Longitudinal wave bending, lateral hydrostatic load variations from waves	Axial response from longitudinal bending, flexural bending and shear from lateral load	Biaxial opposing principal stress	Principal stress* (Munse) for fatigue life Maximum shear* stress (Moyer, Garg) for crack initiation Critical shear plane where principal stresses are out of phase (Lawrence)	Mode I maximum principal stress	-

TABLE 4-1 (continued)

SUMMARY OF METHODS FOR ESTIMATING FATIGUE RESPONSE TO COMPLEX LOADING

	LOADING[1]		RESPONSE	MULTIAXIAL APPROACHES[4]		REL (IN APP
STRUCTURAL DETAIL	GLOBAL	DETAIL[2]	LOCAL STRESS/STRAIN[3]	INITIATION	PROPAGATION	BIAS
Butt Welded Plate	Longitudinal hull wave bending, lateral hydrostatic load variations from waves	Longitudinal primary bending stress, bending and membrane stress from lateral load	Biaxial opposing principal stresses, residual stress present	Principal stress (Lawrence) Principal stress for fatigue life* (Stambaugh)	Maximum principal stress (Lawrence)	1.016
Offshore Structures						
T-T[5], K-K, K-T	Wave loading and out of plane drag and inertial forces from orbital velocities	Axial stress and punching shear, bending stress from out of plane loads	Triaxial opposing principal stresses and shear	Maximum* principal stress or shear stress maximum (BWI, Gurney)	Strain energy release rate for mixed Mode I, II, III (Haung, Rhee) (proposed for T-joints but applicable to more complex joints)	-
Connectors	Bending from current forces	High pre-tension and varying axial stresses in tension legs, axial and torsional loads in risers	Biaxial and triaxial opposing principal stress and strain	Equivalent* stress, Guest law and critical plane approaches for out of phase multiaxial stresses	-	-

TABLE 4-1 (continued)

SUMMARY OF METHODS FOR ESTIMATING FATIGUE RESPONSE TO COMPLEX LOADING

NOTES:

1. Hypothetical loadings are presented.

2. Global detail response refers to nominal state of stress external to notches and local geometry.

3. Actual states of stress vary depending on magnitudes of live and dead loads (e.g., depending on state of mean stress or even direction of encountered waves). This situation can actually change the location of maximum stresses in the detail. See text for additional discussion on stress/strain characteristics.

4. Multiaxial approaches refers to a method for characterizing fatigue response under complex loading conditions.

5. COV are for combined axial and bending on the butt weld, no biaxial stresses.

6. Joints are combined out of plane intersections.

* Approach recommended by the investigators.

indicates that the axial stress is sufficient for characterizing the fatigue response of the hatch corner cutout.

The third example detail is a Center Vertical Keel (CVK). The CVK is subjected to varying axial load along its axis, lateral hydrostatic load with a mean component and often internal static or dynamic (sloshing liquids) loads opposing the hydrostatic load. As indicated, mean stresses are present and often are random over the voyage from one voyage to another. The phase relationship of the axial and lateral load varies in a random nature.

Munse [4-4] suggested that the principal stress correlates the fatigue life (initiation and propagation). Moyar and Garg [4-5] re-analyzed a few tests data and found the octahedral shear stress and maximum shear stress correlated data best. More interestingly, Moyar included a mean stress correction based on the Goodman Diagram.

Recently, Silijander [4-6] introduced a new approach to predict the multiaxial fatigue response in plate bridge girders starting with an analysis of the proportionality of principal stresses in the girder web and flange intersection. They found the critical plane approach of Findley to best correlate the data. However, as discussed in Section 3.0, the application of this technique depends on the availability of information on weld stress concentration factors and cyclic material properties. This type of information may be known by laboratory researchers but is not often available to structural designers.

The transverse butt weld is a fourth example and a very common weld detail, there is little correlatable data for applications similar to those in ship structures. The transverse butt weld is generally located in a biaxial stress field with varying mean stress and fluctuating principal stress proportionality. Tests were conducted on butt welded HY-80 plates subjected to lateral load; however, the biaxial nature of stresses was not considered -- only stresses normal to the weld toe. Similarly, Silijander [4-6] tested butt welds subjected to axial and bending loads and again only considered the maximum stress normal to the weld toe.

are presented in Appendix B. Equivalent shear stress techniques modified for mean stress effects provided very good predictions of fatigue life of a laterally loaded plate with a butt weld based on uniaxial data. It was found, however, that multiaxial approaches based on cyclic strain fatigue properties were not easy to apply because at this time these properties are generally not known for ships and offshore structures. Additionally, stress concentration factors for weld geometry have a large influence on fatigue response depending on the weld profile and quality. The HY-80 plate butt had a quality weld and low weld profile resulting in a stress concentration factor of about 1.15, not a typical stress concentration for welds.

The foregoing discussion illustrates the variability of load characteristics and geometry of ship structures and the different multiaxial fatigue approaches that are applicable to each type of detail.

Additionally, only a few data points were generalized to support the research and are insufficient for use in developing confidence in statistical parameters (bias and scatter) needed for reliability analysis.

4.2 OFFSHORE STRUCTURE APPLICATIONS

Currently there are two distinct approaches for predicting fatigue response in offshore structures. The first is a fatigue life approach based on S-N curves [4-7] developed for various joint configurations. This approach is the most widely used in the industry and uses the maximum stress normal to the weld toe and neglects interaction effects for out-of-plane joint configurations. The British DOE rules [4-8] address complex stress distributions by correlating fatigue data using maximum principal stresses at detail "hotspots" as discussed in Section 2.2.1 of this report. Yung and Lawrence [4-9] have used maximum shear stress and octahedral shear stress to correlate tube and plate high cycle fatigue response under multiaxial loading.

The second approach for fatigue response prediction that is gaining wider acceptance is the fracture mechanics approach. Multiaxial stress distributions are being considered. However, validation efforts are very limited in number, detail

Rhee presented estimates of stress intensity factors for multiaxial loading based on finite element analyses and using an equivalent stress intensity factor as discussed in Section 3.2.2. Figures shown in Section 3.2.2 present the resulting stress intensity factor solutions of two flaw geometries. These plots show that the in-plane bending load cases are predominately mixed mode. All the solutions along the crack front location where the negative Mode I stress intensity factor is developed (for the in-plane bending cases) are not accurate since a negative K_I indicates the crack surface contact and penetration which developed in the analysis were physically impossible. These negative values could be eliminated with a proper multicomponent contact algorithm. A comparison between K_e and any individual component of K_I, K_{II} and K_{III} can indicate the contribution of such an individual component to crack propagation.

While Rhee's K_e is attractively simple, the approach is obviously limited to linear elastic fracture mechanics assumptions in the high cycle range of crack growth. It obviously does not account for crack tip plasticity observed in mixed mode tests as discussed earlier.

Huang [4-10] presented a more simplified approach for calculating mixed mode crack growth in welded tube joints using finite element analysis, line spring elements, and the energy release rate G. Haung compared his approach to experimental results and found good agreement for cases where Mode I dominates and cracks are not long, $0.8 \geq (a/T) \geq 0.2$. This technique is again limited to basic limitations of linear elastic fracture mechanics and does not account for mean and residual stresses, random effects or interactions with corrosive environments. Additionally, there are conflicting reports on the accuracy of assuming Mode I dominant crack growth.

Another type of offshore structure in which multiaxial fatigue considerations are important is riser connectors. These connector systems, illustrated in Figure 4-1, are often subject to high pressure loads and tension and torsion loads that fluctuate about a mean stress. There are several multiaxial fatigue approaches that are good candidates for this type of loading, especially those in which tension and torsion type loads dominate. The automotive and heavy equipment industries have been very active in research on tension and torsion multiaxial fatigue with several specialized approaches investigated by Tipton and Nelson [4-11] which have been described previously. However, these

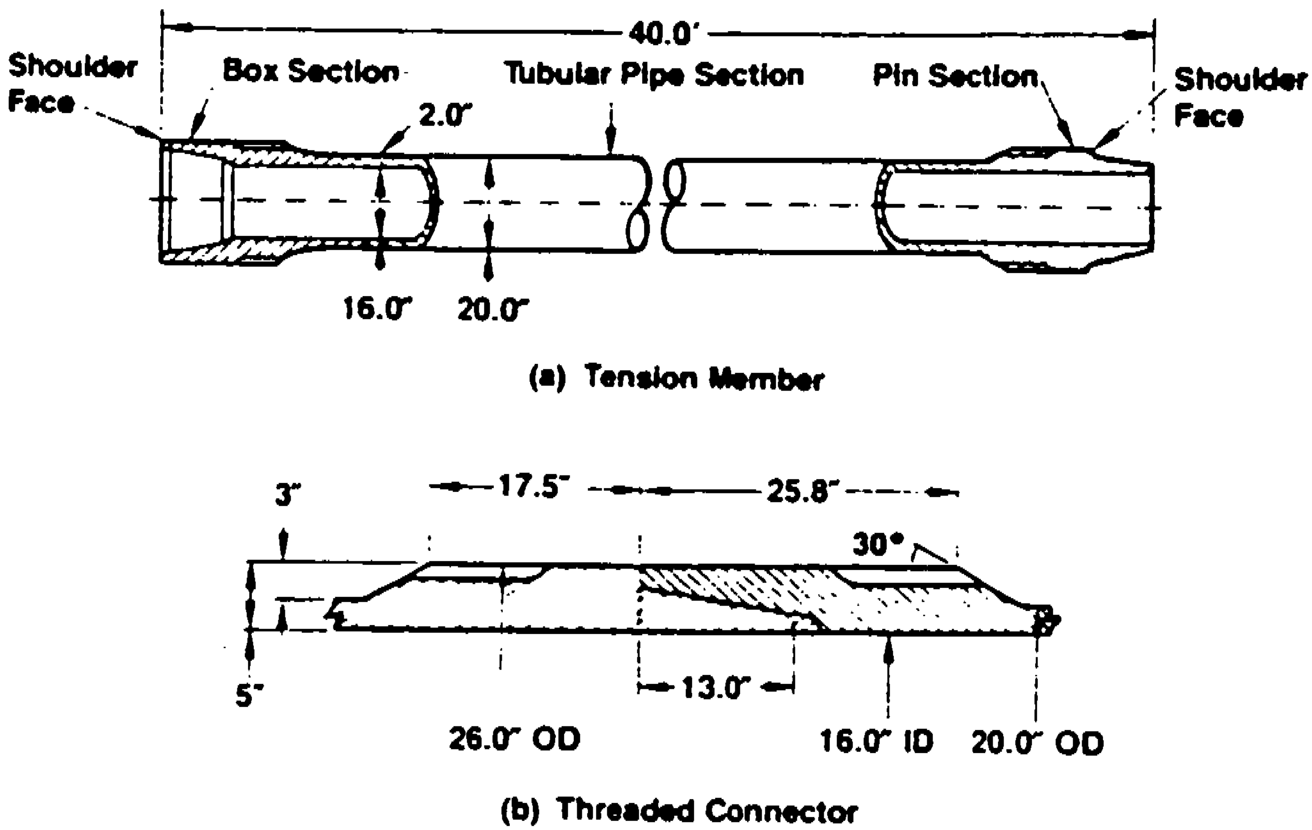


Figure 4-1. Tension Member and Connector with High Pre-Load

approaches should be subject to verification using offshore materials and the exact geometries used in connectors.

The equivalent stress techniques have been used most often because they are easiest to apply and required material data is available. Of the equivalent stress techniques reviewed, maximum shear stress techniques generally predict crack initiation and equivalent principal stress is most useful for predicting fatigue life where crack propagation life dominates.

However, from the foregoing discussion and the summary in Table 4-1, there is no universal technique for predicting multiaxial fatigue response for each varied application in marine structures. This is because marine structural details vary in geometry, applied load, stress state and fabrication procedures. Unlike pressure vessels or rotating machinery (where these approaches have been applied) there are a great number of variables for each structural detail.

4.3 FACTORS INFLUENCING MULTIAXIAL FATIGUE RESPONSE IN MARINE DETAILS

The many factors which influence fatigue response in marine structures were discussed in Section 2.1. The factors that have influence in fatigue response of welded details are discussed here along with the research conducted to determine their influence on multiaxial fatigue response and prediction. These factors include stress gradients, stress proportionality and phasing, elastic and plastic strain relationships, mean and residual stress, random loading, corrosion, thickness effects, materials and fabrication procedures. A data set has been identified where several of these effects were investigated. The data set is from welded HY-80 and HY series steel plates subjected to lateral pressure load. Effects from various sources are easier to separate from one another by comparing different data sets. The intent of this review is to illustrate the influence these factors have on multiaxial fatigue response.

Tests of a variety of welded plate laboratory specimens of HY steels are also summarized in this study. The general configurations of these various laboratory specimens are presented in Table 4-2. A summary of the fatigue resistance of these various members is presented in Table 4-3 and Figures 4-2 through 4-19. Table 4-4 gives a brief description of the test members included in the study of HY steels.

Table 4-2. General Configuration of Various Specimen Types

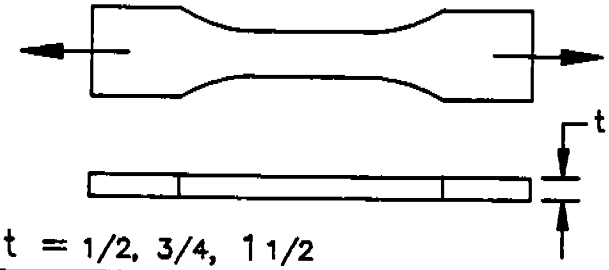
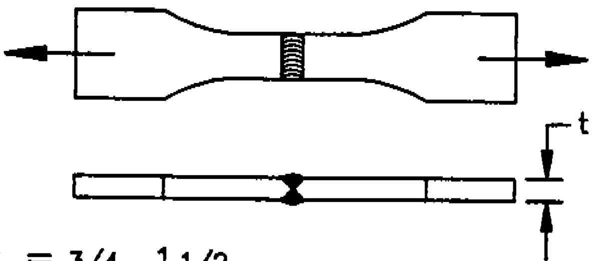
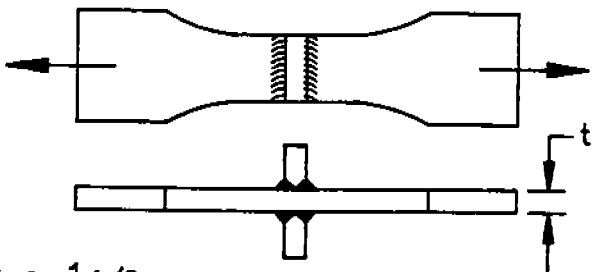
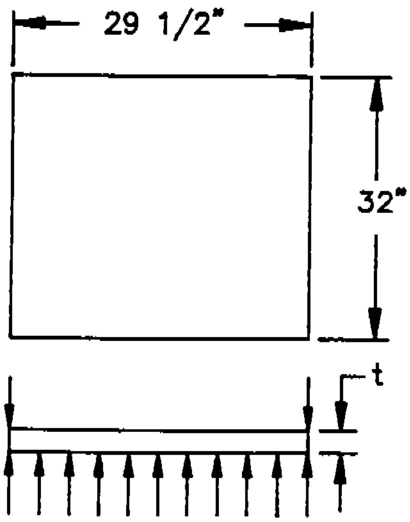
Type	Specimen	Steel	Refs/Notes
I	 t = 1/2, 3/4, 1 1/2	HY-80 HY-100 HY-130/150	4-12, 4-13, 4-14, 4-15, 4-16, 4-17, 4-19
II	 t = 3/4, 1 1/2	HY-80 HY-100	4-12, 4-13 4-14, 4-15 4-16, 4-17, 4-18, 4-19, 4-20, 4-21
III	 t = 1 1/2	HY-80 HY-100	4-12, 4-13, 4-17
IV	 t = 1 1/2, 1 5/8	HY-80 HY-130/150	4-54 IV(A)-Simply supported on 4 edges IV(B)-Simply supported on 2 edges, free on 2 edges

Table 4-2. General Configuration of Various Specimen Types (cont'd.)

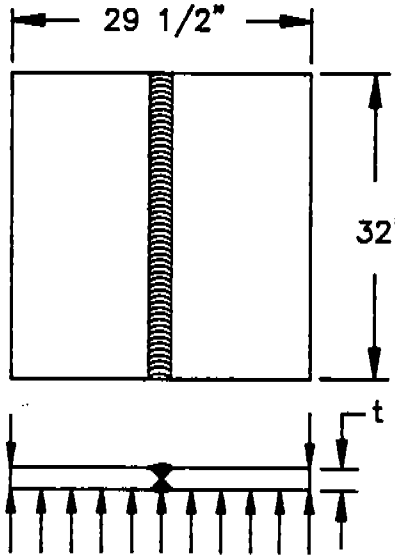
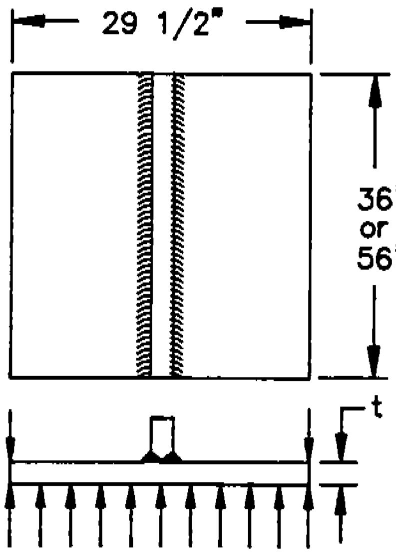
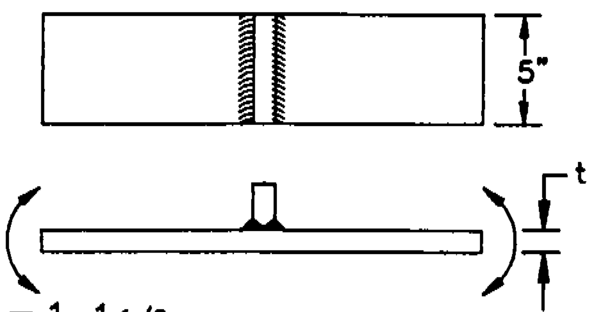
Type	Specimen	Steel	Refs/Notes
V	 t = 1, 1 11/16	HY-80 HY-130/150	4-22, 4-23, 4-24, 4-25, 4-26, 4-27 4-28, 4-29, 4-30, 4-53, 4-32 V(A)-Simply supported on 4 edges V(B)-Simply supported on 2 edges, free on 2 edges
VI	 t = 1, 1 1/2, 1 11/16	HY-80 HY-100	4=33, 4-22, 4-34, 4-35, 4-36, 4-37, 4-38, 4-31, 4-41, 4-42, 4-45, 4-30, 4-55 VI(A)-Simply supported on 4 edges (29 1/2 x 57 1/2 x 1 1/2) VI(B)-Simply supported on 2 edges, free on 2 edges VI(C)-Cast tees (No welding)
VII	 t = 1, 1 1/2	HY-80	4-33, 4-47

Table 4-2. General Configuration of Various Specimen Types (cont'd.)

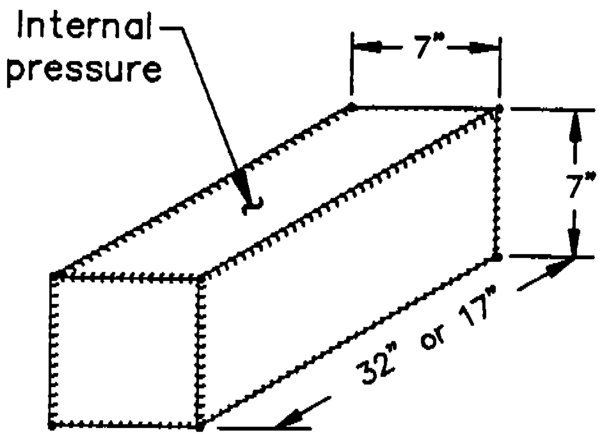
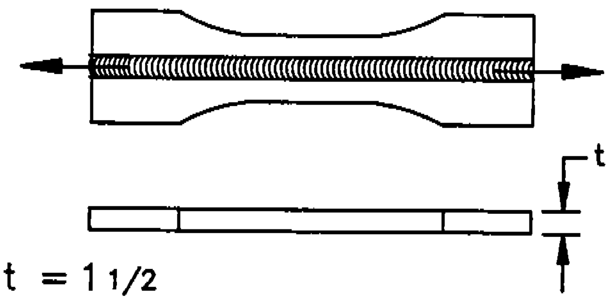
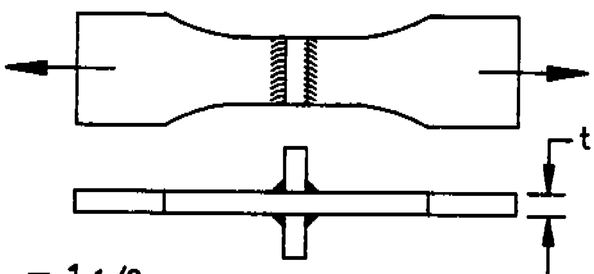
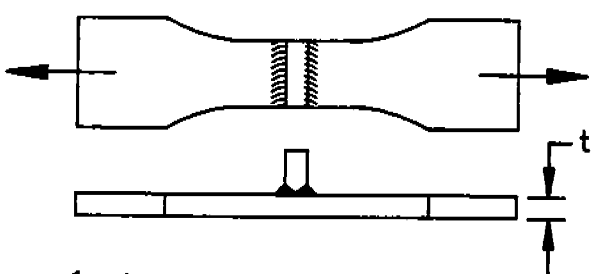
Type	Specimen	Steel	Refs/Notes
VIII	 t = 1, 2	MS (71.2ksi tensile) HY-80 HY-100 HY-140	4-49, 4-50 4-51, 4-52 For 2" Plate box dimensions are 12 1/4"x12 1/4" x 28 3/4"
IX	 t = 1 1/2	HY-80	4-12, 4-13
X	 t = 1 1/2	HY-80	4-12
XI	 t = 1 1/2	HY-80 HY-100	4-12, 4-13 4-17, 4-18

Table 4-2. General Configuration of Various Specimen Types (cont'd.)

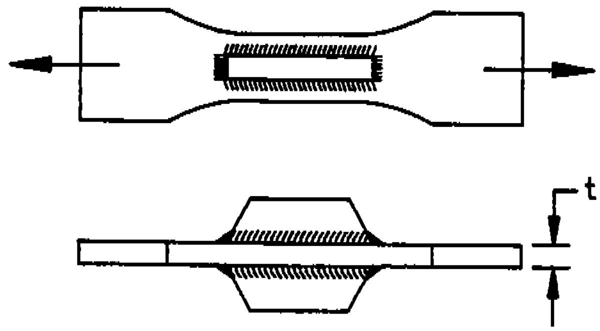
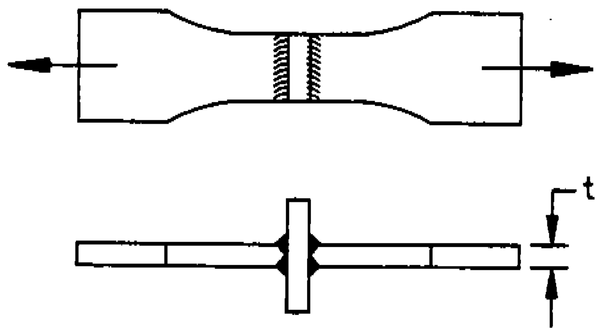
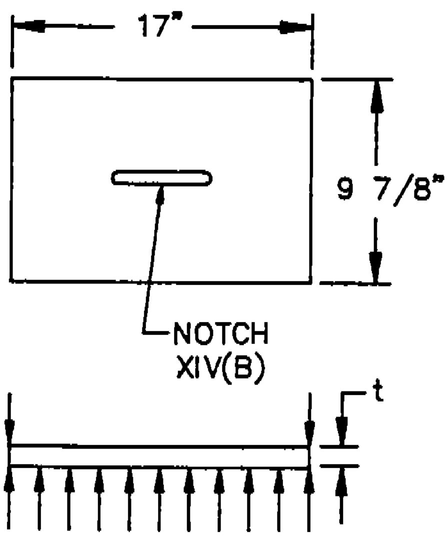
Type	Specimen	Steel	Refs/Notes
XII	 t = 1 1/2	HY-80	4-13
XIII	 t = 3/4, 1 1/2	HY-80 HY-100	4-12, 4-13
XIV	 t = 3/4	A303B 70B	4-55, 4-56, 4-57 Simply supported plate 4 edges XIV(A)-Plain plate XIV(B)-Notched (Charpy-V)

TABLE 4-3

FATIGUE STRENGTHS OF VARIOUS SPECIMEN TYPES

SPEC. TYPE	STRESS RATIO	FATIGUE STRENGTH, KSI, AT LIFE OF 10^4	10^5	10^6	m	MATERIAL	FIGURE NO.
I	R=0	121	72	43	4.45	HY-130/150	4-2, 4-3
I	R=-1	157	100	64	5.16	HY-80, HY-100	
II	R=-1	128	72	40.5	4.00	HY-80, HY-100	
II	R=0	90	50.5	28.3	3.98	HY-80, HY-100	4-4
II	R=+½	65.5	36.2	20.2	3.92	HY-80, HY-100	
III	R=-1	127	43		2.13	HY-80	4-20
III	R=0	90	30		2.10	HY-80	
IV A	(XIV A) R=0	160	86	46	3.71 - 3.68	70B	4-6
IV B	R=0	127 (1)	78 (1)	47 (1)	4.72 - 4.54	HY-80	
IV B	R=0	129 (2)	80 (2)	49 (2)	4.82 - 4.70	HY-80	4-5
V A	R=0	137 (1)	51 (1)		2.33	HY-80	
V A	R=0	143 (2)	44.5 (2)		1.97	HY-80	4-7, 4-8, 4-9
V B	R=0	106 (1)	39.5 (1)		2.33	HY-80	
V B	R=0	110 (2)	43.5 (2)		2.48	HY-80	4-7, 4-8, 4-9
VI A	R=0 (Est.)	103 (2)	52 (2)		3.37	HY-80	4-11, 4-20
VI B	R=0	68.5 (2)	33.6 (2)		3.23	HY-80	4-10, 4-11, 4-20
VI B	R=0	92 (2)	52		4.04	HY-100	4-23
VI C	R=0	113 (2)	82 (2)		7.18	HY-80	4-5, 4-11
VII	R=0	76	35.3		3.00	HY-80	
VII	R=-1	110	50	32.5	2.90	HY-80, HY-140	4-12, 4-13
VIII	R=0	86	47		3.11	HY-100, M	4-14

TABLE 4-3 (continued)

FATIGUE STRENGTHS OF VARIOUS SPECIMEN TYPES

SPEC. TYPE	STRESS RATIO	FATIGUE STRENGTH, KSI, AT LIFE OF					
		10^4	10^5	10^6	m	MATERIAL	FIGURE NO.
IX		92	53	31	4.18 - 4.29	HY-80	4-15
X	R=0	(54) (Est.)	(32) (Est.)		(4.40)	HY-80	4-15
XI	R=1	113	68	40.5	4.53 - 4.44	HY-80	
XI	R=0	82	51	32	4.84 - 4.94	HY-80	4-16
XI	R=+½	62	38	23	4.70 - 4.86	HY-80	
XII	R=-1		(32) (Est.)	(17.5) (Est.)	(3.82)	HY-80	4-17
XIII	R=-1	96	45		3.03	HY-80, HY-100	4-17
XIV B	R=0	76	53.5	37.5	6.56 - 6.48	70B, A302B	4-18(a)
XIV A	R=0	160	85	46	3.64 - 3.75	70B	
XIV A	R=0	120	65	35	3.75 - 3.72	A302B	4-6
VI B	R=0	122 (1)	68 (1)	38 (1)	3.95	HY-80	
VI B	R=0 Toes ground	122 (2)	69 (2)	39 (2)	4.03	HY-80	4-10, 4-11
VI A	R=0	81.5 (1)	39 (1)		3.12	HY-80	4-11
VI B	R=0	60 (1)	30 (1)		3.32	HY-80	4-11
VI B	R=0	82 (1)			3.94	HY-100	4-23
VI C	R=0	112 (1)	80 (1)	57	6.84 - 6.79	HY-80	4-5, 4-10
Shot Peened							
VII	R=0	90	42		3.02	HY-80	4-13
VII	R=-1	118	59		3.32	HY-80	4-13
Ground and Shot Peened							
VII	R=0	105	62		4.37	HY-80	4-13
VII	R=-1	130	90		6.26	HY-80	4-13

(1) 10% increase in deflection of specimen; (2) 100% increase in deflection of specimen

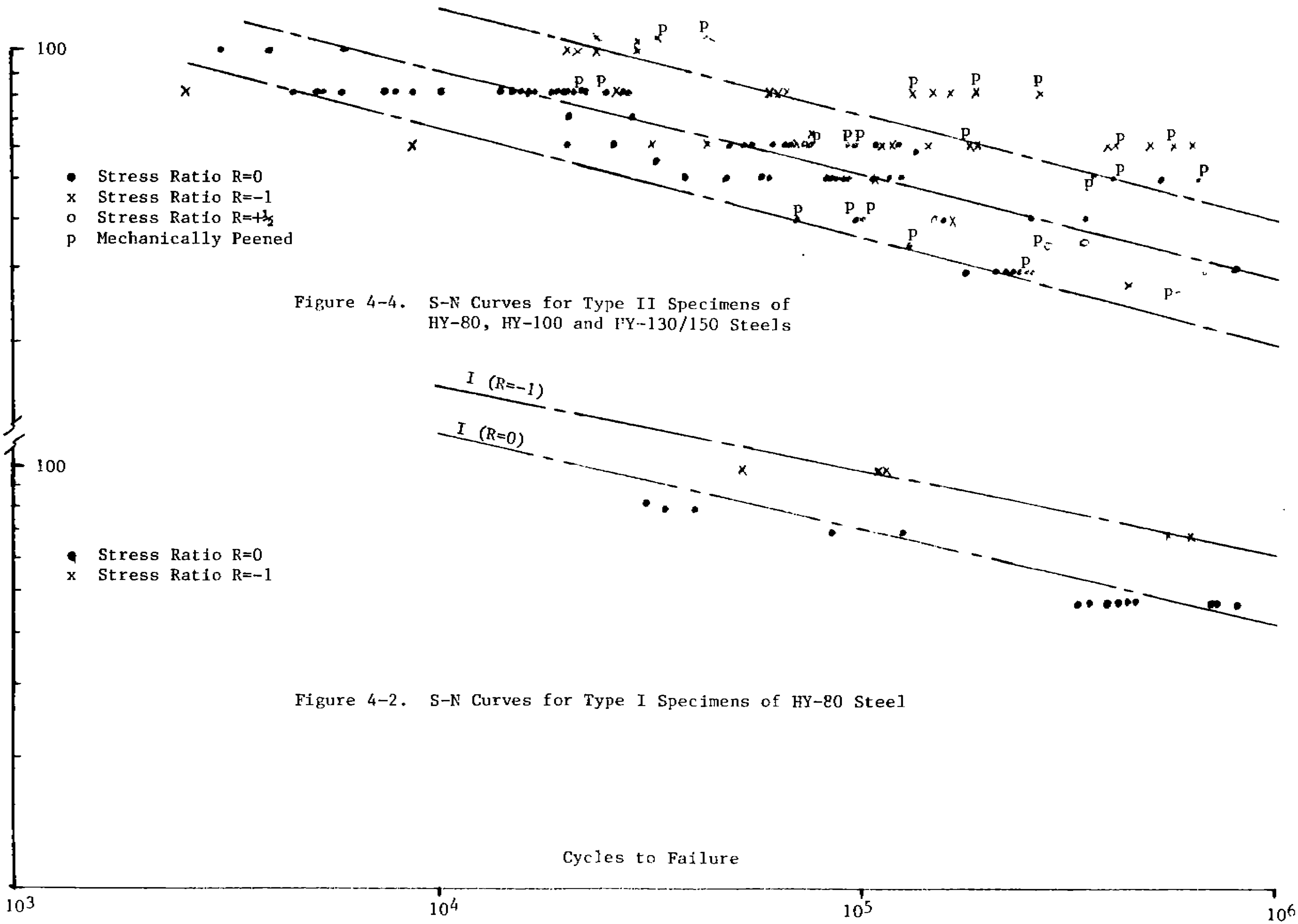


Figure 4-4. S-N Curves for Type II Specimens of HY-80, HY-100 and HY-130/150 Steels

Figure 4-2. S-N Curves for Type I Specimens of HY-80 Steel

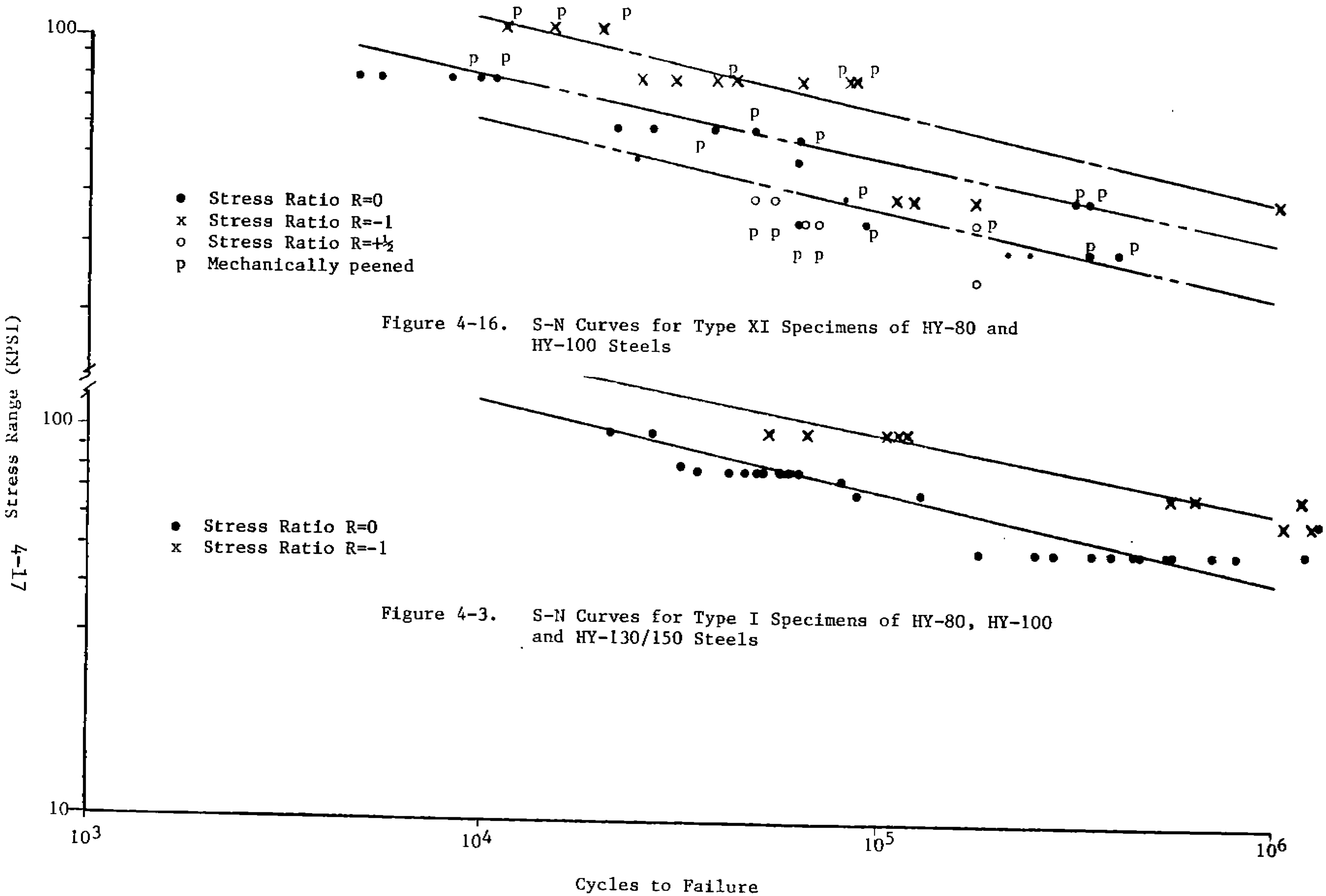


Figure 4-16. S-N Curves for Type XI Specimens of HY-80 and HY-100 Steels

Figure 4-3. S-N Curves for Type I Specimens of HY-80, HY-100 and HY-130/150 Steels

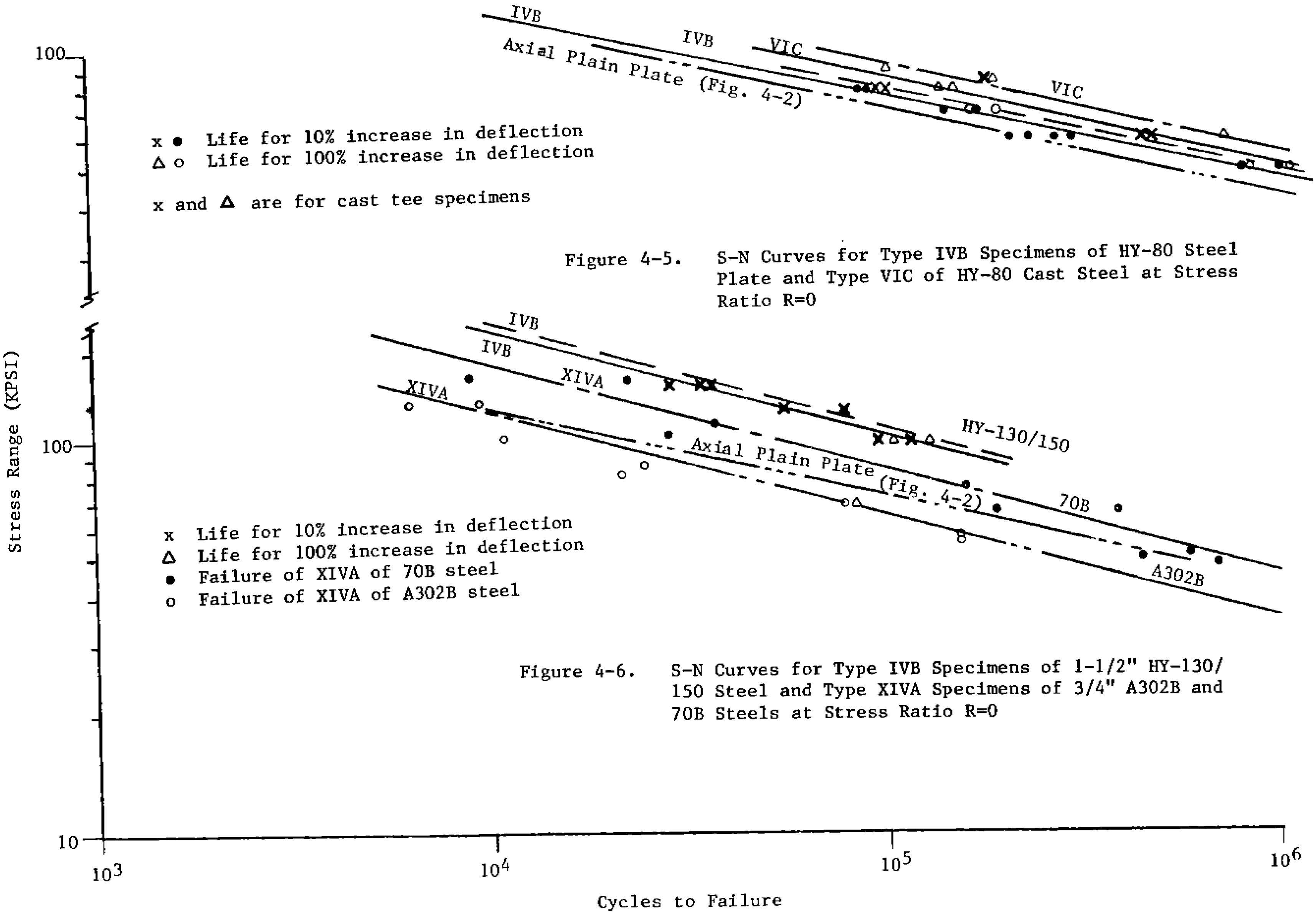


Figure 4-5. S-N Curves for Type IVB Specimens of HY-80 Steel Plate and Type VIC of HY-80 Cast Steel at Stress Ratio R=0

Figure 4-6. S-N Curves for Type IVB Specimens of 1-1/2" HY-130/150 Steel and Type XIVA Specimens of 3/4" A302B and 70B Steels at Stress Ratio R=0

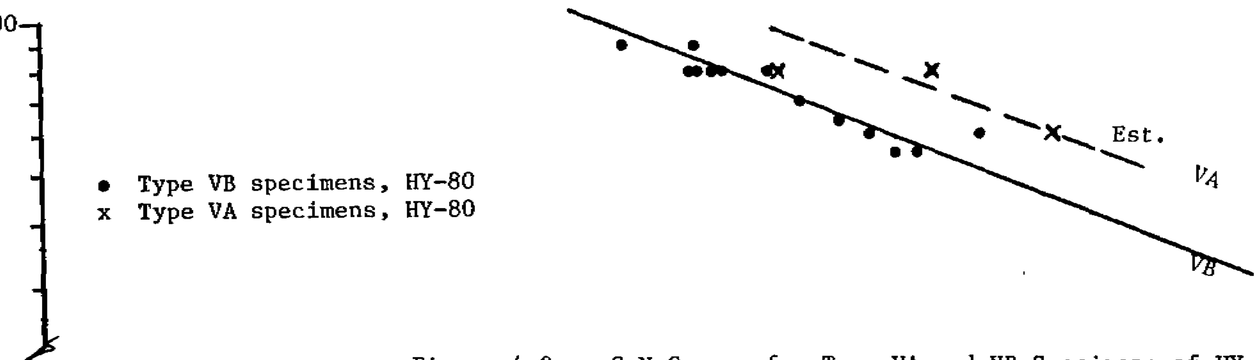


Figure 4-8. S-N Curves for Type VA and VB Specimens of HY-80 Steel at Stress Ratio R=0 (Life at 100% increase in deflection)

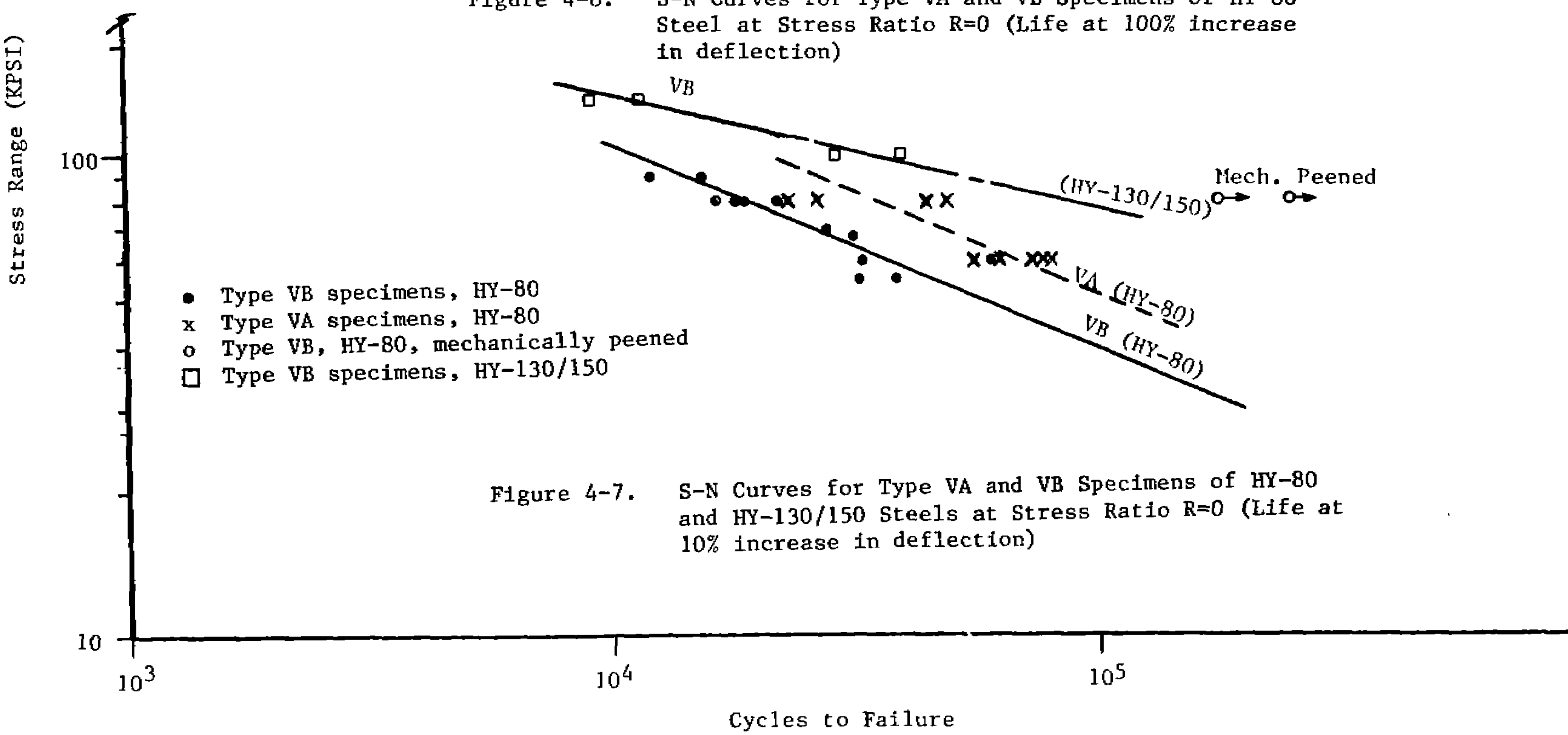


Figure 4-7. S-N Curves for Type VA and VB Specimens of HY-80 and HY-130/150 Steels at Stress Ratio R=0 (Life at 10% increase in deflection)

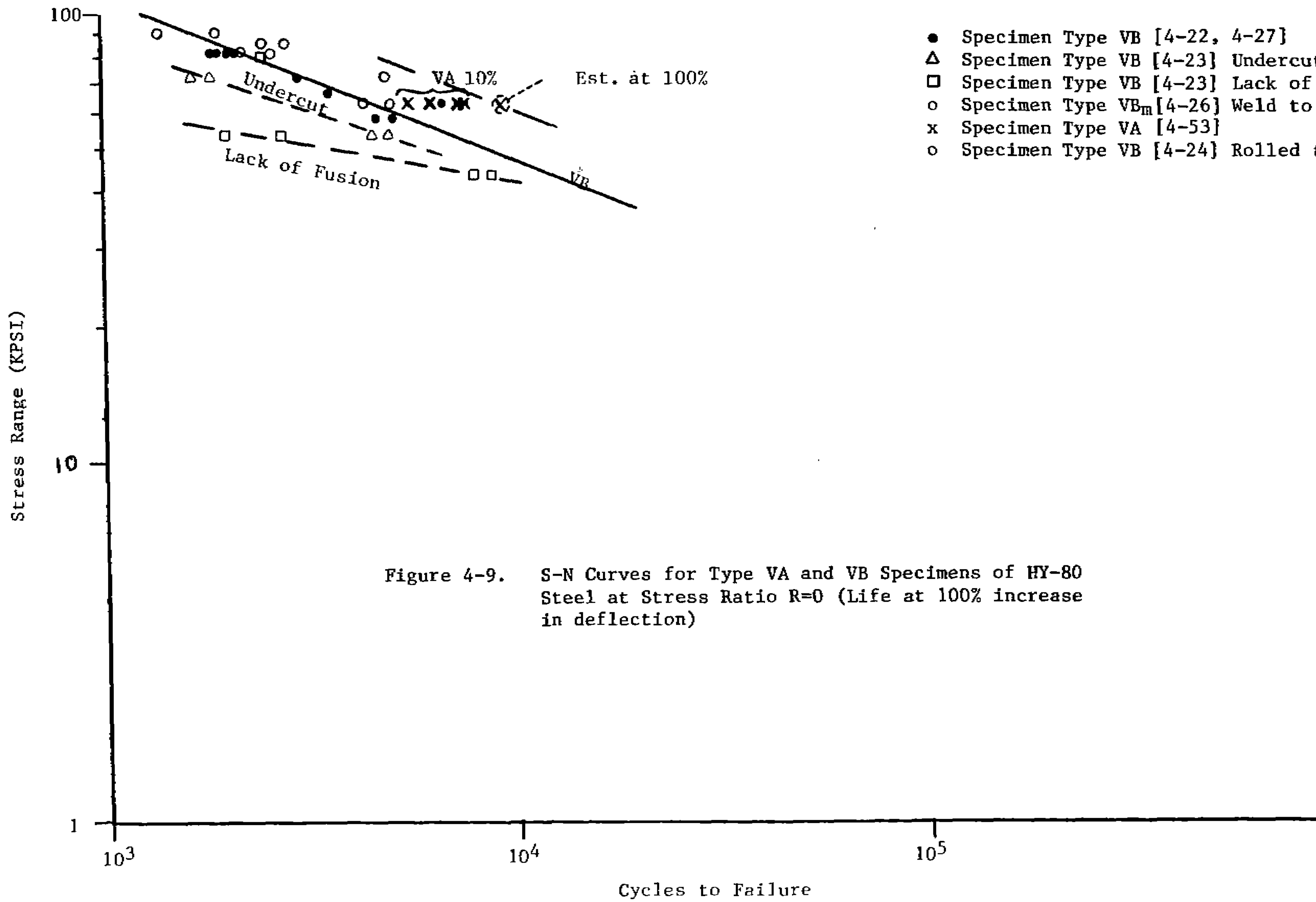


Figure 4-9. S-N Curves for Type VA and VB Specimens of HY-80 Steel at Stress Ratio R=0 (Life at 100% increase in deflection)

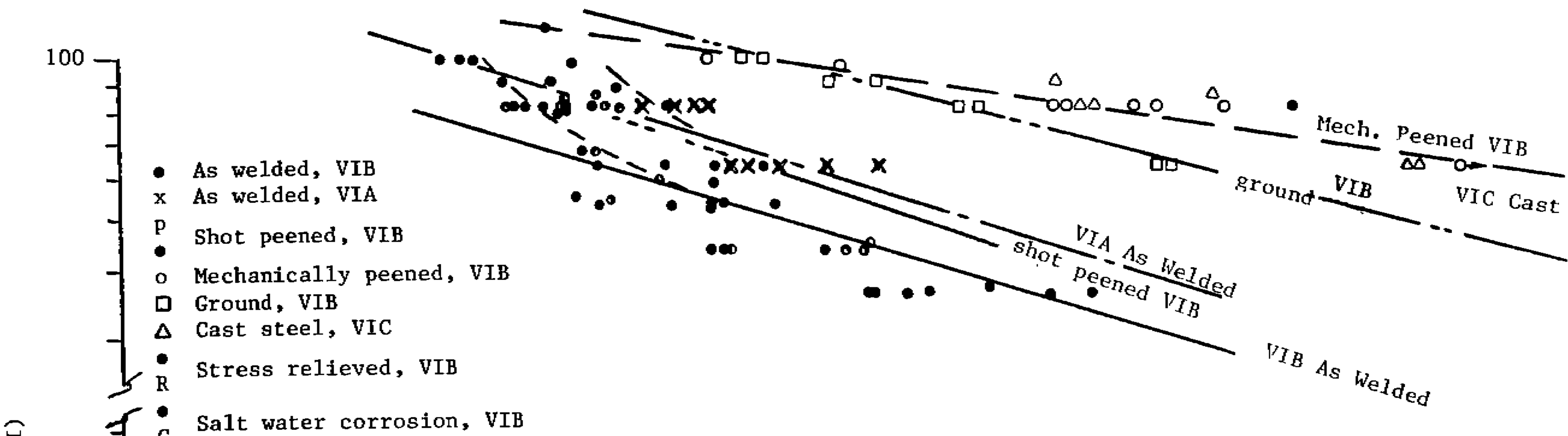


Figure 4-10. S-N Curves for HY-80 Type VI Specimens Under Various Treatments at Stress Ratio R=0 (Life at 10% increase in deflection)

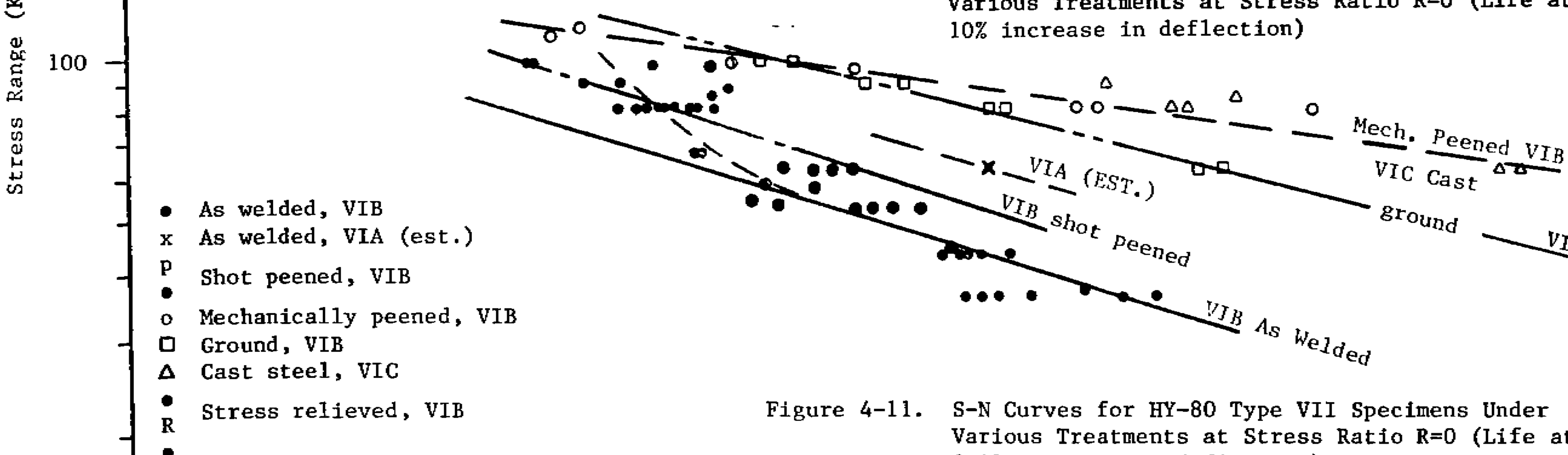


Figure 4-11. S-N Curves for HY-80 Type VII Specimens Under Various Treatments at Stress Ratio R=0 (Life at

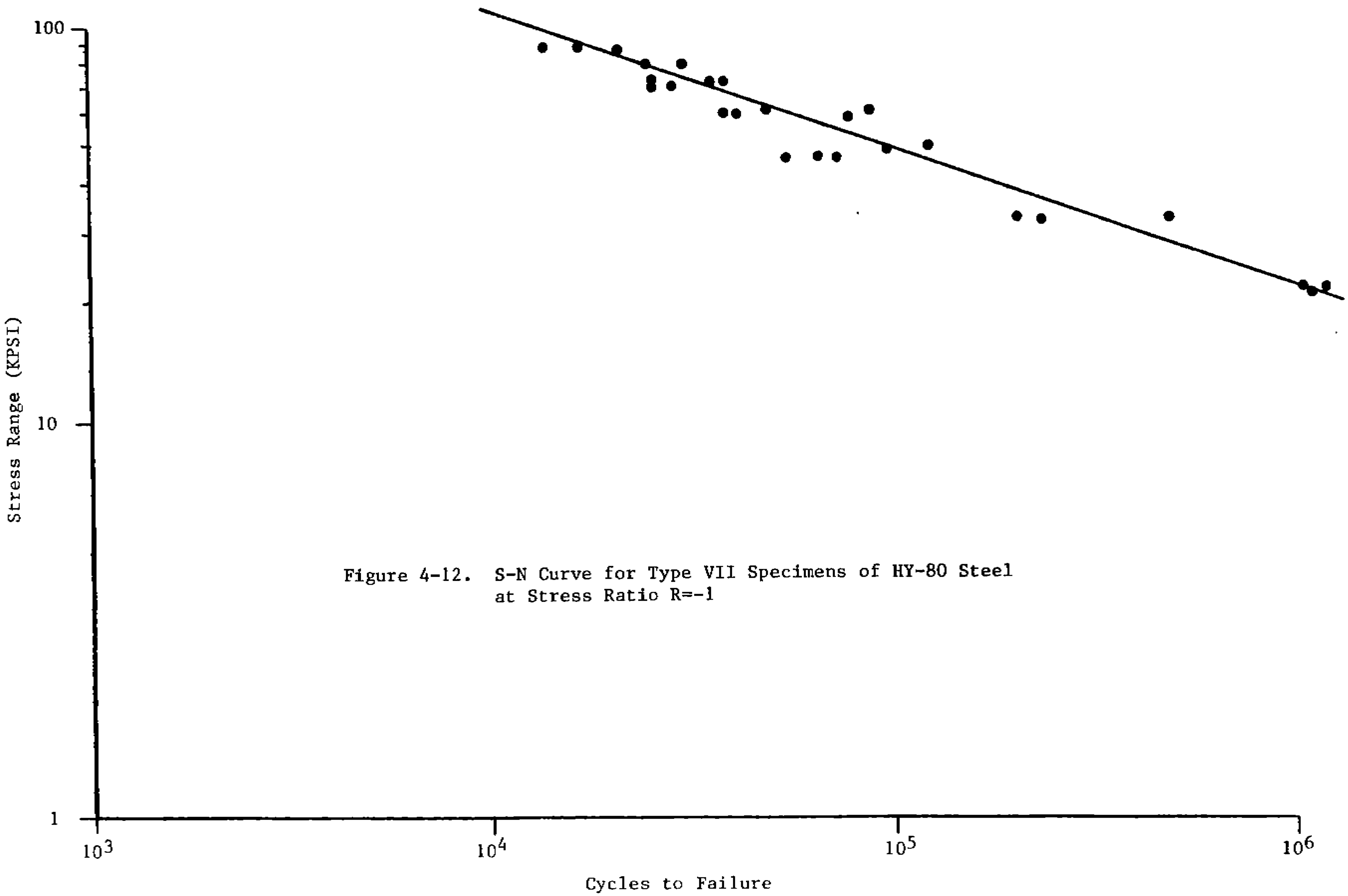


Figure 4-12. S-N Curve for Type VII Specimens of HY-80 Steel at Stress Ratio R=-1

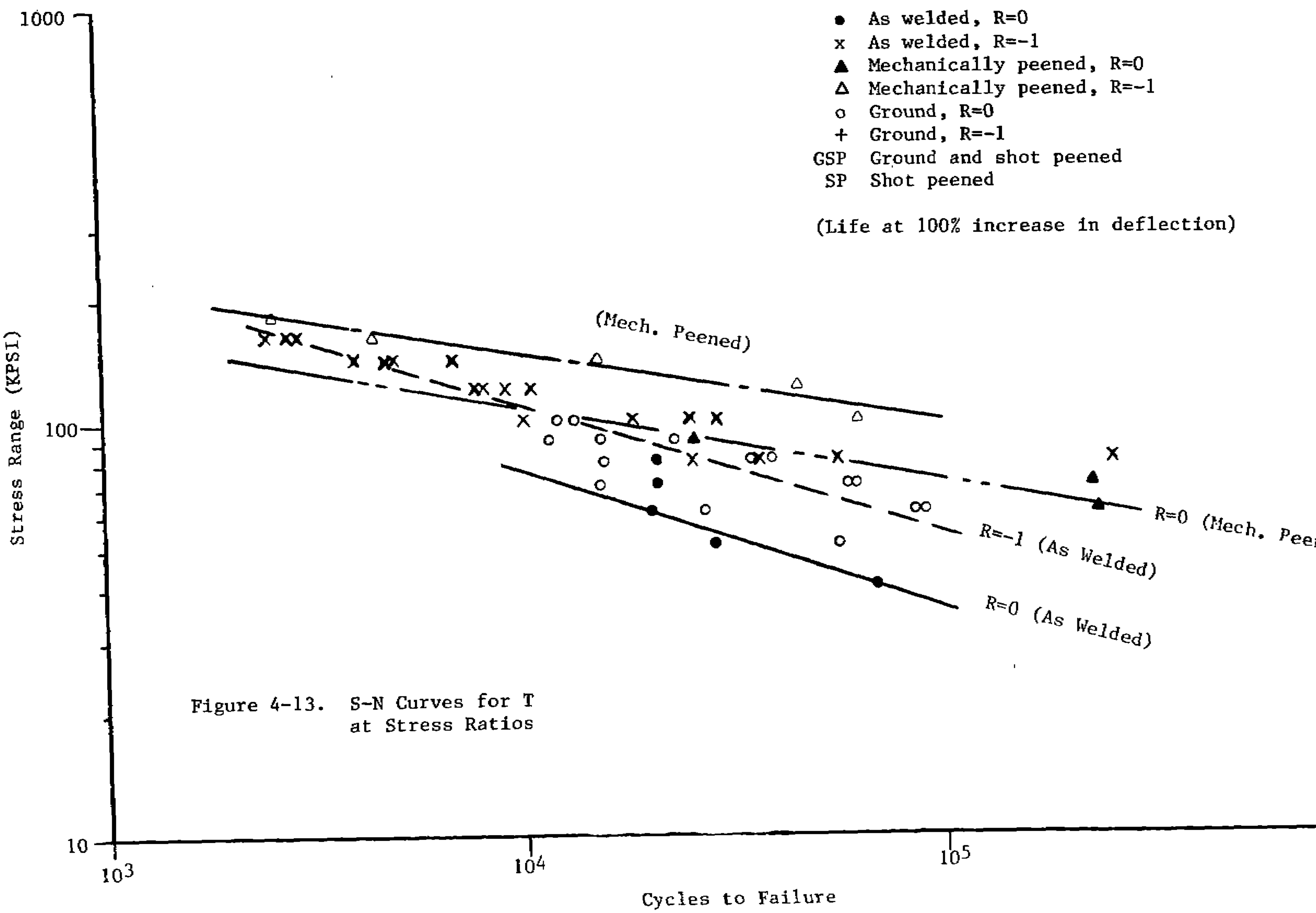


Figure 4-13. S-N Curves for T
at Stress Ratios

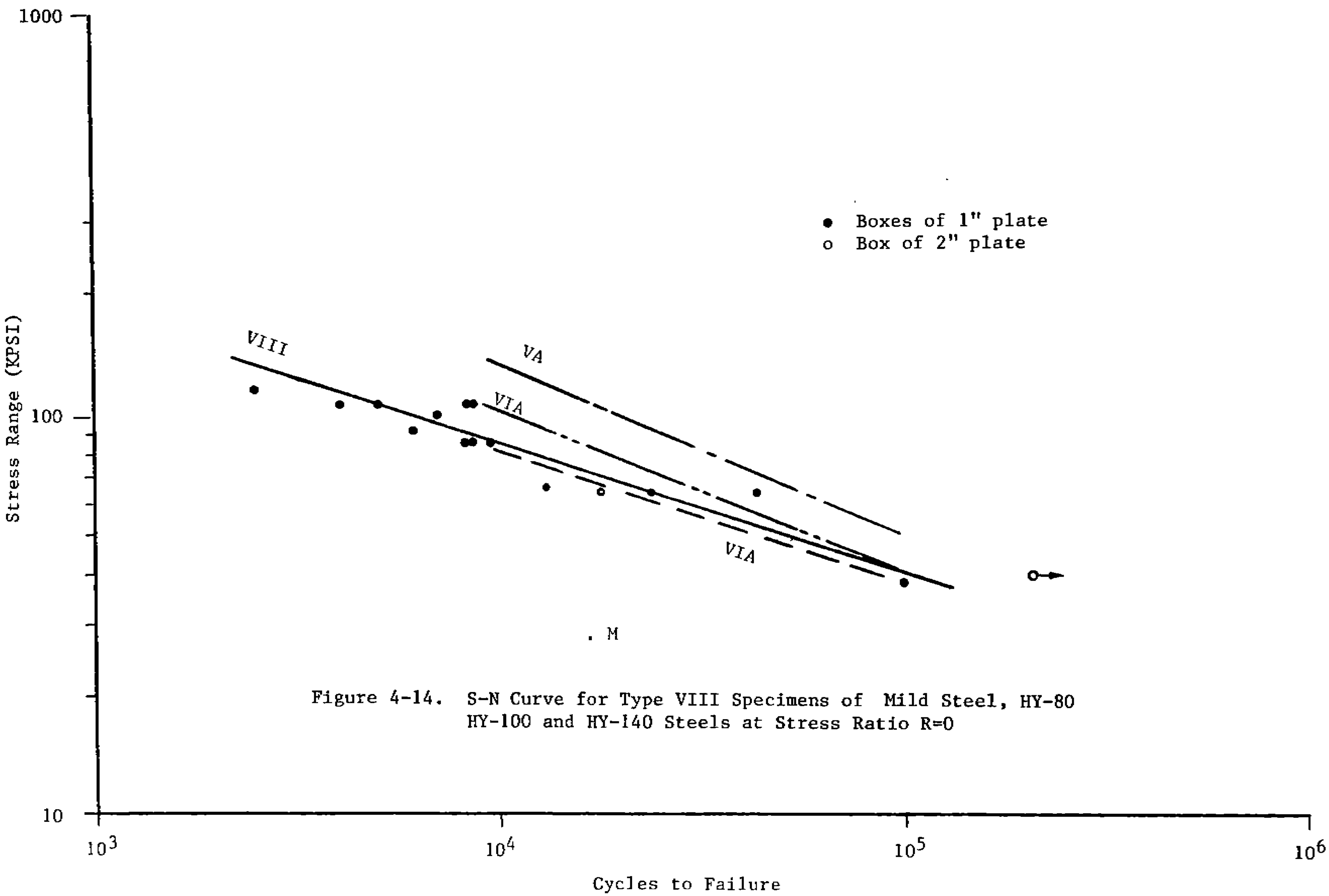


Figure 4-14. S-N Curve for Type VIII Specimens of Mild Steel, HY-80 HY-100 and HY-140 Steels at Stress Ratio R=0

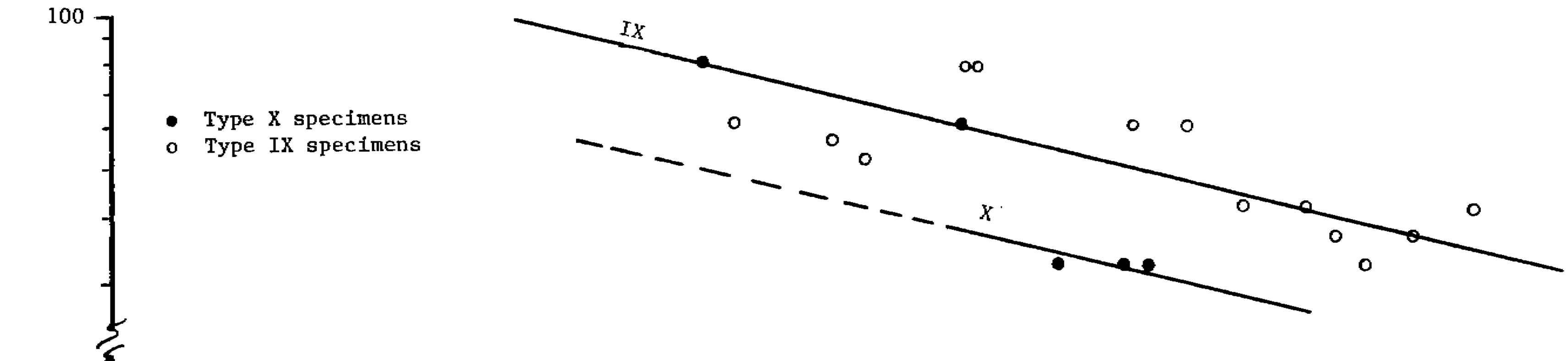


Figure 4-15. S-N Curves for Type IX and X Specimens of HY-80 Steel at Stress Ratio R=0

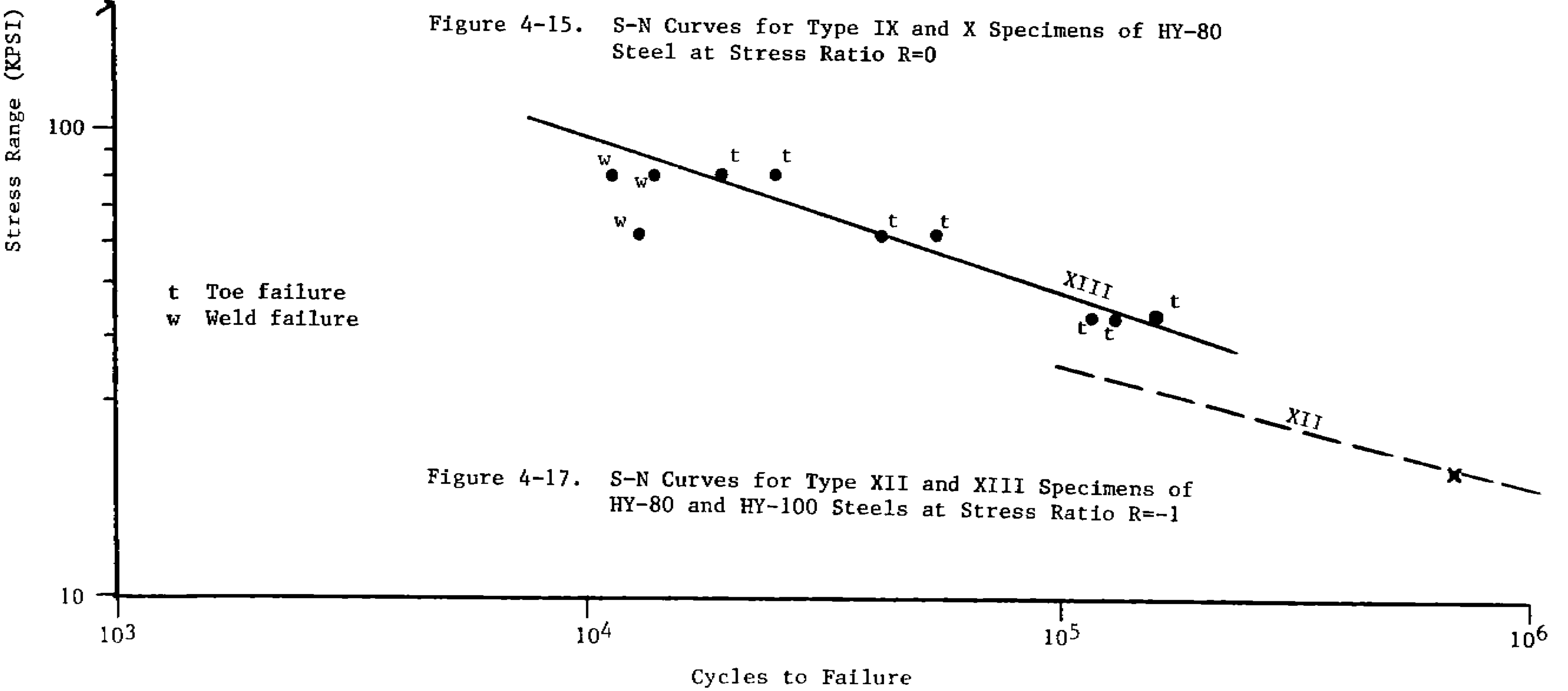


Figure 4-17. S-N Curves for Type XII and XIII Specimens of HY-80 and HY-100 Steels at Stress Ratio R=-1

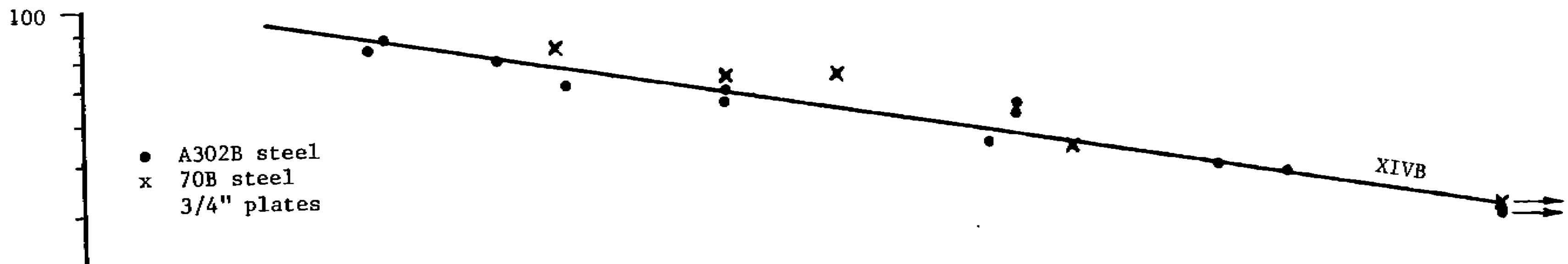


Figure 4-18(a). S-N Curve for Type XIVB Notched Specimens of A302B and 70B Specimens at Stress Ratio R=0

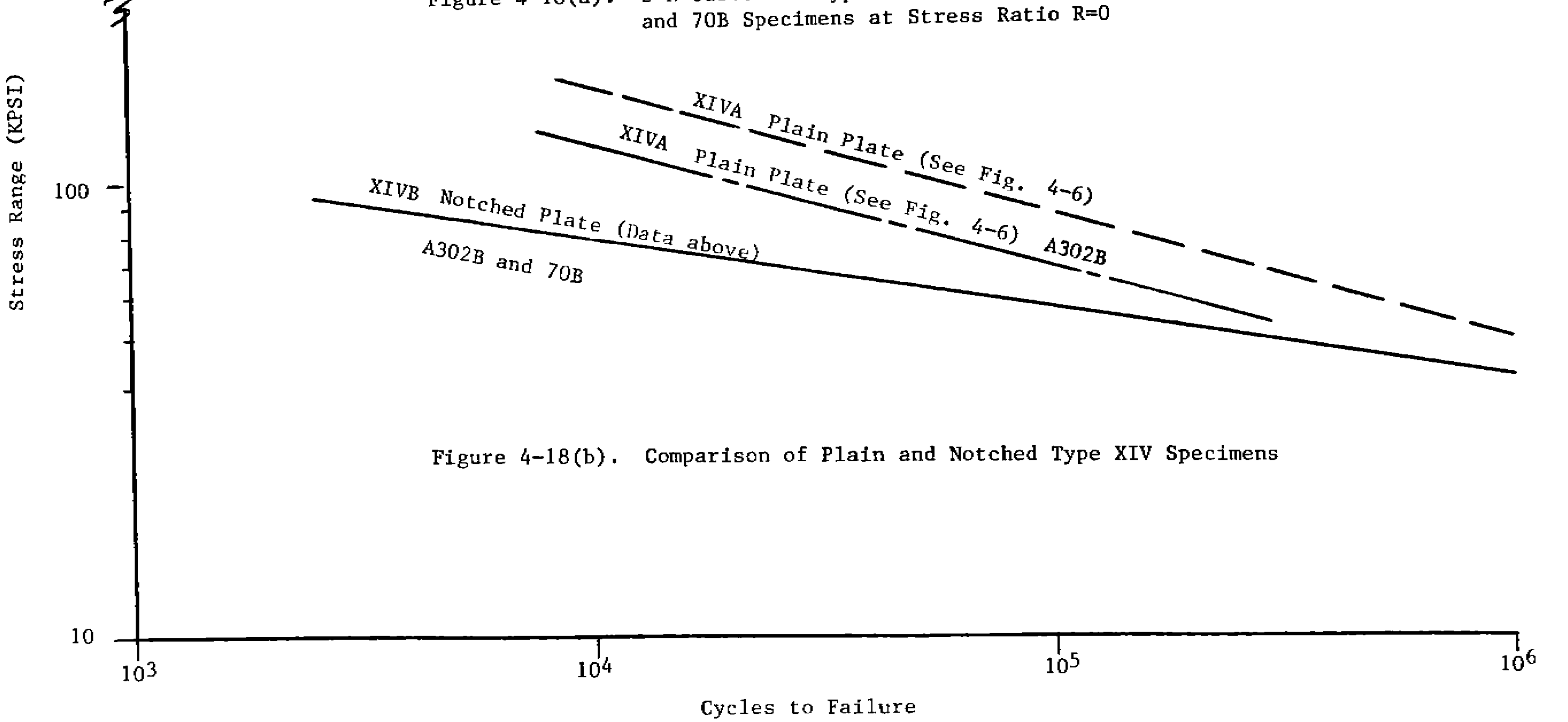


Figure 4-18(b). Comparison of Plain and Notched Type XIV Specimens

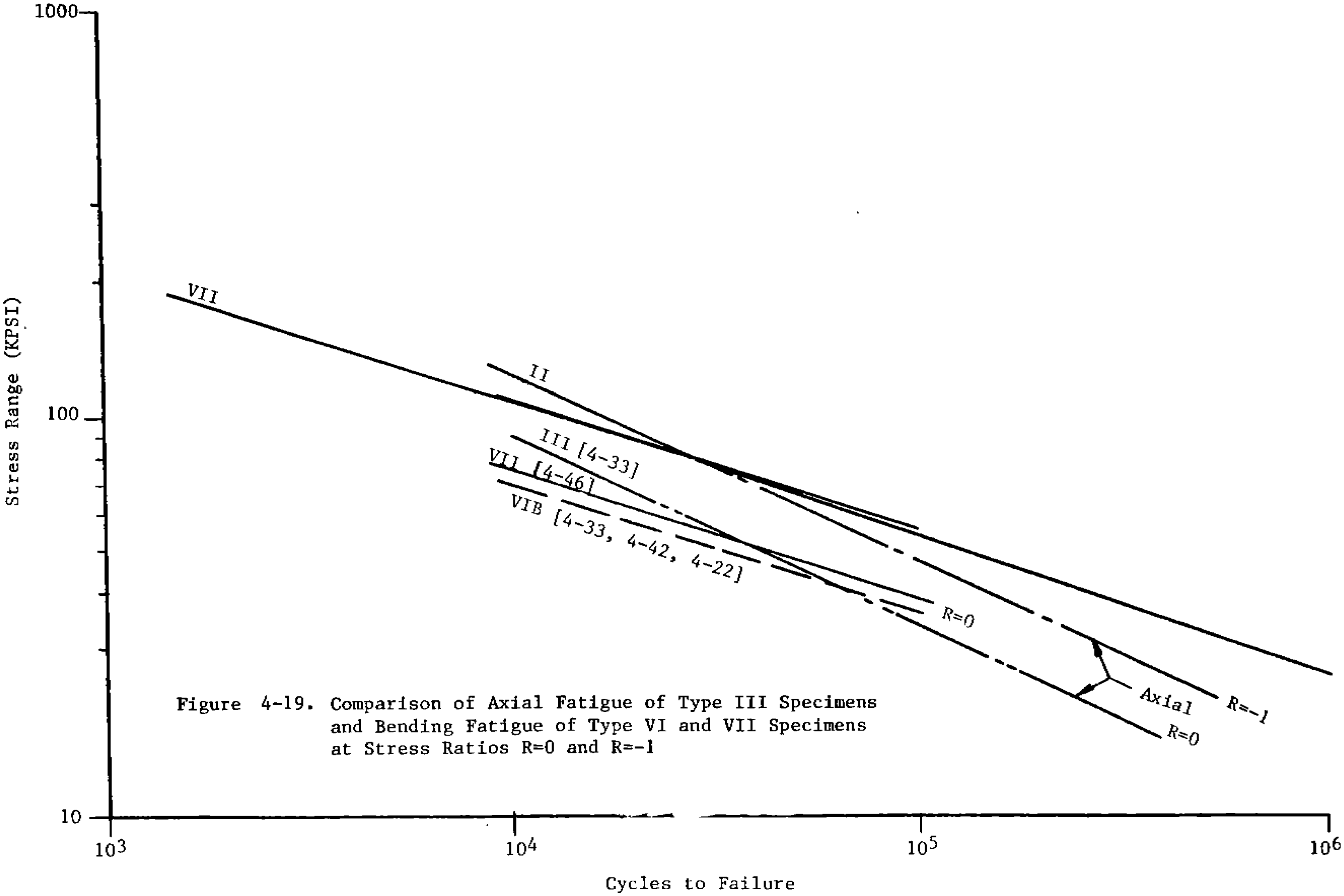


Figure 4-19. Comparison of Axial Fatigue of Type III Specimens and Bending Fatigue of Type VI and VII Specimens at Stress Ratios R=0 and R=-1

TABLE 4-4

HY STEEL TEST MEMBERS USED TO INVESTIGATE MULTIAXIAL FATIGUE RESPONSE

Type I. Axially loaded flat plate specimens in thicknesses of 1/2, 3/4 and 1-1/2-inch. Materials included are HY-80, HY-100 and HY-130/150. The members have been studied under the stress ratios of R=0 and R=-1. The results of the tests are found in Figures 4-2 and 4-3.

Type II. These members are again flat plate members with transverse butt welds and studied under axial loading in thickness of 3/4 and 1-1/2-inches. Members included in the studies are of HY-80 and HY-100 steels and were subjected to stress ratios of R=0 and R=-1, and R=+1/2. The results of the tests are found in Figure 4-4.

Type III. These are also flat plate specimens subjected to axial loadings and fabricated with transverse stiffeners welded to both surfaces of the plate with full penetration welds. Material was 1-1/2 inches thick and test members fabricated of HY-80 and HY-100 steels.

Type IV. The type IV specimens consisted of flat rectangular plates that were subjected to transverse pressure thereby producing a flexural loading of the plates. Type IVA specimens were simply supported on four edges. In the case of type IVB members, the plates were simply supported on two opposite edges and free on the remaining two edges. In this instance, most of the tests have been conducted on HY-80 or HY-130/150 steels. The results of the test can be fund in Figures 4-5 and 4-6.

Type V. These are flat plate members with transverse butt welds that were subjected to transverse bending loads and supported either on four or two edges of the plate. In these studies, primarily conducted on HY-80 and HY-130/150 steels, defects such as undercut and lack of fusion were introduced in the welds. The results of these various studies can be found in Figures 4-7, 4-8 and 4-9.

Type VI. These are flat type specimens which were simply supported on either two or four edges of the plates and subjected to a uniform transverse pressure. However, in this instance a stiffener was welded to the surface of the plate with full penetration welds. In this instance most of the tests were conducted on HY-80 and HY-100 steels. However, a variety of variables have been included in the studies and cast steel specimens have also been studied. The results of these various studies can be found in Figures 4-10 and 4-11.

TABLE 4-4 (continued)

HY STEEL TEST MEMBERS
USED TO INVESTIGATE ULTIAXIAL FATIGUE RESPONSE

Type VII. The Type VII members were beam-type plates on which a transverse stiffener had been welded with full penetration welds. Again, these members are fabricated of HY-80 steel and subjected to a flexural loading which provided a stress ratio of R=1. In a limited number of tests a stress ratio R=0 was also included. The toes of the fillet welds of these members were treated in various ways including mechanically peening, grinding, and shot-peening. The results of these various tests will be found in Figures 4-12 and 4-13.

Type VIII. These specimens consisted of box sections fabricated of plates. The boxes were fabricated of mild steel, HY-100, and HY-140 steels. The various box members were subjected to a stress ratio R=0. Only a limited number of tests of this type have been conducted. Nevertheless, results of these various tests are summarized in Figure 4-14.

Type IX. These members consisted of flat plates specimens with a longitudinal full penetration butt weld in the member. Results of tests on HY-80 steel are presented in Figure 4-15.

Type X. These members consisted of flat plates with transverse attachments on the surfaces of the plate and attached with fillet welds. The tests on HY-80 steel are summarized also in Figure 4-15. However, only a very limited number of tests were conducted on such members.

Type XI. These specimens consisted of flat plates subjected to axial loadings and fabricated with a transverse attachment on one surface attached with full penetration welds. Material was HY-80 or HY-100 steel and the members were subjected to stress ratios of R=-1, R=0 and R=1/2. Results of these various tests are summarized in Figure 4-16.

Type XII. This specimen consisted of a flat plate with longitudinal attachments fillet welded to the surface of the plate. Only one tests of this type was conducted on HY-80 steel and is summarized in Figure 4-17.

Type XIII. These specimens were fabricated as a cruciform T-joint. The flat plates were welded to a transverse bar with full penetration welds and the transverse bar subjected to a loading normal to the surface of the bar. Both HY-80 and HY-100 steels were used in these T-joints. The results of the few tests of this type are presented also in Figure 4-17.

TABLE 4-4 (continued)

**HY STEEL TEST MEMBERS
USED TO INVESTIGATE MULTIAXIAL FATIGUE RESPONSE**

Type XIV. These specimens were flat plate members that were subjected to a transverse pressure and simply supported on four edges. These were smaller than the specimens discussed above under Types IV, V and VI. They were also thinner plates, only 3/4-inch thick. Nevertheless, they were loaded in a manner similar to some of the Type IV, V and VI flat plate specimens. In this instance, however, a Type XIVA member consists of a plain plate and a Type XIVB specimen is plate notched on the surface with a (charpy-V) notch. These results of these various tests will be found in Figures 4-6 and 4-18.

4.3.1 Multiaxial Stress Fields

Tests of plain plates, butt-welded plates, flat plates with attachments, and welded boxes will be examined in detail to obtain a general indication of the effects of multiaxial stresses on the fatigue behavior of these types of structural members.

Flat plate members will be discussed. Tests of Type I, Type IVB and Type XIVA members can be examined to provide some indication of the effect of biaxial stresses on flat plates. The results of fatigue studies on these three types of members are presented in the following tabulation. The detailed data (range of fatigue stress) are presented in Table 4-3 and Figures 4-2 through 4-6.

Fatigue Life of HY Steels

Type	Fatigue Strength at Life of 10^4 cycles	10^5 cycles
I (HY-80)	120 ksi	72 ksi
IVB* (HY-80)	127 ksi	78 ksi
IVB* (HY-130/150)	194 ksi	104 ksi
XIVA (70B)	160 ksi	85 ksi

* Fatigue strength for 10% increase in deflection.

From this tabulation, the axial fatigue strength (Type I) is slightly lower than the fatigue strength of the Type IVB plates in bending for HY-80 steel. The plates in bending appear to have fatigue strengths 5 to 8% higher than that of flat plates subjected to axial loadings. However, when one compares the fatigue resistance of the flat plates of HY-80 and HY-130/150 steels, it is found that the fatigue resistance of the higher strength steel is 33 to 53% higher for the higher strength steel. Thus, the strength of the material has a very significant effect on the fatigue resistance of the flat plates, a much greater effect than the difference found when comparing the bending and axial loading of a given material.

A comparison of the fatigue behavior of the unwelded Type IVB and Type XIVA specimens gives an indication of the effect of one axis bending versus biaxial bending in a flat plate. In this instance, it is found that the fatigue resistance of 3/4-inch plate of the XIVA specimens was 9 to 26% greater than that of the type IVB specimens. However, the materials differed in thickness, the XIVA specimens were 3/4-inch thick and the IVB specimens were 1-1/2-inch thick and they were also of a different material. The IVB specimens were fabricated of HY-80 steel with a tensile strength of 108 ksi whereas the Type XIVA members were fabricated of 70B steel with a tensile strength of 102 ksi. Nevertheless, it appears that flat plates subjected to a transverse loading had a somewhat higher fatigue strength when subjected to biaxial stressing than when subjected to uniaxial stressing.

A second group of members examined consisted of flat plates with a transverse butt weld which were subjected to transverse uniform pressure. These members, Type V, had a lower fatigue strength than that of the plain flat plates of Type IV, previously considered. The S-N curves were also steeper thus indicating a more severe notch effect in the butt welded members which produced the lower fatigue strength.

In the case of the Type V members, both A and B specimens were tested and thereby give some indication of the effect of a biaxial loading. As indicated in Figures 4-7 and 4-8, the biaxial flexural loading again provided an increase in fatigue resistance. In this instance the fatigue strength of the Type VA members was approximately 30% greater than that of the Type VB members.

The third group of flat plate members subjected to lateral loadings were plates with transverse welded attachments (Type VI). The following tabulation presents the results of the Types IV, V and VI test specimens.

Fatigue Life of Different Details

Type	Fatigue Strength at Life of 10^4 cycles	10^5 cycles
IV B	127 ksi	78 ksi
V B	106 ksi	39.5 ksi
VI B	60 ksi	30 ksi
V A	137 ksi	51 ksi
VI A	81 ksi	39 ksi

It is readily evident that the effect of butt welding and the addition of welded attachments has a significant effect on the fatigue resistance of the flat plates subjected to transverse pressures, the greater the stress concentration of the weld detail, the greater the reduction in fatigue resistance.

As in the case of detail Type V, it can be seen that the biaxial stressing in the Type VI members (Type VIA vs. Type VIB) again produced an increase in fatigue resistance of approximately 30%. This is a significant increase in fatigue resistance under the biaxial stressing condition.

Another evaluation can be made by comparing the Type III, Type VI, and Type VII specimens. These include plates subjected to lateral loading, beams subjected to bending, and axial specimens all of which have stiffeners attached to them. Stress ratios R=0 and R=-1 were included for these three types of members. A summary of the results is presented in Figure 4-19. Here it may be seen that at approximately 40,000 cycles of loadings, the fatigue resistance of the three types of members are approximately equal. However, the Type III specimens have a somewhat steeper slope to their S-N curve. This would suggest that the stress concentration for the detail was somewhat more severe for this type of specimen than for the Type VI and VII specimens. It can also be seen that the fatigue resistance is approximately 50% greater under a completely reversed stress cycle than it is for a 0 to tension type of stress cycle.

The fourth type of member to be considered is the welded box of HY-100 steel. These boxes, Type VIII, were subjected to internal pressure and consequently stressed in a manner similar to the butt welded plates or plates with attachments, Types VA and VIA. The stresses in the longitudinal and transverse direction of the boxes are in a ratio of 1 to 2. The results for the various series of tests on boxes are shown in Figure 4-14. Their behavior appears to have been quite similar to that of the stiffened plates, Type VIA. The slopes of the S-N curves appear to be quite similar and their fatigue strengths comparable. The plain butt welded plates, Type VA, appear to have a somewhat higher fatigue strength. In one instance, a mild steel box (identified by M) was tested under internal pressure and appeared to have a somewhat lower fatigue resistance, as shown in the figure.

In summary, it appears that biaxial stressing in ratio of approximately 1 to 2 provides a fatigue strength on the order of 25 to 30% greater than that of members stressed axially or in bending in only one direction.

4.3.2 Mean Stress

The effect of mean stress on the fatigue strength of the various types of specimens can most readily be studied by examining the fatigue behavior of the members at various stress ratios. In Table 4-5, data are available for various stress ratios for details of Type I, II, III, VII, and XI. A summary of the relative fatigue strengths of the members at various stress ratios and at lives of 10^4, 10^5, and 10^6 cycles are presented. In this table it is evident that a significant difference exists in the fatigue strengths at stress ratios of -1, 0, and +1/2.

A visual indication of the effect of stress ratio on fatigue strength can be obtained from the fatigue diagram of Figure 4-20. Here the fatigue strengths for the various details, at various lives, have been plotted and clearly show the stress ranges decrease significantly as the mean stress is increased. It appears that the fatigue resistance can be represented by the following relationship.

$$F_r = F_{ro}(1-0.45R)$$

where F_r = fatigue stress range at stress ratio, R.

F_{ro} = fatigue stress range for a stress ratio of 0 (0-to-tension stress cycle).

R = stress ratio, ratio of minimum to maximum stress in the stress cycle.

In a similar study [4-5], a similar relationship was presented except that the coefficient for R was somewhat smaller. Nevertheless, in both evaluations it is seen that the fatigue resistance is clearly affected by the mean stress to which the members are subjected.

4.3.3 Stress Gradient

Relatively few data are available concerning the effect of stress gradient on the fatigue resistance of welded members. Some data are available on small rotating beam specimens and are reported in various locations in the literature. For

TABLE 4-5

EFFECT OF MEAN STRESS ON FATIGUE STRENGTH OF VARIOUS MEMBERS

SPEC. TYPE	STRESS RATIO	FATIGUE STRENGTH AT VARIOUS LIVES 10^4	10^5	10^6
I	R=0	1.0	1.0	1.0
	R=-1	1.30	1.39	1.49
II	R=+1/2	0.73	0.72	0.71
	R=0	1.0	1.0	1.0
	R=-1	1.42	1.42	1.43
III	R=0	1.0	1.0	-
	R=-1	1.41	1.43	-
VII	R=0	1.0	1.0	-
	R=-1	1.45	1.42	-
XI	R=+1/2	0.76	0.74	0.72
	R=0	1.0	1.0	1.0
	R=-1	1.38	1.33	1.27

NOTE: Values are the ratio of the fatigue strength at the stress ratio shown to the fatigue strength at a stress ratio R=0.

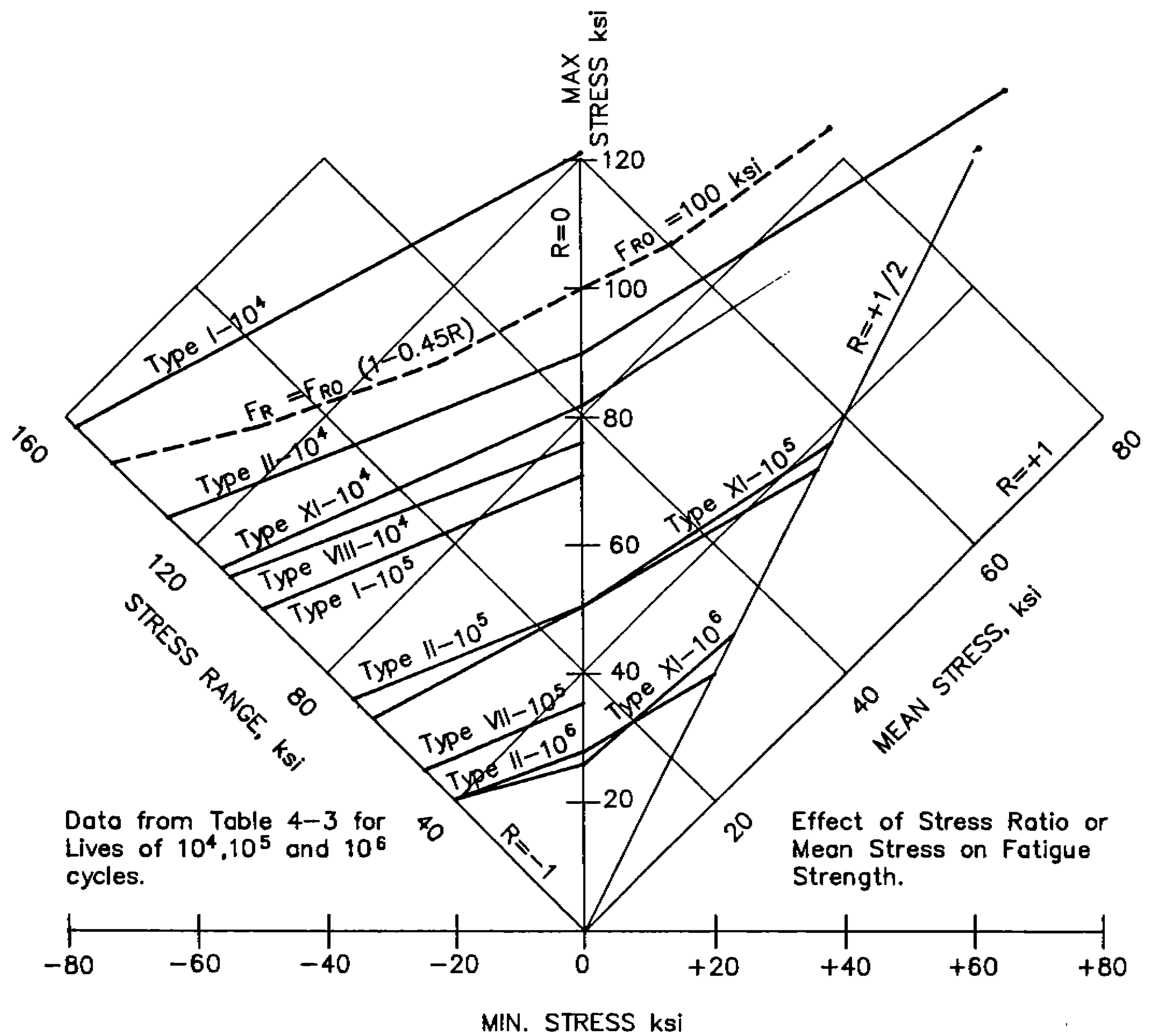


Figure 4-20. Modified Goodman Diagram for HY-80

example, Reference 4-58 indicates that as the diameter of a member increases the fatigue resistance of the material can be expected to decrease under a flexural loading. There are indications that up to a diameter or thickness of approximately 6 inches, the fatigue resistance continues to decrease. Beyond that thickness or depth there is little or no further decrease in fatigue resistance. This is in agreement with the fatigue resistance of 12-in. deep beams. The fatigue resistance of the beams is approximately the same as that of axially loaded plates of the same type of material, indicating that there is no stress gradient effect in the 12-in. beam tests.

If it is assumed that axially loaded members represent the flexural minimum fatigue resistance of deep members or thick material where there is no stress gradient effect, then some general indication of stress gradient effects can be obtained by comparing the results of the data tabulated in Table 4-3. In Figure 4-19, data are compared for Type III, VI, and VII members. This comparison indicates that for a life of approximately 40,000 cycles there is no significant difference in fatigue resistance for the axially loaded members (Type III specimens) and the plates and beams 1 1/2-inches thick that are loaded in flexure (Type VI and VII specimens). However, the S-N curves for the Type III specimens, members with attachments on both sides of the plate, have a steeper slope and consequently a lower fatigue resistances at very long lives.

A similar comparison is shown in Figure 4-21 for butt-welded plates under axial loading and flexural loading. Again, there is a difference in slope of the S-N curves. In this instance however, the S-N curve for the butt-welded plates subjected to flexural loading has a steeper slope than those for the members loaded axially. It is evident that much more information is required to evaluate the question of stress gradient, particularly where multi-axial stresses exist.

4.3.4 Residual Stresses

Another factor considered in this study is the effect of residual stresses on the fatigue behavior of the various types of test members. The data presented in Figures 4-10 and 4-11 clearly show that residual weld stresses had little or no effect on fatigue life of Type VIB members; members that were stress relieved had approximately the same fatigue resistance as members tested in the as-welded condition. This should be expected for the short lives studied in the investigations.

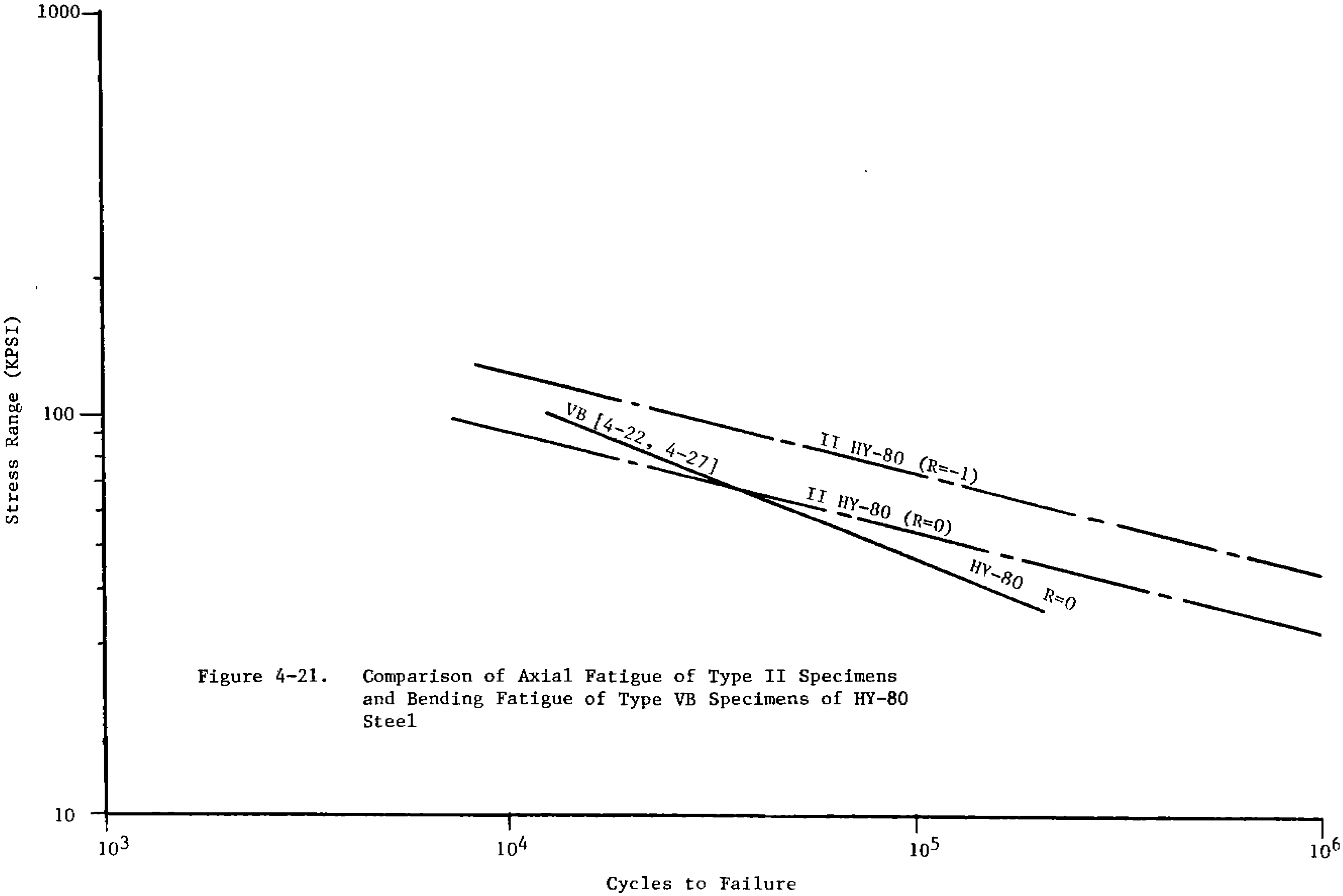


Figure 4-21. Comparison of Axial Fatigue of Type II Specimens and Bending Fatigue of Type VB Specimens of HY-80 Steel

However, members subject to lower stress ranges (long lives) stress relieving can be expected to have an effect on the fatigue resistance of welded test members. If compressive residual stresses are introduced they can have a beneficial fatigue effect on a member, even at the high stress ranges employed in the tests reported herein. This can clearly be seen in the Figures 4-22 and 4-23 studies wherein test members were mechanically peened. This mechanical peening not only introduced residual compressive stresses in the peened region but also changed the geometry at the toe of the weld where this mechanical peening was applied.

Detailed examination of the data in Reference 4-42, wherein members were mechanically peened and then stress-relieved, indicates that the stress-relieving treatment reduced the fatigue resistance because it removed compressive residual stresses. In the cases where a stress-relieving treatment was applied before mechanical peening a higher fatigue resistance was obtained, indicating that the residual compressive stresses remained in the member after mechanical peening in this latter case and the higher fatigue resistance resulted.

From the limited data available, it is evident that the effect of residual stresses on fatigue, whether in uniaxial or multiaxial test, depends upon the sense of the residual stresses, their magnitude, and the magnitude of the applied stresses.

4.3.5 Corrosion

Although corrosion could be a very important factor for ship structures, relatively little information is available concerning the effect of salt water on the fatigue behavior of weldments in ship steel. In Reference 4-22, three corrosion tests were conducted on Type VIB specimens. In these tests salt water produced about an 18% reduction in fatigue resistance and approximately 58% reduction in fatigue life of the test members. However, since these tests were conducted at a frequency of 20 cycles per minute, the test would only have taken approximately 50 hours. This raises many question concerning the effects of corrosion under long life conditions and also many of the other factors that are involved in corrosion fatigue. In Reference 4-59, reference is made to many pertinent factors that affect corrosion fatigue. These include such factors as temperature, oxygen content, electrolytic potential, load ratios, cyclic frequency, etc. It is readily evident that much more needs to be done to evaluate the effects of corrosion in fatigue.

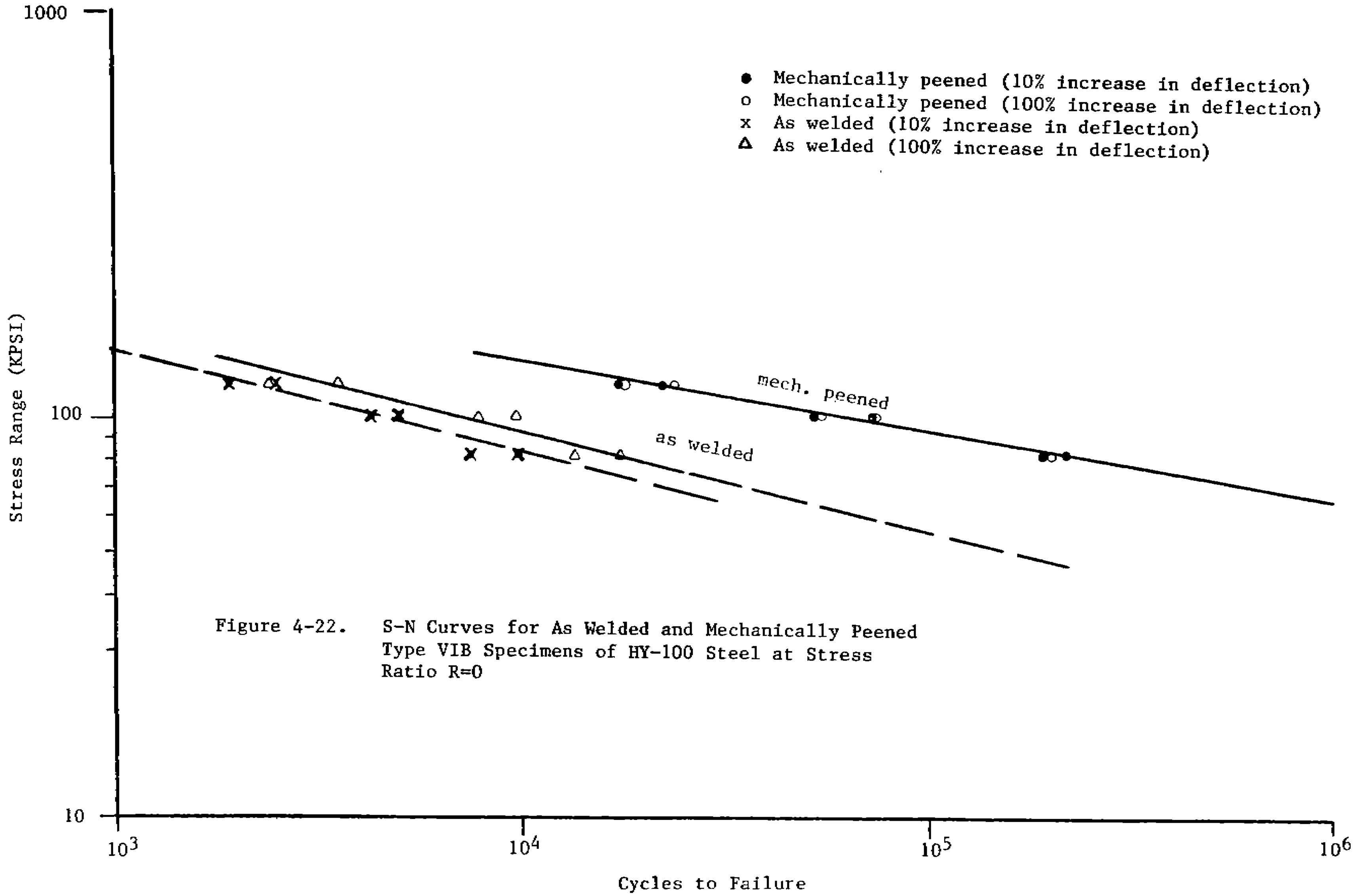


Figure 4-22. S-N Curves for As Welded and Mechanically Peened Type VIB Specimens of HY-100 Steel at Stress Ratio R=0

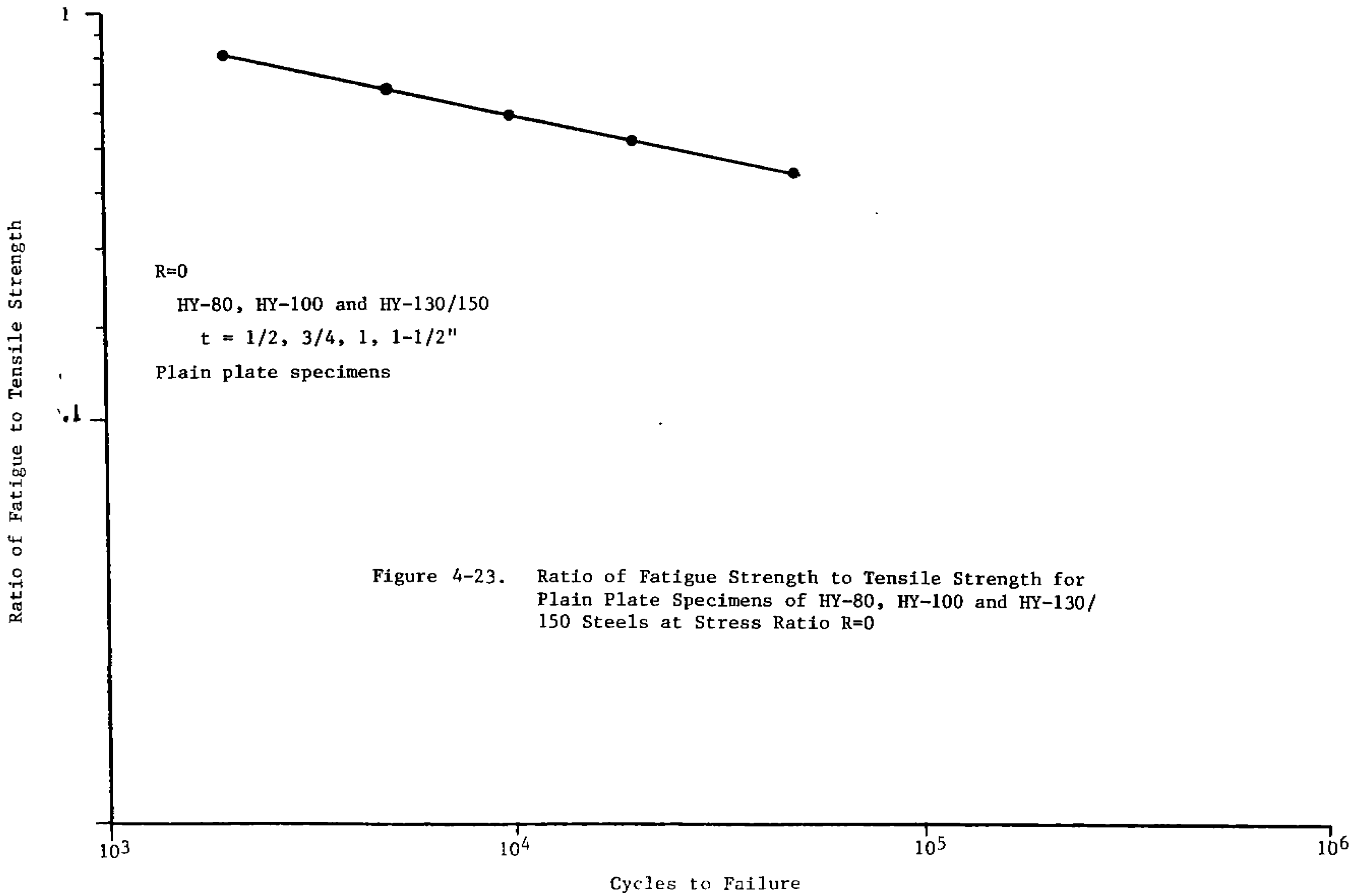


Figure 4-23. Ratio of Fatigue Strength to Tensile Strength for Plain Plate Specimens of HY-80, HY-100 and HY-130/150 Steels at Stress Ratio R=0

4.3.6 Geometry, Fabrication Treatment, Defects

It is well known that the geometry of welded structural members has a significant effect on the fatigue behavior of such members. This is readily evident in the fatigue strengths tabulated in Table 4-3. Here it can be seen that at a life of 10^5 cycles fatigue strengths for a stress ratio R=0 are found to range from 32 to 54 ksi and for a life of 10^6 cycles the fatigue strength range from 17.5 ksi to 57 ksi. Among the highest values are the results from tests on plain plate material and cast members for which the geometry is very simple and with little or no stress concentration.

As noted previously and shown in Figures 4-22 and 4-23, mechanical peening the toes of welds can improve the fatigue resistance of a welded member. However, care must be exercised in evaluating such improvements since much of the improvement obtained in the weldments shown in these figures was a result of the residual compressive stresses produced by the mechanical peening.

Grinding of the weld toes can also provide some benefit to the fatigue resistance. However, in this instance the improvement is a result of the improved geometry.

Another geometrical factor that may affect the fatigue strength of structure weldments is their defect or discontinuity quality. In the case of a 1/16-in. undercut in Type VB and VIB specimens only a small reduction in fatigue strength was observed. However, a lack of fusion was found to produce a greater reduction (see Figures 4-9 through 4-11). See Reference 4-23.

4.3.7 Size - Thickness

In the investigations included this study, materials varying in thickness from 1/2-inch to approximately 1 5/8-inches were included. Nevertheless, there appears to be relatively small effect of member thickness. This effect appears to be overshadowed by such other factors as tensile strength and magnitude of stress applied in the tests.

An examination of thickness in fact was included for specimens of Type I, VI, VII, and VIII. In the case of the Type I specimens, plain flat plates subjected to axial loadings, thicknesses ranging from 1/2-inch to 1 1/2-inches were included. In the case of 3/4 and 1 1/2-inch specimens of HY80 steel, greater lives were obtained for the 1 1/2-inch material than for the 3/4-inch thick material when tested on a stress cycle 0 to 50 ksi. This is contrary to the general observation often made that the fatigue resistance decreases with an increase in thickness. Upon examination of the strength of the two material, it was found that the tensile strength of the thicker material was approximately 8% greater than the tensile strength of the 3/4-inch thick material. This no doubt accounts for the increased fatigue resistance of the thicker material.

In the case the Type VI and Type VII specimens, relatively little information was available concerning the effect of thickness and no firm conclusions could be drawn concerning this factor.

For the Type VIII box type specimens, material 1 inch and 2 inches in thickness were used in the tests. Neglecting the tensile strength of the materials it is found that both thickness provided approximately the same strength for the test members. This may be seen in Figure 4-14.

4.3.8 Material Strength

Material strengths ranging from mild steel to HY-130/150 steel have been included in this study. These various materials have been examined for a variety of specimen type and in general it is found that the fatigue strength tends to increase with the tensile strength. However, the relationship of the fatigue strength to tensile strength is affected by a variety of other factors such as geometry, stress level, etc.

Combining the data for flat plate specimens of HY-80, HY-100, HY-130/150 steels and in thicknesses of 1/2-inch to 1 1/2-inches, Reference 4-19, one finds that the ratio of fatigue strength to tensile strength varies from values of 0.80 to 0.40, depending upon the life of the member. As shown in Figure 4-15, the ratio of 0.80 is obtained for a life of 20,000 cycles and the value of 0.40 at a life of 500,000 cycles. The shortest lives, of course, are obtained at the highest test stress levels and the longest lives at the lower test stress levels. Thus, it would appear that the fatigue to tensile strength ratio is primarily a function of the magnitude of stress to which its members are subjected, this being a more important factor than the strength of the material or the thickness of the material.

The geometry of the specimens also becomes an extremely important factor and may overshadow the effect of the tensile strength of the material. It is evident that material does have some effect on the behavior of the members but other factors such as geometry and stress magnitudes play a much more important role.

4.3.9 Random Loading

While random loading has been proven to have a large effect on the fatigue life of structures, to date little or no data are available concerning random loadings on members subjected to multiaxial fatigue of the type considered in this study. The type of random loading distribution can have a large influence on the effective stress used in the fatigue analysis. We also know that marine structures are subjected to random loading where principal stresses' proportions are also random. Random loading is a factor that must be strongly considered in any future multiaxial fatigue testing.

4.3.10 Stress Phasing

In laboratory studies of multiaxial stresses the repeated stresses are generally applied in a proportional manner, that is, the proportions of flexure to shear is constant or the proportion of tension or flexure to torsion is constant. However, in marine structures such as a ship or an offshore platform, proportions will vary. Consequently, in a stiffened laboratory beam, for example, the direction of principal stress at a given point in the beam will remain constant even though the magnitude of the loading is changed. In marine structures that are subjected to varying flexural and torsional loads, the magnitudes and directions of the principal stresses will change with time.

To date there does not appear to be any information available concerning the above types of stress phasing in welded structural details. However, considerable research has been conducted on notched specimens as discussed in Section 3.2. Until such information becomes available for welded details it will probably be necessary to consider the fatigue of such structural details in terms of the maximum principal tensile stresses without consideration of the oscillation or rotation in the direction in which such stresses might be applied.

5.0 EVALUATION OF MULTIAXIAL FATIGUE RELIABILITY

The evaluation of multiaxial fatigue reliability was performed to quantify the effects of multiaxial fatigue and with the purpose of developing a baseline to be used in judging overall improvements in reliability through the use of multiaxial fatigue design procedures. The effort included a review of existing reliability formats and data to develop a baseline to judge overall improvements in reliability by using multiaxial fatigue design procedures in marine structures.

5.1 RELIABILITY FORMAT FOR EVALUATING MULTIAXIAL FATIGUE IN MARINE STRUCTURES

The purpose of engineering design is to ensure the safety or performance of a given system for a given period of time and/or under a specified loading condition. The absolute safety of the system cannot be guaranteed due to the number of uncertainties involved. In structural design, these uncertainties can be due to randomness of loads, simplifying assumptions in the strength analysis, material properties, etc. However, through probabilistic analysis we can limit the risk of unacceptable consequences. The major benefit of a reliability-based design approach which utilizes probabilistic analysis is that a designer will be able to generate an engineering system which is both efficient and reliable to the level specified or desired.

The concepts of the reliability approaches for most types of structures are relatively simple, but complexity is associated with execution of the procedure.

In general, the structural reliability problem can be considered as one of load and resistance. Failure occurs when the supply (the resistance or strength of the system) is less than the demand (the loading of the system). For a structural system, this can be stated as:

$$\text{Probability of Failure} = P_f = P\ (\text{strength} < \text{load})$$

If both the load and strength are treated as random variables, then the reliability problem can be treated using probabilistic methods. In order to perform a reliability analysis, a mathematical model which relates the load and resistance must be derived. This relationship is expressed in the form of a limit-state equation. For the simple case used above, it would appear as

$$g(x) = R - L$$

where R and L are the resistance and load effect random variables. Failure is represented by the region where g(x) is less than zero while the safe region is where g(x) is greater than zero. The line g(x) = 0 represents the boundary between the regions and is thus defined as the limit-state equation.

The implementation of reliability-based design methods does not mean that all engineers and designers need to be deeply versed in probability theory. Rather, the design criteria they used should be developed in a format which is both familiar to the users and which should produce desired levels of uniformity in safety among groups of structures. This should be accomplished without departing drastically from existing general practice.

One of the more popular formats for including probabilistic information in structural design is the Load and Resistance Factor Design (LRFD) approach as recommended by the National Bureau of Standards [5-1]. This approach uses load amplification factors and resistance reduction factors (partial safety factors) and can be expressed as:

$$\phi R \geq \sum_{i=1}^{n} \delta_i L_i$$

where R is the resistance, e.g. in flexural shear, fatigue, etc.; L_i is the i^{th} load effect, e.g. due to dead, live, wind, earthquake loads, etc.; ϕ is the resistance reduction factor; δ_i is the i^{th} partial load effect amplification factor; and n is the total number of load effects considered in the limit-state design equation.

The implementation of an LRFD format for reliability-based fatigue design has been proposed by Albrecht [5-2], Ang [5-3], and Munse [5-4] for cumulative damage fatigue approaches and by Wirsching [5-5] for the fracture mechanics approach to fatigue propagation.

For fatigue cumulative damage estimates of structures, the resistance is usually represented in terms of the mean and standard deviation of the number of cycles to failure at a given stress range. This information is typically derived from constant amplitude fatigue tests data for specimens or components being investigated. According to this approach, a number of these tests are conducted and the results are provided in the form of stress range versus life (S-N) curves. The question arises whether an equivalent stress or strain approach based on small component tests simulating the load effects of the complex joint can be used to reduce the scatter about the mean line, thus reduce conservatism and improve reliability of the structure.

Before this question can be answered, it is beneficial to review the reliability formats in greater detail to illustrate the role of fatigue data bias and scatter in the reliability formats.

The standard deviation of the fatigue life data can be found. However, the scatter of the data about the mean fatigue line is not the only uncertainty involved in the S-N analysis. A measure of the total uncertainty (coefficient of variation) in fatigue life, V_R, is usually developed to include the uncertainty in fatigue data, errors in the fatigue model, and any uncertainty in the individual stresses and stress effects.

Ang [5-3] and Munse [5-4] suggested that the total COV in terms of fatigue life could be given by:

$$V_R^2 = V_N^2 + V_F^2 + V_C^2 + (mV_S)^2$$

where:

V_R = total COV of resistance in terms of cycles to failure

V_N = variation in fatigue test data about mean S-N line

V_F = variation due to errors in fatigue model and use of Miner's rule

V_C = variation due to uncertainty in mean intercept of the regression line; includes effects of fabrication, workmanship, and uncertainty in slope

V_S = variation due to uncertainty in equivalent stress range; includes effects of error in stress analysis

m = slope of mean S-N regression line.

Values of m and V_N can be obtained from sets of S-N curves for the type of detail being investigated; a number of which are tabulated by Munse [5-4].

Reasonable values for the remaining uncertainties are available in the literature [5-3 through 5-8]. Typically, V_S is taken to be 0.1, V_C is assumed to be 0.4, and V_F is taken as 0.15.

As can be seen from the discussion, there are many more uncertainties other than V_N for scatter. However, this is where the primary improvement in reliability will be derived when using multiaxial fatigue design procedures. Typically, COV for scatter V_N is .6 for welded structural details.

To determine the relationship between load and resistance for fatigue analysis, the value of COV for scatter in fatigue life must be converted to total COV of resistance in terms of stress range. Using the properties of log-normal distributions, resistance transformation in terms of stress range is given by

$$V_R{}' = \left[\left[(1 + V_R^2)^{\frac{1}{m^2}}\right] - 1\right]^{1/2}$$

Reiterating the reliability equation where

$$\phi R' \geq \gamma L'$$

where the load and resistance factors, γ and ϕ, respectively, are expressed in terms of stress range and can be given by

$$\gamma = \exp\ (\delta \beta V_L')$$

$$\phi = \exp\ (-\delta \beta V_R')$$

where δ = a splitting function and β is an acceptable probability of failure [5-9].

The reliability equation can also be used to establish a level of reliability for fracture mechanics fatigue crack growth methodologies. Again, there are numerous uncertainties including initial flaw size, type and orientation, effects of experimental procedures, and geometry of structure to name a few. These, of course, contribute to the bias and scatter of the crack growth rate parameters on the Paris equation discussed in Section 2.

5.2 RELIABILITY OF MULTIAXIAL FATIGUE RESEARCH

With this background we now have a basis for comparing multiaxial fatigue data to existing approaches and determine if there are improvements in bias and scatter resulting in more reliable designs.

The British Department of Energy (DOE) [5-10] published fatigue design rules for offshore structures. Reliability data is presented for T-joint and S-N curve and is said to be applicable to other complex joints as well. The standard deviation of special interest is the use of principal stress and maximum shear stress at the hot spot rather than stress normal to the weld toe as applicable to the API rules [5-11] where the standard deviation is on the order of 2 or more. The resulting improvement in standard deviation is apparent by using a combined stress.

Nokleby [5-12] reviewed criteria for prediction of high cycle fatigue under multiaxial stress conditions including octahedral stress criterion, shear stress criterion, internal friction criterion and shear stress intensity criterion. The tests were conducted on smooth steel specimens with the results as follows.

Criterion	Bias	COV %
Octahedral stress	1.021	9.2
Shear stress	1.080	9.5
Internal friction	1.023	6.8
Shear stress intensity	1.018	6.4

A positive bias indicates a conservative prediction and measurement value. From the paper, it appears that the bias of all four methods are on the conservative side most of the time.

Munse [5-4] investigated the fatigue initiation and crack growth in stiffened plate beam similar to CVK or web frames in ship structures. An S-N curve is presented in which the maximum bending stress on the extreme fiber at the location of failure is used to correlate data. These data are shown in Figure 3-27. It is apparent from this comparison that the maximum bending stress at the fracture section does not provide a consistent relationship in these tests either. A much better correlation is obtained when the data are analyzed on the basis of the maximum principal tensile stress (including the effect of shear) at the point of failure. Data plotted on this latter basis are presented in Figure 3-28. Although there is still scatter in the test results, there is a well-defined scatter band for the entire range of lives.

It appears that the shear and flexural stress in the web of the welded members and the resulting principal tensile stresses, along with stress concentration gradients at stiffener welds, produce the most critical fatigue condition in the weldments with various stiffeners.

Moyar and Garg [5-13] re-analyzed Munse's plate beam data and correlated the data using maximum principal stress, modified maximum range of shear stress, and the maximum octahedral shear stress criteria.

A comparison of the corresponding predictions for the three theories versus the observed cycles to failure for the type B beam specimens is provided in Table 3-4. The mean and standard deviation of the difference of predicted minus observed cycles to failure are presented. The maximum shear range theory appears more satisfactory in that the mean difference in prediction and observation is smaller. Overall, the maximum principal stress theory is the least conservative.

Yung and Lawrence [5-14] evaluated multiaxial fatigue design procedures for tube plate welds under combined bending and torsion. The multiaxial fatigue theories of Lohr and Ellison and Kandil, Brown and Miller gave reasonable non-conservative estimates of fatigue life. Table 3-5 presents the fatigue life data and results of predictions. As indicated, the mean values for maximum shear strain are in best agreement with experimental results. The COV was not calculated, however Figures 3-31 through 3-33 illustrate the scatter in data. Although there are differences in bias for each approach, the scatter is similar for the limited number of points shown.

Lawrence et al [5-15] presented reliability data for their Initiation-Propagation approach for predicting fatigue life of welded details. He compared his approach with that of Munse's and others. The comparison is very interesting in that it shows the potential gains in predicting the stress distribution at the weld toe versus nominal stress in the structural member as is done in Munse's approach. However, there is a key tradeoff between the additional work required to predict that combined stress at the weld toe and having stress data presented as nominal stress with additional inherent data scatter. Also, Lawrence had the advantage of knowing the notch stress concentration factors of his welded specimens -- an item not known to structural designers. This issue is central to the need for multiaxial fatigue research and the possible gain in reliability versus ease of use for designers.

5.3 EVALUATION OF MULTIAXIAL FATIGUE RELIABILITY FOR MARINE STRUCTURES

To put these coefficients of variance data in perspective, it is helpful to estimate the effect on total reliability or safety factor for gains in reliability from using multiaxial techniques. The LRFD format presented by Noland and Albrecht [5-7] is used to make this example relatively simple and easy to follow rather than using more complex variations. The net input will be similar to other techniques by using

$$V_S = .1$$
$$V_C = .4$$
$$V_F = .15$$

and varying V_N (scatter of S-N data).

The impact of total reliability becomes clear. As shown in Table 5-1, large improvements in coefficient of variance in data scatter produce relatively small changes in total reliability. This small change results because the magnitude of COV for the other factors or uncertainties in the total problem of fatigue in marine structures. In other words, there are many large uncertainties in the overall problem. A large improvement in one parameter only produces a small net effect. This finding strongly suggests improvements should be made across the board rather than singly. This conclusion is not completely fair because if a multiaxial fatigue procedure were used it is likely that other uncertainties would be improved. For example, the level of accuracy in stress prediction is likely to be greater, helping to improve the overall level of reliability.

Nevertheless, this is a good illustration of the potential overall benefit if multiaxial fatigue procedures are used.

On a more deterministic note, one of the original questions often asked by structural designers is how to characterize the complex stress distribution for fatigue analysis. With

TABLE 5-1

POTENTIAL IMPROVEMENTS IN OVERALL RELIABILITY USING MULTIAXIAL FATIGUE PROCEDURES

	V_n	V_R^2	V_R'	ϕ	ϕ/γ	% of API	% of Munse/DOE
MAF	.25*	.385	.143	.723	2.07	33	10
Munse /DOE	.6	.703	.184	.653	2.3	20	-
API	1.3	2.033	.27	.545	2.75	-	-

* Representative value based on data presented for welded plate beams and welded tube plate connections.

implementation of the research program described next, these answers will be provided.

6.0 CONCLUSIONS AND RECOMMENDATIONS

General multiaxial fatigue research has been investigated for simple welded structures by several researchers. There are indications that multiaxial fatigue approaches are capable of improving fatigue design in marine structures. However, additional effort is required to fully characterize multiaxial fatigue response in marine structures to a level of accuracy beyond existing techniques that are based on component tests and nominal stress fields. The vast majority of multiaxial fatigue design procedures were developed for unwelded structures where structural loading is not as complex as for marine structures where principal stresses and mean stresses are random -- a factor unaccounted for in most multiaxial fatigue procedures.

Implementation of this technology should certainly not be limited to component testing arena. There will be many "spin-offs" from the advances made or anticipated in multiaxial fatigue research. For example, engineering design relies heavily on analytical procedures such as finite element analyses to aid the design and testing procedures. The results of these analyses are often quite detailed and may be the input to fatigue life predictions made prior to construction. Redefining the critical parameters necessary for fatigue life predictability will certainly affect a large base of existing software, not to mention the actual analytical procedures. The designs that are complex enough to require finite element modeling may be the very applications that require a fatigue life predictive approach using other than the existing component-based criteria.

Additionally, the approach must be used in conjunction with predictive techniques that are capable of predicting stress characteristics in the weld regions in great detail. While there is a trend toward detail in stress predictions, there is considerable additional effort. The designer must choose the approach which best fits the design application and stress conditions. Similarly, when detailed fitness for service assessments are conducted, these multiaxial approaches will be quite useful, especially fracture mechanics approaches that account for mixed mode Stage I crack growth.

6.1 CONCLUSIONS

Conclusions on the multiaxial fatigue approaches are outlined below. These conclusions are presented to summarize the findings cited above and to form a basis for further research recommendations.

1. There is currently no universal parameter for correlating cyclic multiaxial stress/strain with fatigue life for marine structures. Very few have been investigated for welded joints as a group and additional validation

efforts are required before they can be recommended for general application to marine structures.

2. Of the correlation approaches reviewed, the effective or equivalent criteria such as use of maximum shear stress for crack initiation and maximum principal stress for crack growth are found to be potentially useful tools for extrapolating materials' responses from one stress state to another and for life correlation in high cycle multiaxial regimes and appear to be suitable for predicting fatigue initiation and propagation in simple weld configurations (e.g., butt welds in laterally loaded plates). However, additional research is required before these approaches may be integrated into a design approach.

3. There are a number of factors whose influence must be quantified and validated before multiaxial techniques can be applied to welded details with acceptable levels of confidence. These factors include: mean stress, residual stress, stress gradients, and random load and phasing effects.

4. There is currently insufficient data to determine conclusively if there is a significant improvement in fatigue reliability (bias and scatter) over techniques such as Munse's approach presented in SSC-318 or the UK DOE code for offshore structures. By characterizing detailed stress distributions on a local level, additional variables are added that are unquantified and increase uncertainty in overall levels of reliability even though the state of stress may be known on a detail level. From a deterministic perspective the equivalent stress correlations (e.g. principal stress) are useful for structural analysts interested in predicting stresses in welded details; however, many other variables must be considered in any multiaxial fatigue analysis.

5. The stress distributions in ship structural details examined as part of this study are located in stress fields where axial primary stress dominates the principal stress field. There are secondary biaxial stress effects at frames and floors. Cutouts produce axial stress at the free surface in the global biaxial stress fields. The principal stress phase relationship is random. On the other extreme, offshore structures have complex tubular joint configurations subjected to multiaxial stresses and strains with varying out of phase principal stress fields.

6. Both ships and offshore platforms experience both a constant mean stress (e.g. dead load from structural weight) and varying mean stresses that have not been accounted for in existing fatigue design procedures, let alone for multiaxial approaches.

7. Eventually, more extensive characterization of time dependent multiaxial loading spectra of actual marine structural components will be useful in directing the application oriented research efforts.

6.2 RECOMMENDED MULTIAXIAL FATIGUE RESEARCH

Designers of marine structures need information on the fatigue response of complex welded details under loading conditions with variable time-dependent principal stress. Based on the review of multiaxial fatigue it is concluded that, while there are potentially useful tools available, none have been developed or validated to an extent that will provide a reliable design without extensive component testing or analysis.

Although multiaxial fatigue technology has been developed for relatively simple structures this technology will not transfer directly to complex marine structures. Additional research is required to validate multiaxial fatigue technology for marine structures. The first step in the research program is to define the stress parameters, primarily in ship structures; to conduct follow on tests based on realistic stress conditions.

6.2.1. DEFINE SPACIAL AND TEMPORAL CHARACTERISTICS OF PRINCIPAL STRESSES IN SHIP AND OFFSHORE WELDED DETAILS.

Principal stress characteristics and their phase relationships have been shown to have a substantial influence on multiaxial fatigue response and the selection of appropriate prediction technique. These characteristics must be fully defined before multiaxial fatigue technology can be validated for use in marine structures.

The characteristics of principal stresses include magnitudes, phase relationships, gradients and mean stress components. The random nature of each shall be considered. These characteristics shall be identified for typical structural details including a bottom plate butt weld, a CVK bracket and a longitudinal cut out for ships and multiplaner KT joints with overlapping and non-overlapping members for offshore structures.

Typical loading conditions shall be identified and applied to the marine structure and detailed stress calculations shall be performed to a level of detail required to support nominal stress and crack growth studies.

6.2.2 CONDUCT MULTIAXIAL FATIGUE TESTS ON MARINE STRUCTURAL DETAILS

Multiaxial fatigue testing should be conducted to validate approaches using representative stresses and strains identified in the previous project.

This effort should be conducted to validate multiaxial fatigue approaches in marine structures based on their unique loading characteristics. The best approach will likely depend on the specific structure considered. Even in marine structures as a group, principal stress characteristics are different. The associated multiaxial fatigue approach will reflect the stress characteristics identified. Fatigue initiation and crack growth tests shall be conducted. Sufficient tests shall be conducted to estimate bias and scatter for integration into reliability studies. Analytical correlation shall be conducted to validate the various multiaxial fatigue techniques that apply to marine structures including stress and strain based fatigue life criteria and crack growth criteria (e.g. K_{eff}). Studies should concentrate on the high cycle fatigue range.

6.2.3 INTEGRATE MULTIAXIAL FATIGUE RESEARCH INTO A RELIABILITY BASED FORMAT

The objective of this research is to develop analytical approaches for predicting multiaxial fatigue response in marine structures. The approaches will likely be different depending on the marine structural detail. A decision tree or expert system may be appropriate. The tasks for this project should be developed as part of the project described above.

REFERENCES

1-1. Buckley, W.H., "Progress in the Development of Structural Load Criteria for Extreme Waves", Extreme Loads Symposium, SNAME, SSC, 1981.

1-2. Ochi, M.K., "Principles of Extreme Value Statistics and Their Application", Extreme Loads and Response Symposium, SNAME, SSC, 1981.

1-3. White, G.J. and B.M. Ayyub, "Reliability Based Fatigue Design for Ship Structures", ASNE Journal, May 1985.

1-4. Stambaugh, K. and W. Wood, "Ship Fracture Mechanisms Investigation", Final Report for SSC, March 1987.

1-5. Thayamballi, A., Y-K Chen and D. Lin, "Fracture Mechanics Based Assessment of Fatigue Reliability in Ship Structures", Ship Structures Symposium, 1984.

1-6. Munse, W.H., T.W. Wilbur, M.L. Tellalian, K. Nicoll and K. Wilson, "Fatigue Characterization of Fabricated Ship Details for Design", Report No. SSC-318, Department of Civil Engineering, University of Illinois at Urbana-Champain, sponsored by the Ship Structure Committee, 1983.

1-7. American Petroleum Institute, "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms", API RP2A, 7th edition, January 1976.

1-8. British Department of Energy (UK DOE), "Fatigue Design Guidance for Steel Welded Joints in Offshore Structures", London HMSO, 1984.

2-1. Liu, D. and A. Bakker, "Practical Procedures for Technical and Economic Investigations of Ship Structural Details," Marine Technology, January 1981.

2-2. Fricke W. and H. Daetzold, "Application of the Cyclic Strain Approach to the Fatigue Failure of Ship Structural Details," Journal of Ship Research, September 1987.

2-3. Munse, W., "Fatigue Characterization of Fabricated Ship Details for Design," SSC-318, 1983.

2-4. Stambaugh, K. and B. Wood, "SL-7 Research Program Summary Conclusions and Recommendations," SSC 313, 1981.

2-5. Stambaugh, K. and W. Wood, "Ship Fracture Mechanisms Investigation", Final Report for SSC, March 1987.

2-6. Chen, Y., J. Chin and A. Thayamballi, "Validation of Fatigue Life Prediction Using Containership Hatch Corner Stain Measurements," SNAME Trans, 1986.

2-7. Gulati, K.C., W. Wang and D. Kan, "An Analytical Study of Stress Concentration Effects in Multibrace Joints Under Combined Loading," OTC 4407, 1982.

2-8. Marshall, P., "Limit State Design of Tubular connectors," Behavior of Offshore Structures, the Norwegian Institute of Technology, 1976.

2-9. Marshall, P., "Basic Considerations for Tubular Joint Design in Offshore Construction," Welding Research Council Bulletin 193, April 1974.

2-10. Rodabaugh, "Review of Data Relevant to the Design of Tubular Joints for Use in Fixed Offshore Platforms," Welding Research Council Bulletin 256, January 1980.

2-11. Bea R. G., "Reliability of Marine Structures Fundamentals" prepared for SSC seminar on reliability February 1988.

2-12. Miner, M.A., "Cumulative Damage in Fatigue", Journal of Applied Mechanics, Vol. 12, 1945.

2-13. American Petroleum Institute, "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms", API RP2A, 7th edition, January 1976.

2-14. American Welding Society, "Design of New Tubular Structures", Section 10 of D1.1, 1975.

2-15. British Department of Energy (UK DOE), "Fatigue Design Guidance for Steel Welded Joints in Offshore Structures", London HMSO, 1984.

2-16. Potvin, A.B., J.G. Kuang, R.D. Leick and J.L. Kahlich, "Stress Concentration in Tubular Joints", Society of Petroleum Engineers Journal, August 1977.

2-17. Marshall, P.W., Limit State Design of Tubular Connections", Behavior of Offshore Structures, The Norwegian Institute of Technology, 1976.

2-18. M.W. Kellogg Co., "Design of Piping Systems", 2nd Edition, Wiley, 1956.

2-19. Bijlaard, P.P., "Stresses from Local Loadings in Cylindrical Pressure Vessels", Trans. ASME, August 1955.

2-20. Kuang, J.G., A.B. Potvin and R.D. Leick, "Stress Concentration in Tubular Joints", Seventh Annual Offshore Technology Conference, OTC Paper No. 2205, 1975.

2-21. Wordsworth, A.C., "Stress Concentration Factors at K and KT Tubular Joints," Paper 7, Fatigue on Offshore Structural Steel, Institute of Civil Engineers, 1981.

2-22. Gibstein, M.D., "Parametric Stress Analysis of T Joints," Paper 26, European Offshore Steels Research Seminar, The Welding Institute, November 1978.

2-23. Beale, L.A. and A.A. Toprac, "Analysis of In-Plane T, Y, and K Welded Tubular Connections", WRC Bulletin 125, October 1967.

2-24. Reber, J.B., "Ultimate Strength Design of Tubular Joints", Fourth Annual Offshore Technology Conference, OTC Paper No. 1664, 1972.

2-25. Visser, W., "On the Structural Design of Tubular Joints", Sixth Annual Offshore Technology Conference, OTC Paper No. 2117, 1974.

2-26. Caulkins, D.W., "Parameter Study for Frameti Elastic Stress in Tubular Joints", Shell Oil Company, Construction-Design Group, New Orleans, LA, CDG Report 15, September 1966.

2-27. Sikora, J.P., A. Dinsenbacher and J.E. Beach, "A Method for Estimating Lifetime Loads and Fatigue Lives for SWATH and Conventional Monohull Ships", Naval Engineers Journal, ASNE, May 1983, pp. 63-85.

2-28. White, G.J. and B.M. Ayyub, "Reliability Based Fatigue Design for Ship Structures", ASNE Journal, May 1985.

2-29. Park Soko, F.V. Lawrence, "Fatigue Characterization of Fabricated Ship Details for Design, Phase II", Department of Civil Engineering, University of Illinois, for the Ship Structure Committee, 1988.

2-30. Thayamballi, A., Y-K Chen and D. Lin, "Fracture Mechanics Based Assessment of Fatigue Reliability in Ship Structures", Ship Structures Symposium, 1984.

2-31. Rhee, H.C., "Application of Finite Element Alternating Method to Offshore Structural Fatigue Analysis", OTC 5111, 1986.

2-32. Huang, X., "A Finite Element Evaluation of the Stress Intensity Factors of Surface Cracks in a Tubular Joint", OTC 5665, 1986.

2-33. Liu, D. and A. Bakker, "Practical Procedures for Technical and Economic Investigations of Ship Structural Details," Marine Technology, January 1981.

2-34. Stambaugh, K.A., D.L. Edinberg, G.E. Fenton and P.R. Van Mater, Jr., "Structural Analysis of the Island Class Patrol Boats", for USCG Avery Point, CT, December 1988.

3-1. Rolfe, S.T. and J.M. Barsom, "Fracture and Fatigue Control in Structures. Applications of Fracture Mechanics", Prentice-Hall, Inc., 1977.

3-2. Forsyth, P.J.E., "Exudation of Material from Slip Bands at the Surface of Fatigued Crystals of an Aluminum-Copper Alloy", Nature, Vol. 171, 1953, pp. 172.

3-3. Cottrell, A.H. and D. Hull, "Extrusion and Intrusion by Cyclic Slip in Copper", Proceedings of the Royal Society (London), Vol. 242a, 1957, pp. 211.

3-4. Fine, M.E. and R.O. Ritchie, "Fatigue-Crack Initiation and Near Threshold Crack Growth", Fatigue and Microstructure, ASM Materials Science Seminar, St. Louis, MO, 1978, pp. 245-278.

3-5. Laird, C., "Mechanisms and Theories of Fatigue", Fatigue and Microstructure, ASM Materials Science Seminar, St. Louis, MO, 1978, pp. 149-203.

3-6. Boettner, R.C., C. Laird and A.J. McEvily, "Crack Nucleation and Growth in High-Strain - Low Cycle

Fatigue", Transactions of the Metallurgical Society, AIME, Vol. 233, 1965, pp. 379-387.

3-7. Forsythe, P.J.E., "A Two Stage Process of Fatigue Crack Growth", Proceedings of the Symposium on Crack Propagation, Cranfield, England, 1961.

3-8. Fash, J.W., N.J. Hurd, C.T. Hua and D.F. Socie, "Damage Development During Multiaxial Fatigue of Unnotched and Notched Specimens", Low Cycle Fatigue, ASTM STP 942, H.D. Solomon et al, Eds., ASTM, 1988, pp. 874-898.

3-9. Mason, W., "Alternating Stres Experiments", Proceedings of the Institution of Mechanical Engineers (Br.), February 1917, pp. 121-196.

3-10. Mason, W. and W.J. Delaney, "Alternating Combined Stress Experiments", British Association for the Advancement of Science, London, 1921, pp. 329-341.

3-11. Gough, H.J. and H.V. Pollard, "The Strength of Metals Under Combined Alternating Stresses", Proceedings of the Institution of Mechanical Engineers (Br.), Vol. 131, No. 3, November 1935, pp. 3-54.

3-12. Gough, H.J. and H.V. Pollard, "The Effect of Specimen Form on the Resistance of Metals to Combined Alternating Stresses", Proceedings of the Institution of Mechanical Engineers (Br.), Vol. 132, December 1936, pp. 549-573.

3-13. Gough, H.J. and H.V. Pollard, "Properties of Some Materials for Cast Crankshafts with Special Reference to Combined Stresses", Proceedings, Institution of Automobile Engineers (Br.), Vol. 31, March 1937, pp. 821-893.

3-14. Gough, H.J., H.V. Pollard and W.J. Clenshaw, "Some Experiments on the Resistance of Metals to Fatigue Under Combined Stress", Aeronautic Research Council (Br.), R&M 2522, 1951.

3-15. Guest, J.J., "Combined Stress. A Criticism of the Gough and Pollard Law", The Automobile Engineer, September 1937, pp. 326-332.

3-16.Guest, J.J., "Recent Research on Combined Stress", Proceedings of the Institution of Automobile Engineers (Br.), Vol. 35, December 1940, pp. 33-72, 146-171.

3-17.Tipton, S.M. and D.V. Nelson, "Fatigue Life Predictions for a Notched Shaft in Combined Bending and Torsion", Multiaxial Fatigue, ASTM STP 853, K.J. Miller and M.W. Brown, Eds., Am. Soc. for Testing and Materials, 1985, pp. 514-550.

3-18.Findley, W.N., "Combined Stress Fatigue Strength of 76S-T61 with Superimposed Mean Stresses and Corrections for Yielding", Technical Note 2924, National Advisory Committee for Aeronautics, May 1953.

3-19.Sines, G. and G. Ohgi, "Fatigue Criteria Under Combined Stresses or Strains", Journal of Engineering Materials and Technology, Vol. 103, April 1981, pp. 82-90.

3-20.McDiarmid, D.L., Proceedings, Second International Conference on Pressure Vessel Technology, ASME, San Antonio, TX, 1973, pp. 851-862.

3-21.Langer, B.F. "Design of Pressure Vessels for Low-Cycle Fatigue," ASME, January 10, 1962, Journal of Basic Engineering.

3-22.ASME "Criteria of the ASME Boiler and Pressure Vessel Code for Design by Analysis in Sections III and VII, Division 2," ASME 1969.

3-23.Krempl, E., "The Influence of State of Stress on Low-Cycle Fatigue of Structural Materials: A Literature Survey and Interpretive Report", ASTM STP 549, Am. Soc. for Testing and Materials, 1974.

3-24.Grubsic, V. and A. Simburger, "Fatigue Under Combined Out-of-phase Multiaxial Stresses", International Conference on Fatigue Testing and Design, Society of Environmental Engineers, London, 1976, pp. 27.1-27.8.

3-25.ASME Boiler and Pressure Vessel Code, Section III, Division I, Subsection NA, Appendix XIV, ASME, 1974.

3-26."Cases of ASME Boiler and Pressure Code", Code Case N-47-12, ASME, 1978.

3-27. Lee, S-B, "A Criterion for Fully Reversed Out-of-Phase Torsion and Bending", Multiaxial Fatigue, ASTM STP 853, K.J. Miller and M.W. Brown, Eds., ASTM, 1985, pp. 553-568.

3-28. Brown, M.W. and K.J. Miller, "Two Decades of Progress in the Assessment of Low-Cycle Fatigue Life", Low-Cycle Fatigue and Life Prediction, ASTM STP 770, C. Amzallag et al, Eds., ASTM, 1982, pp. 482-499.

3-29. Gonyea, D.C., in Fatigue at Elevated Temperatures, ASTM STP 520, ASTM, 1973, pp. 678-687.

3-30. Jordan, E., M. Brown and K. Miller, "Fatigue Under Severe Nonproportional Loading", Multiaxial Fatigue, ASTM STP 853, Miller and Brown, Eds., 1985.

3-31. Brown, M.W. and K.J. Miller, "A Theory for Fatigue Failure Under Multiaxial Stress-Strain Conditions", Proceedings of the Institution of Mechanical Engineers (Br.), Vol. 187, No. 65, 1973, pp. 745-755 and discussion, pp. D229-D244.

3-32. Brown, M.W. and K.J. Miller, Fatigue of Engineering Materials and Structures, Vol. 1, 1979, pp. 217-229.

3-33. Lohr, R.D. and E.G. Ellison, Fatigue of Engineering Materials and Structures, Vol. 3, 1980, pp. 1-17.

3-34. Kandil, F.A., M.W. Brown and K.J. Miller, "Biaxial Low-Cycle Fatigue Fracture of 316 Stainless Steel at Elevated Temperatures", Book 280, The Metals Society, London, 1982, pp. 203-210.

3-35. Leese, G.E., "Engineering Significance of Recent Multiaxial Research", Low Cycle Fatigue, ASTM STP 942, G.R. Solomon et al, Eds., ASTM, 1988, pp. 861-873.

3-36. Socie, D.F., L.A. Waill and D.F. Dittmer, "Biaxial Fatigue of Inconel 718 Including Mean Stress Effects", Multiaxial Fatigue, ASTM STP 853, K.J. Miller and M.W. Brown, Eds., ASTM, 1985, pp. 463-481.

3-37. Brown, M.W. and K.J. Miller, Fatigue of Engineering Materials and Structures, Vol. 1, 1979, pp. 231-246.

3-38. Konter, A.W.A., G.T.M. Janssen and W. Husslage, "Effect of Biaxial Loading and Geometry on Prediction of Low-

Cycle Fatigue Life", Proceedings, International Conference on Structural Mechanics in Reactor Technology, Berlin, 1979, Division L, Paper 13/1.

3-39. Zamrick, S.Y. and R.E. Frismuth, Experimental Mechanics, Vol. 13, May 1973, pp. 204-208.

3-40. Feltner, C. and J. Morrow, "Microplastic Strain Hysteresis Energy as a Criterion for Fatigue Fracture", Journal of Basic Engineering, ASME, Vol. 83D, March 1961, pp. 15-22.

3-41. Morrow, J., "Cyclic Plastic Strain Energy and the Fatigue of Metals", Internal Friction, Damping and Cyclic Plasticity, ASTM STP 378, ASTM, 1965, pp. 45-87.

3-42. Halford, G.R., "The Energy Required for Fatigue", Journal of Materials, Vol. 1, No. 1, March 1966, pp. 3-18.

3-43. Leis, B.N., "An Energy-Based Fatigue and Creep-Fatigue Damage Parameter", Journal of Pressure Vessel Technology, ASME, Vol. 98, No. 4, 1977, pp. 524-533.

3-44. Garud, Y.S., "A New Approach to the Evaluation of Fatigue Under Multiaxial Loadings", Journal of Engineering Materials and Technology, ASME, Vol. 103, April 1981, pp. 118-125.

3-45. Kanazawa, K., K.J. Miller and M.W. Brown, "Cyclic Deformation of 1% Cr-Mo-V Steel Under Out-of-Phase Loads", Proceedings of the 3rd International Conference on Mechanical Behavior of Materials, ICM-3, Cambridge (England), 20-24 August 1979.

3-46. Lefebvre, D., K.W. Neale and F. Ellyin, "A Criterion for Low-Cycle Fatigue Failure Under Biaxial States of Stress", Journal of Engineering Materials and Technology, ASME, Vol. 103, January 1981, pp. 1-6.

3-47. Ellyin, F. and D. Kujawski, "Plastic Strain Energy in Fatigue Failure", Journal of Pressure Vessel Technology, ASME, Vol. 106, November 1984, pp. 342-347.

3-48. Ellyin, F., "Effect of Tensile-Mean-Strain on Plastic Strain Energy and Cyclic Response", Journal of Engineering Materials and Technology, Vol. 107, April 1985, pp. 119-125.

3-49.Ellyin, F. and K. Golos, "Multiaxial Fatigue Damage Criterion", Journal of Engineering Materials and Technology, ASME, Vol. 110, January 1988, pp. 63-68.

3-50.Smith, E.W. and K.J. Pascoe, "Fatigue Crack Initiation and Growth in a High-Strength Ductile Steel Subject to In-Plane Biaxial Loading", ASTM STP 853, 1985.

3-51.Kitagawa, H., R. Yuuki and T.K. Tandbem, " K-Dependency of Fatigue Growth of Single and Mixed Mode Cracks Under Biaxial Stresses", ASTM STP 853, 1985.

3-52.Gao, G., N. Alagok, M. Brown and K. Miller, "Growth of Fatigue Cracks Under Combined Mode I and Mode II Loads", ASTM STP 853, 1985.

3-53.Ritchie, R.O., F.A. McClintock, E.K. Tscheyg and H.N. Hashemi, "Mode III Fatigue Crack Growth Under Combined Torsional and Axial Loading", ASTM STP 853, 1985.

3-54.Hourlier, F., H. D'Hondt, M. Truchon and A. Pineau, "Fatigue Crack Path Behavior Under Polymodal Fatigue", ASTM STP 853, 1985.

3-55.Pook, L.P., "Comments on Fatigue Crack Growth Under Mixed Modes I and III and Mode III Loading", ASTM STP 853, 1985.

3-56.Kfouri, A.P. and K.J. Miller "Three-Dimensional Constitutive Relations and Ductile Fracture," S. Nemat-Nasser, Ed., Proceedings, Conference, Dourdan, France, 2-5 June 1980, International Union of Theoretical and Applied Mechanics, 1981, pp. 83.

3-57.Erdogan, F. and G.C. Sih, "On the Crack Extension in Plates Under Plane Loading and Transverse Shear", Journal of Basic Engineering, Vol. 85, 1963, pp. 519-525.

3-58.Williams, J.G. and P.D. Ewing, "Fracture Under Complex Stress", International Journal of Fracture Mechanics, Vol. 8, 1972, pp. 441-446.

3-59.Finnie, I. and A. Saith, "A Note on the Angled Problem and Directional Stability of Crack", International Journal of Fracture Mechanics, Vol. 9, 1973, pp. 484-486.

3-60.Swedlow, J.L., Criterion for Growth on the Angled Crack", Cracks and Fracture, ASTM STP 601, 1976, pp. 506-521.

3-61. Williams, J.G., "On the Stress Distribution at the Base of a Stationary Crack", Journal of Applied Mechanics, Vol. 24, 1957, pp. 109-114.

3-62. Woo, C.W. and L.H. Ling, "On Angled Crack Initiation Under Biaxial Loading", Journal of Strain Analysis, Vol. 19, No. 1, 1984, pp. 51-59.

3-63. Zamricks, Y., A. Serbi and D.C. Davis, "Fatigue Crack Initiation and Growth rom an Induced Angled Crack Under Biaxial Bending", Third International Conference on Biaxial, Multiaxial Fatigue, 1989.

3-64. Kfouri, A.P. and K.J. Miller, Fracture 1977, Vol. 3, D.M.R. Taplin, Ed., University of Waterloo Press, U.K., 1978, pp. 241.

3-65. Munse, W.H. and J.E. Stallmeyer, "Influence of Weld Details on Fatigue of Welded Beams and Girders", Symposium on Fatigue of Welded Structures, Cambridge University, March 1960.

3-66. Moyar, G.J. and V.K. Garg, "Biaxial Fatigue Design Procedure Applied to Welded Plate Structures", private communications, August 1983.

3-67. White, G.J. and B.M. Ayyub, "Reliability Based Fatigue Design for Ship Structures", ASNE Journal, May 1985.

3-68. Yung, J-Y and F.V. Lawrence, "Predicting the Fatigue Life of Welds Under Combined Bending and Torsion", University of Illinois, Department of Engineering, April 1986.

3-69. Silijander, A., P. Kurath and F.V. Lawrence, "Proportional and Non-Proportional Multiaxial Fatigue of Tube-to-Plate Weldments"

3-70. Thayamballi, A., Y-K Chen and D. Lin, "Fracture Mechanics Based Assessment of Fatigue Reliability in Ship Structures", Ship Structures Symposium, 1984.

3-71. Bueckner, H.F., "The Propagation of Cracks and the Energy of Elastic Deformation", Trans. ASME, Vol. 80E, August 1958.

3-72. Gurney, T.R., Fatigue of Welded Structures, Cambridge University Press, Cambridge, New York and Melbourne, 2nd Ed., 1979.

3-73. Hellen, T.K., "On the Method of Virtual Crack Extension", Int. J. Num. Meth. Eng., 1975, pp. 187-205.

3-74. McDonald, A. et al, "The Influence of Geometrical and Loading Parameters on the Fatigue Life of Tubular Joints", Proceedings, International Conference on Steel in Marine Structures", Proceedings, International Conference on Steel in Marine Structures, Paris, France, October 1981.

3-75. Huang, X., "A Fracture Mechanics Analysis of the Fatigue Reliability of Tubular Joints", Ph.D. thesis, University of Glasgow, 1987.

3-76. Sih, G.C., "Mechanics of Fracture: Linear Response", Proceedings of the First International Conference on Numerical Methods in Fracture Mechanics, Swansea, U.K., January 1978.

3-77. Rhee, H.L., "Fracture Mechanics Fatigue Life Analysis for a Multiplane K-Joint," Seventh OMAE Conference, February 1988.

3-78. McDonald, A., et. al., "The Influence of Geomettrical and Loading Parameters on the Fatigue Life of Tubular Joints," Proceedings, International Conference on Steel in Marine Structures," Paris, France, October 1981.

3-79. Bilby, B.A. and G.E. Cardew, "The Crack with a Kinked Tip", International Journal of Fracture, Vol. 11, 1975, pp. 708-712.

3-80. Hussain, M.A., S.L. Pu and J. Underwood, "In Fracture Analysis", STP 560, American Society for Testing and Materials, Philadelphia, 1974, pp. 2-28.

3-81. Masahiro, I. and T. Sakae, "A Critical Analysis of the Relationship Between the Energy Release Rate and the Stress Intensity Factor for Non-Coplanar Crack Extension Under Combined Mode Loading", International Journal of Fracture, Vol. 18, 1982, pp. 19-28.

3-82. Huang, X., Z-Z Du and J.W. Hancock, "A Finite Element Evaluation of the Stress Intensity Facotrs of Surface Cracks in Tubular Joints", OTC 5665, May 1988.

3-83. Broek, D. and R.C. Rice, The Fatigue Crack Growth Properties of Rail Steels, DOT-TSC-1076, Battelle Columbus Laboratories, Columbus, OH, July 1977.

3-84. Pook, L.P., "The Fatigue Crack Direction and Threshold Behavior of Mild Steel Under Mixed Mode I and III Loading", International Journal of Fatigue, Vol. 7, No. 1, 1985.

3-85. Nishioka, T. and S.N. Atluri, "A Major Development Towards a Cost-Effective Alternating Method for Fracture Analysis of Steel Reactor Pressure Vessels", Transactions of the 6th International Conference on Structural Mechanics in Reactor Technology, Paper G1/2, Paris, 1981.

3-86. Nishioka, T. and S.N. Atluri, "Analytical Solution for Embedded Elliptical Cracks, and Finite Element Alternating Method for Elliptical Surface Cracks, Subjected to Arbitrary Loadings", Engineering Fracture Mechanics, Vol. 17, No. 3, 1983.

3-87. Vijayakumar, K. and S.N. Atluri, "An Embedded Elliptical Flaw in an Infinite Solid Subject to Arbitrary Crack-Face Tractions", Journal of Applied Mechanics, Vol. 48, 1981.

3-88. Segedin, C.M., "A Note on Geometric Discontinuities in Elastostatics", International Journal of Engineering Science, Vol. 6, 1968.

3-89. Nishioka, T. and S.N. Atluri, "Analysis of Surface Flaws in Pressure Vessels by a New 3-Dimensional Alternating Method", Journal of Pressure Vessel Technology, Vol. 104, No. 4, 1982.

3-90. Nishioka, T. and S.N. Atluri, "An Alternating Mtehod of Analysis of Surface Flawed Aircraft Structural Components", AIAA Journal, Vol. 21, No. 5, 1983.

3-91. Trefftz, E., Handbuch der Physik, Vol. 6, Springer-Verlag, Berlin, 1928, pp. 92.

3-92. Huang, X., "A Fracture Mechanics Analysis of the Fatigue Reliability of Tubular Joints", Ph.D. thesis, University of Glasgow, 1987.

3-93. Rice, J.R. and N. Levy, "The Part-Through Surface Crack in an Elastic Plate", Journal of Applied Mechanics, Vol. 39, 1972, pp. 185-194.

3-94. Parks, D.M., "The Inelastic Line Spring for Elastic-Plastic Fracture Mechanics Parameters for Surface-Cracked Plates and Shells", Journal of Pressure Vessel Technology, Vol. 103, 1981, pp. 246-254.

3-95. Desvaux, G.J., "The Line Spring Model for Surface Flaw, An Extension to Mode 2 and Mode 3", MSc thesis, Massachusetts Institute of Technology, 1985.

3-96. ABAQUS User's Manual, Hibbit, Karlsson and Sorensen, Inc., Providence, RI, 1982.

4-1. Liu, D. and A. Bakker, "Practical Procedures for Technical and Economic Investigations of Ship Structural Details," Marine Technology, January 1981.

4-2. Chen, Y., J. Chin and A. Thayamballi, "Validation of Fatigue Life Prediction Using Containership Hatch Corner Stain Measurements," SNAME Trans, 1986.

4-3. Discussion to Reference 4-2.

4-4. Munse, W.H. and J.E. Stallmeyer, "Influence of Weld Details on Fatigue of Welded Beams and Girders", Symposium on Fatigue of Welded Structures, Cambridge University, March 1960.

4-5. Moyar, G.J. and V.K. Garg, "Biaxial Fatigue Design Procedure Applied to Welded Plate Structures", private communications, August 1983.

4-6. Silijander, A., P. Kurath and F.V. Lawrence, "Proportional and Non-Proportional Multiaxial Fatigue of Tube-to-Plate Weldments."

4-7. American Petroleum Institute, "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms", API RP2A, 7th edition, January 1976.

4-8. British Department of Energy (UK DOE), "Fatigue Design Guidance for Steel Welded Joints in Offshore Structures", London HMSO, 1984.

4-9. Yung, J-Y and F.V. Lawrence, "Predicting the Fatigue Life of Welds Under Combined Bending and Torsion", University of Illinois, Department of Engineering, April 1986.

4-10. Huang, X., "A Fracture Mechanics Analysis of the Fatigue Reliability of Tubular Joints", Ph.D. thesis, University of Glasgow, 1987.

4-11. Tipton. S.M. and D.V. Nelson, "Fatigue Life Predictions for a Notched Shaft in Combined Bending and Torsion", ASTM STP 853, 1985.

4-12. Sahgal, R. K. and W. H. Munse. "Fatigue Behavior of Axially Loaded Weldments in HY-80 Steel," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 204, September 1960.

4-13. Hartmann, A. J. and W. H. Munse. "Fatigue Behavior of Welded Joints and Weldments in HY-80 Steel Subjected to Axial Loadings," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 250, July 1962.

4-14. Zimmerman, J. E., Jr. and W. H. Munse. "Fatigue Behavior of Defective Welded Joints in HY-80 Steel Subjected to Axial Loading," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 252, July 1962.

4-15. Hartmann, A. J., W. H. Bruckner, J. Mooney and W. H. Munse. "Effect of Weld Flaws on the Fatigue Behavior of Butt-Welded Joints in HY-80 Steel," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 275, December 1963.

4-16. Munse, W. H., W. H. Bruckner, A. J. Hartmann and J. B. Radziminski. "Studies of the Fatigue Behavior of Butt-Welded Joints in HY-80 and HY-100 Steels," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 285, November 1964.

4-17. Munse, W. H., W. H. Bruckner, J. B. Radziminiski, R. W. Hinton and J. W. Leibold. "Fatigue of Plates and Weldments in HY-100 and HY-130/150 Steels," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 300, November 1965.

4-18. Radziminski, J. B., R. A. Ridha, H. A. Osman, D. F. Meinheit and W. H. Munse. "Fatigue Behavior of Mechanically Peened HY-80 Weldments Subjected to Axial

4-18. Radziminski, J. B., R. A. Ridha, H. A. Osman, D. F. Meinheit and W. H. Munse. "Fatigue Behavior of Mechanically Peened HY-80 Weldments Subjected to Axial Loading," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 312, August 1966.

4-19. Radziminski, J. B., R. W. Hinton, D. F. Meinheit, H. A. Osman, W. H. Bruckner and W. H. Munse. "Fatigue of Plates and Weldments in High Strength Steel," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 318, February 1967.

4-20. Radziminski, J. B., F. V. Lawrence, S. Mukai, P. Panjwani, R. Johnson, R. Mah and W. H. Munse. "Low Cycle Fatigue of HY-130(T) Butt Welds," SRS No. 342, December 1968.

4-21. Radziminski, J. B., F. V. Lawrence, T. W. Wells, R. Mah and W. H. Munse. "Final Report - Low Cycle Fatigue of Butt Weldments of HY-100(T) and HY-130(T) Steel," Univ. of Ill., SRS 361, July 1970.

4-22. Wolfe, R. J., I. L. Stern, P. Abramov and H. Nagler. "Investigation of Effect of Welding on the Fatigue Properties of HY-80 Steel," New York Naval Shipyard, Material Laboratory Project 6160-2, Progress Report 3, July 1961.

4-23. Stern, I., R. Wolfe, H. Nagler and P. Abramov. "Effects of Various Weld Flaws on Fatigue Properties of HY-80 Butt and Fillet Weld Assemblies," New York Naval Shipyard, Material Laboratory, Project 6160-1, Progress Report No. 2, 1962.

4-24. U. S. Naval Applied Science Laboratory. "Fatigue of Iron Base Alloys HY-80 Steel, Rolled Plate to Cast Plate Butt Welds, etc.," Project No. 6160-2, September 1963.

4-25. U. S. Naval Applied Science Laboratory. "Effect of Embrittled 110-18 Weld Metal on the Fatigue Life of HY-80 Steel Butt Weldments," Project No. 9300-1, Technical Memorandum No. 2, October 1963.

4-26. U. S. Naval Applied Science Laboratory. "Fatigue of Iron Base Alloys HY-80 Steel Cast Tees Butt Welded to Rolled Section Single and Double Weld Joint Design," Project 9300-1, Technical Memorandum No. 19, December 1964.

4-27. U. S. Naval Applied Science Laboratory. "Mechanical Pennding Procedure for Improvement of Fatigue Properties of HY-80 Butt Welds," Project 9300-1, Technical Memorandum No. 34, August 1965.

4-28. U. S. Naval Applied Science Laboratory. "Investigation of Corrosion Fatigue and Stress Corrosion Properties of HY-130/150 Steel," Tech. Memo. No. 51, Lab. Project 9300-1, Brooklyn, NY, Oct. 3, 1966.

4-29. U. S. Naval Applied Science Laboratory. "Fatigue of Large Scale Uniformly Loaded Simply Supported Butt Welded Rectangular Plates," Project 9300-23, SF 013-03-02, Technical Memorandum No. 8, Task 2025, June 1967.

4-30. Gaites, R. A. and M. Rosen. "Fatigue of Structural Elements: Development of a Strain Gage Technique for the Determination of Crack Depth in High Strength Steel Structural Weldments," U. S. Naval Applied Science Lab., Project No. 930-23, Progress Report 5, September 1967.

4-31. Rosen, M. and R. A. Gaites. "Fatigue of Large Scale Uniformly Loaded Simply Supported Butt Welded Rectangular Plates," U. S. Naval Applied Science Laboratory, Project 9300-23, Project Report No. 6, SF 013-03-02, 1968.

4-32. Gaites, R. A. and M. Rosen. "Fatigue of Large Scale Uniformly Loaded Supported Butt Welded Rectangular Plates," U. S. Naval Applied Science Lab.-Material Sciences Division, Lab. Project 930-23, Progress Report No. 9, October 31, 1968.

4-33. Cordiano, H. V., P. Abramov and E. Lewis. "Investigation of the Effect of Welding on the Fatigue Properties of HY-80 Steel," New York Naval Shipyard, Material Laboratory Project 6160-2, Progress Report No. 1, September 1959.

4-34. Naval Applied Science Laboratory. "Fatigue of Iron Base Alloys--HY-80 Steel, Cast Tee," New York Naval Shipyard, Brooklyn, New York, Project No. 6160-2, Progress Report No. 7, June 1963.

4-35. Foster, M. L. and P. Abramov. "Development of NASL Mechanical Peening Procedure for Improvement of Fatigue Properties of HY-80 Welds," U. S. Naval Applied Science Laboratory, Project 9300-1, Progress Report No. 1, April 1964.

4-36.U. S. Naval Applied Science Laboratory. "Effects of Grinding on Fatigue Life of Tee Weldments," Project 9300-1, Technical Memorandum No. 11, July 1964.

4-37.U. S. Naval Applied Science Laboratory. "Effects of Shot Peening on Fatigue Life of HY-80 Tee Fillet Weldments," Project 9300-1, Technical Memorandum No. 15, August 1964.

4-38.U. S. Naval Applied Science Laboratory. "Effect of Grinding and Shot Peening on Fatigue Life of Tee Weldments," Project 9300-1, Technical Memorandum No. 18, September 1964.

4-39.U. S. Naval Applied Science Laboratory. "Investigation of the Effect of Welding on the Fatigue Properties of HY-100 Steel Tee-Fillet Welded Plates in as-Welded Condition," Tech. Memo., U. S. Naval Base, Brooklyn, NY, Feb. 2, 1965.

4-40.Rosen, M. and H. V. Cordiano. "Fatigue of Large Scale Uniformly Loaded Simply Supported Tee-Fillet Welded Rectangular Rib-Stiffened Plates," U. S. Naval Applied Science Laboratory, Project 9300-23, Technical Memorandum No. 6, SF013-03-02, December 1966.

4-41.U. S. Naval Applied Science Laboratory. "Fatigue of Structural Elements--Initial Studies on the Effect of Overstrain on Residual Stresses and Fatigue," Material Science Division, Project 9300-23, Technical Memorandum No. 5, SF013-03-02, Task 2025, May 1966.

4-42.Abramov, P. and F. Ginsberg. "Effects of Stress Relieving on Manual Metal Arc (MMA) and Metal Inert Gas Spary Arc (MIG) HY-80 Tee Weldment," U. S. Naval Applied Science Laboratory, Project 9300-1, Progress Report No. 6, SF 020-01-01, Task 0722, April 1967.

4-43.Rosen, M. "Fatigue of Large Scale Uniformly Loaded Simply Supported Tee-Fillet Welded Rectangular Rib-Stiffened Plates," U. S. Naval Applied Science Laboratory, Project 9300-23, Technical Memorandum No. 7, SF 013-03-02, Task 2025, March 1967.

4-44.Abramov, P. and F. Ginsberg. "Effects of Stress Relieving on Manual Metal Arc (MMA) and Metal Inert Gas Spary Arc (MIG) HY-80 Tee Weldment," U. S. Naval Applied Science Laboratory, Project 9300-1, Progress Report No. 6, SF 020-01-01, Task 0722, April 1967.

4-45. Gaites, R. A. and M. Rosen. "Fatigue of Large Scale Uniformly Loaded Simply Supported Tee-Fillet Welded Rectangular Rib-Stiffened Plates," U. S. Naval Applied Science Laboratory, Project 930-23, Progress Report No. 8, June 7, 1968.

4-46. Rosen, M. and H. V. Cordiano. "Fatigue of Structural Elements, Development of Economical Beam Type Structural Elements for Simulated Fatigue Evaluations," U. S. Naval Apoplied Science Laboratory, Project No. 9300-23, Progress Report No. 4, June 1967.

4-47. Rosen, M. and H. V. Cordiano. "Fatigue of Structural Elements, Development of Economical Beam Type Structural Elements for Simulated Fatigue Evaluations," U. S. Naval Applied Science Laboratory, Project No. 9300-23, Progress Report No. 4, June 196

4-48. Gross, M. R. and R. E. Heise, Jr. "Low-Cycle Fatigue Behavior of Internally Pressurized Boxes," American Society of Mechanical Engineers, Transactions, Journal of Engineering for Industry, Vol. 66, WA/Unt-1, May 1966.

4-49. Heise, R. E. and M. R. Gross. "Low-Cycle Fatigue Behavior of an Internally Pressurized Box," U. S. Navy Marine Engineering Laboratory, Research and Development Phase Report 86 108C, Sub-Project S-R007 01 01 Task 0856, December 1963.

4-50. Gross, M. R. and R. E. Heise, Jr. "Low-Cycle Fatigue Behavior of Internally Pressurized Boxes," American Society of Mechanical Engineers, Transactions, Journal of Engineering for Industry, Vol. 66, WA/Unt-1, May 1966.

4-51. Gross, M. R. "Low-Cycle Fatigue Behavior of Internally Pressurized Boxes," U. S. Naval Marine Engineering Laboratory, Assignment 86-108, Report 5/67, March 1967.

4-52. Gross, M. R. "Low-Cycle Fatigue Behavior of a Double-Box Structure," Naval Ship Research and Development Center, Report No. 2568, Decmeber 1967.

4-53. Rosen, M. and R. A. Gaites. "Fatigue of Large Scale Uniformly Loaded Simply Supported Butt Welded Rectangular Plates," U. S. Naval Applied Science Laboratory, Project 9300-23, Project Report No. 6, SF 013-03-02, 1968.

4-54. Abramov, P. and F. Ginsberg. "Investigation of Fatigue Life of HY-130/150 Steel," Tech. Memo., No. 43, Lab. Project 9300-1, U. S. Naval Applied Science Lab., Brooklyn, NY, June 14, 1966.

4-55. Bowman, C. E. and T. J. Dolan. "Biaxial Fatigue Properties of Pressure Vessel Steels," Welding Journal Research Supplement, Vol. 32, No. 11, pp. 529s-37s, November 1953.

4-56. Bowman, C. E. and T. J. Dolan. "Studies of Biaxial Fatigue Properties of Pressure Vessel Steels," Welding Journal, pp. 51s, January 1955.

4-57. Bowman, C. E. and T. J. Dolan. "Resistance of Low-Alloy Steel Plates to Biaxial Fatigue," Welding Journal, Vol. 35, No. 2, pp. 102s-109s, February 1956.

4-58. Munse, W. H. "Fatigue of Welded Steel Structures," Welding Research Council, 1964.

4-59. Burnside, O. H., Hudak, S. J., Jr., Oelkers, E., Chan, K. and Dexter, R. J. "Long-Term Corrosion Fatigue of Welded Marine Steels," Ship Structure Committee, U.S.C.G. Hdqts., (G-M/TP 13), Washington, DC, 1984. SR-1276. SSC-326.

5-1. Harris, D.O., E.Y. Lim and D.D. Dedhia, "Probability of Pipe Fracture in the Primary Coolant Loop of a PWR Plant", Science Applications, Inc., Prepared for the U.S Nuclear Regulatory Commission, Washington, DC, August 1981.

5-2. Albrecht, P., "S-N Fatigue Reliability Analysis of Highway Bridges", Probabilistic Fracture Mechanics and Fatigue Methods: Applications for Structural Design and Maintenance, ASTM STP 798, J.M. Bloom and J.C. Ekvall, Eds., American Society of Testing and Materials, 1983, pp. 184-204.

5-3. Ang, A.H-S. and W.H. Munse, "Practical Reliability Basis for Structural Fatigue", Meeting Reprint 2492, ASCE National Structural Engineering Conference, April 14-18, 1975.

5-4. Munse, W.H., T.W. Wilbur, M.L. Tellalian, K. Nicoll and K. Wilson, "Fatigue Characterization of Fabricated Ship Details for Design", SSC-318, 1983.

5-5. Wirsching, P.H., "Probability-Based Fatigue Design Criteria for Offshore Structures", Final Project Report, API-PRAC Project No. 80-15, Department of Aerospace and Mechanical Engineering, University of Arizona, Tucson, Arizona, January 1983.

5-6. Albrecht, P. and Ch. Nolan, "Load and Resistance Factor Design of Steel Structures for Fatigue", Civil Engineering Report, University of Maryland, College Park, MD, June 1983.

5-7. Nolan, Ch. and P. Albrecht, "Load and Resistance Factor Design for Fatigue of Steel Structures", ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability, Berkeley, CA, January 1984.

5-8. Albrecht, P., "Review of Fatigue Design Methods for Highway Bridges", Civil Engineering Report, University of Maryland, College Park, MD, May 1986.

5-9. Ang, A.H-S and C.A. Cornell, "Reliability Bases for Structural Safety and Design", Journal of the Structural Division, ASCE, Vol. 100, No. ST9, Proc. Paper 10777, September 1974, pp. 1755-1769.

5-10. British Department of Energy (UK DOE), "Fatigue Design Guidance for Steel Welded Joints in Offshore Structures", London HMSO, 1984.

5-11. American Petroleum Institute, "Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms", API RP2A, 7th edition, January 1976.

5-12. Nokleby, "Fatigue Under Multiaxial Stress Conditions", MD 81001, Division of Machine Elements, The Norwegian Institute of Technology, Trondheim, Norway.

5-13. Moyar, G.J. and V.K. Garg, "Biaxial Fatigue Design Procedure Applied to Welded Plate Structures", private communications, August 1983.

5-14. Yung, J-Y and F.V. Lawrence, "Predicting the Fatigue Life of Welds Under Combined Bending and Torsion", University of Illinois, Department of Engineering, April 1986.

APPENDIX A

BIBLIOGRAPHY ON MULTIAXIAL FATIGUE

1. COMBINED STRESSES

Yung, J-Y, and F.V. Lawrence, "Predicting the Fatigue Life of Welds Under Combined Bending and Torsion," University of Illinois, Department of Engineering, April 1986.

Tipton, S.M., and D.V. Nelson, "Fatigue Life Predictions for a Notched Shaft in Combined Bending and Torsion," Multiaxial Fatigue, ASTM STP 853, K.J. Miller and M.W. Brown, Eds., Am. Soc. for Testing and Materials, 1985, pp. 514-550.

Tipton, S.M., "Fatigue Life Predictions for a Notched Shaft in Combined Bending and Torsion," Interim Report No. 1. Mechanical Engineering Department, Stanford University, Stanford, CA, December 1982.

Tipton, S.M., and D.V. Nelson, "Fatigue Life Predictions for a Notched Shaft in Combined Bending and Torsion," ASTM STP 853, 1982.

Gau, H., N. Alagok, M. Brown, and K. Miller, "Growth of Fatigue Cracks Under Combined Mode II and Mode II Loads," ASTM STP 853, 1982.

Ritchie, R.O., F.A. McClintock, and E.K. Tschegy, "Mode III Fatigue Crack Growth Under Combined Torsional and Axial Loading," ASTM STP 853, 1982.

Pook, L.P., "Comments on Fatigue Crack Growth Under Mixed Modes I and III and Pure Mode III Loading," ASTM STP 853, 1982.

Sines, G., and G. Ohgi, "Fatigue Criteria Under Combined Stresses or Strains," Journal of Engineering Materials and Technology, Vol. 103, April 1981, pp. 82-90.

Hashin, Z., "Fatigue Failure Criteria for Combined Cyclic Stress," Naval Air Systems Command, Office of Naval Research, 1979.

"Fatigue Under Complex Loading Analysis and Experiments," Advances in Engineering, Vol. 6, Society of Automotive Engineers, Warrendale, PA, 1977.

Grubsic, V., and A. Simburger, "Fatigue Under Combined Out-of-phase Multiaxial Stresses," International Conference on Fatigue Testing and Design, Society of Environmental Engineers, London, 1976, pp. 27.1-27.8.

Krempl, E., "The Influence of State of Stress on Low Cycle Fatigue of Structural Materials," ASTM 549, American Society For Testing and Materials, Philadelphia, 1974.

McDiarmid, D.L., Proceedings, Second International Conference on Pressure Vessel Technology, ASME, San Antonio, TX, 1973, pp. 851-862.

Ronay, Maria, "Interaction of Cyclic Torsion with Axial Loads," Symposium on Structural Fatigue in Aircraft, ASTM STP No. 404, Fifth Pacific Area Meeting Papers, pp. 176-189, November 1966.

Ronay, M., "Conditions of Interaction of Cyclic Torsion with Axial Loads," Columbia University, Institute for Study of Fatigue and Reliability, Report No. 25, August 1965.

Sines, G., "The Prediction of Fatigue Fracture Under Combined Stresses at Stress Concentration," [J] Trans. Soc. Mech. Engrs. Japan, Vol. 26, No. 170, Oct. 1960, pp. 1340-1347.

Frost, N. E., "Notch Effects and the Critical Alternating Stress Required to Propagate a Crack in an Aluminum Alloy Subject to Fatigue Loading," Journal Mechanical Engineering Science, Vol. 2, 1960.

Findley, W. N., "Final Report on Basic Research on Fatigue Failures Under Combined Stress," Tech. Rep. Div. Engng, Engng Mater. Res. Lab. Brown Univ. No EMRL-13 [Tech. Rep. U.S. Army Ordnance Corps Project No. 7], Dec. 1958, pp. 20-24.

Findley, W. N., P. N. Mathur, E. Szczepanski, and A. O. Temel, "Energy Versus Stress Theories for Combined Stress - A Fatigue Experiment Using a Rotating Disc," Tech. Rep. Div. Engng, Engng Mater. Res. Lab. Brown Univ. No. EMRL-13 [Tech. Rep. U.S. Army Ordnance Corps Project No. 7], Dec. 1958, pp. 1-19.

Findley, W. N., "A Theory for the Effect of Mean Stress on Fatigue of Metals Under Combined Torsion and Axial Load or Bending," Tech. Rep. Div. Engng Mater. Res. Lab. Brown Univ. No. EMRL-12 [Tech. Rep. U.S. Army Ordnance Corps Project No. 6], March 1958, pp. 23.

Ono, M., "Test to Determine the Fatigue Limit Under Combined Repeated Stress," [J] Yamaguchi Daigaku Koˆgakubu gakuhoˆ, Vol. 8, No. 1, Dec. 1957, pp. 65-72.

Findley, W. N., "Fatigue of Metals Under Combinations of Stresses," Transactions of the American Society of Mechanical Engineers, Vol. 79, No. 6, Aug. 1957, pp. 1337-1348.

Yokobori, T., "A Theoretical Criterion for the Fracture of Metals Under Combined Alternating Stresses," Journal of Applied Mechanics, Vol. 24, No. 1, March 1957, pp. 77-80.

Coleman, J. J., and W. N. Findley, "A Theory of the Non-Linear Influence of Normal Stress on Fatigue Under Combined Stresses," Proc. 2nd Confer. on the Mechanics of Elasticity and Plasticity, U. S. Army, Office of Ordnance Research, Feb. 1957, pp. 214-243.

Murphy, J. J., C. R. Soderberg, and D. B. Rossheim, "Considerations Affecting Future Pressure Vessel Codes," Welding Journal, Easton, Vol. 35, No. 12, Dec. 1956, pp. 582s-596s.

Coleman, J. J., and W. N. Findley, "A Theory of the Non-Linear Influence of Normal Stress on Fatigue Under Combined Stresses," Tech. Rep. Div. Engng, Engng Mater. Res. Lab. Brown Univ. No. 4, Nov. 1956, pp. 28.

Findley, W. N., "Theories Relating to Fatigue of Materials Under Combinations of Stress," Tech. Rep. Div. Engng, Engng Mater. Res. Lab. Brown Univ. No. 2, June 1956, pp. 12.

Findley, W. N., "Fatigue of Metals Under Combinations of Stress," Tech. Rep. Div. Engng, Engng Mater. Res. Lab. Brown Univ. No. 3, June 1956, pp. 25.

Findley, W. N., J. J. Coleman, and B. C. Hanley, "Theory for Combined Bending and Torsion Fatigue with Data for SAE 4340 Steel," Rep. Engng Mater. Res. Lab. Brown Univ. No. ERMRL-4, April 1956, pp. 23.

Findley, W. N., "Theories Relating to Fatigue of Materials Under Combinations of Stress," Colloquium on Fatigue. [Editors: W. Weibull and F. K. G. Odqvist]. Berlin-Gottingen-Heidelberg, Springer-Verlag, 1956, pp. 35-42.

Findley, W. N., J. J. Coleman, and B. C. Hanley, "Theory for Combined Bending and Torsion Fatigue With Data for SAE 4340 Steel," Proc. Int. Conf. Fatigue Metals. London, Instn Mech. Engrs, 1956, pp. 150-157.

Findley, W. N., and P. N. Mathur, "Modified Theories of Fatigue Failure Under Combined Stress," Proc. Soc. Exp. Stress Analysis, Vol. 14, No. 1, 1956, pp. 35-46.

Marin. J., "Interpretation of Fatigue Strengths for Combined Stresses," Proc. Int. Conf. Fatigue Metals. London, Instn Mech. Engrs, 1956, pp. 184-194.

Sines, G., "Failure of Materials Under Combined Repeated Stresses with Superimposed Static Stresses," NACA Technical Note 3495, National Advisory Committee for Aeronautics, November 1955, pp. 69.

Findley, W. N., and P. N. Mathur, "Modified Theories of Fatigue Failure Under Combined Stress," Tech. Rep. No. 5 Office of Ordinance Research Contract No. DA-11-022-ORD-995, Proj. No. TB2-0001(313). Rep. Univ. Ill. Dep. Theoret. Appl. Mech., 1954, pp. 23.

Thurston, R. C. A., and J. E. Field, "The Fatigue Strength Under Bending, Torsional, and Combined Stresses of Steel Test Pieces With Stress Concentrations," Proc. Instn Mech. Engrs, Vol. 168, No. 31, 1954, pp. 785-796.

Findley, W. N., P. N. Mathur, and D. E. Martin, "Fatigue Failure Under Combinations of Stresses," Final Rep. Office of Ordnance Research Contract No. DA-11-022ORD-995, Proj. No. TB2-0001(313). Rep. Univ. Ill. Dep. Theoret. Appl. Mechan. No. 61, May 1954, pp.19.

Findley, W. N., W. I. Mitchell, and D. E. Martin, "Combined Bending and Torsion Fatigue Tests of 25S-T Aluminum Alloy," Department of Theoretical and Applied Mechanics, Univ. of Ill., Urbana, January 1954.

Findley, W. N., "Effect of Range of Stress on Fatigue of 76S-T61 Aluminum Alloy Under Combined Stresses Which Produce Yielding," Journal of Applied Mechanics, Vol. 20, No. 3, Sept. 1953, pp. 365-374.

Findley, W.N., "Combined Stress Fatigue Strength of 76S-T61 with Superimposed Mean Stresses and Corrections for Yielding," Technical Note 2924, National Advisory Committee for Aeronautics, May 1953.

Findley, W. N., "Fatigue of 76S-T61 Aluminum Alloy Under Combined Bending and Torsion," Preprint, American Society for Testing Materials, No. 97, 1952.

Gough, H. J., H. V. Pollard, and W. J. Clenshaw, "Some Experiments on the Resistance of Metals to Fatigue Under Combined Stresses," Re. Memo. Aeronaut. Res. Coun., No. 2522, 1951, pp. 141.

Sag, N., "Design of Machine Parts Subjected to Fluctuating Combined Bending and Tension," Engineer, Lond., Vol. 192, No. 4985, Aug. 10, 1951, pp. 172-176.
Nishihara, T., and M. Kawamoto, "The Strength of Metals Under Combined Alternating Bending and Torsion With Phase Difference," Mem. Coll. Engng Kyoto, vol. 11, no. 5, June 1945, pp. 85-112.

Nishihara, T., and M. Kawamoto, "A New Criterion for the Strength of Metals Under Combined Alternating Stresses," Mem. Coll. Engng Dyoto, vol. 11, no. 4, July 1944, pp. 65-83.

Nishihara, T., and M. Kawamoto, "The Strength of Metals Under Combined Alternating Bending and Torsion," Mem. Coll. Engng Kyoto, vol. 10, no. 6, 1941, pp. 117-201. Abstr.: Metallurg. Abstr., vol. 10, pt. 4, April 1943, p. 124.

McAdam, D. J., "The Influence of the Combination of Principal Stresses in Fatigue of Metals," Proc. Amer. Soc. Test. Mater., vol. 42, 1942, pp. 576-594.

Marin, J., "Strength of Steel Subjected to Biaxial Fatigue Stresses," Welding Journal, Easton, Pa., vol. 21, no. 11, Nov. 1942, pp. 554s-559s.

Marin, J., "Designing for Bending, Twisting, and Axial Loads," Mach. Design, vol. 14, no. 9, Sept. 1942, pp. 79-83, and no. 10, Oct. 1942, pp. 78-80, 260 and 262.

Marin, J., "Interpretation of Experiments on Fatigue Strength of Metals Subjected to Combined Stresses," Welding Journal Research Supplement, May 1942, pp. 245s-248s.

Guest, J.J., "Recent Research on Combined Stress," Proceedings of the Institution of Automobile Engineers (Br.), Vol. 35, December 1940, pp. 33-72, 146-171.

Guest, J.J., "Combined Stress: A Criticism of the Gough and Pollard Law," The Automobile Engineer, September 1937, pp. 326-332.

Gough, H.J., and H.V. Pollard, "Properties of Some Materials for Cast Crankshafts with Special Reference to Combined Stresses," Proceedings, Institution of Automobile Engineers (Br.), Vol. 31, March 1937, pp. 821-893.

Gough, H.J., and H.V. Pollard, "The Effect of Specimen Form on the Resistance of Metals to Combined Alternating Stresses," Proceedings of the Institution of Mechanical Engineers (Br.), Vol. 132, December 1936, pp. 549-573.

Terazawa, K., "Fatigue Resistance and Allowable Stress of Ductile Materials Under Combined Alternating Stresses," J. Soc. Nav. Archit., Japan, vol. 59, Dec. 1936, pp. 15-44.

Gough, H.J., "The Strength of Metals Under Combined Alternating Stresses," Iron Steel Ind., vol. 9, Jan. 1936, pp. 132-137, and Feb. 1936, pp. 177-178.

Gough, H.J., and H. V. Pollard, "The Strength of Metals Under Combined Alternating Stresses," Proc. Instn Mech. Engrs. Lond., vol. 131, Nov. 1935, pp. 3-103. Engineering, Lond., vol. 140, no. 3643, Nov. 8, 1935, pp. 511-513, and no. 3645, Nov. 22, 1935, pp. 565-567. Mech. World, vol. 98, no. 2551, Nov. 22, 1935, pp. 501-502, and no. 2552, Nov. 29, 1935, pp. 527-528, 536. Metallurgia, Manchr, vol. 13, no. 73, no. 73, Nov. 1935, pp. 17-20. Iron Steel Ind., vol. 9, Jan. 1936, pp. 132-137, and Feb. 1936, pp. 177-178.

Gough, H.J., and H.V. Pollard, "The Strength of Metals Under Combined Alternating Stresses," Proceedings of the Institution of Mechanical Engineers (Br.), Vol. 131, No. 3, November 1935, pp. 3-54.

Kohn, P., "Zusammengesetzte Beanspruchung und Sicherheit bei Statischer und Wechselnder Belastung," [Complex Stresses and Safety Under Static and Fatigue Loads.] Schweiz. Bauztg., Vol. 102, no. 17, Oct. 21, 1933, pp. 203-205.

Hohenemser, K., and W. Prager, "Zur Frage der Ermu╗dungsfestigkeit bei Mehrachsigen Spannungszusta╗nden," [On the Question of Fatigue Strength Under Complex Stress Conditions.] Metallwritschaft, vol. 12, no. 24, June 16, 1933, pp. 342-343.

Mason, W., and W.J. Delaney, "Alternating Combined Stress Experiments," British Association for the Advancement of Science, London, 1921, pp. 329-341.

2. MULTIAXIAL STRESSES

Zamrick, Y., A. Serbi, and D.C. Davis, "Fatigue Crack Initiation and Growth From an Induced Angled Crack Under Biaxial Bending," Third International Conference on Biaxial, Multiaxial Fatigue, 1989.

Huang, X., Z-Z Du, and J.W. Hancock, "A Finite Element Evaluation of the Stress Intensity Factors of Surface Cracks in Tubular Joints," OTC 5665, May 1988.

Ellyin, F., and K. Golos, "Multiaxial Fatigue Damage Criterion," Journal of Engineering Materials and Technology, ASME, Vol. 110, January 1988, pp. 63-68.

Fash, J.W., N.J. Hurd, C.T. Hua, and D.F. Socie, "Damage Development During Multiaxial Fatigue of Unnotched and Notched Specimens," Low Cycle Fatigue, ASTM STP 942, H.D. Solomon et al, Eds., ASTM, 1988, pp. 874-898.

Leese, G.E., "Engineering Significance of Recent Multiaxial Research," Low Cycle Fatigue, ASTM STP 942, G.R. Solomon et al, Eds., ASTM, 1988, pp. 861-873.

Rajpathak, S. S., and W. H. Hartt, "Fatigue Crack Initiation of Selected High Strength Steels in Sea Water," Proceedings of the Seventh International Conference on Offshore Mechanics and Arctic Engineering, Houston, 1988.

Huang, X., "A Fracture Mechanics Analysis of the Fatigue Reliability of Tubular Joints," Ph.D. thesis, University of Glasgow, 1987.

Ellyin, F., "Effect of Tensile-Mean-Strain on Plastic Strain Energy and Cyclic Response," Journal of Engineering Materials and Technology, Vol. 107, April 1985, pp. 119-125.

Chang, P. Y., "Effects of Stress Concentration and Reinforcement on the Fatigue Strength of Rectangular Openings," Marine Technology, October 1985.

Desvaux, G.J., "The Line Spring Model for Surface Flaw, An Extension to Mode 2 and Mode 3," MSc thesis, Massachusetts Institute of Technology, 1985.

Lee, S-B, "A Criterion for Fully Reversed Out-of-Phase Torsion and Bending," Multiaxial Fatigue, ASTM STP 853, K.J. Miller and M.W. Brown, Eds., ASTM, 1985, pp. 553-568.

Pook, L.P., "The Fatigue Crack Direction and Threshold Behavior of Mild Steel Under Mixed Mode I and III Loading," International Journal of Fatigue, Vol. 7, No. 1, 1985.

Smith, E.W., and K.J. Pascoe, "Fatigue Crack Initiation and Growth in a High-Strength Ductile Steel Subject to In-Plane Biaxial Loading," Multiaxial Fatigue, ASTM, 1985, pp. 111-134.

Kitagawa, H., R. Yuuki, and T.K. Tandbem, " K-Dependency of Fatigue Growth of Single and Mixed Mode Cracks Under Biaxial Stresses," ASTM STP 853, 1985.

Hua, G., N. Alagok, M. Brown, and K. Miller, "Growth of Fatigue Cracks Under Combined Mode I and Mode II Loads," ASTM STP 853, 1985.

Ritchie, R.O., F.A. McClintock, E.K. Tscheyg, and H.N. Hashemi, "Mode III Fatigue Crack Growth Under Combined Torsional and Axial Loading," ASTM STP 853, 1985.

Hourlier, F., H. D'Hondt, M. Truchon, and A. Pineau, "Fatigue Crack Path Behavior Under Polymodal Fatigue," ASTM STP 853, 1985.

Pook, L.P., "Comments on Fatigue Crack Growth Under Mixed Modes I and III and Mode III Loading," ASTM STP 853, 1985.

Ellyin, F., and D. Kujawski, "Plastic Strain Energy in Fatigue Failure," Journal of Pressure Vessel Technology, ASME, Vol. 106, November 1984, pp. 342-347.

Ho, N. J., and F. V. Lawrence., "The Fatigue of Weldments Subjected to Complex Loadings," Fracture Control Program, College of Engineering, University of Illinois, FCP Report No. 45, January 1983.

Nishioka, T., and S.N. Atluri, "An Alternating Method of Analysis of Surface Flawed Aircraft Structural Components," AIAA Journal, Vol. 21, No. 5, 1983.

Nishioka, T., and S.N. Atluri, "Analytical Solution for Embedded Elliptical Cracks, and Finite Element Alternating Method for Elliptical Surface Cracks, Subjected to Arbitrary Loadings," Engineering Fracture Mechanics, Vol. 17, No. 3, 1983.

ABAQUS User's Manual, Hibbit, Karlsson and Sorensen, Inc., Providence, RI, 1982.

Brown, M.W., and K.J. Miller, "Two Decades of Progress in the Assessment of Low-Cycle Fatigue Life," Low-Cycle Fatigue and Life Prediction, ASTM STP 770, C. Amzallag et al, Eds., ASTM, 1982, pp. 482-499.

Smith, E. W., and K. J. Pascoe, "Fatigue Crack Initiation and Growth in a High-Strength Ductile Steel Subject to In-Plane Biaxial Loading," ASTM STP 853, 1982.

Kitagawa, H., R. Yuuki, K. Tohgo, and M. Tanabe, " K-Dependency of Fatigue Growth of Single and Mixed Mode Cracks Under Biaxial Stresses," ASTM STP 853, 1982.

Jacquelin, B., F. Hourlier, and A. Fineau, "Crack Initiation Under Low-Cycle Multiaxial Fatigue," ASTM STP 853, 1982.

Leis, B. N., J. Ahmad, and M. F. Kanninen, "Effect of Local Stress State on the Growth of Short Cracks," ASTM STP 853, 1982.

Leese, G. E., and J. Morrow, "Low Cycle Fatigue Properties of a 1045 Steel in Torsion," ASTM STP 853, 1982.

Fash, J. W., D. F. Socie, and D. L. McDowell, "Fatigue Life Estimates for a Simple Notched Component Under Biaxial Loading," ASTM STP 853, 1982.

Jordan, E. H., M. W. Brown, and K. J. Miller, "Fatigue Under Severe Nonproportional Loading," ASTM STP 853, 1982.

Sonsino, C. M., and V. Grubisic, "Fatigue Behavior of Cyclically Softening and Hardening Steels Under Multiaxial Elastic-Plastic Deformation," ASTM STP 853, 1982.

Nokleby, J. O., "Prediction of High Cycle Fatigue Under Multiaxial Stress Conditions," Det Norske Veritas, ABSE Colloquium Fatigue of Steel and Concrete Structures, Lausanne, 1982.

Nokleby, J. O., and A. U. Walden, "Prediction of Fatigue at Multiaxial Stress--How Good is the Criterion of the ASME Code," Det Norske Veritas, 1982.

Nishioka, T., and S.N. Atluri, "Analysis of Surface Flaws in Pressure Vessels by a New 3-Dimensional Alternating Method," Journal of Pressure Vessel Technology, Vol. 104, No. 4, 1982.

Vaughan, H., "Fatigue and Fracture of Structural Elements Under Random Loads," Royal Institution of Naval Architects, 1982.

Smith, I.J., I.F.C. Smith, and S.J. Hurworth, "A 3-D Analysis of the Longitudinal Non-Load-Carrying Fillet Welding Fatigue Specimen," Research Report, The Welding Institute, 184/1982, May 1982.

Garud, Y.S., "A New Approach to the Evaluation of Fatigue Under Multiaxial Loadings," Journal of Engineering Materials and Technology, ASME, Vol. 103, April 1981, pp. 118-125.

Kfouri, A.P., and K.J. Miller "Three-Dimensional Constitutive Relations and Ductile Fracture," S. Nemat-Nasser, Ed., Proceedings, Conference, Dourdan, France, 2-5 June 1980, International Union of Theoretical and Applied Mechanics, 1981, pp. 83.

Nishioka, T., and S.N. Atluri, "A Major Development Towards a Cost-Effective Alternating Method for Fracture Analysis of Steel Reactor Pressure Vessels," Transactions of the 6th International Conference on Structural Mechanics in Reactor Technology, Paper G1/2, Paris, 1981.

Parks, D.M., "The Inelastic Line Spring for Elastic-Plastic Fracture Mechanics Parameters for Surface-Cracked Plates and Shells," Journal of Pressure Vessel Technology, Vol. 103, 1981, pp. 246-254.

Vijayakumar, K., and S.N. Atluri, "An Embedded Elliptical Flaw in an Infinite Solid Subject to Arbitrary Crack-Face Tractions," Journal of Applied Mechanics, Vol. 48, 1981.

Marloff, R. H., and R. L. Johnson, "The Influence of Multiaxial Stress on Low Cycle Fatigue of Cr-Mo-V Steel at 1000°F," Welding Research Council Bulletin 264, 1980.

Lohr, R.D., and E.G. Ellison, Fatigue of Engineering Materials and Structures, Vol. 3, 1980, pp. 1-17.

Kanazawa, K., K.J. Miller, and M.W. Brown, "Cyclic Deformation of 1% Cr-Mo-V Steel Under Out-of-Phase Loads," Proceedings of the 3rd International Conference on Mechanical Behavior of Materials, ICM-3, Cambridge (England), 20-24 August 1979.

Brown, M.W., and K.J. Miller, Fatigue of Engineering Materials and Structures, Vol. 1, 1979, pp. 217-229.

Konter, A.W.A., G.T.M. Janssen, and W. Husslage, "Effect of Biaxial Loading and Geometry on Prediction of Low-Cycle Fatigue Life," Proceedings, International Conference on Structural Mechanics in Reactor Technology, Berlin, 1979, Division L, Paper 13/1.

Liu, A. F., and D.F. Dittmer, "Effect of Multiaxial Loading on Crack Growth," Northrop Corporation Report to the Air Force Flight Dynamics Laboratory AFFDL-TR-78-175, December 1978.

Kfouri, A.P., and K.J. Miller, Fracture 1977, Vol. 3, D.M.R. Taplin, Ed., University of Waterloo Press, U.K., 1978, pp. 241.

Blass, J. J., and S. Y. Zamrik, "Multiaxial Low-Cycle Fatigue of Type 304 Stainless Steel--Discussion of paper by Manson, C. S., and Halford, G. R.," ASME Transactions, Vol. 99, Ser. J., p. 283-85, July 1977.

Broek, D., and R.C. Rice, "The Fatigue Crack Growth Properties of Rail Steels," DOT-TSC-1076, Battelle Columbus Laboratories, Columbus, OH, July 1977.

Leis, B.N., "An Energy-Based Fatigue and Creep-Fatigue Damage Parameter," Journal of Pressure Vessel Technology, ASME, Vol. 98, No. 4, 1977, pp. 524-533.

Parsons, M.W., and K.J. Pascoe, "Observations of Surface Performation, Crack Initiation and Crack Growth in Low-Cycle Fatigue Under Biaxial Stress," Materials Science and Engineering, Vol. 22, 1976, pp. 31-50.

Swedlow, J.L., "Criterion for Growth on the Angled Crack," Cracks and Fracture, ASTM STP 601, 1976, pp. 506-521.

Ohura, K., K. Ohji, and Y. Ohkubo., "Fatigue Crack Growth Under Biaxial Loading," International Journal of Fracture, Vol. 10, pp. 609-610, Reprint, 1974.

"Classification of Welded Bridge Details for Fatigue Loading," National Cooperative Highway Research Program, National Academy of Sciences, 1974.

Gonyea, D. C., "Method for Low-Cycle Fatigue Design Including Biaxial Stress and Notch Effects," ASTM STP 520, p. 678, August 1973.

Zamrick, S.Y., and R.E. Frismuth, Experimental Mechanics, Vol. 13, May 1973, pp. 204-208.

Brown, M.W., and K.J. Miller, "A Theory for Fatigue Failure Under Multiaxial Stress-Strain Conditions," Proceedings of the Institution of Mechanical Engineers (Br.), Vol. 187, No. 65, 1973, pp. 745-755, and discussion, pp. D229-D244.

Finnie, I., and A. Saith, "A Note on the Angled Problem and Directional Stability of Crack," International Journal of Fracture Mechanics, Vol. 9, 1973, pp. 484-486.

Evans, W. J., "Deformation and Failure Under Multiaxial Stress--A Survey of Laboratory Techniques and Experimental Data," Note No. NT833, National Gas Turbine Establishment, Hants, U.K., 1972.

Williams, J.G., and P.D. Ewing, "Fracture Under Complex Stress," International Journal of Fracture Mechanics, Vol. 8, 1972, pp. 441-446.

Rice, J.R., and N. Levy, "The Part-Through Surface Crack in an Elastic Plate," Journal of Applied Mechanics, Vol. 39, 1972, pp. 185-194.

Segedin, C.M., "A Note on Geometric Discontinuities in Elastostatics," International Journal of Engineering Science, Vol. 6, 1968.

Christensen, R. H., and M.B. Harmon, "Fatigue Crack Propagation," ASTM STP 415, American Society for Testing and Materials, Philadelphia, 1967, p. 5.

Yokobori, T., and T. Yoshimura, "A Criterion for Fatigue Fracture Under Multi-axial Alternating Stress State," Rep. Res. Inst. Strength and Fracture of Materials, Tohoku University, Sendai, Japan, Dec. 1966, Vol. 2, No. 2, pp. 45-54.

Halford, G.R., "The Energy Required for Fatigue," Journal of Materials, Vol. 1, No. 1, March 1966, pp. 3-18.

Findley, W. M., and J. F. Tracy, "Fatigue Under Pulsating Hydrostatic Pressure," Proceedings of the First International Conference on Fracture, Volume 3. The Japanese Society for Strength and Fracture of Materials, September 12-17, 1965, pp 1343-1385.

Morrow, J., "Cyclic Plastic Strain Energy and the Fatigue of Metals," Internal Friction, Damping and Cyclic Plasticity, ASTM STP 378, ASTM, 1965, pp. 45-87.

Welter, G., and J.A. Choquet, "Triaxial Tensile Stress Fatigue Testing," Welding Journal Research Supplement, Vol. 42, No. 12, pp. 565s-70s, December 1963.

Erdogan, F., and G.C. Sih, "On the Crack Extension in Plates Under Plane Loading and Transverse Shear," Journal of Basic Engineering, Vol. 85, 1963, pp. 519-525.

Feltner, C., and J. Morrow, "Microplastic Strain Hysteresis Energy as a Criterion for Fatigue Fracture," Journal of Basic Engineering, ASME, Vol. 83D, March 1961, pp. 15-22.

Lehrer, W. M., and H. Schwartzbart, "Static and Fatigue Strength of Metals Subjected to Triaxial Stresses," Proc. Am. Soc. Test. Mater., Vol. 60, 1960, pp. 610-625. [Paper presented at 28th ann. Meet. Instit. Aeronaut. Sciences, New York, Jan. 25-27, 1960. Paper No. 60-12, pp. 17].

Matthaes, K., "Die Festigkeit bei mehrachsiger Beanspruchung," [The Strength Under Multi-Axial Stressing.] Metall, Vol. 14, No. 5, May 1960, pp. 413-420.

Mozberg, R. K., "X-Ray Investigation of the Fatigue Process in Mild Steel Under Complex Stress Conditions," [R] Tallinna Poluteh. Inst. Toim. Seer. A, No. 156, 1959, pp. 41-56.

Sines, G., "Behaviour of Metals Under Complex Static and Alternating Stresses," Metal Fatigue. [Editors: G. Sines and J. L. Waisman]. New York, McGraw-Hill Book Co. Inc., 1959, pp. 145-169.

Lehrer, W., and H. Schwartzbart, "Investigation of Static and Fatigue Strength of Metals Subjected to Triaxial Stresses," W.A.D.C. Tech. Rep. No. 58-430, Dec. 1957, pp. 66.

Welter, G., and A. Choquet, "Nouvelle Machine a Essais Statiques et en Fatigue par Tensions Triaxiales," [A new machine for static and fatigue tests in triaxial tension.] Ingenieur, Montreal, Vol. 42, No. 167, 1956, pp. 14-22. Metaux Corros. Inds, Vol. 32, No. 383/384, July-Aug. 1957, pp. 300-312.

Morrison, J. L., B. Crossland, and J.S.C. Parry, "Fatigue Under Triaxial Stress - A Testing Machine and Preliminary Results," Engineering, Lond., Vol. 183, No. 4752, April 5, 1957, pp. 428-432.

Williams, J.G., "On the Stress Distribution at the Base of a Stationary Crack," Journal of Applied Mechanics, Vol. 24, 1957, pp. 109-114.

Parry, J.S.C., "Further Results of Fatigue Under Triaxial Stress," Proc. Int. Conf. Fatigue Metals. London, Instn Mech. Engrs, 1956, pp. 132-137.

Morrison, J.L.M., B. Crossland, and J.S.C. Parry, "Fatigue Under Triaxial Stress: Development of a Testing Machine and Preliminary Results," Chart. Mech. Engr, Vol. 3, No. 3, March 1956, pp. 153-155.

Bundy, R. W., and J. Marin, "Fatigue Strength of 14S-T4 Aluminum Alloy Subjected to Biaxial Stresses," Proc. Am. Soc. Test. Mater., Vol. 54, 1954, pp. 755-768.

Gol'Tsev, D. I., "On the Strength Under Conditions of Variable Load and Complex Stressing." [R] Voprosy Dianmiki i Dinamicheskoi Prochnosti [Problems of Dynamics and Dynamical Strength]. Riga, Izd-vo Akad. Nauk Latviiskoi SSSR, Vol 1, 1953, pp. 47-88.

Serensen, S. V., "On the Fatigue Resistance Under A Complex Stress Condition and Symmetrical Cycle." [R] Nekotorye voprosy ustalostnoi prochnosti stali [Some Problems of the Fatigue Strength of Steel]. [Editor: N. N. Davidenkov]. Leningrad, Mashgiz, 1953, pp. 102-115.

Sigwart, H., "Schadenfalle an Konstruktionsteilen bei mechrachsigem Spannungszustand," [Damage conditions in structural members under multiaxial stress conditions.] Maschinenschaden, Vol. 25, No. 9/10, 1952, pp. 118-124.

Marin, J., "Biaxial Tension-Tension Fatigue Strengths of Metals," J. Appl. Mech., Vol. 16, No. 4, Dec., 1949, pp. 383-388, and [discussion] Vol. 17, No. 2, June 1950, p. 222.

Marin, J., and W. Shelson, "Biaxial Fatigue Strength of 24ST Aluminum Alloy," Tech. Note Nat. Adv. Comm. Aero., Wash., No. 1889, May 1949, pp. 41.

Fowler, F. H., "On Fatigue Failures Under Triaxial Static and Fluctuating Stresses and A Statistical Explanation of Size Effect," Trans. Amer. Soc. Mech. Engrs, vol. 67, no. 3, April 1945, pp. 213-215.

Trefftz, E., Handbuch der Physik, Vol. 6, Springer-Verlag, Berlin, 1928, pp. 92.

Nokleby, "Fatigue Under Multiaxial Stress Conditions," MD 81001, Division of Machine Elements, The Norwegian Institute of Technology, Trondheim, Norway.

Silijander, A., P. Kurath, and F.V. Lawrence, "Proportional and Non-Proportional Multiaxial Fatigue of Tube-to-Plate Weldments"

3. PLATES IN BENDING - BIAXIAL STRESSES

Sakane, M., M. Ohnami, and M. Sawada, "Fracture Modes and Low Cycle Biaxial Fatigue Life at Elevated Temperature," Journal of Engineering Materials and Technology, October 20, 1986.

Woo, C.W., and L.H. Ling, "On Angled Crack Initiation Under Biaxial Loading," Journal of Strain Analysis, Vol. 19, No. 1, 1984, pp. 51-59.

Jong-Yeong, Yung, "The Effects of Variable and Biaxial Loadings on the Fatigue of Weldments," Thesis proopsal, Dept. of Metallurgy and Mining Engineering, Dec. 1983. University of Illinois.

Moyar, G.J., and V.K. Garg, "Biaxial Fatigue Design Procedure Applied to Welded Plate Structures," private communications, August 1983.
Miller, K. J., "Mode I Fatigue Crack Growth Under Biaxial Stress at Room and Elevated Temperatures," ASTM STP 853, 1982.

McDiarmid, D. L., "Designing for High-Cycle Biaxial Fatigue Using Surface Strain Records," ASTM STP 853, 1982.

Smith, E. W., and K.J. Passcoe, "Fatigue Crack Initiation and Growth in HY103 Steel Subject to In-Plane Biaxial Loading," Technical Report CUED/C-MAT.TR.95, Cambridge University Engineering Department, Cambridge, 1982.

McDiarmid, D. L., "Fatigue Under Out-Of-Phase Biaxial Stresses of Different Frequencies," ASTM STP 853, 1982.

Kandil, F.A., M.W. Brown, and K.J. Miller, "Biaxial Low-Cycle Fatigue Fracture of 316 Stainless Steel at Elevated Temperatures," Book 280, The Metals Society, London, 1982, pp. 203-210.

Lefebvre, D., K.W. Neale, and F. Ellyin, "A Criterion for Low-Cycle Fatigue Failure Under Biaxial States of Stress," Journal of Engineering Materials and Technology, ASME, Vol. 103, January 1981, pp. 1-6.

Adams, W. R., "Uniaxial and Biaxial Low Cycle Fatigue Behaviour Under Isothermal and Temperature Cycling Conditions," Ph.D. Thesis, University of Nottingham, U.K., 1972.

Rosen, M., and R. A. Gaites, "Fatigue of Large Scale Uniformly Loaded Simply Supported Butt Welded Rectangular Plates," U. S. Naval Applied Science Laboratory, Project 9300-23, Project Report No. 6, SF 013-03-02, 1968.

Gaites, R. A., and M. Rosen, "Fatigue of Large Scale Uniformly Loaded Simply Supported Butt Welded Rectangular Plates," U. S. Naval Applied Science Lab.-Material Sciences Division, Lab. Project 930-23, Progress Report No. 9, October 31, 1968.

Gaites, R. A., and M. Rosen, "Fatigue of Large Scale Uniformly Loaded Simply Supported Tee-Fillet Welded Rectangular Rib-Stiffened Plates," U. S. Naval Applied Science Laboratory, Project 930-23, Progress Report No. 8, June 7, 1968.

Rosen, M., and H. V. Cordiano, "Fatigue of Structural Elements, Development of Economical Beam Type Structural Elements for Simulated Fatigue Evaluations," U. S. Naval Applied Science Laboratory, Project No. 9300-23, Progress Report No. 4, June 1967.

U. S. Naval Applied Science Laboratory, "Fatigue of Large Scale Uniformly Loaded Simply Supported Butt Welded Rectangular Plates," Project 9300-23, SF 013-03-02, Technical Memorandum No. 8, Task 2025, June 1967.

U. S. Naval Applied Science Laboratory, "Fatigue of Structural Elements Study of Residual Stresses in Linearly Varying Biaxial Stress Fields," Lab. Project 9300-23, Progress Report No. 2, Brooklyn, NY, April 26, 1967.

Rosen, M., "Fatigue of Large Scale Uniformly Loaded Simply Supported Tee-Fillet Welded Rectangular Rib-Stiffened Plates," U. S. Naval Applied Science Laboratory, Project 9300-23, Technical Memorandum No. 7, SF 013-03-02, Task 2025, March 1967.

Blatherwick, A. A., and N. D. Viste, "Fatigue Damage During Two-Level Biaxial-Stress Tests," Air Force Materials Laboratory, AFML-TR-66-355, Wright-Patterson Air Force Base, Ohio, February 1967.

Rosen, M., and H. V. Cordiano, "Fatigue of Large Scale Uniformly Loaded Simply Supported Tee-Fillet Welded Rectangular Rib-Stiffener Plates," U. S. Naval Applied Science Laboratory, Project 9300-23, Technical Memorandum No. 6, SF013-03-02, December 1966.

Abramov, P., and F. Ginsberg, "Investigation of Fatigue Life of HY-130/150 Steel," Tech. Memo, #43, Lab. Project 9300-1, U. S. Naval Applied Science Lab., Brooklyn, NY, June 14, 1966.

U. S. Naval Applied Science Laboratory, "Fatigue of Structural Elements--Initial Studies on the Effect of Overstrain on Residual Stresses and Fatigue," Material Sciences Division, Project 9300-23, Technical Memorandum No. 5, SF013-03-02, Task 2025, May 1966.

U. S. Naval Applied Science Laboratory, "Crack Growth Properties of Welds in HY-80, Hy-100, and HY-130/150 High Strength Steels in a Sea Water Environment," Project 9300-1, Technical Memorandum No. 39, September 1965.

Cordiano, H. V., W. Werchniak, and B. S. Silverman, "Fatigue of Structural Elements Development of Theory and Measurement of Residual Stresses at Tee Fillet Welds in 1-1/2 in. HY-80 Steel," U. S. Naval Applied Science Lab, September 1965.

U.S. Naval Applied Science Laboratory, "Mechanical Peening Procedure for Improvement of Fatigue Properties of HY-80 Butt Welds," Project 9300-1, Technical Memorandum No. 34, August 1965.

U.S. Naval Applied Science Laboratory, "Investigation of the Effect of Welding on the fAtigue Properties of HY-100 Steel, Tee Fillet Welded Plates, Mechanically Peened," Project 9300-1, Technical Memorandum No. 31, June 1965.

U.S. Naval Applied Science Laboratory, "Investigation of the Effect of Welding on the Fatigue Properties of HY-100 Steel Tee-Fillet Welded Plates in As-Welded Condition," Technical Memorandum, U. S. Naval Base, Brooklyn, NY, Feb. 2, 1965.

Anonymous, "Effects of Fatigue on the Explosion Bulge Properties of HY-80 Butt Weldments," U. S. Naval Applied Science Laboratory Lab Project 9300-I, Tech. Memorandum 7, 1964.

Anonymous, "Fatigue of Structural Elements Measurement of Residual Stresses at Tee-Fillet Welds in HY-80 Steel," U.S. Naval Applied Science Laboratory - Materials Sciences Division, Lab Project 9300-23, 1964.

U.S. Naval Applied Science Laboratory, "Fatigue of Iron Base Alloys HY-80 Steel cast Tees But Welded to Rolled Section Single and Double Weld Joint Design," Project 9300-1, Technical Memorandum No. 19, December 1964.

U.S. Naval Applied Science Laboratory, Lab. Project 9300-1, Tech. Memo #19, "Fatigue of Iron Base Alloys HY-80 Steel Cast Tees Butt Welded to Rolled Section Single and Double Weld Joint Design," U. S. Naval Appl. Sci. Lab., Naval Base, Brooklyn, NY. Dec. 14, 1964.

U.S. Naval Applied Science Laboratory, Lab. Project 9300-1, Tech. Memo. #18, "Effects of Grinding and Shot Peening on Fatigue life of Tee Weldments," Technical Memorandum No. 18, U.S. Naval Appl. Sci. Lab., Naval Base, Brooklyn, NY, September 30, 1964.

U.S. Naval Applied Science Laboratory, "Effects of Shot Peening on Fatigue Life of HY-80 Tee Fillet Weldments," Project 9300-1, Technical Memorandum No. 15, August 1964.

U.S. Naval Applied Science Laboratory, Lab. Project 9300-I, Tech. Memo #11, "Effects of Grinding on Fatigue Life of Tee Weldments," U. S. Naval Applied Sci. Lab., Naval Base, Brooklyn, NY, Project 9300-1, Technical Memorandum No. 11, July 8, 1964.

Foster, M. L., and P. Abramov, "Development of NASL Mechanical Peening Procedure for Improvement of Fatigue Properties of HY-80 Welds," U.S. Naval Applied Science Laboratory, Project 9300-1, Progress Report No. 1, April 1964.

Cordiano, H. V., "Submarine Structural Fatigue Program Design Procedures for Low-Cycle Fatigue," Technical Memorandum #1, U. S. Naval Applied Science Laboratory, Project 9300-23, SF013-03-02, Task 2025, March 1964.

U.S. Naval Applied Science Laboratory, "Effect of Embrittled 110-18 Weld Metal on the Fatigue Life of HY-80 Steel Butt Weldments," Project No. 9300-1, Technical Memorandum No. 2, October 1963.

U.S. Naval Applied Science Laboratory, "Fatigue of Iron Base Alloys HY-80 Steel, Rolled Plate to Cast Plate Butt Welds, etc.," Project No. 6160-2, September 1963.

U.S. Naval Applied Science Laboratory, "Fatigue of Iron Base Alloys--NY-80 Steel, Cast Tee," New York Naval Shipyard, Brooklyn, New York, Project No. 6160-2, Progress Report No. 7, June 1963.

Anonymous, "Investigation of the Effect of Welding on the Fatigue Properties of HY-80 Steel; Virgin Plate - Report on Fatigue of SR007-01-01, Task 0856," New York Naval Shipyard-Material Lab., Project 6160-2, April 1962.

Wolfe, R. J., I. L. Stern, P. Abramov, and H. Nagler, "Investigation of Effect of Welding on the Fatigue Properties of HY-80 Steels," New York Naval Shipyard, Material Laboratory Project 6160-2, Progress Report 3, July 1961.

Bowman, C. E., and T. J. Dolan, "Biaxial Fatigue Studies of High Strength Steels Clad with Stainless Steel," T.A.M. Report No. 164, Dept. of Theoretical and Applied Mechanics, Univ. of Ill., Urbana, May 1960.

Anonymous, "Effects of Welding on Fatigue Properties of HY80 Steel," New York Naval Shipyard-Materials Lab., Project 6160-2, Progress Report 2, April 1960.

Stern, I. L., and H. V. Cordiano, "Fatigue Properties of Welds, In HY-80 Steel," Proceedings (HY-80 Steel Fabrication in Submarine Construction) Bureau of Ships Conference, Washington, DC, March 1960.

Cordiano, H. V., P. Abramov, and E. Lewis, "Investigation of the Effect of Welding on the fatigue Properties of HY-80 Steel," New York Naval Shipyard, Material Laboratory Project 6160-2, Progress Report No. 1, September 1959.

Cordiano, H. V., P. Abramov, and E. Lewis, "Investigation of the Effect of Welding on the Fatigue Properties of HY-80 Steel," Technical Report, Material Laboratory, New York Naval Shipyard, Brooklyn, NY, September 9, 1959.

Bowman, C. E., and T. J. Dolan, "Resistance of Low-Alloy Steel Plates to Biaxial Fatigue," Welding Journal, Vol. 35, No. 2, pp. 102s-109s, February 1956.

Bowman, C. E., and Dolan, T. J., "Studies of the Biaxial Fatigue Properties of Pressure Vessel Steels," Welding Journal, Easton, Vol 34, No. 1, Jan. 1955, pp. 51s-59s, and [discussion] No. 4, April 1955, pp. 179s-182s.

Dolan, R.J., "Fatigue as a Factor in Pressure Vessel Design," Weld. J., Easton, Vol. 33, No. 6, June 1954, pp. 265s-275s. Tech. Rep. Univ. Ill. Dep. Theoret. Appl. Mech. No. 40 on Behaviour of Materials Under Repeated Stress, June 1954, pp. 19.

Bowman, C.E., "Design of Hydraulically Operated Biaxial Fatigue Testing Machine," Appl. Hydraul. Pneumat., Vol. 6, No. 12, Dec. 1953, pp. 61-64.

Bowman, C.E., and T. J. Dolan, "Biaxial Fatigue Properties of Pressure Vessel Steels," Welding Journal Research Supplement, Vol. 32, No. 11, pp. 529s-37s, November 1953.

Marin, J., and W.P. Hughes, "Fatigue Strengths of 14S-T4 Aluminum Alloy Subjected to Biaxial Tensile Stresses," Tech. Notes Natn. Advis. Comm. Aeronaut., Wash., No. 2704, June 1952, pp. 24.

Blaser, R.U., and J. T. Tuchker, Jr, "Biaxial Fatigue Tests on Flat-Plate Specimens," Welding Journal Research Supplement, Vol. 31, No. 3, March 1952, pp. 161s-168s.

Morikawa, G. K., and L. V. Griffis, "Biaxial Fatigue Strength of Low-Carbon Steels," Welding Journal, Vol. 24, No. 3, pp. 167s-174s, March 1945.

Ahmad, J., B.N. Leis, and M.R. Kanninen, "Analysis of Fatigue Crack Propagation Under Biaxial Loading Using an Inclined Strip Yield Representation of Crack Tip Plasticity," "Fatigue of Engineering Materials and Structures," to appear.

4. PRINCIPAL STRESSES IN FLEXURAL MEMBERS - TENSION FIELD

Stickel, G. D., "Fatigue Tests of Welded Transverse Stiffener Details: Summary, Analysis, and Comparison of Results," Master of Science Thesis in Civil Engineering, University of Pittsburgh, 1985.

Moyar, G.J., and V.K. Garg, "Biaxial Fatigue Design Procedure Applied to Welded Plate Structures," private communications, August 1983.

Fisher, J. W., B.M. Barthelemy, D.R. Merta, J.A. Edinger, "Fatigue Behavior of Full-Scale Welded Bridge Attachments," National Cooperative Highway Research Program Report #227, 1980.

Albrecht, P., and Ian M. Friedland, "Fatigue-Limit Effect on Variable-Amplitude Fatigue of Stiffeners," ASCE Journal of the Structural Division Proceedings of the American Society of Civil Engineers, Vol. 150, No. ST12, December 1979.

Bardell, G.R., and G. L. Kulak, "Fatigue Behavior of Steel Beams with Welded Details," Department of Civil Engineering, University of Alberta, Structural Engineering Report No. 72, September 1978.

Maeda, Y., S. Marsui, and Y. Matsuda, "Fatigue Test of Open-Web Expanded Beams," Technology Reports of Osaka University, Vol. 25, No. 1246, 1975.

Fisher, Albrecht, Yen, Klingerman, and McNamee, "Fatigue Strength of Steel Beam with Welded Stiffeners and Attachments," National Cooperative Highway Research Program Report 147, 1974.

Frost, R. W., and R. E. Leffler, "Fatigue Tests of Beams with Rectangular Web Holes," American Society of Civil Engineers, Journal of the Structural Division, Vol. 97, No. ST2, pp. 509-27, February 1971.

Parsanejad, S., and A. Ostapenko, "On the Fatigue Strength of Unsymmetrical Steel Plate Girders," Welding Research Council, Bulletin No. 156, pp. 48-59, November 1970.

Toprac, A. A., "Fatigue Strength of Hybrid Plate Girders," Welding Journal Research Supplement, Vol. 48, No. 5, pp. 195s-202s, May 1969.

Mueller, J. A., and B. T. Yen, "Fatigue Strength of Plate Girders; 1) Girder Web Boundary Stresses and Fatigue, 2) Fatigue Tests of Large-Size Welded Plate Girders," Steel Research for Construction, Bulletin No. 17, April 1969, Lehigh Univ. American Iron and Steel Institute.

Mueller, J. A., and B. T. Yen, "Girder Web Boundary Stresses and Fatigue," Welding Research Council, Bulletin No. 127, January 1968.

Yen, B. T., and J. A. Mueller, "Fatigue Tests of Welded Plate Girders in Shear," Lehigh Univ., Institute of Research, Fritz Engr. Lab., Report No. 303.6, 1964.

Hall, L. R., and J. E. Stallmeyer. Thin Web Girder Fatigue Behavior as Influenced by Boundary Rigidity," Structural Research Series No. 278. Univ of Ill., January 1964.

Munse, W. H., and J. E. Stallmeyer, "Fatigue in Welded Beams and Girders," Highway Research Board, Bulletin No. 315, 1962.

Gurney, T. R., and C. C. Woodley, "Investigation into the Fatigue Strength of Welded Beams: High Tensile Steel Beams with Stiffeners Welded to the Webs," Part III in a Series, British Welding Journal, Vol. 9, No. 9, September 1962.

Munse, W. H., and J.E. Stallmeyer, "Behavior of Welded Built-In Beams Under Repeated Loads," University of Illinois, April, 1960.

Munse, W.H., and J.E. Stallmeyer, "Influence of Weld Details on Fatigue of Welded Beams and Girders," Symposium on Fatigue of Welded Structures, Cambridge University, March 1960.

Munse, W. H., and J. E. Stallmeyer, "Influence of Weld Details on Fatigue of Welded Beams and Girders," British Welding Journal, Vol. 7, No. 3, March 1960.

Tada, Y. M. Konatsubara, K. Kamoto, Y. Moriwaki, and Y. Tabuchi, "Some Considerations About the Shear Fatigue Strength of Welded Beams," I.I.W. Document XIII-220-60, February 1960.

Douba, N. G., and J. E. Stallmeyer, "The Behavior of Stiffened Beams Under Repeated Loads," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 173, April 1959.

Fisher, W. E., and J. E. Stallmeyer, "Behavior of Welded Built-Up Beams Under Repeated Loads," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 147, March 1958.

Stallmeyer, J.E., W.H. Munse, and B.J. Goodal, "Behaviour of Welded Built-Up Beams Under Repeated Loads," Weld. J., Easton, Vol. 36, No. 1, Jan. 1957, pp. 27s-36s. Civ. Engng Stud. Univ. Ill. Struct. Res. Ser. Rep. No. 135, Jan. 1957, pp. 12.

de Leiris, H., "Etude Experimentale de la Resistance a la Traction Cyclique de Liaisons Soudees pour Radisseurs de Fond et de Point de Petroliers," [Experimental Investigation of the Cyclic Tensile Strength of Welded Joints in Bottom and Deck Stiffeners of Oil Tankers.] Bull. Ass. Tech. Marit. Aeronaut., Vol. 53, Mem. No. 1104, 1954, pp. 647-663.

Weck, R., "Fatigue Strength of Panels With Welded Angle Stiffeners," Welding Journal, Easton, Vol. 31, No. 7, July 1952, pp. 338s-349s.

Weck. R., "Fatigue Strength of Panels With Welded Angle Stiffeners. Weld. Res., Lond., Vol. 5, No. 5, Oct. 1951, pp. 219r-249r.

Maeda, Y., "Fatigue Cracks of Deep Thin-Walled Plate Girders," Bridge Engineering, Vol. 1, Transportation Research record 664, Transportation Research Board, National Academy of Science, pp. 120-128.

5. **CROSS GIRDERS**

Braithwaite, A. B. M., and T. R. Gurney, "Fatigue Tests on Cross Girder Connections," British Welding Research Association, Report D7/352/65, 1965. Also see British Welding Journal, pp. 71-80, February 1967.

6. **PRESSURE VESSELS - TUBES AND MATERIALS**

"Long Range Plan for Pressure Vessel Research--Eight Edition," Welding Research Council Bulletin 327, 1987.

Wellman, G.W., S.T. Rolfe, and R.H. Dodds, "Failure Prediction of Notched Pressure Vessels Using the CTOD Approach," Welding Research Council Bulletin 299, 1984.

Jacquelin, B., F. Hourlier, and A. Pineau, "Transactions," American Society of Mechanical Engineers, Journal of Pressure Vessel and Technology, Vol. 106, 1983, pp. 138-143.

Huther, M., F. Benoit, and J. Poudret, "Fatigue Analysis Method for LNG Membrane Tank Details," Paper to be presented at Extreme Loads Response Symposium, Arlington, VA 10/19-20/81. Society of Naval Architects and Marine Engineers.

ASME Boiler and Pressure Vessel Code, Section III, Code Case N-47-12, American Society of Mechanical Engineers, New York, 1980.

Yamaguchi, Y., S. Hattori, and H. Yanai, "Fatigue Fracture of Thick-Walled Cylinders," Research and Development, Kobe Steel Engineering Reports, Vol 28, No. 2, pp. 47-51, April 1978.

Barsom, J. M., "Fatigue Behavior of Pressure-Vessel Steels," Welding Research Council Bulletin 194, 1974.

ASME Boiler and Pressure Vessel Code, Sec. III, Div. 1., Subsection NA, American Society of Mechanical Engineers, New York, 1974.

Ardentov, V.V., T.I. Ivanova, V.A. Ignatov, G.P. Karsov, M.P. Rosanov, and B.T. Timofeev, "Some Problems of Establishing Acceptance Levels for Defects in Welded Pressure Vessels," I.I.W. Annual Assembly 1972, IIW General Reports.

Riccardella, P.C. and T.R. Mager, "Fatigue Crack Growth Analysis of Pressurized Water Reactor Vessels," Proceedings, 1971 National Symposium on Fracture Mechanics, STP 513, Stress Analysis and Growth of Cracks, American Society for Testing and Materials, 1971.

Langer, B. F., "Pressure Vessel Engineering Technology," Elsevier, Amsterdam-London-New York, 1971, pp. 59-100.

Gregory, S. C., W. R. Blackstone, and R. C. DeHart, "Studies of the Fatigue Strength of Pressure Vessels," Technical Summary Report, Southwest Research Institute, Project No. 03-1275, U. S. Atomic Energy Commission, Contract No. AT(11-1)-1228, pp. 1-85, December 15, 1970.

Haslam, G. H., "The Fatigue Strength of Thick-Walled Steel Cylinders Subjected to Pulsating Internal Pressures," N.E.L. Report No. 444. February 1970.

ASME, "Criteria of the ASME Boiler and Pressure Vessel Code for Design by Analysis in Sections III and VII, Division 2," ASME, 1969.

Geilenkeuser, H., K. Wellinger, and M. Liebrich, "The Behavior of Cold Worked Welded Line Pipe Under Repetitive Internal Stressing Conditions," Welding Research Abroad, Vol. 15, No. 8, pp. 33-51, October 1969.

Stockman, G., and A. Sijs, "Results of Fatigue Tests on Tangetial Nozzles, part 2," Welding Research Abroad, Vol. 15, No. 1, pp. 2-24, January 1969.

Wellinger, K., and D. Sturn, "Performance of Pressure Vessels With Nozzles Under Pulsating Internal Pressure," I.I.W. Document No. XIII-518-68, pp. 1-14, 1968.

Pickett, A. G., and S. C. Grigory, "Cyclic Pressure Tests of Full-Size Pressure Vessels," Welding Research Council, Bulletin No. 135, November 1968.

Taylor, T. E., "Low-Cycle Fatigue Behavior of Low Alloy Steel Pressure Vessels," Welding Research Abroad, Vol. 14, No. 2, pp. 10-12, February 1968.

Salkin, R. V., "How Low Cycle Fatigue Embrittles Pressure Vessel Steel," I.I.W. XIII-451-67, 1967.

Taylor, T. E., "Low Cycle Fatigue Behavior of Low Alloy Steel Pressure Vessels," British Welding Journal, pp. 641-48, December 1967. Also see British Welding Research Association, Report E/7/66.

Gross, M. R., "Low-Cycle Fatigue Behavior of a Double-Box Structure," Naval Ship Research and Development Center, Report No. 2568, December 1967.

Taylor, T. E., "Effect of Test Pressure on the Fatigue Performance of Mild Steel Cylindrical Pressure Vessels Containing Nozzles," British Welding Journal, Vol. 14, No. 3, March 1967 (British Welding Research Assoc. Report E/6/65).

Gross, M. R., "Low-Cycle Fatigue Behavior of Internally Pressurized Boxes," U.S. Naval Marine Engineering Laboratory, Assignment 86 108, Report 5/67, March 1967.

Taylor, T. E., "Effect of Test Pressure on the Fatigue Performance of Mild Steel Cylindrical Pressure Vessels Containing Nozzles," British Welding Journal, pp. 117-20, March 1967.

Wood, D. S., "The Effect of Creep on the High Strain Fatigue Behavior of a Pressure Vessel Steel," Welding Journal Research Supplement, Vol. 45, No. 2, pp. 90s-96s, February 1966.

Kudrjavcev, I. V., and A. D. Chudnovstij, "Carrying Capacity of Welded Vessels Under Internal Pressure with Low Frequency Loading," International Institute of Welding, Commission XIII, Doc. XIII-388-65, USSR National Welding Committee, Moscow, 1965.

International Institute of Welding, "Brittle Fracture of a Pressure Vessels (Regenerator)," IIW Doc XIII-400-65.
Bevitt, E., A. Cowan, and A. L. Stott, "Failure Tests on Cylindrical Steel Vessels Containing Axial Faults," Welding Research Abroad, Vol. 11, December 1965, pp. 23-37.

Gross, M. R., and R. E. Heise, Jr., "Low-Cycle Fatigue Behavior of Internally Pressurized Boxes," American Society of Mechanical Engineers, Transaction, Journal of Engineering for Industry, Vol. 66, WA/Unt-1, May 1966. Also see U.S. Naval Marine Engineering Laboratory, Assignment 86 108, Phase Report 68/65, February 1965.

Heise, R. E., and M. R. Gross, "Low-Cycle Fatigue Behavior of an Internally Pressurized Box," U.S. Navy Marine Engineering Laboratory, Research and Development Phase Report 86 108C, Sub-Project S-R007 01 01 Task 0856, December 1963.

Peterson, R. E., "Fatigue of Metals: Part 3--Engineering and Design Aspects," Materials Research and Standards, Vol. 3, 1962, pp. 116.

Heywood, R.B., Designing Against Fatigue of Metals, Royal Aircraft Establishment, 1962.

Heywood, R. B., Designing Against Fatigue of Metals, Reinhold Publishing Corporation, New York, 1962.

Koositra, L. F., and M. M. Lemcoe, "Low Cycle Fatigue Research on Full-Size Pressure Vessels," Welding Journal Research Supplement, Vol. 41, No. 7, p. 297s, July 1962.

Langer, B. F., "Design of Pressure Vessels for Low-Cycle Fatigue," ASME, January 10, 1962, Journal of Basic Engineering.

Dolan, T. J., "Significance of Fatigue Data in Design of Pressure Vessels," Pressure vessel and piping design - collected papers 1927-1959. New York, Am. Soc. Mech. Engrs, 1960, pp. 541-546.

Markl, A.R.C., "Fatigue Test of Welding Elbows and Comparable Double-Mitre Bends," Pressure vessel and piping design - collected papers 1927-1959. New York, American Society of Mechanical Engineers, 1960, pp. 371-381.

Markl, A.R.C., "Fatigue Tests of Piping Components," Pressure vessel and piping design - collected papers 1927-1959. New York, American Society of Mechanical Engineers, 1960, pp. 91-101.

de Leiris, H., "Le Comportment aux Mises en Pression Re'pe'tees des Bouteilles en Alliage le[6]ger Affectee's d'Accidents de Surface," [The behaviour of light-alloy compressed air cylinders subjected to repeated applications of pressure and having surface defects.] Bull. Ass. Tech. Marit. Aeronaut., Vol. 59, 1959, pp. 231-240.

Lane, P.H.R., "Essais de Fatigue par Pression Pulsatoire sur Divers Types d'Assemblages de Tabulures sur Recipient a Pression," [Pulsating pressure fatigue tests on various types of tubular joints on a pressure vessel.] Soudage Tech. Conn., Vol. 13, No. 5/6, May-June 1959, pp. 197-202.

Lankford, W. T. Pressure vessel high strain fatigue and information required to design against it. Docums Int. Inst. Weld., No. IIW/IIS-34-59, April 22, 1959, pp. 18.

Rose, R.T., "Pressure vessel design," Engng Mater. Des., Vol. 2, No. 1, Jan. 1959, pp. 33-37.

Lane, P.H.R., "Fatigue Tests on Pressure Vessel Branch Connections," Welding Research Abroad, Vol. 4, No. 8, pp. 19-24, October 1958.

Hoare, R. G., and H. Stott, "Investigation Into the Life of Aluminum Alloy Cylinders Under Repeated Application of Internal Hydraulic Pressure (part I and II)," Scient. Tech. Memo. No. 16/58, Sept. 1958, pp. 5.

Kerkhof, W.P., "Sterkteberekening Van Vaten Voor Hoge Inwendige Druk," [New developments in the design of pressure vessels.] Ingenieur, 's Grav., Vol 70, No. 33, Aug. 15, 1958. pp. Ch47-Ch56.

Morcock, D.S., "Hydrostatic Fuselage Pressure Fatigue Test Program Lockheed C-130A Hercules," Can. Aeronaut. J., Vol. 4, No. 6, June 1958, pp. 180-184.

Zver'Kov, B.V., "Machine for Investigating the Fatigue Strength of Pipes Under Pressure Combined With Cyclic Bending Tests," [R] Zav. Lab., Vol. 24, No. 12, Dec. 1958, pp. 1514-1517. English translation: Ind. Lab. Vol. 24, No. 12, Feb. 1960, pp. 1656-1658.

Kooistra, L. F., "Fatigue-Effect of Plastic Fatigue of Pressure Vessel Material and Design," Welding Journal Research Supplement, Vol. 36, 1957, pp. 120s-120s.

Murphy, J. J., and D. B. Rossheim, "Effect of Plastic Fatigue on Pressure-Vessel Materials and Design," (Discussion), Welding Journal, September 1957, p. 429s.

Welter, C., and J. Dubuc, "Fatigue Resistance of Simulated Nozzles in Model Pressure Vessels," Welding Journal Research Supplement, Vol. 36, No. 6, pp. 271s-274s, June 1957.

Harman, C. R, General Editor, Handbook for Welding Design, Volume 1, The Institute of Welding, 1956.

Murphy, J. J., C.R. Soderberg, and D.B. Rossheim, "Considerations Affecting Future Pressure Vessel Codes," Weld. J., Easton, Vol. 35, No. 12, Dec. 1956, pp. 582s-596s.

Peters, R. W., and N.F. Dow, "Failure Characteristics of Pressurized Stiffened Cylinders," Tech. Notes Natn. Advis. Comm. Aeronaut., Wash. No. 3851, Dec. 1956, pp. 18.

Dubuc, J., and G. Welter, "Fatigue Tests of Model Pressure Vessels," Welding Journal, Vol. 35, pp. 329s-337s, July 1956.

Dolan, T. J., "Significance of Fatigue Data in Design of Pressure Vessels," Weld. J., Easton, Vol. 35, No. 5, May 1956, pp. 255s-260s.

Markl, A.R.C., H.H. George, and E.C. Rodabaugh, "Pressure-Pulsation Tests Shown Why Branch Connections Fail," Pipe Line Ind., Vol. 3, No. 1, July 1955, pp. 32-38.

Gross. J. H., and R. D. Stout, "Plastic Fatigue Properties of High-Strength Pressure-Vessel Steels," Welding Journal Research Supplement, Vol. 34, No. 4, April 1955, pp. 161s-166s.

Dolan, R.J., "Fatigue As A Factor in Pressure Vessel Design," Weld. J., Easton, Vol. 33, No. 6, June 1954, pp. 265s-275s. Tech. Rep. Univ. Ill. Dep. Theoret. Appl. Mech. No. 40 on Behaviour of Materials Under Repeated Stress, June 1954, pp. 19.

Narduzzi, E. D., and G. Welter, "Fatigue Tests of Pressure Vessels," Welding Journal Research Supplement, Vol. 33, pp. 230s-238s, May 1954.

Hunt, M.S., and B.N. Mason, "Repeated Pressure Tests on "Apollo" Centre Fuselage," Scient. Tech. Memo No. 7/53, March 1954, pp. 6.

Gross, J.H., D.E. Gucer, and R.D. Stout, "The Plastic Fatigue of Pressure Vessel Steels," Welding Journal Research Supplement, March 1954, pp. 128s.

Sag, N., "Design of Machine Parts Subjected to Fluctuating Combined Bending and Tension," Engineer, Lond., Vol. 192, No. 4985, Aug. 10, 1951, pp. 172-176.

Cassidy, P.R., "Higher Design Stresses for Pressure Vessels," Wel. J., Easton. Vol. 30, No. 3, March 1951, pp. 123s-124s.

Kooistra, L. F., and Blaser, R. V., "Experimental Technique in Pressure-Vessel Testing," Trans. Amer. Soc. Mech. Engrs, Vol 72, July 1950, pp. 579-589.

Maier, A.F., "Wechselbeanspruchung von Rohren unter Innerdruck" [Alternating stressing of tubes under internal pressure.] Stahl u. Eisen, vol. 54, no. 50, Dec. 13, 1934, pp. 1289-1291. English asbtr.: J. Iron St. Inst., vol. 131, no. 1, 1935, p. 484. English translation: Associated Technical Services Inc., Trans. no. 596-GJ.

Japan Pressure Vessel Research Council, Subcommittee on Significance of Weld Discontinuities, "Influence of Weld Discontinuities on Fatigue Behavior," Fabrication Division of the Japan Pressure Vessel Research Council.

7. SECONDARY STRESSES IN GIRDER WEBS

Goodpasture, D. W., and J. E. Stallmeyer, "Fatigue Behavior of Welded Thin Web Girders as Influenced by Web Distortion and Boundary Rigidity," Univ. of Ill., Civil Engineering Studies, Structural Research Series No. 328, August 1967.

Yen, B. T., and J. A. Mueller, "Fatigue Tests on Large-Size Welded Plate Girders," Welding Research Council Bulletin No. 118, November 1966.

Corrado, T. A., J. A. Mueller, and B. T. Yen, "Fatigue Tests of Welded Plate Girders," Lehigh Univ. Inst. of Research, Fritz Engr. Lab., Report No. 303.9, May 1965.

8. SHIP STRUCTURES

Stambaugh, K.A., D.L. Edinberg, G.E. Fenton, and P.R. Van Mater, Jr., "Structural Analysis of the Island Class Patrol Boats," for USCG Avery Point, CT, December 1988.

Fricke W., and H. Daetzold, "Application of the Cyclic Strain Approach to the Fatigue Failure of Ship Structural Details," Journal of Ship Research, September 1987.

Hong, D.P., O.H. Kin, and Y.S. Less, "Analysis of Structural Damage of a Large Ore/Coal Carrier," SNAME Spring Meeting/Star Symposium, 1987.
Chen, Y., J. Chin, and A. Thayamballi, "Validation of Fatigue Life Prediction Using Containership Hatch Corner Stain Measurements," SNAME Trans, 1986.

Munse, W.H., Discussion of "Validation of Fatigue Life Prediction Using Containership Hatch-Corner Strain Measurements," (1986).

Chen, Y.-K., J.-W. Chiou, and A. K. Thayamballi, "Validation of Fatigue Life Prediction Using Containership Hatch-Corner Strain Measurements," The Society of Naval Architects and Marine Engineers, No. 10, Advance copy to be presented at the Annual Meeting, New York, NY, November 19-22, 1986.

Tomita, Yasumitsu; Yukio Fujimoto, "An Analysis for Fatigue Failure of Ship Structural Members," Naval Architecture and Ocean Engineering, Vol. 22, 1984, pp. 215-226.

Thayamballi, A., Y-K. Chen, and D. Lin, "Fracture Mechanics Based Assessment of Fatigue Reliability in Ship Structures," Ship Structures Symposium, 1984.

Munse, W.H., T.W. Wilbur, M.L. Tellalian, K. Nicoll, and K. Wilson, "Fatigue Characterization of Fabricated Ship Details for Design," Report No. SSC-318, Department of Civil Engineering, University of Illinois at Urbana-Champaign, Sponsored by the Ship Structure Committee, 1983.

Cyclic Fatigue of Steel Ship Welded Joints, Bureau Veritas, 31, Rue Henri-Rochefort 75021, Paris Cedex 17. Guidance Note NI 188-October 1983.

Lereim, J., Fracture Mechanics in Design Influence of Cracks on the Convential Static Strength Criteria, Det Norske Veritas, 1982.

Petershagen H., Fatigue Testing in Ship Building, Material Testing Members as Well as Working Load Simulation, Material Engineering Press Karlsruhe, 1981, pp. 211-220. (Text in German)

Ushirokawa, O., Fatigue Strength of Ship Structures: Practical Application for Design, Royal Institution of Naval Architects, 1981.

Liu, D., and A. Bakker, "Practical Procedures for Technical and Economic Investigations of Ship Structural Details," Marine Technology, January 1981.

Jordan, C.R., and L.T. Knight, "Further Survey of In-Service Performance of Structural Details," Ship Structure Committee, SSC.294, 1980.

Nibbering, J.J.W., "Recent Work in Belgium and the Netherlands on Fatigue and Brittle Fracture of Interest for Marine Structures," Laboratorium Voor Scheepsconstructies - Technische Hogeschool - Delft 1979. Report No. 215.

Nibbering, J.J.W., "Design Against Fatigue and Fracture for Marine Structures," Laboratorium Voor Scheepsconstructies - Technische Hogeschool - Report No. 220, Delft 1979.

Iida, K., and M. Matoba, "Japanese Studies on the Fatigue Strength of Slotted Parts in Ship's Hull Members," Welding in the World, Vol. 16, No. 5/6, 1978.

Chen, Y.N., and J.W. Chiou, "Analysis of Fatigue Life of Hatch Corner Details of Sea-Land McLean Based on Measured Stress Data," Ocean Engineering Division, American Bureau of Shipping, July 1978, (Proprietary Report Subject to Restricted Distribution).

Nagano, T., T. Nitta, and H. Yashima, "Study on the Fatigue Strength of Local Parts of Ship Structures - First report Strength of Side Frame and Stiffener Snip Ends.," Shipbuilding and Marine Engineering, Vol 11, No. 3, p. 21, Japan, 1977.

El Gammal, M. M., "A New Method for Estimating the Fatigue Life of Ship Structures," Int. Ship. Progress, Vol. 22, 1975.

Iida, K., "Concepts for Fatigue Design of Welded Marine Structures," NAUT Report No. 9009, July 1974.

Meek, M., R. Adams, J.C. Chapman, I.H. Reibel, and I.P. Wieske, "The Structural Design of the O.C.L. Container Ships," Transactions of the Royal Institution of Naval Architects, P. 241, 1972.

Kjellander, S. L., "Hull Damages on Large Swedish-Built Ships," Kockums Shipyard, Stockholm, Report No. 16/33/1, December 1972.

Mori, Matoba, Kawasaki, Nakajima, Hiroe, and Umesaki. Application of Program Fatigue Test to Member Joints of Hulls," Mitsubishi Technical Bulletin No. 78, July 1972.

Anonymous, "Application of Program Fatigue Test to Member Joints of Hulls," Mitsubishi Heavy Industries LTD, Mitsubishi Technical Bulletin No. 78, pp. 1-23, July 1972.

Akita, Y., K. Uchino, and J. Hamanaka, "Influence of the Radius of Curvature of the Curved Knee Inner Flange on the Fatigue Strength (Study fatigue strength in cor. connect. in box struc.)." Prog. Rpt. 1, Res. Inst. Ishikawajima-Harima Heavy Ind. Co., Ltd., Tokyo, Japan, IIW Doc. XIII-571-70, July 1970.

9. TUBULAR OR OFFSHORE STRUCTURES

Bea, R.G., "Reliability of Marine Structures Fundamentals" prepared for SSC - seminar on reliability February 1988.

Rhee, H.L., "Fracture Mechanics Fatigue Life Analysis for a Multiplane K-Joint," Seventh OMAE Conference, February 1988.

Park Soko, F.V. Lawrence, "Fatigue Characterization of Fabricated Ship Details for Design, Phase II," Department of Civil Engineering, University of Illinois, for the Ship Structure Committee, 1988.

Gowda, S.S., and A. Helenius, "Fatigue Crack Growth Analysis of Welded Tubular Joints of Offshore Structures," Proceedings of the Seventh International Conference on Offshore Mechanics and Arctic Engineering, Houston, 1988.

Hsu, T. M., "A Simplified Method for Calculating the Remaining Fatigue Life of Cracked Structures," Proceedings of the Seventh International Conference on Offshore Mechanics and Arctic Engineering, Houston, 1988.

Huacan, F., and C. Znenxin, "Experimental Research of Fatigue Crack Propagation of Offshore Platform Joints Under Random Loading," Proceedings of the Seventh International Conference on Offshore Mechanics and Arctic Engineering, Houston, 1988.

Chang, R. W., "Low Cycle Fatigue Properties of Welded Joints for Offshore Structural Steels," Proceedings of the Seventh International Conference on Offshore Mechanics and Arctic Engineering, Houston, 1988.

Yee, R., D.J. Burns, U.H. Mehaupt, R. Bell, and O. Vosikovsky, "Thickness Effect and Fatigue Crack Development on Welded Joints," Proceedings of the Seventh International Conference on Offshore Mechanics and Arctic Engineering, Houston, 1988.

Sablok, A., and W.H. Hartt, "Fatigue of High Strength Steels in Seawater," Proceedings of the Seventh International Conference on Offshore Mechanics and Arctic Engineering, Houston, 1988.

Rodabaugh, E. C., "Accuracy of Stress Intensification Factors for Branch Connections," Welding Research Council Bulletin 329, 1987.

Rhee, H.C., "Application of Finite Element Alternating Method to Offshore Structural Fatigue Analysis," OTC 5111, 1986.

Huang, X., "A Finite Element Evaluation of the Stress Intensity Factors of Surface Cracks in a Tubular Joint," OTC 5665, 1986.

Raju, P. P., "A Parametric Three-Dimensional Finite Element Study of 45° Lateral Connections," Welding Research Council Bulletin 301, 1985.

Burnside, O.H., S.J. Hudak, Jr., E. Oelkers, K. Chan, and R.J. Dexter, "Long-Term Corrosion Fatigue of Welded Marine Steels," Ship Structure Committee, USCG HQ, (G-M/TP 13), Wash. D.C., 1984. SR-1276. SSC 326.

Mang, F., O. Bucak, and G. Steidl, "The Influence of Residual Stresses on the Fatigue Strength of Hollow Section Joints," IIW Doc. XIII-1122-84, Versuchsanstalt fur stahl, Holz und Steine, Universitat (TH) Karlsruhe.

Hudak, S.J. Jr., O.H. Burnside, and K.S. Chan, "Analysis of Corrosion Fatigue Crack Growth in Welded Tubular Joints," Sixteenth Annual Offshore Technology Conference, May 7-9, 1984, Houston, Texas.

Baba, Shunsuke, Kohki Nonomiya, and Tateo Kajita, "Low-Cycle Fatigue of Stiffened Tubular Joints," Journal of Structural Engineering, Vol. 110, No. 2, Feb. 1984, pp. 301-315. ASCE.

British Department of Energy (UK DOE), "Fatigue Design Guidance for Steel Welded Joints in Offshore Structures," London HMSO, 1984.

Gulati, K.C., W. Wang, and D. Kan, "An Analytical Study of Stress Concentration Effects in Multibrace Joints Under Combined Loading," OTC 4407, 1982.

Wardenier, J., "The Fatigue Behavior of Hollow Section Joints," Ch. 8 in Hollow Section Joints by J. Wardenier, Delft University Press, 1982, pp. 367-419.

Larsen, J.M., Some Aspects of Fatigue of Tubular Joints, Det Norske Veritas, 1982.

Foss, G., and M.G. Michelsen, Determination of SCF in Crown Position for the Connection Between Chord and Brace by Means of Photoelastic Model Tests, Det Norske Veritas, 1982.

Dover, W.D., and S. Dharmavasan, "Fatigue Fracture Mechanics Analysis of T and Y Joints," Offshore Technology Conference, 1982.

Almar-Naess, A., P.J. Haagensen, B. Lian, T. Moan, and Simonsen, "Investigation of the Alexander L. Kielland Failure--Metallurgical and Fracture Analysis," Offshore Technology Conference, 1982.

Louvenssen A., and D. Dijkstra, "Fatigue Tests on Large Post Weld Heat Treated and Welded Tubular T-Joints," Offshore Technology Conference, May 1982.

Bernites, M.M., "Problems in Marine Riser Design," Marine Technology, Vol. 19, No. 1, January 1982.

Wordsworth, A.C., "Stress Concentration Factors at K and KT Tubular Joints," Paper 7, Fatigue on Offshore Structural Steel, Institute of Civil Engineers, 1981.

Baba, S., K. Ninomiya, K. Kajita, and M. Nakamura, "Low-Cycle Fatigue of Stiffened Tubular Joints," Manuscript for Publication in ASCE Proc. Str. Div., 1981.

Moguerou, A., R. Vassal, G. Vassiere, and J. Bahuaud, Low-Cycle Fatigue Strength of Tubes Under Biaxial Tension/Compression and Internal/External Pressure Loading, Material Testing and Testing of Structural Members as Well as Working Load Simulation, Material Engineering Press, Karlsruhe, 1981, pp. 55-64.

De Back, J., (Rapporteur's Notes) "Session X-Testing Tubular Joints," Institute TNO for Building Materials and Building Structures, Stevin Lab., U. of Technology, Delft, Netherlands, 1981, ECSC-Offshore Steels Programme.

De Back, J., "Plenary Session 7, Strength of Tubular Joints," Delft U. of Technology, Stevin Lab, Netherlands - ECSC-Offshore Steels Programme, 1981.

Dijkstra, O.D., and J. De Back, "Fatigue Strength of Tubular X- and T-Joints. (Dutch Tests)," Paper T. S. 8.4, Institute TNO for Building Materials and Building Structures, Stevin Lab., U. of Technology, Delft, The Netherlands, 1981.

Wildschut, H., J. De Back, W. Dortland, and J.L. Van Leeuwen, "Fatigue Behavior of Welded Joints in Air and Sea Water," Paper No.5, Institute TNO for Building Materials and Building Structures, Stevin Lab, U. of Technology, Delft, Netherlands, 1981.

Dijkstra, O.D., W. Visser, and G.T.M. Janssen, "Comparison of Strain Distributions in Three X-Joints Determined by Strain Gauge Measurements and Finite Element Calculations," Paper T. S. 6.2, Institute TNO for Building Materials and Building Structures, Stevin Lab., U. of Technology, Delft, Netherlands, 1981.

Ogle, R.B., and G.L. Kulak, "Fatigue Strength of Trusses Made From Rectangular Hollow Sections," Structural Engineering Report No. 102, Dept. of Civil Engineering, Univ. of Alberta, Edmonton, Nov. 1981.

Gibstein, M.B., and E.T. Moe, "Numerical and Experimental Stress Analysis of Tubular Joints with Inclined Braces," International Conference Steel in Marine Structures, Paris, October 1981.

Gibstein, M.B., "Fatigue Strength of Welded Tubular Joints Tested at Det Norske Veritas Laboratories," International Conference Steel in Marine Structures, Paris, October 1981.

Van Leeuwen, J.L., J. De Back, and G.H.G. Vaessen, "Constant Amplitude Fatigue Tests on Welded Steel Joints Performed in Air and Seawater," Technical Session 2, Internal. Confr., Steel in Marine Structures, Commission of the European Communities, Paris, France, Oct. 5-8, 1981. (Institute de Recherches de la Siderurgie Francaise).

Scholte, H.G., and H. Widschut, "Technical Session 5, Fatigue Crack Propagation Tests on Welded Specimens in Air and Sea Water," Internatl. Confr., Steel in Marine Structures, Institut de Recherches de la Siderurgie Francaise, Commission of the European Communities, Oct. 5-8, 1981.

Zwaans, M.H.J.M., P.A.M. Jonkers, and J.L. Overbeeke, "The Endurance of a Welded Joint Under Two Types of Random Loading in Air and Seawater," Technical Session 7, Paper 7.3, Internatl. Confr., Steel in Marine Structures, Institut de Recherches de la Siderurgie Francaise, Commission of the European Communities, Paris, Oct. 5-8, 1981.

Noordhoek, C., "ECSC-CIDECT Fatigue Tests on Square Hollow Section Joints and the Comparison with the ECSC-Offshore Tubular Joint Results," Technical Session 10, Paper 10.5, Internatl. Confr. Steel in Marine Structures, Communities, Paris, Oct. 5-8, 1981.

Burnside, O.H., P.K. Nair, S.J. Hudak, D.L. Eck, and J.R. Maison, "Long-Term Corrosion Fatigue of Welded Marine Steels - First Year Assessment," Southwest Research Institute - Interim Report, Contract No. DTCG-80-C-20028, SWRI Project No. 02-6292, Aug. 1981.

Asano, K., K. Iida, K. Ishikawa, and M. Toyofuku, "A Proposal for Fatigue Design Procedure for Offshore Tubular Connections," 1981 Annual Meeting of International Institute of Welding, NAUT Report No. 9031, July, 1981.

Wardenier, J., and J. De Back, "Considerations in Static and Fatigue Design of Tubular Joints," Presented at the May 11-15, 1981 ASCE International Convention and Exposition, New York.

Dijkstra, O.D., and J. DeBack, "Fatigue Strength of Tubular T- and X-Joints," OTC 3696, Offshore Technology Conference, 12th Annual OTC, Houston, TX, May 5-8, 1981.

De Back, J., and G.H.J. Vaessen, "Fatigue and Corrosion Fatigue Behavior of Offshore Steel Structures," Foundation for Materials Research in the Sea, ECSC Convention 7210-KB/6/602 (J.7.1 f/76) Final Report, Delft U. of Technology, April 1981.

Wardenier, J., and D. Dutta, "The Fatigue Behavior of Lattice Girder Joints in Square Hollow Sections," Reprint of paper presented at the Conference on "Joints in Structural Steelwork," Teeside, April 6-9, 1981, IIW Annual Assembly 1981, Oporto, DOC XV-493-81, XIII-1005-81.

Baba, S., Y. Arizumi, and M. Naruoka, "Low-Cycle Fatigue Test of Welded Tubular Joints," Journal of the Structural Division, March 1981, Vol. 107, No. ST 3, pp. 487-506.

Rodabaugh, E. C., "Review of Data Relevant to the Design of Tubular Joints for Use in Fixed Offshore Platforms," Welding Research Council Bulletin 256, 1980.

Rajkotia, D.P., and W.C. Schnobrich, "A Stress Analysis of Circular Cylindrical Shell Intersections, Including the Influence of Reinforcement, Cycling Plasticity and Fatigue," University of Illinois, SRS No. 483, Dec. 1980.

Noordhoek, C., J. Wardenier, and D. Dutta, "The Fatigue Behavior of Welded Joints in Square Hollow Sections, Part 1," (test results and S-N curves), Institute TNO for Building Materials and Building Structures, Stevin-Lab, Delft University of Technology, ESCS Convention 6210 KD-1-103, Stevin Report 6-79-11, May 1980.

Yamasaki, Tokuya, Noboru Yamamoto, and Junichi Kudoh, "Studies of Stress Concentrations and Tolerances for Weld Defects in Full-Scale Tubular Joints," 12th Offshore Technology Conference, Houston, TX, May 5-8, 1980.

Noordhoek, C., J. Wardenier, and D. Dutta, "The Fatigue Behavior of Welded Joints in Square Hollow Sections, Part 2, Analysis," Institute TNO for Building Materials and Building Structures, Stevin-Lab, Delft Univ. of Technology, ECSC Convention 6210 KD-1-103, Stevin Report 6-10-4, March 1980.

Dutta, D., J. Wardenier, and C. Noordheok, "Creep and Fatigue of Simple Welded Truss Work Joints From Round Sections (Zeit-und Dauerfestigkeit von Einfachen Geschweissten Fachwerk-Knoten aus Rundhohlprofilen)," Stevin-Lab, Delft U. of Technology, Institute TNO for Building Materials & Building Structures, EGKS-Vertrag No. 6210-KD-1-103, March 1980.

Rodabaugh, "Review of Data Relevant to the Design of Tubular Joints for Use in Fixed Offshore Platforms," Welding Research Council Bulletin 256, January 1980.

Yamasaki, T., S. Takazawa, and M. Komatsu, "Static Fatigue Tests on Large-Size Tubular T-Joints," Offshore Technology Conference, 11th Annual OTC in Houston, TX, April 30-May 3, 1979.

Vaessen, G.H.G., and J. De Back, "Fatigue Behavior of Welded Steel Joints in Air and Seawater," OTC 3421, 11th Annual Offshore Technology Confr., Houston, TX, April 30-May 3, 1979.

Riggs, R. P., "Fatigue Considerations for Semisubmersible Structures," Marine Technology, January 1979.

Gibstein, M. D., "Parametric Stress Analysis of T Joints," Paper 26, European Offshore Steels Research Seminar, The Welding Institute, November 1978.

"Section 10. Design of New Tubular Structures," in Structural Welding Code AWS D1.1-79, American Welding Society, 1978, p. 145.

Iida, K., H. Sakurai, and S. Hayaski, "Fatigue Strength of Welded Tubular K-Joints of 88N/MM Class High Strength Steel," I.I.W. Document XIII-872-78, May 1978.

Jaske, C.E., D. Brock, J.E. Slater, and W.E. Anderson, "Corrosion-Fatigue of Structural Steels in Seawater and for Offshore Applications," ASTM STP 642, p.19, February 1978.

K nang, A. B., et al, "Stress Concentration in Tubular Joints," Soc. Pet. Eng. Journal, August 1977.

Sharp, M.L., and G.E. Nordmark, "Fatigue Strength of Welded Tubular Aluminum Truss," ASCE, Journal of the Structural Division, p. 1619, August 1977.

Potvin, A.B., J.G. Kuang, R.D. Leick, and J.L. Kahlich, "Stress Concentration in Tubular Joints," Society of Petroleum Engineers Journal, August 1977.

Dijkstra, O.D., and G.H.G. Vaessen, "The Fatigue Behavior of Joints in Jacket Constructions," OTC 2852, 9th Annual OTC, Houston, TX, May 2-5, 1977.

Kuang, J.G., A.B. Potvin, and R.D. Leick, "Stress Concentration in Tubular Joints," Seventh Annual Offshore Technology Conference, OTC Paper No. 2205, 1975. A private communication from N.R. Maddox to E.C. Rodabaugh, March 15, 1977, states that seven of the formulas presented in Table 2 have been modified. The modified equations are contained herein.

American Petroleum Institute, "Recommended Practice for Planning, Designing, and Constructing Fixed Offshore Platforms," API RP2A, 7th edition, January 1976.

Marshall, P.W., "Limit State Design of Tubular Connections," Behavior of Offshore Structures, The Norwegian Institute of Technology, 1976.

Anonymous (I.I.W.), "Welded Joints in Tubular Structures," I.I.W. Annual Assembly in Sydney - Document XV-382-76, 1976.

Nolte, K.G., and J.E. Hansford, "Closed-Form Expressions for Determining the Fatigue Damage of Structure Due to Ocean Waves," Offshore Technology Conference, Paper No. OTC 2606, 1976.

Williams, A.K., and J.E. Rinne, "Fatigue Analysis of Steel Offshore Structures," Proceedings, November 1976.

American Welding Society, Design of New Tubular Structures, Section 10 of D1.1, 1975.

Marshall, P. W., "Basic Considerations for Tubular Joint Design in Offshore Construction," Welding Research Council Bulletin 193, April 1974.

Kurobane, Y, and M. Konomi, "Fatigue Strength of Tubular K-Joints - S-N Relationships Proposed as Tentative Design Criteria," Welding Research Abroad, Vol XX, No. 4, April 1974.

Visser, W., "On the Structural Design of Tubular Joints," Sixth Annual Offshore Technology Conference, OTC Paper No. 2117, 1974.

Matoba, Teramoto, Kawasaki, and Kaminokado, "Fatigue Strength of Welded Tubular Joints with Various Stiffeners in Offshore Structures," I.I.W. Document No. XIII-687-73, 1973.

Becker, J.M., W.W. Gerberich, and J.G. Bouwkamp, "Fatigue Failure of Welded Tubular Joints," American Society of Civil Engineers, Journal of the Structural Division, Vol. 98, ST1, pp. 37-59, January 1972.

Reber, J.B., "Ultimate Strength Design of Tubular Joints," Fourth Annual Offshore Technology Conference, OTC Paper No. 1664, 1972.

Akita, Y., A. Matsui, K. Uchino, and T. Kamiyama, "Experimental Study on the Fatigue Strength of Welded Tube to Gusset-Joints, Progress Report No. 1," Ishikawajima-Harima Heavy Ind. Co., Ltd. Research Inst., Tokyo, Japan, IIW Doc. No XIII-573-70.

Mall, G., and R. Zirn, "Load Carrying Behavior of Tube Intersections Under Tensile and Dynamic Loading Conditions," Welding Research Abroad, Vol. 16, No. 9, pp. 6-18, November 1970. Also see Schweissen and Schneiden, December 1969.

Kurobane, Y., "Effects of Low-Cycle Alternating Loads on Tubular K-Joints," Metal Construction and British Welding Journal, Vol. 1, No. 6, June 1969.

Wellinger, Karl, Helmut Gassman, and Harald Zenner, "The Fatigue Behavior of Welded Pipes Under Combined Pulsating Stresses in Relation to the Angle of Inclination of the Weld Seam," Report Issued by Staatliche Materilaprufungsanstalt an der Univ. Stuttgart, German, IIW Doc. XIII-519-68.

Natarajan, M., and A.A. Toprac, "The Fatigue Strength of Tubular T-Joints," Univ. of Texas, Structures Fatigue Research Laboratory, November 1968.

Kurobane, Y., M. Natarajan, and A.A. Toprac, "Fatigue Tests of Tubular T-Joints," Univ. of Texas, Structures Fatigue Research Laboratory, November 1967.

Beale, L.A., and A.A. Toprac, "Analysis of In-Plane T-, Y-, and K-Welded Tubular Connections," WRC Bulletin 125, October 1967.

Caulkins, D.W., "Parameter Study for Frameti Elastic Stress in Tubular Joints," Shell Oil Company, Construction-Design Group, New Orleans, LA, CDG Report 15, September 1966.

Kampschaefer, G.E. Jr., F.E. Havens, and J.P. Bruner, "Engineering Data for the Design and Fabrication of Offshore Drilling Platforms with Heat-Treated Steels," American Society of Mechanical Engineers, Vol. 87, 1965.

Sabothy, B., "Tubular Connection - Welded Tubular Connection Subjected to Dynamic Loads (Design, Carrying Capacity)," International Institute of Welding, Yugoslav Delegation, Doc. XIII-386-65, 1965.

Schlegel, E., "Zur Konstruktion von geschweiГten Rohrverzweigungen" [The design of welded pipe branch connections.] Stahlbau, Vol. 28, No. 1, Jan. 1959, pp. 13-21.

M.W. Kellogg Co., Design of Piping Systems, 2nd edition, Wiley, 1956.

Bijlaard, P.P., "Stresses from Local Loadings in Cylindrical Pressure Vessels," Trans. ASME, August 1955.

Miner, M.A., "Cumulative Damage in Fatigue," Journal of Applied Mechanics, Vol. 12, 1945.

10. DESIGN METHODS AND CRITERIA

Recommended Practice for Planning, Designing, and Constructing Fixed Offshore Platforms, American Petroleum Institute, Washington, D.C., 1987.

Madsen, H.O., R.K. Skjong, A.G. Tallin, and F. Kirkemo, "Probabilistic Fatigue Crack Growth Analysis of Offshore Structures, with Reliability Updating Through Inspection," SNAME Marine Structural Reliability Symposium, 1987.

Wirsching, P. H., "Probability-Based Fatigue Design Criteria for Offshore Structures," Project Report API-PRAC Project #81-15, Engineering Experiment Station, Univ. of Arizona, Tucson, AZ 85721, January 1983.

Asano, K., K. Iida, K. Ishikawa, and M. Toyofuku, "A Proposal for Fatigue Design Procedure for Offshore Tubular Connections," 1981 Annual Meeting of International Institute of Welding, NAUT Report No. 9031, July, 1981.

Ellingwood, B.R., T.V. Galambos, J.L. MacGregor, and A.C. Corwell, "Development of a Probability Based Load Criterion for American National Standard A58," National Bureau of Standards, Special Publication NO. 577, June 1980.

Light, M., and P.E. Wirsching, "Probability-Based Fatigue Design Criteria for Offshore Structures," Engineering Experiment Station, University of Arizona, API-Prac Project #15, November 1979.

Nibbering, J.J.W., and D. Faulkner, "Fracture-Safe Design of Maritime Structures," Paper presented at International Symposium "Integrity of Offshore Structures," The Institution of Engineers and Shipbuilders in Scotland, April 6-7, 1978.

Shiraishi, T., "ISSC's Tentative Fatigue Design Guide and Its Comments, English Translation," I.I.W. Document XIII-800-76, 1976.

Socie, D.F., and JoDean M. Morrow, "Review of Contemporary Approaches to Fatigue Damage Analysis," FCP Report No. 24, College of Engineering, University of Illinois, Urbana, IL, December 1976.
Gurney, T. R., "Fatigue Design Rules for Welded Steel Joints," British Welding Institute Research Bulletin, p. 115, May 1976.

Mann, J. Y., "Fatigue in Structural Design," Australian Institute of Metals, Proceedings of Symposium, March 1976.

Gurney, T. R., and J. G. Hicks, "On Fatigue Design Rules for Welded Structure," Offshore Technology Conference, Paper No. OTC 1907, 1973.

Gurney, T. R., "A Comparison of Fatigue Design Rules," Fatigue of Welded Structures Conference, The Welding Institute, Paper #1, pp. 1-17, July 1970.

Ogle, M. H., "Implications of Current Fatigue Design Rules," Fatigue of Welded Structures Conference, The Welding Institute, Paper #5, pp. 53-72, July 1970.

Smith, D. W., and W. I. Halse, "Fatigue Design Specification - What the Designer Needs," Fatigue of Welded Structures Conference, The Welding Institute, Paper #27, pp. 391-404, July 1970.

Garrison D. R., and M. L. Salive, "Correlation of the Bement-Pohler Fatigue Theory with Experimental Fatigue Data and Its Extension to Notched Specimens," Naval Ship Research and Development Center Report 3343, June 1970.

Kawano, K., and H. Nisitani, in "The Thirteenth Japan Congress on Materials Research-Metallic Materials," March 1970, pp. 73-76.

Nisitani, H., and K. Kawano, in "The Eleventh Japan Congress on Materials Research-Metallic Material," March 1968, pp. 49-51.

11. FAILURES

Stambaugh, K., and W. Wood, "Ship Fracture Mechanisms Investigation," SSC, 1987.

Fricke, W., and H. Paetzold, "Application of the Cyclic Strain Approach to the Fatigue Failure of Ship Structural Details," Journal of Ship Research, September 1987.

Naess, Almar, et al, "Investigation of the Alexander L. Kielland Failure - Metallurgical and Fracture Analysis," OTC 4236, 1982.

Vedeler, G., "To What Extent do Brittle Fracture and Fatigue Interest Shipbuilders Today?" Houdremont Lecture, IIW Annual Assembly, June 1962, Sveiseteknikk, No. 3.

International Institute of Welding, Commission XIII, "Observations on Fractures of Welded Water Tanks of Locomotive Tenders," IIW Doc. IIS/IIw-40-60, December 1960.

International Institute of Welding, Commission XIII, "Failures of the Bottom Plating Observed on Low-Tonnage Motor Vessels," IIW Doc. IIS/IIW-39-60, December 1960.

DeLeiris, H., "Fatigue Cracks Found in the Stern Structure of a Cargo Ship," Welding Research Abroad, October 1955, pp. 27-31.

Nibbering, J. J. W., "Fracture Mechanics and Ships," Ghent University, Belgium/Deift University, The Netherlands.

12. RELIABILITY (GENERAL)

Mansour, A. E., "Tutorial Summary on Structural Reliability Theory Directed at the Marine Industry--Draft Final Report," Mansour Engineering, Berkeley, CA, 1988.

Paliou, C., M. Shinozuka, and Y.N. Chen, "Reliability and Durability Analysis of Marine Structures," SNAME Marine Structural Reliability Symposium, 1987.

White, G.J., and B.M. Ayyub, "A Reliability-Conditioned Approach for the Fatigue Design of Marine Structures," SNAME Marine Structural Reliability Symposium, 1987.

Madsen, H.O., R.K. Skjong, A.G. Tallin, and F. Kirkemo, "Probabilistic Fatigue Crack Growth Analysis of Offshore Structures, with Reliability Updating Through Inspection," SNAME Marine Structural Reliability Symposium, 1987.

Kaneqaonkar, Hari B., and Achintya Haldar, "Non-Gaussian Response of Offshore Platforms: Fatigue," ASCE - Journal of Str. Division, Vol 113, No. 9, Sept. 1987, pp. 1899-1908.

Paliou, C., M. Shinozuka, and Y.N. Chen, "Reliability and Durability of Marine Structures," ASCE - Journal of Str. Division, Vol. 113, No. 6, June 1987, pp. 1297-1314.

Albrecht, P., "Review of Fatigue Design Methods for Highway Bridges," Civil Engineering Report, University of Maryland, College Park, MD, May 1986.

Vrouwenvelder, A.C.W.M., and E.M. Gostelie, "Reliability Analysis for the Fatigue Limit State of the ASTRID Offshore Platform," HERON, Vol. 31, No. 1, 1986.

Wirsching, Paul H., "Fatigue Reliability for Offshore Structures," Journal of Structural Engineering, Vol. 110, No. 10, October 1985 (Paper No. 19235). ASCE.

Knapp, Alphia E., and Bernhard Stahl, "Offshore Platform Fatigue Cracking Probability," Journal of Structural Engineering, ASCE, Vol. 111, No. 8, Aug. 1985, pp. 1657-1660.

White, G.J., and B.M. Ayyub, "Reliability Based Fatigue Design for Ship Structures," ASNE Journal, May 1985.

Nolan, Ch., and P. Albrecht, "Load and Resistance Factor Design for Fatigue of Steel Structures," ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability, Berkeley, CA, January 1984.

Ang, A.H., and R.M. Bennett, "Reliability of Structural Systems," SNAME Ship Structure Symposium, 1984.

Ferro, G., and D. Cervetto, "Hull Girder Reliability," SNAME Ship Structure Symposium, 1984.

Thayamballi, A., Y-K Chen, and D. Lin, "Fracture Mechanics Based Assessment of Fatigue Reliability in Ship Structures," Ship Structures Symposium, 1984.

Albrecht, P., and Ch. Nolan, "Load and Resistance Factor Design of Steel Structures for Fatigue," Civil Engineering Report, University of Maryland, College Park, MD, June 1983.

Albrecht, P., "S-N Fatigue Reliability Analysis of Highway Bridges," Probabilistic Fracture Mechanics and Fatigue Methods: Applications for Structural Design and Maintenance, ASTM STP 798, J.M. Bloom and J.C. Ekvall, Eds., American Society of Testing and Materials, 1983, pp. 184-204.

Albrecht, P., "S-N Fatgue Reliability analysis of Highway Bridges," Probabalistic Fracture Mechanics and Fatigue Methods. Applications for Structural Design and Maintenance, ASTM STP 799, J. M. Bloom and J. C. Ekvall, Eds., ASTM 1983.

Dowling, N. E., "Torsional Fatigue Life of Power Plant Equipment Rotating Shafts," DOE RA/29353-1, U.S. Department of Energy, Washington, DC, September 1981.

Harris, D.O., E.Y. Lim, and D.D. Dedhia, "Probability of Pipe Fracture in the Primary Coolant Loop of a PWR Plant," Science Applications, Inc., Prepared for the U.S. Nuclear Regulatory Commission, Washington, DC, August 1981.

Kinra, R. K., and P.W. Marshall, "Fatigue Analysis of the Cognac Platform," Journal of Petroleum Technology, 1979 Offshore Technology Conference, pp. 374-386, 1979.

Light, M., and P.E. Wirsching, "Probability-Based Fatigue Design Criteria for Offshore Structures," Engineering Experiment Station, University of Arizona, API-Prac Project #15, November 1979.

Lohne, Per W., "Fatigue Analysis of Welded Joints in Offshore Structures," Metal Construction and British Welding, Vol. II, No. 8, August, 1979.

Ang, A.H-S., and W.H. Munse, "Practical Reliability Basis for Structural Fatigue," Meeting Reprint 2492, ASCE National Structural Engineering Conference, April 14-18, 1975.

Ang, A.H-S, and C.A. Cornell, "Reliability Bases for Structural Safety and Design," Journal of the Structural Division, ASCE, Vol. 100, No. ST9, Proc. Paper 10777, September 1974, pp. 1755-1769.

Nokleby, "Fatigue Under Multiaxial Stress Conditions," MD 81001, Division of Machine Elements, The Norwegian Institute of Technology, Trondheim, Norway.

APPENDIX B

EXAMPLES OF MULTIAXIAL FATIGUE LIFE PREDICTIONS FOR A LATERALLY LOADED PLATE WITH A TRANSVERSE BUTT WELD

EXAMPLES OF MULTIAXIAL FATIGUE LIFE PREDICTIONS FOR A LATERALLY LOADED PLATE WITH A TRANSVERSE BUTT WELD

In this section several examples will be developed using both uniaxial and multiaxial approaches to illustrate and compare the methodologies. The example case is based on tests of HY-80 test specimens, each 28" x 56" x 1.50" thick welded on the centerline of the long dimension. The test specimens were simply supported (on the upper surface) and subjected to a uniform pressure loading (on the lower surface) from zero to maximum pressure which yielded a maximum calculated stress at the center of the panel of 80 ksi. The tests were conducted at the Naval Applied Science Laboratory, Brooklyn, New York in 1968. Results are reported in Reference [B-1]. A sketch of the panel is shown in Figure B-1. A summary of the results is given in Table B-1. The methodologies used for comparison of stress-based methods are:

(a) maximum principal stress
(b) maximum shearing stress
(c) maximum octahedral shearing stress

The principal stresses at the center of an unwelded plate of the dimensions shown in Figure B-1 have been calculated by the methods of Timonshenko and Woinowsky-Krieger [B-2]. The results for all four test articles are:

$$\sigma_1 = \sigma_x = 80{,}000 \text{ psi}$$
$$\sigma_2 = \sigma_y = 36{,}520 \text{ psi}$$
$$\tau_{max} = 40{,}000 \text{ psi}$$

The objective is to predict fatigue life for the butt welded plate using simple uniaxial test data. The S-N curve for unwelded HY-80 is plotted on the basis of data given for Type I specimens. These specimens are standard flat plate uniaxial fatigue test specimens. The data is plotted in Figure B-2.

Each methodology will now be outlined briefly and the results compared with the lives shown in Table B-1. The means of the observed lives shown there are:

Fatigue life to first observation of cracking,
N_{f1} = 13,950 cycles

Fatigue life to 10% increase in deflection,
N_{f2} = 36,138 cycles.

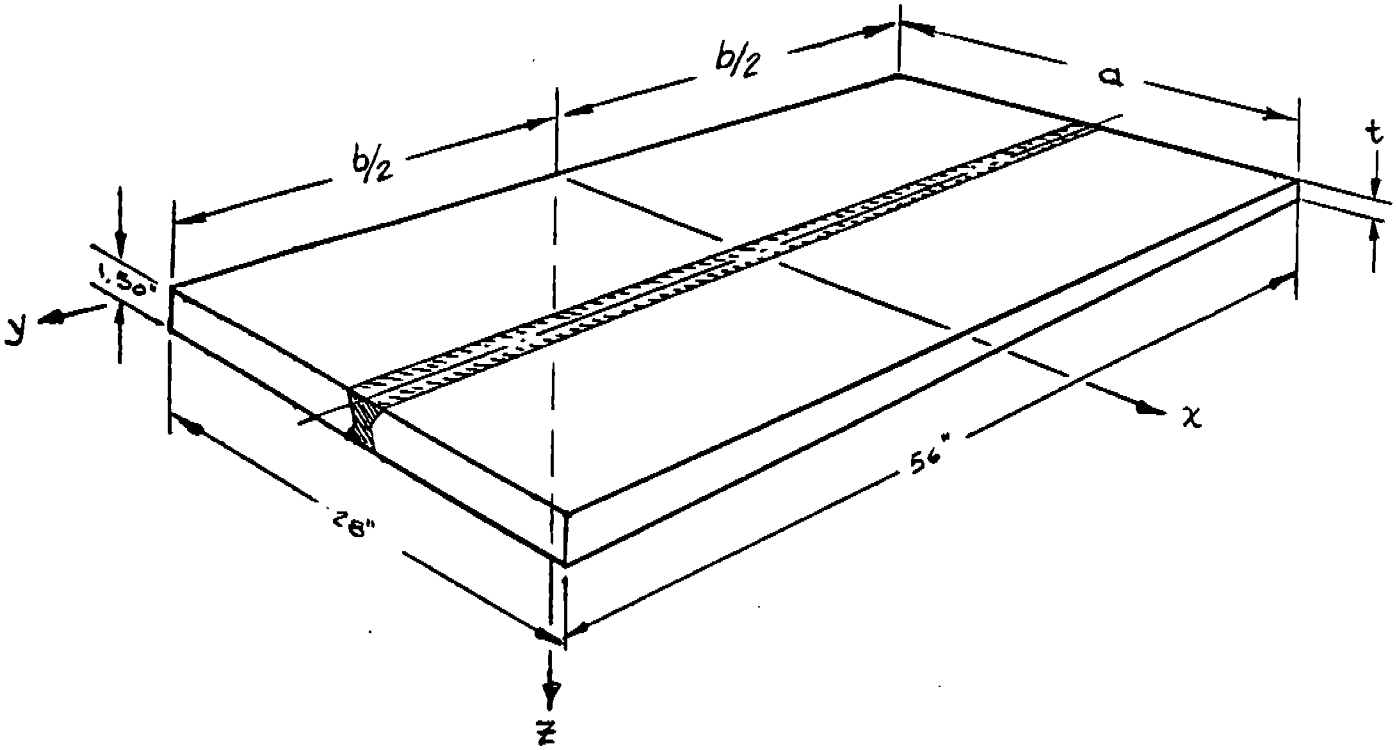


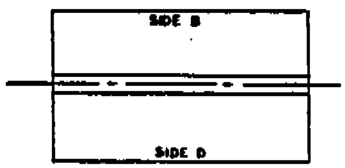
FIGURE B-1

TYPICAL 56 BY 28 BY 1 1/2 in. HY-80 STEEL BUTT WELDED PLATE ELEMENTS

TABLE B-1

SUMMARY OF FATIGUE RESULTS

ELEMENT NUMBER	NOMINAL CYCLIC STRESS (PSI)	AVERAGE THICKNESS AT CRITICAL SECTION (IN.)		APPLIED UNIFORM PRESSURE (PSI)	NUMBER OF STRESS CYCLES TO			LOCATION OF INITIAL CRACK
		SIDE B	SIDE D		FIRST OBSERVATION OF CRACK SIDE B	FIRST OBSERVATION OF CRACK SIDE D	10 % INCREASE IN DEFLECTION	
2-1B	80,000	1.496	1.493	374	16,100	25,200	49,700	SIDE B - ON CENTER SIDE D-2½" OFF CENTER
2-2B	80,000	1.508	1.502	379	19,100	14,800	45,250	SIDE B-1½" OFF CENTER SIDE D-5¼" OFF CENTER
2-3B	80,000	1.497	1.492	374	7,200	4,700	22,900	SIDE B - 2" OFF CENTER SIDE D - 3" OFF CENTER
2-4B	80,000	1.478	1.474	364	15,500	9,000	26,700	SIDE B - 1" OFF CENTER SIDE D - 1" OFF CENTER



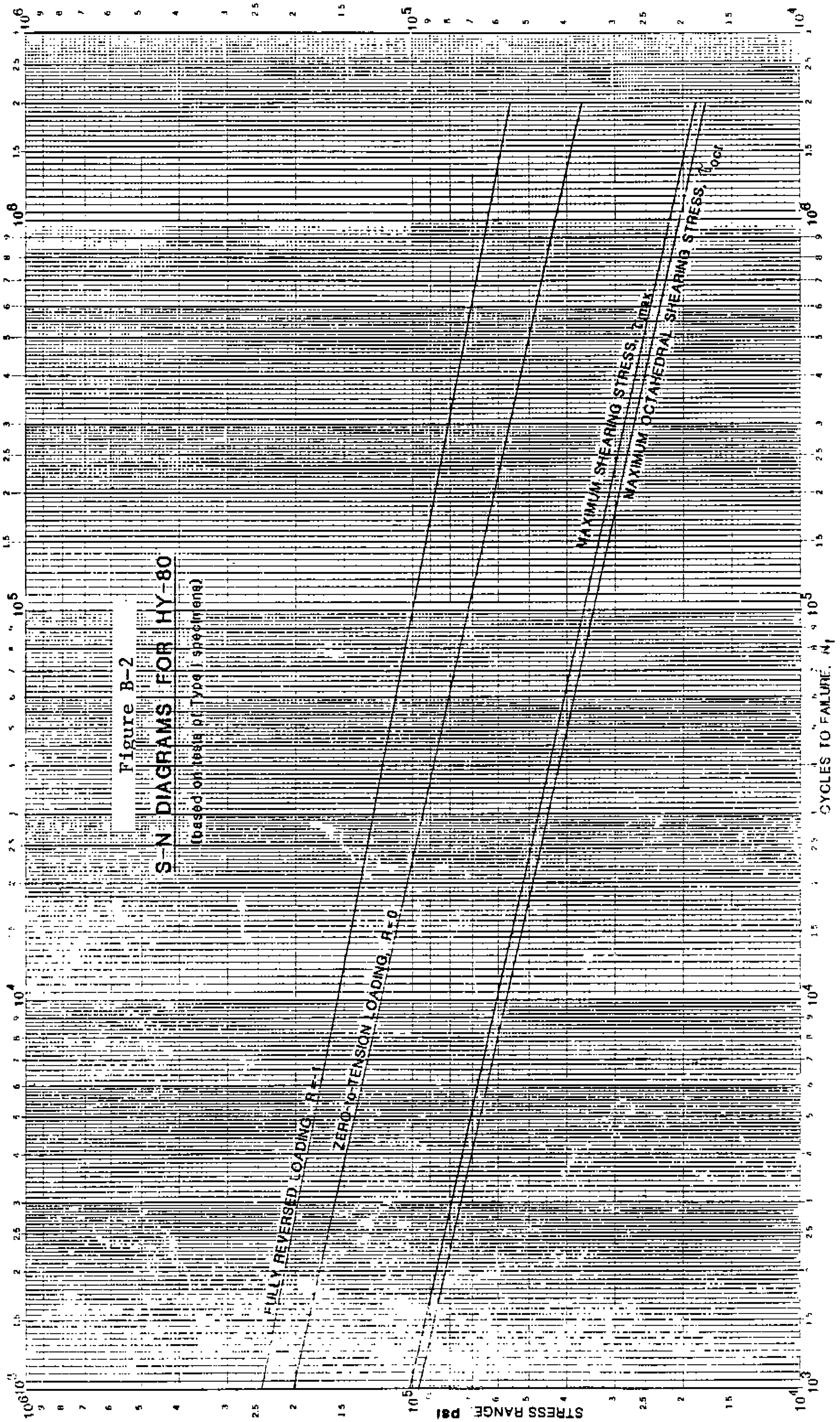


Figure B-2
S-N DIAGRAMS FOR HY-80
(based on tests of Type I specimens)

(a) <u>Maximum principal stress</u>. This method assumes that fatigue lives will correlate when the maximum principal stress, corrected for the presence of stress concentrations, in the structure is the same as that in the uniaxial tests at the point of failure.

The fatigue notch factor, K_f of Reference B-3, was used to account for stress concentration due to the longitudinal weld.

$$K_f = 1 + \frac{K_t - 1}{1 + a/r}$$

where

$$K_t = \beta \left[1 + \alpha \, (t/r)^2\right]$$

From Reference B-3,

$$x = 0.165 \, (\tan\phi)^{.167} \text{ for butt welds in bending}$$

$$\beta = 1.0$$

$$\alpha = 0.50$$

From Reference B-3,

$$\phi_m = 18.25^\circ \text{ and } (\tan\phi)_m^{.167} = .8274$$

Assume the notch root radius, r = .025 in. and that a, the crack length, is also .025 in. Then $K_t = 2.057$ and

$$K_f = 1.529$$

The stress at the midpoint of the panel and adjacent to the weld will be

$$\sigma_f = (1.529)(80{,}000) = 122{,}320 \text{ psi}$$

The pressure loading on the panel varies from zero to maximum, thus enter the S-N curve for R = 0 with σ_f = 122,320 psi and find that N_f = 9,400 cycles.

(b) <u>Maximum shearing stress</u>. The maximum shearing stress criteria assumes that failure in the structure will occur at the same life that would be observed in a uniaxial fatigue test at the same level of maximum shearing stress.

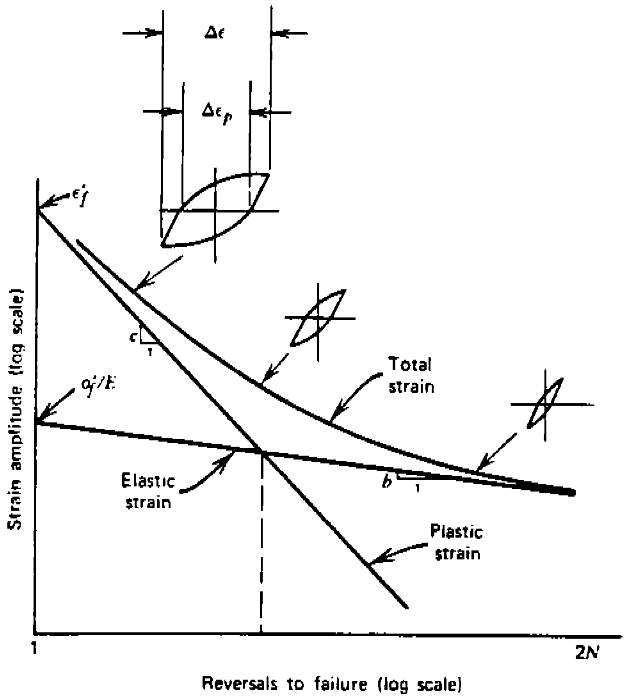


Figure B-3. Strain-Life Curves Showing Total Elastic and Plastic Strain Components (taken from Reference B-5)

For the plate and for the uniaxial test specimen,

$$\tau_{max} = \frac{\sigma_1}{2}$$

Applying this relationship to the R=0 S-N curve gives the curve labeled "τ_{max}" in Figure B-2. Using the same value of K_f found in (a) above yields

$$\tau_{max} = (1.529)(40,000) = 61,160 \text{ psi}$$

In this case the same life is obtained as in case (a)

$$N_f = 9,400 \text{ cycles}$$

(c) <u>Maximum octahedral shearing stress</u>. This criterion assumes that failure will occur at the same lives when the maximum octahedral stress in the structure is the same as that in the uniaxial fatigue test. Octahedral shearing stress is given by

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2\right]^{1/2}$$

For the uniaxial test specimen $\sigma_2 = \sigma_3 = 0$ this reduces to

$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_1 = .471\ \sigma_1$$

Applying this constant to the S-N curve yields the curve labeled "τ_{oct}" in Figure B-2.

For the plate $\sigma_1 = 80,000$ psi, $\sigma_2 = 36,520$ psi, $\sigma_3 = 0$ which yields

$$\tau_{oct} = 32,710 \text{ psi}$$

Applying the value of K_f found in (a) to τ_{oct}, the fatigue life is $N_f = 18,000$ cycles.

A comparison of methods is presented below.

	Equivalent N_f, Stress, ksi	Cycles
Maximum principal stress	122.	9,400
Maximum shearing stress	61.	9,400
Maximum octahedral shearing stress	32.	18,000
No. of cycles to initiation from tests	-	13,950

Thus it appears that the shear stress and principal stress provides the best albeit slightly conservative predictions when using uniaxial data to predict biaxial fatigue in a laterally loaded plate with butt weld. Octahedral stress is closest in magnitude but under predicts fatigue life, an undesirable outcome from a designers viewpoint. The effects of stress concentration factors for weld details has a large influence in these predictions and must be considered carefully in prediction of multiaxial fatigue response in welded details.

<u>Strain-based methods</u>

The original intent of this section was to make a direct comparison of stress-based methods and strain-based methods using the results of large scale fatigue tests on a shipbuilding steel, thus the choice of the Naval Applied Science Laboratory (NASL) tests of a 56" by 28" HY-80 butt-welded specimen. The methods to be considered were:

(a) maximum principal strain
(b) maximum shear strain
(c) Lohr and Ellison critical plane method
(d) Kandil, Brown and Miller critical plane method.

Each of the above involves a variant of the <u>strain-life equation</u>:

$$\frac{\Delta\epsilon}{2} = \frac{\sigma_f'}{E}(2N)^b + \epsilon_f'(2N)^c$$

where $\frac{\Delta\epsilon}{2}$ = total strain amplitude

σ_f' = fatigue strength coefficient

b = fatigue strength exponent

ϵ_f' = fatigue ductility coefficient

c = fatigue ductility exponent

2N = number of stress reversals.

A graphical interpretation of the cyclic constraints σ_f', ϵ_f', b and c is shown in Figure B-3 taken from Reference B-5. The same reference contains a summary of the monotonic and cyclic strain properties of selected engineering alloys -- a list that is about as complete as any available from other public domain sources. While the properties of a number of steels are included, HY-80 is not among them, and therein lies one of the problems in applying strain methods -- lack of a comprehensive and complete tabulation of multiaxial fatigue properties. In the absence of reliable information on these properties for HY-80 it was not possible to proceed further in the comparison of stress-based with strain-based biaxial life predictions.

REFERENCES FOR APPENDIX B

B-1. Gaites, R.A. and M. Rosen, "Fatigue of Large Scale Uniformly Loaded Simply Supported Butt Welded Rectangular Plates", Naval Applied Science Laboratory, Brooklyn, NY, Lab Project 930-23, Progress Report 9, SF 35-422-324, Task 2025, 31 October 1968.

B-2. Timoshenko, S. and S. Woinowsky-Krieger, "Theory of Plates and Shells", 2nd edition, McGraw-Hill.

B-3. Yung, J.Y. and F.V. Lawrence, "Analytical and Graphical Aids for the Fatigue Design of Weldments", American Welding Society, Proceedings on Conference on Fitness of Purpose for Welded Construction, May 14-16, 1985.

B-4. Yung, J.Y. and F.V. Lawrence, "A Comparison of Methods for Predicting Weldment Fatigue Life Under Variable Load Histories", College of Engineering, University of Illinois at Urbana-Champaign, Materials Engineering - Mechanical Behavior, Report No. 117/UILU-ENG85-3601, January 1985.

B-5. Fuchs, H.O. and R.I. Stepheus, "Metal Fatigue in Engineering", John Wiley & Sons, 1980.

COMMITTEE ON MARINE STRUCTURES

Commission on Engineering and Technical Systems

National Academy of Sciences - National Research Council

The COMMITTEE ON MARINE STRUCTURES has technical cognizance over the interagency Ship Structure Committee's research program.

Stanley G. Stiansen (Chairman), Riverhead, NY
Mark Y. Berman, Amoco Production Company, Tulsa, OK
Peter A. Gale, Webb Institute of Naval Architecture, Glen Cove, NY
Rolf D. Glasfeld, General Dynamics Corporation, Groton, CT
William H. Hartt, Florida Atlantic University, Boca Raton, FL
Paul H. Wirsching, University of Arizona, Tucson, AZ
Alexander B. Stavovy, National Research Council, Washington, DC
Michael K. Parmelee, Secretary, Ship Structure Committee, Washington, DC

LOADS WORK GROUP

Paul H. Wirsching (Chairman), University of Arizona, Tucson, AZ
Subrata K. Chakrabarti, Chicago Bridge and Iron Company, Plainfield, IL
Keith D. Hjelmstad, University of Illinois, Urbana, IL
Hsien Yun Jan, Martech Incorporated, Neshanic Station, NJ
Jack Y. K. Lou, Texas A & M University, College Station, TX
Naresh Maniar, M. Rosenblatt & Son, Incorporated, New York, NY
Solomon C. S. Yim, Oregon State University, Corvallis, OR

MATERIALS WORK GROUP

William H. Hartt (Chairman), Florida Atlantic University, Boca Raton, FL
Fereshteh Ebrahimi, University of Florida, Gainesville, FL
Santiago Ibarra, Jr., Amoco Corporation, Naperville, IL
Paul A. Lagace, Massachusetts Institute of Technology, Cambridge, MA
John Landes, University of Tennessee, Knoxville, TN
Mamdouh M. Salama, Conoco Incorporated, Ponca City, OK
James M. Sawhill, Jr., Newport News Shipbuilding, Newport News, VA

SHIP STRUCTURE COMMITTEE PUBLICATIONS

SSC-336 Liquid Slosh Loading in Slack Ship Tanks; Forces on Internal Structures & Pressures by N. A. Hamlin 1986

SSC-337 Part 1 - Ship Fracture Mechanisms Investigation by Karl A. Stambaugh and William A. Wood 1987

SSC-337 Part 2 - Ship Fracture Mechanisms - A Non-Expert's Guide for Inspecting and Determining the Causes of Significant Ship Fractures by Karl A. Stambaugh and William A. Wood 1987

SSC-338 Fatigue Prediction Analysis Validation from SL-7 Hatch Corner Strain Data by Jen-Wen Chiou and Yung-Kuang Chen 1985

SSC-339 Ice Loads and Ship Response to Ice - A Second Season by C. Daley, J. W. St. John, R. Brown, J. Meyer, and I. Glen 1990

SSC-340 Ice Forces and Ship Response to Ice - Consolidation Report by C. Daley, J. W. St. John, R. Brown, and I. Glen 1990

SSC-341 Global Ice Forces and Ship Response to Ice by P. Minnick, J. W. St. John, B. Cowper, and M. Edgecomb 1990

SSC-342 Global Ice Forces and Ship Response to Ice - Analysis of Ice Ramming Forces by Yung-Kuang Chen, Alfred L. Tunik, and Albert P-Y Chen 1990

SSC-343 Global Ice Forces and Ship Response to Ice - A Second Season by P. Minnick and J. W. St. John 1990

SSC-346 Fatigue Characterization of Fabricated Ship Details - Phase 2 by K. K. Park and F. V. Lawrence, Jr. 1988

SSC-349 Development of a Generalized Onboard Response Monitoring System (Phase I) by F. W. DeBord, Jr. and B. Hennessy 1987

SSC-350 Ship Vibration Design Guide by Edward F. Noonan 1989

SSC-351 An Introduction to Structural Reliability Theory by Alaa E. Mansour 1990

SSC-355 Relation of Inspection Findings to Fatigue Reliability by M. Shinozuka 1989

None Ship Structure Committee Publications - A Special Bibliography 1983