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REPORT

ON

A STUDY ON THE STRUCTURAL ACTION OF SUPERSTRUCTURES ON SHIPS

BY

DR. HANS H. BLEICH
Columbia University
Under Bureau of Ships Contract NObs-50538
(Index No. NS-731-034)

FOR

SHIP STRUCTURE COMMITTEE

Convened by
The Secretary of the Treasury

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Dear Sir:

Herewith is a copy of the report covering a preliminary study of "The Structural Action of Superstructures on Ships" by Dr. Hans H. Bleich. This investigation was conducted by Dr. Bleich of Columbia University for the Ship Structure Committee.

Any questions, comments, criticism or other matters pertaining to the Report should be addressed to the Secretary, Ship Structure Committee.

This Report is being distributed to those individuals and agencies associated with and interested in the work of the Ship Structure Committee.

Yours sincerely,

K. K. Cowart

K. K. COWART
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure Committee

R E P O R T

on

A Study on the Structural Action of Superstructures on Ships

by

Dr. Hans H. Bleich
Associate Professor of Civil Engineering
Columbia University

Bureau of Ships
Contract NObs 50538
Index No. NS-731-034

June 1950.

PREFACE

The Navy Department through the Bureau of Ships is distributing this report for the SHIP STRUCTURE COMMITTEE to those agencies and individuals who were actively associated with the research work. This report represents results of part of the research program conducted under the Ship Structure Committee's directive "to improve the hull structures of ships by an extension of knowledge pertaining to design, materials and methods of fabrication".

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1. INTRODUCTION

From model tests, as well as from measurements on ships, it has recently become apparent that Navier's hypothesis does not always hold true for the midship section of a ship with a long deck house. The tests made by Holt[1)] indicate that the strain distributions in the hull and in the deck house, each are straight lines, but that there is a break at the deck level, where the superstructure is offset. A similar result was found in the tests on the S. S. "President Wilson",[2)] where a very pronounced break in the strain distribution occurs at the promenade deck level where the deck house is offset. It is further significant that good agreement with Navier's theory was found in tests by Holt on a different model, where the superstructure was not offset at the strength deck.

It appears from these tests that Navier's theory is not valid for the combined section of hull and deck house, if the sides of the deck house are offset from the sides of the hull. The tests seem to indicate that, instead, the hull and the deck house act as two separate beams, for each of which Navier's hypothesis applies; these two beams are, of course, not independent of each other, but forced to act together to a certain extent by horizontal shear forces and by vertical forces which act between the hull and the deck house.

Starting from the assumption that the hull and the deck house may be considered as beams, to each of which, separately, Navier's hypothesis is applicable, this report will derive expressions for the deflections and stresses in the hull and deck house assuming constant section of hull and house; it will be seen that the theoretical stress distributions found are of the type observed in the tests.

1) M. Holt, Structural Tests of Models Representing a Steel Ship Hull with Aluminum Alloy and Steel Superstructures, paper presented at the March 1949 Meeting, New England Section of the Soc. of Naval Arch. and Marine Engrs., p. 13 and 14, Figs. 8 and 9.

2) J. Vasta, Structural Tests on the Passenger Ship S. S. President Wilson - Interaction Between Superstructure and Main Hull Girder, paper presented at the Nov. 1949 Meeting, Soc. of Naval Architects and Marine Engrs., p. 17, Fig. 27.

The results of this theory concerning the stresses in the mid-ship section can be arranged in simple tables which permit the prediction of the deviation from the conventionally assumed straight-line stress distribution. The method is equally applicable if a part or all of the deck house consists of aluminum.

Before analyzing the full problem we will consider in Section 2 a simplified ship's structure in which the action of vertical forces between hull and deck house is neglected. This simplified structure does not describe the actual conditions in a ship, but because of its relative simplicity it is easier to study the play of forces; the understanding gained is of value in treating the full problem in Section 3. The simplified approach in Section 2 may be considered a generalization of Hovgaard's theory of a beam attached to a deck plate subjected to tension or compression due to the bending of the hull.[1)]

Sections 4 and 5 are devoted to the solution of the general differential equation for two special loading cases, and Section 6 derives an approximate method for using the results of the preceding sections for any type of loading.

Section 7 contains a table of coefficients and a list of the formulas required for the determination of the stresses in the mid-ship section, together with a numerical example.

The final Section 8 contains a review of the theoretical results obtained, discusses a test program to check these theoretical results, and considers futher research necessary to formulate design standards for ship superstructures.

1) W. Hovgaard, Trans. Inst. Naval Architects, Vol. 73, 1931.

2. ANALYSIS OF A SIMPLIFIED TWO-CELL STRUCTURE.

We consider the structure shown in Fig. 1a, a hollow box beam with two cells. The lower box, the hull, is of length L, while the upper box, the deck house, is shorter, and of length l ; both boxes are assumed to be of constant cross section. The cross sectional area and the moment of inertia of the upper section, Fig. 1c, and of the lower section, Fig. 1d, are A_1, I_1, and A_2, I_2, respectively; the distances of the respective centers of gravity from each other and from the deck are a, $\alpha_1 a$ and $\alpha_2 a$, respectively, see Figs. 1b, c and d.

In this Section we make the important simplifying assumption that the deck A B, Fig. 1b, and its supports have no stiffness, and ~~that it~~ will not resist any relative vertical movements between hull and deck house. This assumption is of course not justified in any real ship and we will abandon this assumption in the next Section.

We consider the structure just described under the action of vertical loads and buoyancy acting on the hull only, producing bending moments M in the vessel. We do not assume any external loads to act on the deck house.

Take a section at the distance z from the center of the deck house, and consider the free body diagrams for the deck house and hull, Fig. 2a. The moment and direct forces in the deck house and hull are M_1, N_1 and M_2, N_2 respectively. Moments M_1 and M_2 are positive if they produce compression on top of deck house or hull, and direct forces N_1 and N_2 are positive if they create tension. The external loads and buoyancy acting on the hull to the left of the section have a moment M; further, a shear force T of unknown magnitude will act on the underside of the deck house, and a similar force T will act in the opposite direction on the hull. The shear force T is counted positive if it acts as shown in Fig. 2a. Equilibrium of the portions of deck house and hull in Fig. 2a requires the relations

$$N_1 = -T, \qquad M_1 = -a\alpha_1 T, \tag{1a}$$

$$N_2 = T, \qquad M_2 = M - a\alpha_2 T \tag{1b}$$

Due to the assumption that Navier's hypothesis is valid for the deck house and hull separately, we can determine the stress at any point at a distance x_1 or x_2 from the respective center of gravity. Counting tension stresses σ as positive, we have in the deck house

$$\sigma_1 = -\frac{T}{A_1} + \frac{a\alpha_1 T}{I_1} x_1 \quad , \tag{2a}$$

and in the hull

$$\sigma_2 = \frac{T}{A_2} - \frac{M}{I_2} x_2 + \frac{a\alpha_2 T}{I_2} x_2 \tag{2b}$$

The stresses σ_1 and σ_2 at the junction of house and hull must be alike, and Eqs. (2) furnish, with $x_1 = -a\alpha_1$, $x_2 = a\alpha_2$

$$-\frac{T}{A_1} - \frac{a^2\alpha_1^2 T}{I_1} = \frac{T}{A_2} - \frac{M}{I_2} a\alpha_2 + \frac{a^2\alpha_2^2 T}{I_2} \tag{3}$$

Eq. (3) determines the value of T,

$$T = \frac{a\alpha_2 I_1}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2} + a^2\left(\alpha_2^2 I_1 + \alpha_1^2 I_2\right)} M. \tag{4}$$

Introducing this value of T into Eqs. (1) and (2) we can determine moments and stresses at any point.

T was defined as the total horizontal shear force acting between the left end of the deck house and the section at z. According to Eq. (4) T is proportional to the moment M, and the unit horizontal shear $\frac{dT}{dz}$ which will be transferred by rivets or welds from the hull to the deck house, will be

$$\frac{dT}{dz} = \frac{a\alpha_2 I_1}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2} + a^2\left(\alpha_2^2 I_1 + \alpha_1^2 I_2\right)} V \tag{5}$$

where $V = \frac{dM}{dz}$ is the shear in the structure. However it will be noticed that at the end C of the deck house the shear T is not zero, but equal to

$$T_c = \frac{a\alpha_2 I_1}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2} + a^2\left(\alpha_2^2 I_1 + \alpha_1^2 I_2\right)} M_c \tag{6a}$$

At a point slightly to the left of point C in Fig. 2a there is no deck house and therefore $T = 0$; this means that in addition to the distributed shear $\frac{dT}{dz}$ according to Eq. (5) there must be a concentrated horizontal shear force T_c according to Eq.

(6a) at point C, and a similar concentrated shear force

$$T_D = -\frac{a\alpha_2 I_1}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2} + a^2(\alpha_2^2 I_1 + \alpha_1^2 I_2)} \quad M_L \qquad (6b)$$

at the other end D of the deck house. The distribution of the forces $\frac{dT}{dz}$ is indicated in Fig. 2c; the minus sign in Eq. (6b) is due to the definition of the direction of positive shear forces in Fig. 2a.

It is obvious that the concentrated shear forces T_c and T_D can not exist in any actual structure, their occurrence is due to the fact that shear lag effects were neglected when we assumed Navier's hypothesis to be correct for the full length of the deck house. In reality the forces T_c and T_D will distribute themselves over a finite distance, presumably of the magnitude of the depth of the deck house. This distribution is indicated by the broken line in Fig. 2c, the shaded areas representing T_c and T_D. This means that the stresses in the vicinity of the end of the house found from Eq. (2) are incorrect; but according to St. Venant's Theorem the effect of the simplification will not affect the stresses in the center portions of the structure.

We will now proceed and obtain expressions for the moments M_1, M_2 and direct forces N_1 and N_2. These expressions become somewhat simpler if we introduce the following notations:

$$I_A = a^2 \frac{A_1 A_2}{A_1 + A_2}\,; \qquad (7a)$$

$$\mu = \frac{I_1 + \alpha_1 I_A}{I_2 + \alpha_2 I_A}\,. \qquad (7b)$$

μ will be called the size factor; it is a measure of the size of the deck house in relation to the hull. We will also require the moment of inertia I of the total section consisting of hull and deck house; I can be expressed[1)] by the moments of inertia I_1, I_2 and by the constant I_A :

$$I = I_1 + I_2 + I_A \qquad (8)$$

1) See Appendix, Eq. (a).

Making use of these notations the following expressions are derived in the appendix:

$$N_2 = -N_1 = T = \frac{MI_A}{aI} - \frac{MI_A}{a}\frac{\mu(\alpha_1 - \mu\alpha_2)}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (9a)$$

$$M_1 = M\frac{I_1}{I} - MI_1\frac{\mu}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (9b)$$

$$M_2 = M\frac{I_2}{I} + MI_2\frac{\mu^2}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (9c)$$

These expressions could be used to determine the stresses σ_1 and σ_2 in the deck house and hull; each of the three expressions consists of two terms, the first term being the value of the respective N or M if Navier's Theory would be applicable to the entire section.[1] Instead of using Eq. (9), we can therefore express the the actual stresses as the sum of the stresses σ_N according to Navier, and a correction $\Delta\sigma$,

$$\sigma = \sigma_N + \Delta\sigma \qquad (10)$$

Navier's stresses σ_N can be found from the conventional equation

$$\sigma_N = -\frac{M}{I}x \qquad (11)$$

where x counts from the centroid of the entire section, Fig. 3. The corrective stresses $\Delta\sigma_1$, and $\Delta\sigma_2$ in the deck house and hull, respectively, are

$$\Delta\sigma_1 = \frac{\Delta N_1}{A_1} - \frac{\Delta M_1}{I_1}x_1 \quad , \qquad \Delta\sigma_2 = \frac{\Delta N_2}{A_2} - \frac{\Delta M_2}{I_2}x_2 \qquad (12)$$

where ΔN and ΔM are corrections of N and M given by the second terms of Eqs. (9):

$$\Delta N_1 = -\Delta N_2 = \frac{MI_a}{a}\frac{\mu(\alpha_1 - \mu\alpha_2)}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (13a)$$

$$\Delta M_1 = -MI_1\frac{\mu}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (13b)$$

$$\Delta M_2 = MI_2\frac{\mu^2}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (13c)$$

1) See Appendix, Eqs. (b) and (c).

The computation of the stresses σ_N and $\Delta\sigma$ from Eqs. (11), (12) and (13) is a simple matter. Fig. 4 shows these stresses, separately, and also the total stress computed for the example in Section 7.

Due to the fact that the structure analyzed in this section was simplified by omitting vertical forces acting between hull and deck house the results are of limited significance; the typical break in the stress distribution at deck level is, however, already there. The value of the above analysis lies in the fact that the more accurate analysis presented in the next Section shows that the actual stresses, can be expressed in the form $\sigma = \sigma_N + \Phi \Delta\sigma$ where Φ is a numerical factor depending on the various dimensions of hull and deck house and on the stiffness of the bulkheads.

It might be added at this point that the reasoning presented would be fully applicable also if hull anddeck house would not be of constant section. All formulas derived in this Section remain unchanged, except Eq. (5) for the unit shear $\frac{dT}{dz}$; when deriving this equation by differentiation of Eq. (4) the fraction on the right hand side would no longer be a constant, resulting in an added term in Eq. (5). The important result, that the stress at the midship section can be expressed as the sum of the stress σ_N , according to Navier's theory, and the correction $\Delta\sigma$ remains valid.

3. GENERAL ANALYSIS OF TWO-CELL STRUCTURE.

We consider again the structure indicated in Fig. 1; hull and deck house are assumed to be of constant section as in Section 2, but we now want to take account of the fact that the deck house cannot move freely in the vertical direction in relation to the hull; instead, we introduce the more realistic assumption that any relative displacement of deck house will be resisted by internal vertical forces required to deflect bulkheads or transverse beams supporting the deck house; in other words, we consider the deck house as beam on elastic supports.

Under the action of external vertical loads and buoyancy the structure shown in Fig. 5 will deflect, and we can describe the deformations by the deflections y_1 and y_2 of the center lines of the deck house and hull, respectively, Fig. 5. In order to exclude motions of the entire vessel as a rigid body, we define y_1 and y_2 not as the absolute displacements, but as the relative displacements measured from a straight line CD rigidily connected to the hull. As result of this definition the displacement y_2 of the centroid of the hull at points C and D must always be zero.

We assume further that the stiffness of bulkheads or deck beams resisting relative vertical displacements of the deck house is constant for the full length of the deck house, the magnitude of the stiffness being given by a spring constant K. K is defined as the force per unit length of deck house required to produce a relative deflection equal to one unit of length, Fig. 6; the vertical reaction between hull and deck house will therefore be $K(y_1 - y_2)$ per unit of length. In an actual ship the bulk heads or deck beams will have a spacing s, and the constant K will be the force required to deflect one of the bulk heads or deck beams, divided by this spacing s.

The structure analysed here consists therefore of two beams having areas A_1, A_2 and moments of inertia I_1 and I_2; the two beams are connected along CD in such a way that both, horizontal shear forces and vertical reactions can be transferred. Fig. 7 indicates a general type of loading for the vessel, including the shear and moment diagrams. We assume Navier's hypothesis to be valid for the hull and for the deck house separately, and there is no problem concerning the determination of stresses fore and aft of the deck house; we can, therefore, restrict our analysis to the center portion CD of the structure. This center portion, indicated in Fig. 8, will be under the action of vertical loads p_1 on the deck house, p_2 on the hull (which includes buoyancy), shear forces S_C, S_D, and moments M_C and M_D.

We will now proceed to obtain the differential equation for the two deflections y_1 and y_2 describing the deformation of the structure; these differential equations can conveniently be obtained from the Theorem of Stationary Potential Energy[1]; this theorem states that the deformations of any structure are such that the total potential energy U of the system is a minimum. In the present case the potential energy U consists of the internal strain energy V, and the potential U_W of the external forces p_1, p_2, S_C, S_D, M_C and M_D. The total potential energy $U = V + U_W$ is,[2]

$$U = \frac{1}{2}\int_{-\frac{l}{2}}^{\frac{l}{2}} \left[EI_1 y_1''^2 + EI_2 y_2''^2 + EI_A(\alpha_1 y_1'' + \alpha_2 y_2'')^2 + K(y_1 - y_2)^2 - 2p_1 y_1 - 2p_2 y_2 \right] dz + \left[M y_2' \right]_{-\frac{l}{2}}^{\frac{l}{2}} - \left[S y_2 \right]_{-\frac{l}{2}}^{\frac{l}{2}} \tag{14}$$

Using the symbol δ, and the rules of the calculus of variation,[3] U will be a minimum if

$$\delta U = 0 \tag{15}$$

from which we obtain Euler's equations,[4] which are in this case two simultaneous differential equations of the fourth order for the unknown functions y_1 and y_2:

$$E(I_1 + \alpha_1^2 I_A) y_1^{IV} + K y_1 + \alpha_1 \alpha_2 E I_A y_2^{IV} - K y_2 = p_1 \tag{16a}$$

$$\alpha_1 \alpha_2 E I_A y_1^{IV} - K y_1 + E(I_2 + \alpha_2^2 I_A) y_2^{IV} + K y_2 = p_2 \tag{16b}$$

1) F. Bleich, The Buckling Strength of Metal Structures, Chapter IV.

2) See appendix, Eqs. (1) and (σ).

3) The Buckling Strength of Metal Structures, Appendix to Chapter IV contains a presentation of the calculus of variation.

4) See Appendix, Article 5.

The process of variation also furnishes the following boundary conditions,[1] which are required to determine the 8 arbitrary constants which will appear in the general solutions of the differential equations (16). For $z = +\frac{l}{2}$ and $z = -\frac{l}{2}$:

$$y_2 = 0 \tag{17a}$$

$$E(I_1 + \alpha_1^2 I_A)\, y_1'' + \alpha_1 \alpha_2 E I_A\, y_2'' = 0 \tag{17b}$$

$$\alpha_1 \alpha_2 E I_A\, y_1'' + E(I_2 + \alpha_2^2 I_A)\, y_2'' = -M \tag{17c}$$

$$E(I_1 + \alpha_1^2 I_A)\, y_1''' + \alpha_1 \alpha_2 E I_A\, y_2''' = 0 \tag{17d}$$

The meaning of the first of these boundary conditions is obvious, but the other three require physical interpretation. The moments M_1 and M_2 of the longitudinal stresses in the deck house and hull can be expressed by the usual relationships $M_1 = -EI_1 y_1''$ and $M_2 = -EI_2 y_2''$; Eqs. (17b) and (17c) can therefore be re-arranged:

$$M_1 = \alpha_1 E I_A (\alpha_1 y_1'' + \alpha_2 y_2'') \tag{18a}$$

$$M_2 = M + \alpha_2 E I_A (\alpha_1 y_1'' + \alpha_2 y_2'') \tag{18b}$$

These equations indicate that the moments M_1 and M_2 at the end of the deck house are not equal to zero, and M, respectively, as might have been expected. In the appendix[2] it is shown that the horizontal shear T between deck house and hull, acting as indicated in Fig. 2a, is

$$T = -\frac{E I_A}{a} (\alpha_1 y_1'' + \alpha_2 y_2'') \tag{19}$$

Eqs. (18) become therefore

$$M_1 = -a\alpha_1 T, \tag{20a}$$

$$M_2 = M - a\alpha_2 T, \tag{20b}$$

and these equations are identical with Eqs. (1) for the simplified structure considered in Section 2. It must be remembered that Eqs. (20) apply only at the end-points of the deck house; but at these points the moments and stresses as determined in Section 2 occur and the finding that concentrated horizontal shear forces must be presumed to act at these points is valid. The magnitude of these forces is

1) See Appendix, Article 5.

2) Article 6.

given by Eq. (19) for $z = \pm \ell/2$.

The fourth boundary condition, Eq. (17d) expresses the fact that the shear force at the end of the deck house must vanish; this can be seen by comparison with the expression for the shear force, Eq. (t), derived in the appendix.

4. SOLUTION OF THE DIFFERENTIAL EQUATION FOR CONSTANT MOMENT M.

We consider first the simple case that the loads p_1, p_2 and the shears S_c and S_D are zero, the only loads being $M_c = M_D = M$. From Eq. (16) we obtain the two simultaneous homogeneous differential equations of the fourth order:

$$E(I_1 + \alpha_1 I_A)\, y_1^{IV} + K y_1 + \alpha_1 \alpha_2 E I_A\, y_2^{IV} - K y_2 = 0 \tag{21a}$$

$$\alpha_1 \alpha_2 E I_A\, y_1^{IV} - K y_1 + E(I_2 + \alpha_2^2 I_A)\, y_2^{IV} + K y_2 = 0 \tag{21b}$$

The general solution of these equations contains eight arbitrary constants to satisfy the eight boundary conditions (17). The problem considered in this section is symmetrical with respect to the origin of the coordinate z, and using only symmetrical functions, the general symmetrical solution will contain only four arbitrary constants. This general solution is

$$y_1 = C_1 + C_2 z^2 + C_3 \sin\gamma z \sinh\gamma z + C_4 \cos\gamma z \cosh\gamma z \tag{22a}$$

$$y_2 = C_1 + C_2 z^2 - \mu C_3 \sin\gamma z \sinh\gamma z - \mu C_4 \cos\gamma z \cosh\gamma z \tag{22b}$$

where

$$\gamma = \sqrt[4]{\frac{K}{4E} \, \frac{1+\mu}{\alpha_2 I_1 + \mu \alpha_1 I_2}} \, , \tag{23}$$

while μ is the size factor previously defined, Eq. (7b). The fact that Eqs. (22) are solutions of (21) can be established by substitution.

Introduction of Eq. (22) into the boundary conditions (17) leads to 4 linear equations for the constants C_1 to C_4. The values of C_2, C_3, C_4 are:

$$C_2 = -\frac{M}{2EI} \tag{24a}$$

$$C_3 = \frac{1}{2\gamma^2} \, \frac{\mu \Phi_1 M}{(1+\mu) E (\alpha_2 I_1 + \mu \alpha_1 I_2)} \tag{24b}$$

$$C_4 = \frac{1}{2\gamma^2} \, \frac{\mu \Psi_1 M}{(1+\mu) E (\alpha_2 I_1 + \mu \alpha_1 I_2)} \tag{24c}$$

The value of C_1 is not listed as it will not be required for the purposes of this paper. Φ_1 and Ψ_1 in Eqs. (24) are defined by

$$\Phi_1 = \frac{\sin u \cosh u + \cos u \sinh u}{\sin u \cos u + \sinh u \cosh u} \qquad (25a)$$

$$\Psi_1 = \frac{\cos u \sinh u - \sin u \cosh u}{\sin u \cos u + \sinh u \cosh u} \qquad (25b)$$

where

$$u = \frac{\gamma l}{2} = \frac{l}{2}\sqrt[4]{\frac{K}{4E}\,\frac{1+\mu}{\alpha_2 I_1 + \mu\alpha_1 I_2}} \qquad (26)$$

The stresses in the mid ship section can be computed from the expressions for the moments M_1, M_2 and direct forces N_1, N_2 :

$$M_1 = -EI_1 y_1'' \; , \qquad M_2 = -EI_2 y_2'' \; , \qquad (27a)$$

$$N_1 = -N_2 = \frac{EI_A}{a}\left(\alpha_1 y_1'' + \alpha_2 y_2''\right) \qquad (27b)$$

(for derivation of Eq. (27b) see appendix, Eq. (γ).

At the midship section, we have $z = 0$, and differentiation of Eq. (22) furnishes

$$y_1'' = 2C_2 + 2\gamma^2 C_3 \qquad (28a)$$

$$y_2'' = 2C_2 - 2\mu\gamma^2 C_3 \qquad (28b)$$

By substitution of Eqs. (24) into (28), and Eqs. (28) into (27), the following expressions are obtained:

$$N_2 = -N_1 = M\frac{I_A}{aI} - \Phi_1 M\frac{I_A}{a}\,\frac{\mu(\alpha_1 - \mu\alpha_2)}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (29a)$$

$$M_1 = M\frac{I_1}{I} - \Phi_1 M I_1 \frac{\mu}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (29b)$$

$$M_2 = M\frac{I_2}{I} + \Phi_1 M I_2 \frac{\mu^2}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \qquad (29c)$$

Comparing Eqs. (29) and Eqs. (9), found in Section 2, we see as only difference the factor Φ_1 appearing in the second term of each of the Eqs. (29). In Section 2 we had found that the first term of each Eq. (9) represented the result of Navier's theory, while the second term was a correction. The refined theory in this Section furnishes a similar result, but the "correction" found in Section 2 is to be multiplied by a factor Φ_1 . To compute the stresses according to the refined theory we can use the relationship.

$$\sigma = \sigma_N + \Phi_1 \Delta\sigma \quad , \tag{30}$$

where Navier's stresses σ_N , the corrective stresses $\Delta\sigma_1$, $\Delta\sigma_2$ and the corrective forces ΔN_1 , ΔN_2 and moments ΔM_1 , ΔM_2 are to be computed from Eqs. (11), (12) and (13) in Section 2.

This result , Eq. (30), is surprisingly simple; it indicates that the deviation of the stress distribution from Navier's is indicated by the value of the non-dimensional factor Φ_1 which we will call "Deviation Factor". Fig. 9 shows Φ_1 as function of the parameter u defined by Eq. (26). u is a function of the dimensions of hull and deck house and of the stiffness factor K of the bulkheads. u is proportional to the length of the deckhouse, and increases with rising value of K. According to Fig. 9, $\Phi_1 = 1$ for $u = 0$ and decreases for rising values of u ; for $u > 2$ the factor Φ_1 is a small positive or even negative number, indicating that for such values Navier's stress distribution at the mid-ship section is approximately correct.

Eq. (30) was derived for the mid-ship section, $z = 0$. The solution of the differential equations found above permits the computation of the stresses at any other section too, and a similar relationship

$$\sigma = \sigma_N + \Phi_1(z)\, \Delta\sigma \tag{31}$$

exists all along the deckhouse; however, the value of the deviation factor is not the same as at the mid-ship section $z = 0$; $\Phi_1(z)$ is a function of u and also of the ratio z/l , defining the location of the cross section. Fig. 10 shows $\Phi_1(z)$ as function of the ratio z/l for several values of u . The value of the deviation factor at the end of the deck house $z/l = 0.5$ is 1 for all values of u ; for large values of u , $\Phi_1(z)$ is very small everywhere, except near the end of the deckhouse, indicating the validity of Navier's theory in the center portions of the deck house.

5. SOLUTION OF DIFFERENTIAL EQUATION FOR EQUALLY DISTRIBUTED LOADS.

In this section we consider the case of equally distributed loads p_1 and p_2, acting on deck house and hull, respectively, while the moments at the end of the deck house are $M_C = M_D = 0$. Equilibrium requires external shear forces

$$S_C = - S_D = \frac{l}{2} (p_1 + p_2)$$

at the ends C and D. The moment in the mid-ship section due to the loads p_1 and p_2 is

$$M_p = \frac{p_1 + p_2}{8} l^2 \tag{32}$$

The loading being symmetrical, the general symmetrical solutions of Eqs. (16) are:

$$y_1 = C_1 + C_2 z^2 + C_3 \sin \gamma z \sinh \gamma z + C_4 \cos \gamma z \cosh \gamma z + \frac{p_1 + p_2}{24 EI} z^4 + \frac{p_1}{(1+\mu) K} \tag{33a}$$

$$y_2 = C_1 + C_2 z^2 - \mu C_3 \sin \gamma z \sinh \gamma z - \mu C_4 \cos \gamma z \cosh \gamma z + \frac{p_1 + p_2}{24 EI} + \frac{\mu p_2}{(1+\mu) K} \tag{33b}$$

where γ is defined in Eq. (23).

The boundary conditions (17)[1)] furnish the values of the arbitrary constants C in Eqs. (33). To compute the stresses only C_2, C_3 and C_4 are required:

$$C_2 = - \frac{M_p}{2 EI}, \tag{34a}$$

$$C_3 = \frac{1}{2\gamma^2} \frac{\mu \Phi_2 M_p}{(1+\mu) E (\alpha_2 I_1 + \mu \alpha_1 I_2)}, \tag{34b}$$

$$C_4 = \frac{1}{2\gamma^2} \frac{\mu \Psi_1 M_p}{(1+\mu) E (\alpha_2 I_1 + \mu \alpha_1 I_2)}. \tag{34c}$$

where M_p is the moment at midspan given by Eq. (32), and

1) It should be noted that M in Eq. (17c) is in this case zero.

$$\Phi_2 = \frac{2}{u} \frac{\sin u \sinh u}{\sin u \cos u + \sinh u \cosh u} , \tag{35a}$$

$$\Psi_2 = \frac{2}{u} \frac{\cos u \cosh u}{\sin u \cos u + \sinh u \cosh u} , \tag{35b}$$

$u = \frac{\gamma l}{2}$ is defined by Eq. (26).

EqS. (34) are quite similar to EqS. (24) and the further computation follows the pattern of the preceding section; the only difference is that instead of the deviation factor Φ_1 a factor Φ_2 appears. The stresses at the mid-ship section $z=0$ are:

$$\sigma = \sigma_N + \Phi_2 \, \Delta\sigma \tag{36}$$

Eq. (11), (12) and (13) are to be used to compute σ; the moment M in these computations is given by Eq. (32).

Fig. 9 shows Φ_1 and Φ_2 as functions of the parameter u. In the important range $u < 3$, Φ_2 is larger than Φ_1 indicating that equally distributed load produces larger deviations from Navier's stresses than a constant moment. There is only a quantitative difference between the two loading cases considered in this and the preceding section; the spanwise variation of the deviation factor for distributed load will be similar to the one shown in Fig. 10 for constant moment.

One result of the computations in this section deserves attention. While the expressions (33) for the deflections contain terms depending on the loads p_1 and p_2, separately,[1] the stresses apparently only depend on the sum $p_1 + p_2$, which alone is required to compute the moment M according to Eq. (32). The distribution of the equally distributed load between deck house and hull does not affect the stress distribution; it does, however, influence the values of the deflections y_1 and y_2. A transfer of equally distributed load from deckhouse to hull produces only a change of the relative deflection $y_1 - y_2$ of the hull in relationship to the deck house without any change in bending stresses in deck house or hull.

1) The loads p_1 and p_2 act on the deck house and hull, respectively.

6. DETERMINATION OF STRESSES AT MIDSHIP-SECTION FOR ANY LOADING.

In the preceding sections we have determined the stresses for two simple loading cases; we are now going to show how a combination of these two cases can be used for approximate determination of the stresses for any type of loading.

Fig. 11 a shows hull and deck house of a ship, and Fig. 11b a general type of moment diagram due to external loads and buoyancy. We assume that hull and deck house are of constant section between points C and D, while the section of the hull outside these points may vary.

The law of superposition being applicable, we may divide the total loading of the ship in three parts which produce moments in the ship's structure as shown in Figs. 11c, d, and e, respectively. The first part shall produce a constant moment $M_I = \frac{1}{2}(M_c + M_D)$ for the full length of the deck house; the second part shall be such that the moment diagram is a straight line between points C and D, the moments at these points being $\frac{1}{2}(M_c - M_D)$ and $-\frac{1}{2}(M_c - M_D)$, respectively; and the third part shall be the remainder of the loading, such that the sum of the moment diagrams in Fig. 11 c, d and e is equal to the actual moment diagram, Fig. 11b. Because of the choice of the moments in Fig. 11 c and d, the moments in the last diagram, Fig. 11 e, at points C and D must always be zero.

We can now determine the stresses at the midship section for each of the three parts separately, and add the results to get the total stresses.

For the first part of the loading the moment between C and D is of constant value M_I; this being just the loading case considered in Section 4, we can obtain the stresses σ_I due to this loading from Eq. (30).

$$\sigma_I = \sigma_{NI} + \Phi_1 \Delta \sigma_I \, , \tag{37}$$

where the subcripts I indicate that σ_N and $\Delta\sigma$ are determined from Eqs. (11), (12) and (13) using a value M_I for the moment M, appearing in these equations.

Proceeding to the second moment diagram Fig. 11d, we notice that the moment at the midship section is actually zero, and from the fact that the moment curve is antisymmetric we can conclude that the stresses at this section must vanish. This part of the load gives no contribution to the stresses at the midship section.

The third part of the moment diagram is of the type produced by the equally distributed load considered in Section 5, where the moment diagram would be a parabola between points C and D. If we approximate the actual moment diagram by a parabola with the same moment $M_{II} = M - M_{I}$ at the midship section, we can find the stresses at the midship section from Eq. (36),

$$\sigma_{II} = \sigma_{NII} + \Phi_2 \, \Delta \sigma_{II} \tag{38}$$

where the subscript II indicates that the moment M_{II} is to be used when computing σ_N and $\Delta\sigma$ from Eq. (11), (12) and (13).

The entire stress at the midship section will be

$$\sigma = \sigma_{I} + \sigma_{II} = \sigma_{NI} + \sigma_{NII} + \Phi_1 \, \Delta \sigma_{I} + \Phi_2 \, \Delta \sigma_{2} \tag{39}$$

We can simplify this expression by introducing the values σ_N and $\Delta\sigma$ due to the total moment M at the midship section. It is obvious from Eqs. (11), (12) and (13) that

$$\sigma_{NI} = \frac{M_I}{M} \sigma_N \, , \qquad \Delta \sigma_I = \frac{M_I}{M} \Delta \sigma \, ,$$
$$\sigma_{NII} = \frac{M_{II}}{M} \sigma_N \, , \qquad \Delta \sigma_{II} = \frac{M_{II}}{M} \Delta \sigma \, ,$$

Observing $M = M_I + M_{II}$, substitution in Eq. (39) leads to

$$\sigma = \sigma_N + \frac{\Phi_1 M_1 + \Phi_2 M_{II}}{M} \Delta \sigma \tag{40}$$

The values of M_I and M_{II} were defined at the beginning of this section,

$$M_I = \frac{M_c + M_D}{2} \tag{41a}$$

$$M_{II} = M - M_I = M - \frac{M_c + M_D}{2} \tag{41b}$$

Substituting Eqs. (41) into Eq. (40) we obtain finally

$$\sigma = \sigma_N + \Phi \, \Delta \sigma \tag{42}$$

where the deviation factor Φ is given by

$$\Phi = \Phi_2 - \left(\Phi_2 - \Phi_1\right) \frac{M_c + M_D}{2M} \qquad (43)$$

The values of Φ_1 and Φ_2 are defined by Eqs. (25a) and (35a); their numerical values are also given in Table I in the next section.

The approximation used in this section permits the determination of the deviation factor Φ for a general type of loading from two basic factors Φ_1 and Φ_2, resulting in a very simple computation procedure. An example of this procedure is shown in the following Section 7.

From Table 1 in the next section we can draw a conclusion on the degree of approximation to be expected from the procedure leading to Eq. (43). This table shows the values of the deviation factors for two distinctly different moment diagrams, Φ_1 for constant moment, and Φ_2 for parabolic moments. In spite of this pronounced difference, the numerical difference between Φ_1 and Φ_2 never exceeds 0.11; in the expression for the final stresses, $\sigma = \sigma_N + \Phi\,\Delta\sigma_N$, such a variation may produce variations of possibly 20-25% of the total stress, see Fig. 4. Considering the fact that the difference between the actual moment diagram and a parabola will be very much smaller than the difference between a parabola and a straight line, we can conclude that the error due to the approximation can be expected to be less than 5%.

7. TABLE AND FORMULAS FOR STRESSES AT MIDSHIP SECTION, AND NUMERICAL EXAMPLE.

Given the properties of the deck house and hull, and the moments in the ship's structure, i.e. the values:

- A_1 area of deck house
- I_1 moment of inertia of deck house
- A_2 area of hull
- I_2 moment of inertia of hull
- I total moment of inertia of hull and deck house together
- a distance between centroids of hull anddeck house, see Fig. 1b
- $\alpha_1 a$ distance of centroid of deck house from deck, see Fig. 1c.
- $\alpha_2 a$ distance of centroid of hull from deck, see Fig. 1d.
- l length of deck house
- K spring constant expressing rigidity of bulk heads or deck beams. K is the force per unit length of deck house required to produce a relative deflection of one unit of length between hull and deck house.
- M moment in ship's structure at center of deck house
- M_C moment at forward end of deck house
- M_D moment at aft end of deck house

The stresses σ_N according **to** Navier's theory are

$$\sigma_N = -\frac{M}{I}x \qquad (I)$$

where x is the distance of any fiber from the centroid of the section, Fig. 3.

After determining the values

$$I_A = a^2\frac{A_1 A_2}{A_1 + A_2}, \qquad I = I_1 + I_2 + I_A \qquad (II)$$

$$\mu = \frac{I_1 + \alpha_1 I_A}{I_2 + \alpha_2 I_A}, \qquad (III)$$

determine "corrective" moments and forces in deck house and hull

$$\left.\begin{aligned} \Delta N_1 = -\Delta N_2 &= \frac{M I_A}{a} \frac{\mu(\alpha_1 - \mu\alpha_2)}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \\ \Delta M_1 &= -M I_1 \frac{\mu}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \\ \Delta M_2 &= M I_2 \frac{\mu^2}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \end{aligned}\right\} \qquad (IV)$$

The corrective stresses $\Delta\sigma_N$ are:

$$\begin{aligned} &\text{in the deck house} \quad \Delta\sigma_1 = \frac{\Delta N_1}{A_1} - \frac{\Delta M_1}{I_1} x_1 , \\ &\text{in the hull} \quad \Delta\sigma_2 = \frac{\Delta N_2}{A_2} - \frac{\Delta M_2}{I_2} x_2 , \end{aligned} \qquad (V)$$

where x_1 and x_2 are the distances of the fibers from the centroids of deck house and hull, Figs. 1c and d.

After computing the constant u,

$$u = \frac{l}{2} \sqrt[4]{\frac{K(1+\mu)}{4E(\alpha_2 I_1 + \mu\alpha_1 I_2)}} \qquad (VI)$$

the factors Φ_1 and Φ_2 can be read from Table I. The "deviation factor" Φ can then be computed

$$\Phi = \Phi_2 - (\Phi_2 - \Phi_1) \frac{M_c + M_D}{2M} \qquad (VII)$$

The stress distribution at the midship section (at center of deck house) is

$$\sigma = \sigma_N + \Phi \Delta\sigma \qquad (VIII)$$

TABLE 1. VALUES OF DEVIATION FACTORS Φ_1 AND Φ_2 .

u	Φ_1	Φ_2	u	Φ_1	Φ_2
0	1.000	1.000	2.0	0.144	0.249
0.2	1.000	1.000	2.2	0.054	0.165
0.4	0.996	0.997	2.4	-0.009	0.103
0.6	0.979	0.982	2.6	-0.050	0.059
0.8	0.935	0.944	2.8	-0.074	0.029
1.0	0.852	0.872	3.0	-0.084	0.009
1.2	0.728	0.764	3.5	-0.078	-0.012
1.4	0.573	0.629	4.0	-0.052	-0.014
1.6	0.411	0.487	4.5	-0.026	-0.010
1.8	0.264	0.357	5.0	-0.009	-0.004

Numerical Example

We consider the model of a ship's structure shown in Fig. 12, supported at the ends, and loaded by two concentrated loads near the center. The section properties are:[1)]

Deck house: $A_1 = 2.95$ in.^{2}, $I_1 = 11.4$ in.^{4},

$\alpha_1 = 0.372$, $a\alpha_1 = 3.48$ in., $l = 70$ in.

Hull: $A_2 = 7.89$ in.^{2}, $I_2 = 160.9$ in.^{4}

$\alpha_2 = 0.628$, $a\alpha_2 = 5.88$ in., $K = 20,000$ lbs./in.^{2}

General: $E = 29 \times 10^6$ lbs./in.^{2}; $a = 9.36$ in., $M = 375,000$ in.lbs.

$M_C = 150,000$ in.lbs., $M_D = 225,000$ in.lbs.

Centroid of hull and deck house combined: $e = 3.33$ in. (See Fig. 12)

From Eqs. (II): $I_A = 188.1$ in.^{4} $I = 360.4$ in.^{4}

Eq. (I) furnishes with $x = 9.33, 3.33, -6.67$ the stresses σ_N listed in Table 2. From Eq. (III) we obtain

1) The properties used in this example agree with those of the center portion of one of the models used by Holt, except for the value of K which could not be ascertained.

$$\mu = \frac{11.4 + 0.372 \times 188.1}{160.9 + 0.628 \times 188.1} = 0.292 ,$$

and using the values

$$\alpha_1 - \mu \alpha_2 = 0.189 , \qquad \alpha_2 I_1 + \mu \alpha_1 I_2 = 24.64$$

we obtain from Eqs. (IV):

$$\Delta N_1 = -\Delta N_2 = 13,100 \text{ lbs}, \quad M_1 = -39,200 \text{ in. lbs.}, \quad M_2 = 161,600 \text{ in. lbs.}$$

Introducing these values in Eq. (V), and using $x_1 = 2.52$, -3.48, and $x_2 = 5.88$, -4.12, respectively, the values $\Delta\sigma$ in Table 2 were computed.

Using Eq. (VI) we find the constant u,

$$u = \frac{70}{2} \sqrt[4]{\frac{20,000 \times 1.292}{4 \times 29 \times 10^6 \times 24.65}} = \frac{70}{2} \times .0548 = 1.92 ,$$

and with this values we obtain from Table 1:

$$\Phi_1 = 0.192 , \qquad \Phi_2 = 0.292 .$$

We can now compute Φ from Eq. (VII)

$$\Phi = 0.292 - (0.292 - 0.192) \frac{150 + 225}{2 \times 375} = 0.242$$

Referring to Eq. (VIII), the fourth column of Table 2 contains the values $\Phi \Delta\sigma = 0.242 \Delta\sigma$ and the last column the final computed stresses.

TABLE 2

Stress at	Navier's stresses σ_N	$\Delta\sigma$	$\Phi\,\Delta\sigma$	Computed stresses σ
top of deck house	-9700	13080	3160	-6540
top of hull	-3460	-7560	-1830	-5290
bottom of hull	6940	2480	600	7540

Fig. 13 shows the computed stresses and the stresses measured by Holt on a model of similar cross section. The close agreement between the computed and measured stresses should not be construed as quantitative confirmation of the theory presented, because unfortunately the value $K = 20{,}000$ lbs./in.^{2} used in the computation could not be obtained accurately. The stiffness of diaphragms used in the test could not be determined, and the value K used is an average value, estimated from the measured vertical deflections and vertical direct stresses.[1)] It should also be noted that the cross section of the deck house of the test model changed near the ends, and that the length $l = 70$ in. used in the computation is a median value only. However, one need not dismiss the agreement between the theory and the tests entirely, because the test confirms, at least, that the theory furnishes the type of stress distribution actually found in the tests, particularly, the theory shows the characteristic kink in the stress distribution on the deck level.

1) Figs. 7 and 12 of Holt's paper quoted on page 1.

8. CONCLUSION AND OUTLOOK.

The theory presented in this report is based on the concept that the hull and the deck house act as individual beams which are forced to act together by their connections at the deck level. These connections transfer shear stresses such that the longitudinal stresses in deck house and hull at deck level are alike; these connections also transfer vertical reactions, but because of the flexibility of the bulkheads the vertical deflections of deck house and hull will not be alike. Depending on the elasticity of the bulk heads two extremes are possible: For infinitely rigid bulk heads hull and deck house will deflect as a unit resulting in Navier's stress distribution. For very flexible bulk heads only horizontal shear forces are transferred from hull to deck house, the deflections of hull and deck house will be different, and the stress distribution found in Section 2 will occur. The actual condition will lie between these extremes.

A. Results obtained.

The most important single result obtained is the fact that the longitudinal stress distribution at any cross section of the vessel, except close to the ends of the deck house, is characterized and defined by the value of the "Deviation Factor" Φ , which is a function of the non-dimensional parameter

$$u = \frac{l}{2} \sqrt[4]{\frac{K(1+\mu)}{4E(\alpha_2 I_1 + \mu \alpha_1 I_2)}} \qquad (A)$$

and of the shape of the moment diagram. The parameter u characterizes the type of interaction between deck house and hull for any vessel. The analysis indicates that the various properties of the vessel combine into this non-dimensional parameter, which will therefore play a controlling role in the interpretation of tests results and in any future design specification.

It was found that the stress distribution at any point of the vessel may be expressed by Navier's stresses σ_N and an added correction $\Phi \Delta \sigma$ in the form

$$\sigma = \sigma_N + \Phi \Delta \sigma \qquad (B)$$

The stresses σ_N and $\Delta\sigma$ only depend on the bending moment and on the properties of the cross section of the vessel at the point where the stresses are to be found, while the deviation factor Φ contains all other effects in a single package, it expresses:

1. The length l of the deck house.
2. The stiffness of bulk heads or deck beams.
3. The type of moment diagram and loading.
4. It is also dependent on the elastic properties of the cross section of the vessel.

The existence of relation (B) is of great value, because it expresses the longitudinal stress distribution in a non-dimensional manner by the single factor Φ . Test results on different structures with various sizes of deck house and hull can be interpreted by the Φ -concept on a common basis.

The lengthwise distribution of the deviation factor Φ as indicated in Fig. 10 defines the longitudinal stresses and it can be shown that an equation similar to Eq. (B) defines the shear forces T acting between hull and deck house.

The above results where obtained under the following assumptions:

a) Navier's hypothesis of straight line stress and straindistribution is assumed to be valid for the hull and the deck house separately, but not for the entire section.

b) The cross section of the deck house is constant, and the cross section of the hull is constant for the length of the deck house, but not foreand aft of the deck house.

c) The stiffening effect of bulk heads and deck beams expressed by the spring constant K is assumed to be constant and equally distributed for the full length of the deck house.

d) The effect of shear lag has been neglected. This is indirectly implied in assumption a), as Navier's hypothesis is never satisfied if the shear deformations are substantial.

The stress and strain distribution given by Eq. (B) consists of two straight lines with a break on the level of the deck, as shown in Fig. 13. This agrees qualitatively with the results of model and full scale tests. Because of assumption d), Eq. (B) cannot be valid and should not be applied in the vicinity of the ends of the deck house where shear lag must be a controlling factor.

Table 1 in Section 7, can be used for the quick numerical determination of the deviation factor Φ (as function of the parameter **u**) for the stresses at the center of the deck house. Similar tables can be computed for other points, e.g. third or quarter-points of the deck house.

The theory is fully applicable if parts of the deck house are of aluminum instead of steel.

B. Application of the theory to actual ship's structures.

The simplifying assumptions b) and c) stated above are not satisfied in a actual ship's structure, and the qeustion arises whether and to what extent the results obtained apply or can be extended to vessels having neither constant cross section nor equally distributed bulk heads.

The principle of the theory presented in this report is fully applicable if assumptions b) and c) are not made, but different and more complicated mathematical methods for solving the differential equation may have to be used. It is of considerable importance that the stress distribution can again be expressed by an equation of the form[1)]

$$\sigma = \sigma_N + \Phi \Delta \sigma \qquad (B)$$

1) See Appendix, Section 7.

containing a deviation factor Φ , the numerical value of which will of course be different from before. As shown in the appendix equation (B) is a direct result of assumption a).

Thus we know that even in the more general case of variable sections and stiffness the state of stress in the vessel can be described by a curve indicating the values of Φ for the length of the deck house similar to the curves shown in Fig. 10. We can expect that the shape of the Φ curves will again depend on the parameter

$$u = \frac{l}{2} \sqrt[4]{\frac{K(1+\mu)}{4E(\alpha_2 I_1 + \mu \alpha_1 I_2)}} \quad ,$$

where the values of I_1, I_2, μ, etc. are average values of these properties.

The effect of variations in cross section of hull and deck house will express itself in the shape of the deflection curves of the hull and the deck house. Deflection curves generally being not very sensitive to variations of the cross sections it is to be expected that a reasonable approximation of the actual case of variable sections can be obtained by using constant average or median values for A and I.

The fact that the bulk heads and deck beams act at certain points instead of providing a continuous effect, as assumed in assumption c), will not affect the overall stress distribution as long as the bulk heads are spread reasonably equal over the length of the deck house, and as long as there are at least five in number. This is concluded from the fact that the moments and deflections of any beam due to 5 or more equidistant concentrated loads, and due to equally distributed load of equal magnitude are nearly alike.

To provide for the possibility of a concentration of stiffening bulk heads near the ends of the deck houses an additional analysis can be made, and it is believed that this effect can be treated as a correction to the analysis made in Sections 3 - 7.

The emphasis in the preceding paragraphs was on justifying the use of the analysis presented in this report as theoretical basis for actual design. It appears that this analysis contains all the essential features affecting the stress distribution in a real ship and if suitably employed should furnish approximations of sufficient accuracy. It must be kept in mind that Navier's theory gives stresses in the deck house which may be considerably in error, and that further refinement is not necessary if the proposed theory cuts this error to say 10% of the actual stresses. After all, if the experiments quoted earlier had shown only differences of this magnitude with the conventional theory this theory would have been considered good for all practical purposes.

C. Proposed tests to confirm analysis.

The starting point for the analysis presented in this report were the tests quoted on page 1 which indicated that the conventional theory is not applicable. Having completed an analysis which appears to be rational and to take account of all the essential parameters of the problem, it is proper to stop and confirm this analysis by tests.

The primary purpose of these tests being to confirm the analysis presented, the tests should be made with models of constant cross section and of equally distributed bulk head stiffness K in accord with the assumption on which the theory is based. The models could be somewhat similar in size and section to those used in Holt's tests, except that the bulk heads would have to be designed in such manner that there rigidity can be determined beyond doubt.

According to the theory the stress distribution is defined by the single parameter

$$u = \frac{l}{2}\sqrt[4]{\frac{K(1+\mu)}{4E(\alpha_2 I_1 + \mu\alpha_1 I_2)}}$$

In order to test the whole range of results, experimental Φ curves for $u = 1$, 2 and 4 should be obtained, see Fig. 10. It is suggested that two models be used having different length l of the deck house. These models can be made adjustable for two values K each, one model having $u = 1$ or 2, the other $u = 2$ or 4, giving altogether 4 results, of which two, for $u = 2$, should be identical if interpreted by the non-dimensional Φ concept.

It is of importance to observe the longitudinal stresses at close intervals near the end of the deck house in order to determine the extent of the area influenced by shear lag. At the center of the deck house sufficient stress readings should be taken to be able to check the assumed straight-line stress distribution. It is also necessary to obtain the relative movements of deck house and hull for comparison with the theory.

The test results should be evaluated by computing values for the observed stresses and deflections from the theory.

D. Proposed additional theoretical work.

Assuming that the suggested experiments confirm the theory for the simplified structure, the next step should be to analyze numerically a typical vessel of variable cross section and bulk head stiffness. This will provide a basis for judging the error to be expected from using any simplified theory. It appears highly desirable to use the data for the passenger ship S. S. President Wilson for this theoretical investigation because the full scale test results on this vessel provide a possibility for an ultimate check of the theory. It would not be sensible to make tests with refined models simulating a ship structure having variable sections and stiffness; if the tests proposed under C. agree with the theory, model tests with variable sections will necessarily again agree with the theory (excluding the possibility of errors in the mathematical computations) because there is no fundamental difference between constant and variable sections. On the other hand, the analysis of full scale tests might disclose effects which do not occur in the simplified small scale models.

In addition to this analysis of a special case, the theory presented in this report should be extended, as discussed in B, to allow for increased bulk head stiffness at the ends of the deck house. It is expected that this can be done without materially increasing the numerical work in determining stresses as demonstrated on the example in Section 7.

It is also desirable to make an analysis of the shear lag effect near the end of the deck house. The purpose of this analysis would be to determine how far the shear lag effect reaches, and to obtain a simple conclusion for design purposes.

E. Derivation of design rules.

The theory presented in this report, together with the proposed additional experimental and theoretical work is expected to be sufficient to deduct design rules and compute tables or charts for use in actual design, all of which would be based on the non-dimensional parameter u.

It is expected that tables or charts pertaining to the following data will be required:

Deviation factors ξ at equidistant points along the deck house, 0.125 l apart, permitting the determination of bending and shear stresses in the vessel.

Reactions of deck house on the bulk heads at center and at ends of deck house.

Effective moment of inertia of vessel, - required for computation of the deflections of the entire vessel and of its natural frequencies.

APPENDIX

1. Properties of section if hull and deck house act integrally according to Navier.

Upper part: area A_1, moment of inertia I_1

lower part: area A_2, moment of inertia I_2

Location of center of gravity (Fig. 14)

$$e = a\,\frac{\alpha_2 A_2 - \alpha_1 A_1}{A_1 + A_2}$$

Total moment of inertia I:

$$I = I_1 + I_2 + (a\alpha_1 + e)^2 A_1 + (a\alpha_2 - e)^2 A_2$$

$$a\alpha_1 + e = a\,\frac{\alpha_1 A_1 + \alpha_1 A_2 + \alpha_2 A_2 - \alpha_1 A_1}{A_1 + A_2} = \frac{a A_2}{A_1 + A_2}$$

$$a\alpha_2 - e = a\,\frac{\alpha_2 A_1 + \alpha_2 A_2 - \alpha_2 A_2 + \alpha_1 A_1}{A_1 + A_2} = \frac{a A_1}{A_1 + A_2}$$

and

$$I = I_1 + I_2 + a^2\,\frac{A_1 A_2^2 + A_1^2 A_2}{(A_1 + A_2)^2} = I_1 + I_2 + \frac{a^2 A_1 A_2}{A_1 + A_2} = I_1 + I_2 + I_A \qquad (a)$$

where I_A is defined by Eq. (7a)

The resultants N_1 and M_1, N_2 and M_2 of the stresses in the upper and lower portions are determined as follows: The stress σ can be expressed as function of x, see Fig. 14.

$$\sigma = -\frac{M}{I}\,x$$

In the deck house the stress can be expressed alternatively as function of M_1, N_1 and x_1

$$\sigma = \frac{N_1}{A_1} - \frac{M_1}{I_1}\,x_1$$

Because: $x = x_1 + a\alpha_1 + e = x_1 + \dfrac{a A_2}{A_1 + A_2}$, we have

$$-\frac{M}{I}\left(x_1 + \frac{a A_2}{A_1 + A_2}\right) = \frac{N_1}{A_1} - \frac{M_1}{I_1}\,x_1$$

As this must be correct for any value of x_1, we have

$$M_1 = M\,\frac{I_1}{I}, \qquad N_1 = -M\,\frac{a A_1 A_2}{I(A_1 + A_2)} = -M\,\frac{I_A}{a I} \qquad (b)$$

Similarly, we find for the hull

$$M_2 = M\,\frac{I_2}{I}, \qquad N_2 = M\,\frac{I_A}{a I} \qquad (c)$$

2. Derivation of Eqs. (9).

From Eq. (4)

$$T = \frac{a\alpha_2 I_1 M}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2} + a^2(\alpha_2^2 I_1 + \alpha_1^2 I_2)} = \frac{a\alpha_2 I_1 M}{a^2 I_1 I_2 \frac{A_1 + A_2}{a^2 A_1 A_2} + a^2(\alpha_2^2 I_1 + \alpha_1^2 I_2)}$$

Using Eq. (7a) and (8),

$$T = \frac{M I_A}{aI}\,\frac{\alpha_2 I_1 I}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A}\,, \tag{d}$$

$$T = \frac{M I_A}{aI}\left[1 - 1 + \frac{\alpha_2 I_1 (I_1 + I_2 + I_A)}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A}\right],$$

$$T = \frac{M I_A}{aI}\left[1 - \frac{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A - \alpha_2 I_1 (I_1 + I_2 + I_A)}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A}\right],$$

$$T = \frac{M I_A}{aI}\left[1 - \frac{(I_1 + \alpha_1 I_A)(\alpha_1 I_2 - \alpha_2 I_1)}{(I_2 + \alpha_2 I_A)(\alpha_2 I_1 + \mu\alpha_1 I_2)}\right]$$

where according to Eq. (7b)

$$\mu = \frac{I_1 + \alpha_1 I_A}{I_2 + \alpha_2 I_A}\,.$$

Considering

$$\frac{\alpha_1 I_2 - \alpha_2 I_1}{I} = \frac{\alpha_1(I_2 + \alpha_2 I_A) - \alpha_2(I_1 + \alpha_1 I_A)}{I_1 + \alpha_1 I_A + I_2 + \alpha_2 I_A} = \frac{\alpha_1 - \mu\alpha_2}{1 + \mu}\,,$$

T may be written finally

$$T = \frac{M I_A}{aI} - \frac{M I_A}{aI}\,\frac{\mu(\alpha_1 I_2 - \alpha_2 I_1)}{\alpha_2 I_1 + \mu\alpha_1 I_2} = \frac{M I_A}{aI} - \frac{M I_A}{a}\,\frac{\mu(\alpha_1 - \mu\alpha_2)}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \tag{e}$$

Substituting Eq. (d) in Eq. (1a)

$$M_1 = -a\alpha_1 T = M\frac{I_1}{I}\,\frac{-\alpha_1\alpha_2 I I_A}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A}\,,$$

$$M_1 = M\frac{I_1}{I}\left[1 - 1 - \frac{\alpha_1\alpha_2(I_1 + I_2 + I_A) I_A}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A}\right]$$

$$M_1 = M\frac{I_1}{I}\left[1 - \frac{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A + \alpha_1\alpha_2 I_A (I_1 + I_2 + I_A)}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A}\right]$$

$$M_1 = M\frac{I_1}{I_2}\left[1 - \frac{(I_1 + \alpha_1 I_A)(I_2 + \alpha_2 I_A)}{(I_2 + \alpha_2 I_A)(\alpha_2 I_1 + \mu\alpha_1 I_2)}\right] = M\frac{I_1}{I} - M\frac{I_1}{I}\,\frac{\mu(I_2 + \alpha_2 I_A)}{\alpha_2 I_1 + \mu\alpha_1 I_2}$$

Considering

$$\frac{I_2 + \alpha_2 I_A}{I} = \frac{I_2 + \alpha_2 I_A}{I_1 + \alpha_1 I_A + I_2 + \alpha_2 I_A} = \frac{1}{1 + \mu}$$

we obtain finally

$$M_1 = M\frac{I_1}{I} - M I_1 \frac{\mu}{(1+\mu)(\alpha_2 I_1 + \mu\alpha_1 I_2)} \tag{f}$$

Substituting Eq. (d) in Eq. (1b),

$$M_2 = M - a\alpha_2 T = M - M \frac{\alpha_2^2 I_1 I_A}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A} ,$$

$$M_2 = M \frac{I_1 I_2 + \alpha_1^2 I_2 I_A}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A} = M \frac{I_2}{I}\left[1 - 1 + \frac{I(I_1 + \alpha_1^2 I_A)}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A}\right] ,$$

$$M_2 = M \frac{I_2}{I}\left[1 + \frac{-I_1 I_2 - \alpha_2^2 I_1 I_A - \alpha_1^2 I_2 I_A + I_1^2 + I_1 I_2 + I_1 I_A + \alpha_1^2 I_1 I_A + \alpha_1^2 I_2 I_A + \alpha_1^2 I_A^2}{I_1 I_2 + \alpha_2^2 I_1 I_A + \alpha_1^2 I_2 I_A}\right]$$

$$M_2 = M \frac{I_2}{I}\left[1 + \frac{(I_1 + \alpha_1 I_A)^2}{(I_2 + \alpha_2 I_A)(\alpha_2 I_1 + \mu \alpha_1 I_2)}\right] = M \frac{I_2}{I}\left[1 + \frac{\mu(I_1 + \alpha_1 I_A)}{\alpha_2 I_1 + \mu \alpha_1 I_2}\right]$$

Considering

$$\frac{I_1 + \alpha_1 I_A}{I} = \frac{I_1 + \alpha_1 I_A}{I_1 + \alpha_1 I_A + I_2 + \alpha_2 I_A} = \frac{\mu}{1+\mu}$$

we obtain finally,

$$M_2 = M \frac{I_2}{I} + M I_2 \frac{\mu^2}{(1+\mu)(\alpha_2 I_1 + \mu \alpha_1 I_2)} \qquad (g)$$

3. Derivation of strain energy of structure.

Denoting by ϵ_1 and ϵ_2 the average longitudinal strain in the deck house and hull, respectively, the strain energy of the longitudinal stresses will be,

in the deck house: $\displaystyle \frac{E}{2}\int_{-\frac{l}{2}}^{\frac{l}{2}} (A_1 \epsilon_1^2 + I_1 y_1''^2)\, dz$,

in the hull: $\displaystyle \frac{E}{2}\int_{-\frac{l}{2}}^{\frac{l}{2}} (A_2 \epsilon_2^2 + I_2 y_2''^2)\, dz$.

The strains ϵ are counted positive if they represent elongation.

In addition to the strain energy of the longitudinal stresses, there will be energy stored in the bulkheads or deck beams which resist the relative vertical displacements of deck house and hull; this part of the strain energy can be expressed by the spring constant K in the form

$$\frac{1}{2}\int_{-\frac{l}{2}}^{\frac{l}{2}} K (y_1 - y_2)^2\, dz$$

The strain energy of the shear stresses will be neglected; it is small because we consider the case of long deck houses only. The total strain energy V is

$$V = \frac{E}{2}\int_{-\frac{l}{2}}^{\frac{l}{2}}\left[A_1\epsilon_1^2 + I_1 y_1''^2 + A_2\epsilon_2^2 + I_2 y_2''^2 + \frac{K}{E}\left(y_1 - y_2\right)^2\right] dz \qquad (h)$$

The stresses in the deck house and hull can be expressed by the average strains ϵ_1 and ϵ_2, and by the second derivatives y_1'' and y_2'',

in Deck house: $\sigma = E\epsilon_1 + E y_1'' x_1$,

in Hull: $\sigma = E\epsilon_2 + E y_2'' x_2$,

where x_1 and x_2 are as shown in Fig. 1. Because hull and deck house are connected by rivets or welds, the stresses on main deck level must be alike and we have for $x_1 = -a\alpha_1$, $x_2 = a\alpha_2$,

$$E\epsilon_1 - Ea\alpha_1 y_1'' = E\epsilon_2 + Ea\alpha_2 y_2'' . \qquad (i)$$

Further, the longitudinal resultant of all stresses in the Deck house, N_1, must be the average strain ϵ_1 times EA_1 ; similarly $N_2 = EA_2\epsilon_2$. The resultant of all longitudinal forces in the structure consisting of hull and deck house will be obviously $N_1 + N_2$; as the structure is in bending only this resultant must vanish,

$$N_1 + N_2 = E\left(\epsilon_1 A_1 + \epsilon_2 A_2\right) = 0 . \qquad (j)$$

By means of the two eqs. (i) and (j) ϵ_1 and ϵ_2 can be expressed by the curvatures y_1'' and y_2'' :

$$\epsilon_1 = \frac{a A_2}{A_1 + A_2}\left(\alpha_1 y_1'' + \alpha_2 y_2''\right), \qquad \epsilon_2 = -\frac{a A_1}{A_1 + A_2}\left(\alpha_1 y_1'' + \alpha_2 y_2''\right). \qquad (k)$$

Substituting these values into eq. (h), we obtain

$$V = \frac{E}{2}\int_{-\frac{l}{2}}^{\frac{l}{2}}\left[I_1 y_1''^2 + I_2 y_2''^2 + I_A\left(\alpha_1 y_1'' + \alpha_2 y_2''\right)^2 + \frac{K}{E}\left(y_1 - y_2\right)^2\right] dz \qquad (l)$$

4. Potential U_w of external forces.

The potential U_w of any load P is equal to the negative of the work done by this force when the structure deflects. Applying this to the forces p acting on the structure we find that we have to make a distinction between forces p_1 which act on the deck house and forces p_2 which act on the hull. Counting p_1 and p_2 positive if acting downwards, and assuming p_1 and p_2 to be functions of the longitudinal coordinate z , their potential energy will be

$$-\int_{-\frac{l}{2}}^{\frac{l}{2}} (p_1 y_1 + p_2 y_2)\, dz. \tag{m}$$

The shear forces S_c and S_D and the moments M_c and M_D act immediately outside points C and D, and their potential energy will depend on the vertical displacements y_{2C} and y_{2D} and on the rotations of the end surfaces of the hull, y'_{2C} and y'_{2D} . Taking into account the direction of the shears and moments shown in Fig. 8 , their potential energy will be

$$- S_D y_{2D} + S_c y_{2C} + M_D y'_{2D} - M_c y'_{2D}$$

Noting by S and M the shear and moment in the structure, both being functions of the longitudinal cordinate z , we can write this expression in the abbreviated form

$$-\Big[S y_2\Big]_{-\frac{l}{2}}^{\frac{l}{2}} + \Big[M y_2'\Big]_{-\frac{l}{2}}^{\frac{l}{2}} \tag{n}$$

The total potential energy of the external load is the sum of eqs. (m) and (n)

$$U_w = \Big[M y_2'\Big]_{-\frac{l}{2}}^{\frac{l}{2}} - \Big[S y_2\Big]_{-\frac{l}{2}}^{\frac{l}{2}} - \int_{-\frac{l}{2}}^{\frac{l}{2}} (p_1 y_1 + p_2 y_2)\, dz \tag{o}$$

5. Derivation of Euler's equations and boundary conditions, eqs. (17).[1)]

From Eq. (14) we derive by the process of variation

$$\delta U = \Big[M \delta y_2'\Big]_{-\frac{l}{2}}^{\frac{l}{2}} - \Big[S \delta y_2\Big]_{-\frac{l}{2}}^{\frac{l}{2}} + \int_{-\frac{l}{2}}^{\frac{l}{2}} \Big[EI_1 y_1'' \delta y_1'' + EI_2 y_2'' \delta y_2'' + EI_A (\alpha_1 y_1'' + \alpha_2 y_2'')(\alpha_1 \delta y_1'' + \alpha_2 \delta y_2'')$$
$$+ K (y_1 - y_2)(\delta y_1 - \delta y_2) - p_1 \delta y_1 - p_2 \delta y_2\Big]\, dz$$

Performing integration by parts twice on the first three terms under the integral, results in the exoression

1) See note 3 on page 9.

$$\delta U = \Big[EI_1 y_1''\delta y_1' + EI_2 y_2''\delta y_2' + EI_A\left(\alpha_1 y_1'' + \alpha_2 y_2''\right)\left(\alpha_1\delta y_1' + \alpha_2\delta y_2'\right) + M\,\delta y_2'\Big]_{-\frac{l}{2}}^{\frac{l}{2}} -$$

$$-\Big[EI_1 y_1'''\delta y_1 + EI_2 y_2'''\delta y_2 + EI_A\left(\alpha_1 y_1''' + \alpha_2 y_2'''\right)\left(\alpha_1\delta y_1 + \alpha_2\delta y_2\right) + S\,\delta y_2\Big]_{-\frac{l}{2}}^{\frac{l}{2}} +$$

$$+\int_{-\frac{l}{2}}^{\frac{l}{2}}\Big[EI_1 y_1^{IV}\delta y_1 + EI_2 y_2\,\delta y_2 + EI_A\left(\alpha_1 y_1^{IV} + \alpha_2 y_2^{IV}\right)\left(\alpha_1\delta y_1 + \alpha_2\delta y_2\right) + K(y_1 - y_2)(\delta y_1 - \delta y_2) - p_1\delta y_1 - p_2\delta y_2\Big]dz.$$

Rearranging this expression, we obtain

$$\begin{aligned}
U = \;& \Big[\left(EI_1 y_1'' + E\alpha_1^2 I_A y_1'' + E\alpha_1\alpha_2 I_A y_2''\right)\delta y_1'\Big]_{-\frac{l}{2}}^{\frac{l}{2}} + \\
&+\Big[\left(E\alpha_1\alpha_2 I_A y_1'' + EI_2 y_2'' + E\alpha_2^2 I_A y_2'' + M\right)\delta y_2'\Big]_{-\frac{l}{2}}^{\frac{l}{2}} - \\
&-\Big[\left(EI_1 y_1''' + E\alpha_1^2 I_A y_1''' + E\alpha_1\alpha_2 I_A y_2'''\right)\delta y_1\Big]_{-\frac{l}{2}}^{\frac{l}{2}} - \\
&-\Big[\left(E\alpha_1\alpha_2 I_A y_1''' + EI_2 y_2''' + E\alpha_2^2 I_A y_2''' + S\right)\delta y_2\Big]_{-\frac{l}{2}}^{\frac{l}{2}} + \\
&+\int_{-\frac{l}{2}}^{\frac{l}{2}}\Big[EI_1 y_1^{IV} + E\alpha_1^2 I_A y_1^{IV} + E\alpha_1\alpha_2 I_A y_2^{IV} + K(y_1 - y_2) - p_1\Big]\delta y_1\,dz + \\
&+\int_{-\frac{l}{2}}^{\frac{l}{2}}\Big[E\alpha_1\alpha_2 I_A y_1^{IV} + EI_2 y_2^{IV} + E\alpha_2^2 I_A y_2^{IV} - K(y_1 - y_2) - p_2\Big]\delta y_2\,dz.
\end{aligned} \qquad (p)$$

The equation $\delta U = 0$ will be satisfied if each of the six terms vanishes. The two integrals will vanish if the terms in brackets are zero, which furnishes Euler's equations (16). The vanishing of the four other terms, at both boundaries $z = l/2$ and $z = -l/2$, furnishes 8 more conditions which are the boundary conditions of the problem.

Due to the definition of y_1 and y_2 as relative displacements, we have $y_2 = 0$ for $z = \pm l/2$; y_2 having a definite value at $z = \pm l/2$ means that the variation δy_2 at this point will be zero, and the fourth term in eq. (o) vanishes.

The values of δy_1, $\delta y_1'$ and $\delta y_2'$ at $z = \pm l/2$ do not vanish; the first three terms in Eq. (p) will vanish only if each of the expressions by which δy_1, $\delta y_1'$ and $\delta y_2'$ are multiplied will be zero at the boundaries $z = \pm l/2$; after rearrangement the boundary conditions, Eqs. (17), are obtained.

6. Expressions for longitudinal forces and horizontal and vertical shears.

The resultants N_1 and N_2 of the longitudinal stresses in deck house and hull are $N_1 = EA_1\epsilon_1$ and $N_2 = EA_2\epsilon_2$, or with reference to Eqs. (k) and (7a)

$$N_1 = \frac{EI_A}{a}\left(\alpha_1 y_1'' + \alpha_2 y_2''\right)$$
$$N_2 = -\frac{EI_A}{a}\left(\alpha_1 y_1'' + \alpha_2 y_2''\right) \qquad (q)$$

Defining by T the total shear force from the left end of the deck house to any point having the coordinate z, equilibrium requires

$$T = -N_1 = -\frac{EI_A}{a}\left(\alpha_1 y_1'' + \alpha_2 y_2''\right) \quad . \qquad (r)$$

The unit horizontal shear will be

$$\frac{dT}{dz} = -\frac{EI_A}{a}\left(\alpha_1 y_1''' + \alpha_2 y_2'''\right) . \qquad (s)$$

To obtain the expression for the vertical shear V_1 in the deck house, consider an element of the deck house of length dz, Fig. 15. Equilibrium of moments with respect to the centroid requires

$$V_1 - a\alpha_1 \frac{dT}{dz} = \frac{dM_1}{dz} .$$

Introducing $M_1 = -EI_1 y_1''$ and Eq. (s), we obtain

$$V_1 = -EI_1 y_1''' - E\alpha_1 I_A\left(\alpha_1 y_1''' + \alpha_2 y_2'''\right) = -E\left(I_1 + \alpha_1^2 I_A\right) y_1''' - \alpha_1\alpha_2 EI_A y_2''' \qquad (t)$$

Similarly the vertical shear in the hull is

$$V_2 = -\alpha_1\alpha_2 EI_A y_1''' - E\left(I_2 + \alpha_2^2 I_A\right) y_2''' \qquad (u)$$

7. Type of Stress Distribution if Cross Sections are not Constant.

It is possible to derive Eq. (B), (page 27) without any lengthy analysis, demonstrating that it applies even if the cross sections of hull and deck house vary and the bulk head stiffness is not constant. The one and only assumption which must be made is that Navier's theory is applicable to the hull and deck house separately.

According to this assumption the strains in the hull and in the deck house must vary linearly, and the stress distribution must therefore consist of straight lines, Fig. 16, between the values σ_1, σ_2 and σ_3 of the stresses at the top of the deck house, at deck level, and at the bottom of the hull, respectively. The internal stresses σ must be in equilibrium with the external loads, which are at any particular section: the moment M, and the longitudinal force N= o. We have therefore two conditions

$$\int_A \sigma\, dA = o\,, \qquad\qquad \int_A x\,\sigma\, dA = M \qquad\qquad (v)$$

The stresses σ in eqs. (v) can be expressed by the three values σ_1, σ_2 and σ_3 and without actually making this computation, we know that the two eqs. (v) will permit expression of the two stresses σ_2 and σ_3 as functions of the third, σ_1. It appears, therefore, that equilibrium alone restricts the possible stress distributions in such a way that all stresses are defined if one, σ_1, is given.

We want to demonstrate that all stress distributions which satisfy equations (v) can be expressed in the form

$$\sigma = \sigma_N + \Phi\, \Delta\, \sigma \qquad\qquad (w)$$

where σ_N and $\Delta\sigma_N$ are defined in Section 2, and Φ is a numerical factor which may have any value.

It is obvious that the stresses given by equation (w) must satisfy the equilibrium conditions (v) because the stress distribution (w) was determined as the actual one for some structure. On the other hand by varying the value of

Φ in eq. (w) the stress at the top of the deck house may be made equal to any given value, and as only one stress distribution exists which satisfies Eqs. (v) and for which the stress is σ_1 on top of the deck house, this stress distribution can be expressed by equation (w). This conclusion is valid whether the cross section of the vessel remains constant or not. The actual numerical value of Φ can, of course, not be obtained by this simple consideration.

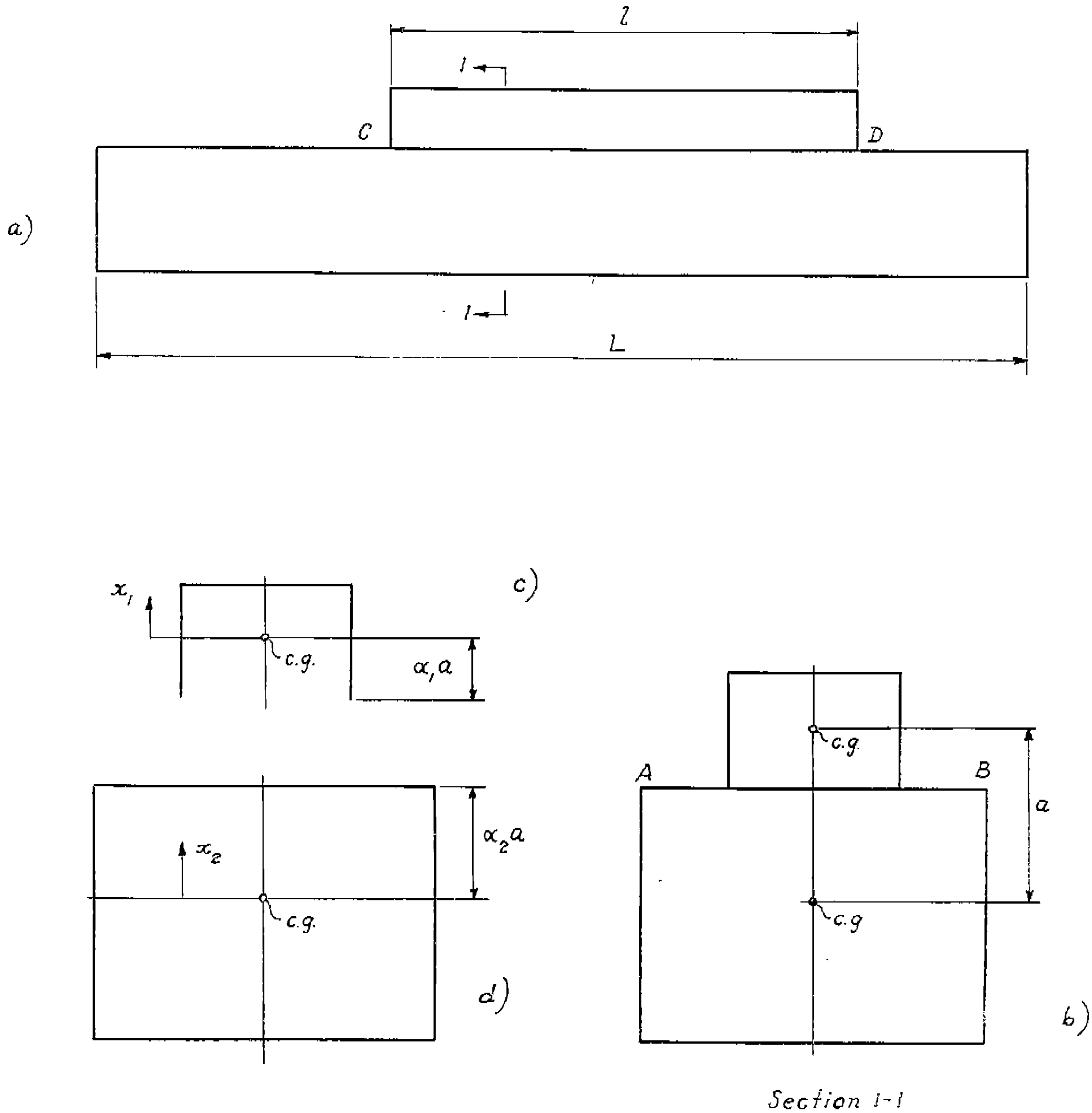
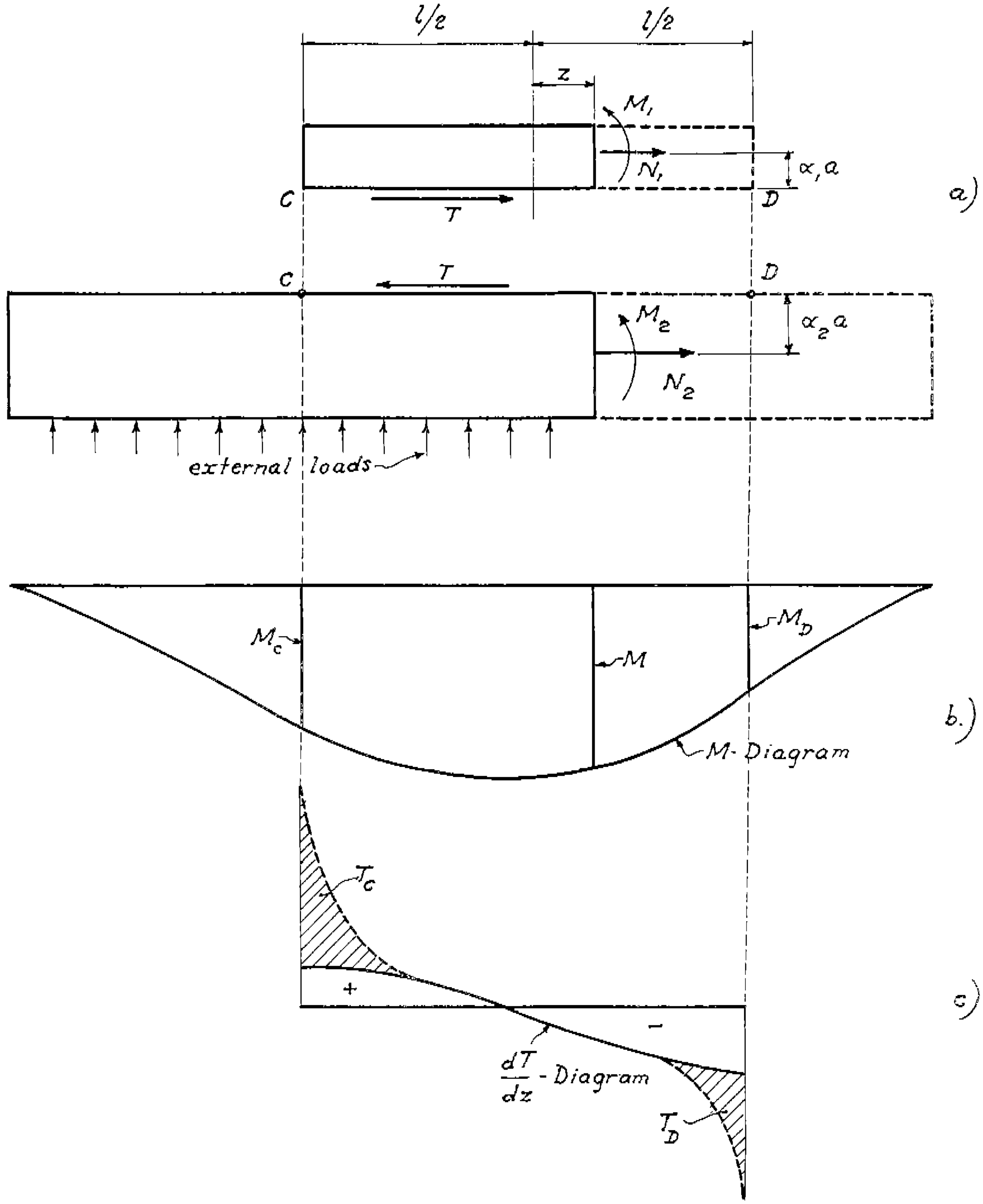


Fig. 1



l/2
l/2
z
M1
N1
α1 a
C
D
T
a)
C
T
D
M2
α2 a
N2
external loads
Mc
M
MD
b.)
M-Diagram
Tc
+
c)
−
dT/dz - Diagram
TD

Fig. 2

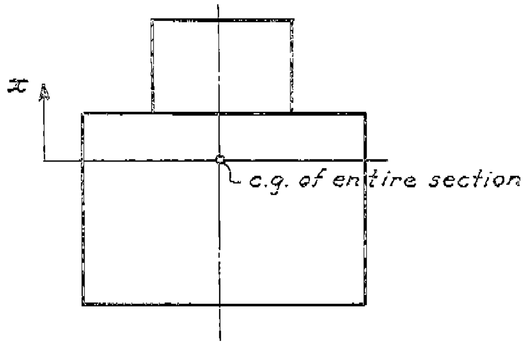


Fig. 3

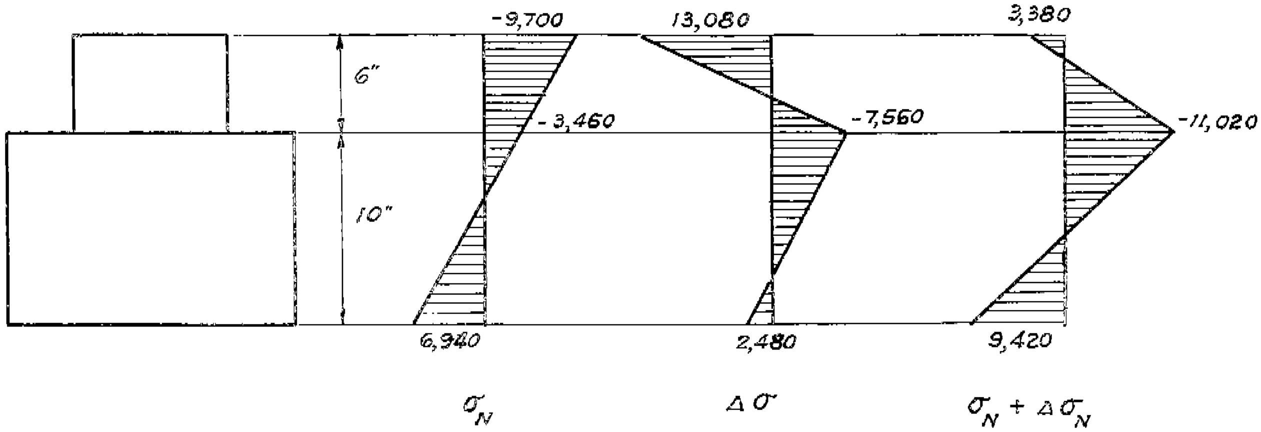


Fig. 4

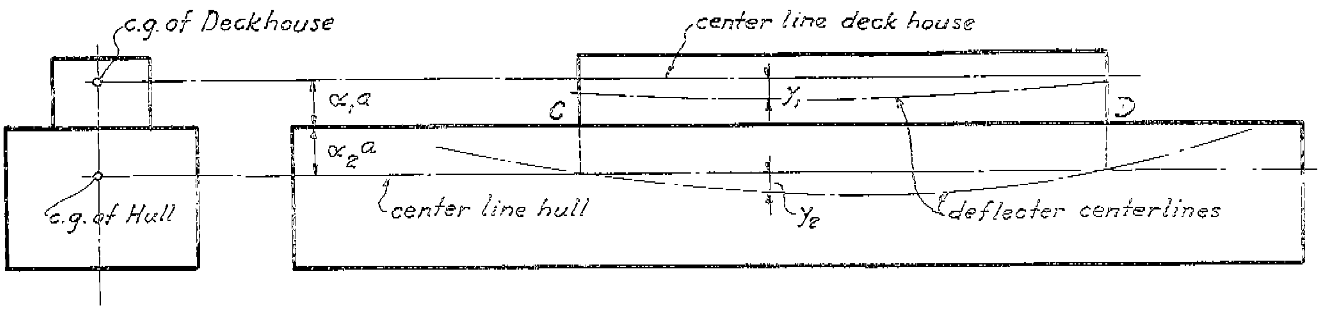


Fig. 5

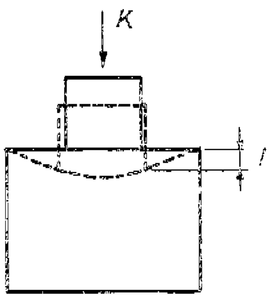


Fig. 6

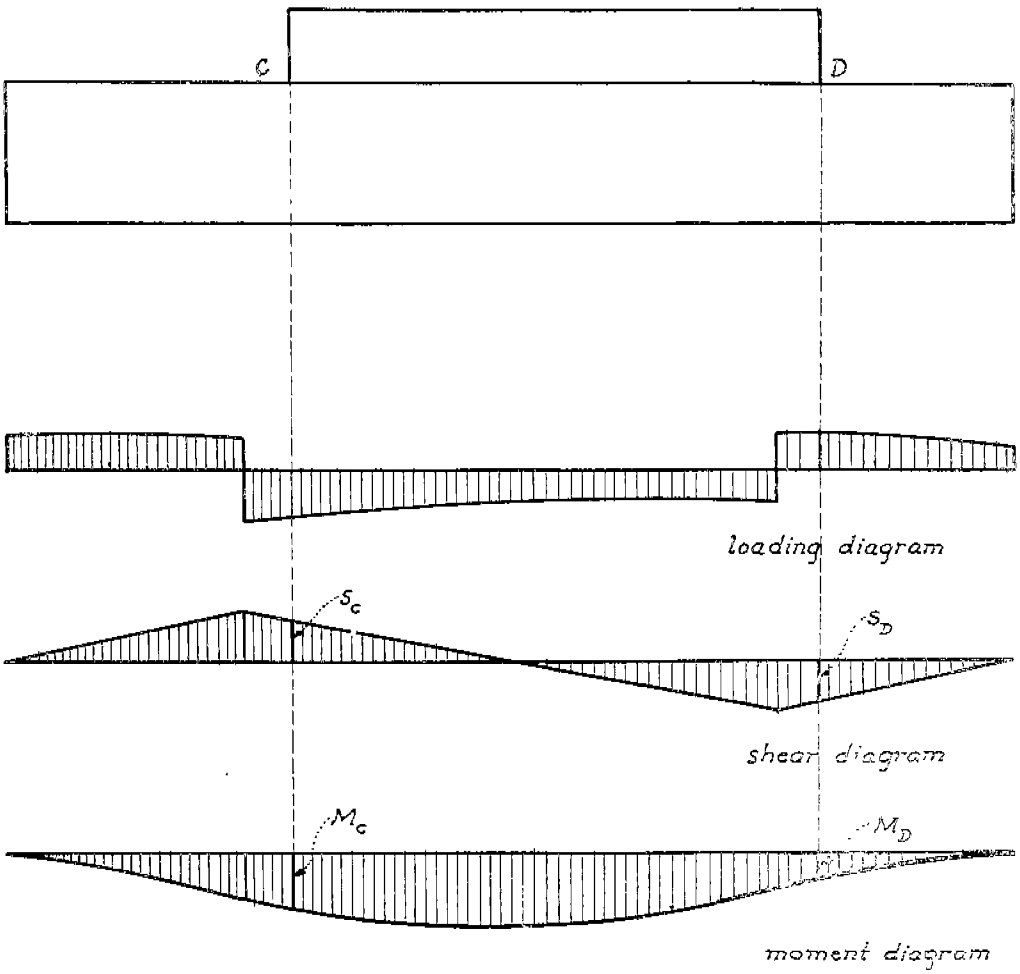


Fig. 7

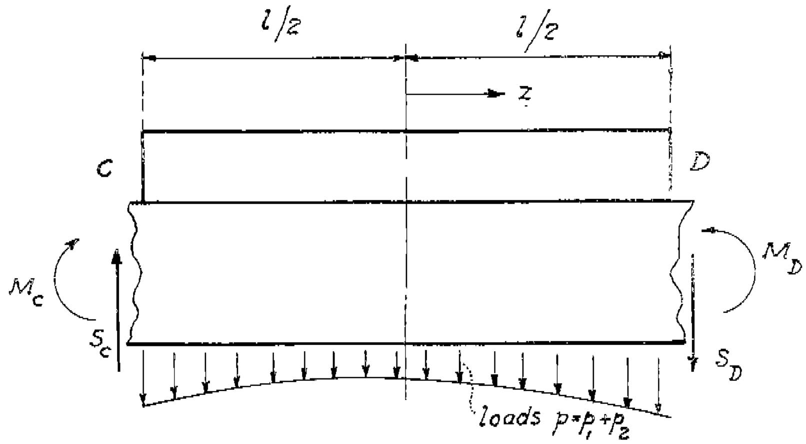
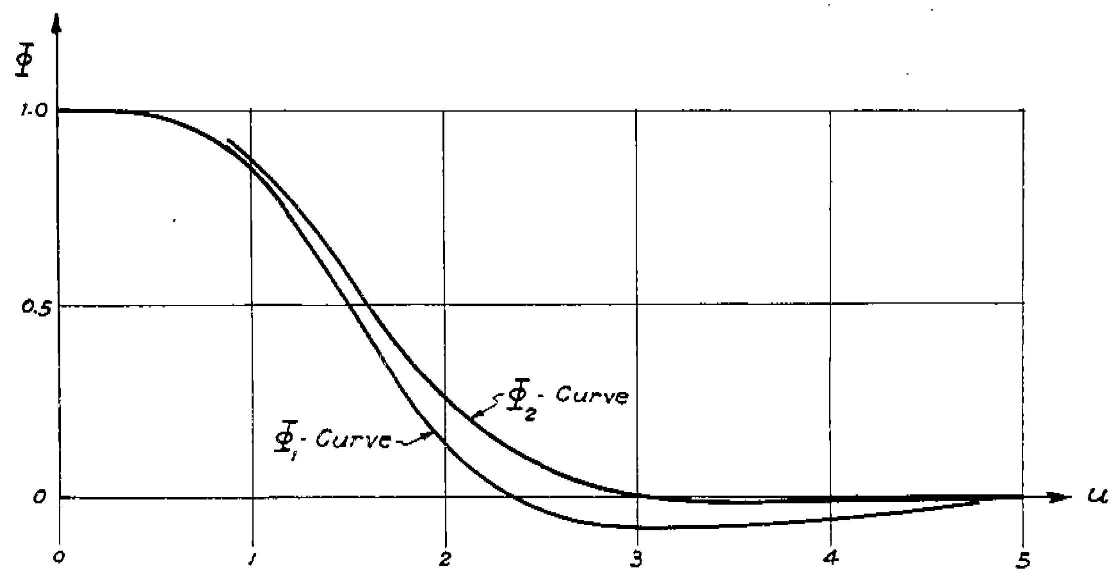
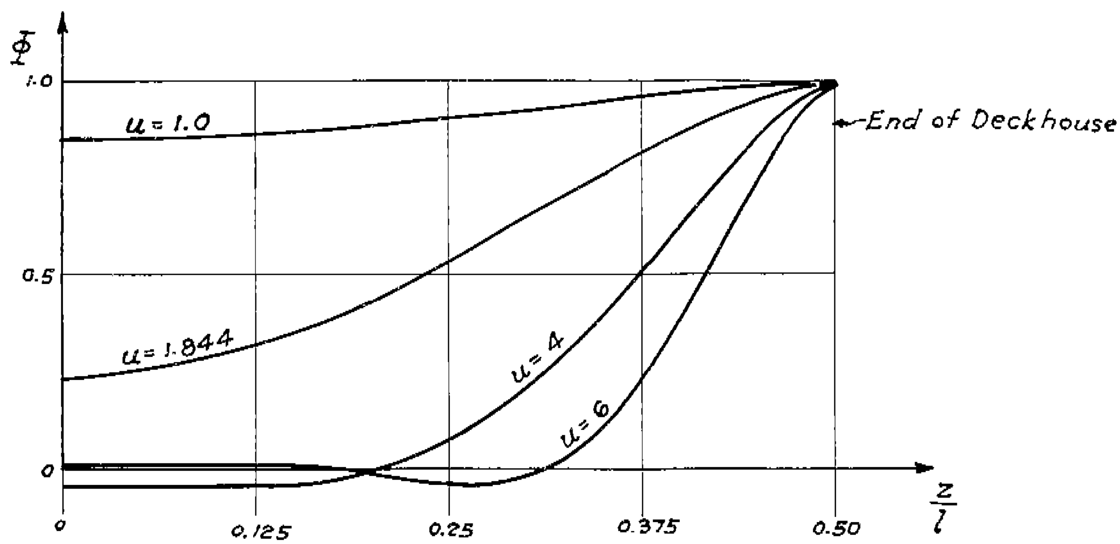


Fig. 8



Φ
1.0
0.5
0
Φ_2 - Curve
Φ_1 - Curve
u
0
1
2
3
4
5

Fig. 9



Φ
1.0
0.5
0
$u = 1.0$
End of Deckhouse
$u = 1.844$
$u = 4$
$u = 6$
0
0.125
0.25
0.375
0.50
$\frac{z}{l}$

Fig. 10

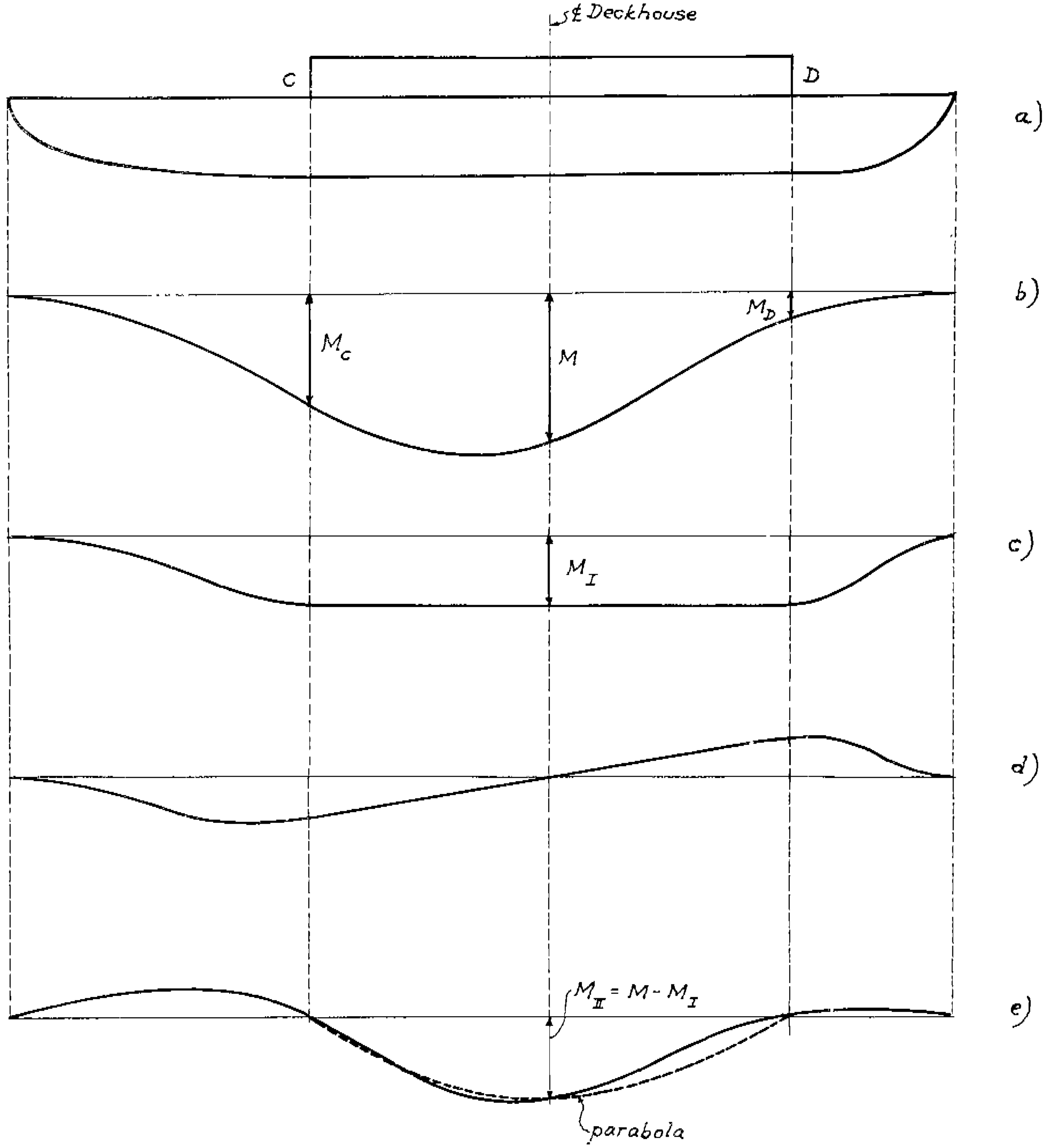
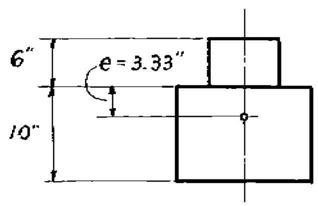


Fig. 11


6"
e = 3.33"
10"

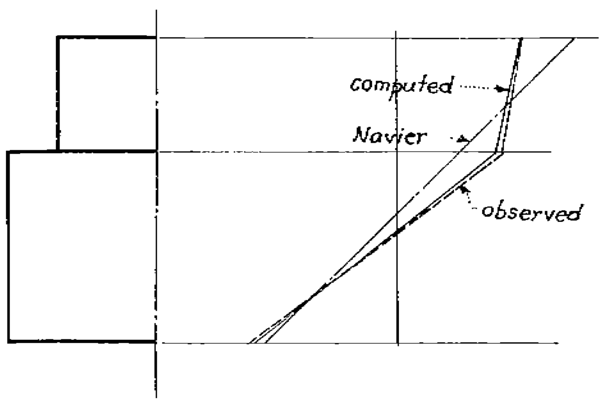

computed
Navier
observed

Fig. 13

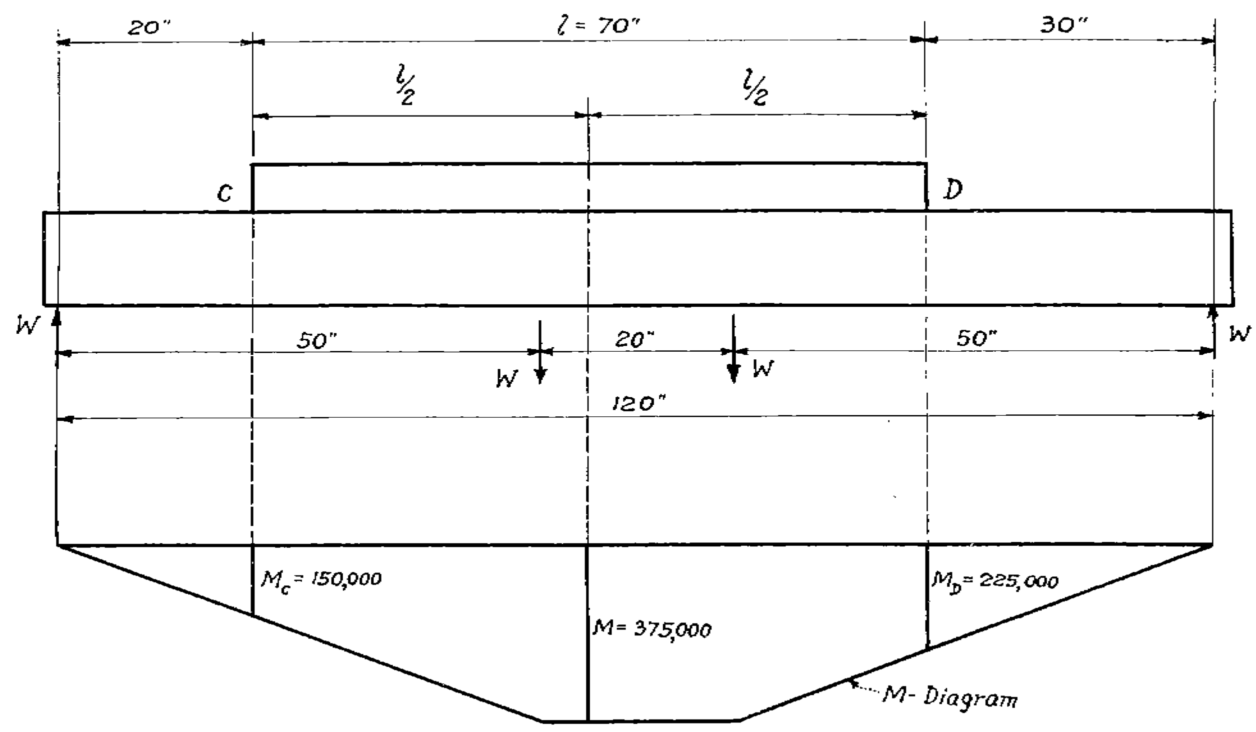

20"
l = 70"
30"
l/2
l/2
C
D
W
50"
20"
50"
W
W
W
120"
M_c = 150,000
M = 375,000
M_D = 225,000
M- Diagram

Fig. 12

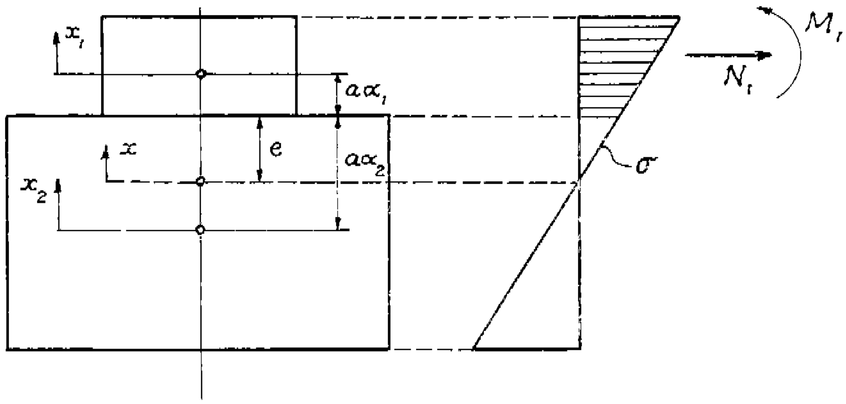


Fig. 14

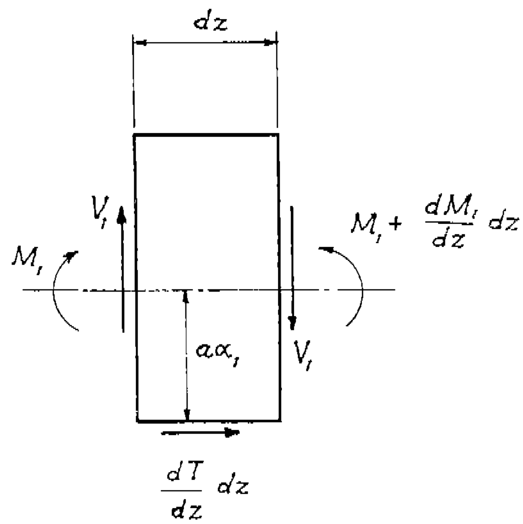


Fig. 15

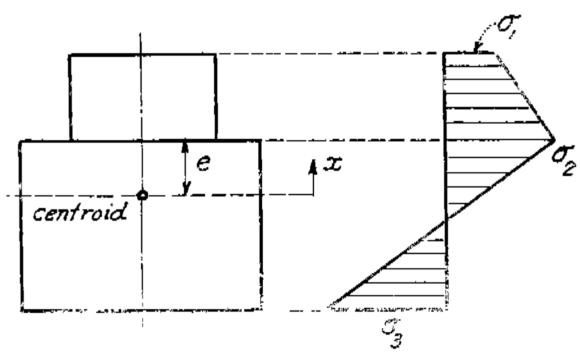


Fig. 16