

FINAL REPORT

(Project SR-119)

on

**WELDED REINFORCEMENT OF OPENINGS IN
STRUCTURAL STEEL TENSION MEMBERS**

by

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D. VASARHELYI and R. A. HECHTMAN
University of Washington

for

SHIP STRUCTURE COMMITTEE

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The Secretary of the Treasury

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Bureau of Ships, Dept. of Navy
Military Sea Transportation Service, Dept. of Navy
United States Coast Guard, Treasury Dept.
Maritime Administration, Dept. of Commerce
American Bureau of Shipping

Address Correspondence To:

Secretary
Ship Structure Committee
U. S. Coast Guard Headquarters
Washington 25, D. C.

SERIAL NO. SSC-75
BuShips Project NS-731-034

MARCH 21, 1955

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SECRETARY
SHIP STRUCTURE COMMITTEE
U. S. COAST GUARD HEADQUARTERS
WASHINGTON 25, D. C.

March 21, 1955

Dear Sir:

As part of its research program related to the improvement of hull structures of ships, the Ship Structure Committee has sponsored an investigation on the welded reinforcement of openings in structural steel members at the University of Washington. Herewith is a copy of the Final Report, SSC-75, of the investigation, entitled "Welded Reinforcement of Openings in Structural Steel Tension Members" by D. Vasarhelyi and R. A. Hechtman.

Comments concerning this report are solicited and should be addressed to the Secretary, Ship Structure Committee.

This report is being distributed to those individuals and agencies associated with and interested in the work of the Ship Structure Committee.

Yours sincerely,



K. K. Cowart
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure
Committee

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(Project SR-119)

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STRUCTURAL STEEL TENSION MEMBERS

by

D. Vasarhelyi and R. A. Hechtman

University of Washington

under

Department of the Navy
Bureau of Ships Contract NObs-50238
BuShips Project No. NS-731-034

for

SHIP STRUCTURE COMMITTEE

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WELDED REINFORCEMENT OF OPENINGS IN STRUCTURAL STEEL TENSION MEMBERS

I. SYNOPSIS

The purpose of this research has been the investigation of some of the geometric factors which affect the performance of plates with reinforced openings, such as the shape of opening, the type and amount of reinforcement, and the width and thickness of the body plate. Some of the tests were repeated at low temperatures to bring in the factor of cleavage fracture. In the course of the project, a considerable amount of work was directed toward determining the nature of the plastic flow which precedes the initiation of fracture and the conditions which precipitate fracture. Specific recommendations based on the findings of the investigation have been made with respect to the design of openings and their reinforcement. Many of the results of the research are applicable to welded structures in general.

The extensive test work required the use and development of somewhat new research methods and techniques. The applicability of Nadai's octahedral strain energy method^(1,4), the plastic stress computation^(2,6), and the resistance-wire grid system of measurements for plastic strain studies might be mentioned as particularly useful.

II. INTRODUCTION

1. Problem of the opening in a structural member. The introduction of an opening in a structural member under tension decreases its effectiveness by reducing its net cross section area and producing a region of stress concentration. The purpose of the reinforcement is the restoration to the greatest possible degree of the characteristics of the member which existed before the opening was present. Some of the more important factors which must be considered in the development of design standards for the welded reinforcement of openings are:

- a) Shape of the opening.
- b) Cross section shape of the reinforcement and the notches present in welded reinforcement because of abrupt changes in section.
- c) Deformability of the region around the opening as it affects the action of the whole member as part of a statically indeterminate structure such as a ship.
- d) Mechanical properties of the steel.
- e) Nature of the loading.
- f) Low-temperature cleavage fracture.

This investigation has been concerned with the first three factors which are related principally to the geometric shape of the opening and its reinforcement.

The load carrying capacity of a member containing an opening can be made equal to that of the intact member by restoration of cross sectional area through suitable reinforcement around the opening. However, as this report will show, only a fraction of the energy absorbing capacity in the plastic range of a member is restored by such reinforcement because the reinforcement cannot improve the stress distribution sufficiently to remove the stress raising effect of the opening. The greatest capacity to absorb energy would exist in a member in which all elements were stressed uniformly up to the point where failure would begin. A plain plate with parallel sides loaded concentrically represents such a member. In contrast, an opening, because of its stress raising effect which results in a nonuniform distribution of strain, prevents the most efficient utilization of the potential capacity of the material to deform plastically. The best that any good design of reinforcement for an opening can assure is the recovery of a fraction of the energy absorbing capacity of the plate without an opening. Since the tendency towards brittle fracture at low temperatures is closely related to the capacity of a structural steel member to absorb energy, the degree to which good design can bring about this restoration is very important in certain cases.

Another point to be considered is that the addition of common types of welded reinforcement increases the thickness and rigidity of the member around the opening and introduces abrupt changes in cross section. It is quite possible to increase the stress raising effect of an opening by the addition of reinforcement and thereby worsen the condition rather than improve it.

Thus it may be seen that the design of an opening in a structural member and the reinforcement therefor is not a simple problem. It is one in which the deformation of the member, as well as its ability to carry stress, must be considered, for only by adequacy in both of these respects can the member carry its proper share of the load as a part of the structure and have the capacity to absorb sufficient energy to prevent fracture in the face of adverse conditions. The objective of the design must be greater efficiency in transmitting the applied forces through the member. Because openings in structural members of all types, including the details of ships, have been the source of many failures, it may be assumed in this problem that the only satisfactory design for an opening is the one which provides the greatest ability to carry load and absorb the energy of deformation.

The purpose of this project has been an investigation of welded reinforcement for openings in plate members to determine

ways in which the design of openings and their reinforcement may be improved. The factors which were varied were the body plate thickness and width, the shape of the opening, the type and amount of reinforcement, and the testing temperature. Forty-one large plate specimens, each having a centrally located opening with or without reinforcement, and two plain plates without an opening were tested. Most of these tests were made at room temperature and resulted in shear fractures. A few specimens were tested at temperatures sufficiently low to produce brittle cleavage fractures. Since failure occurs subsequent to general yielding of the material, an investigation of the plastic deformation which preceded fracture was carried out to establish the manner in which this deformation was related to the geometry of the specimen and the testing temperature.

While a considerable amount of research was accomplished in the course of the project, it did not lessen the need for more work in the future because this problem is a large one and only few variables have been investigated--and none of these exhaustively.

Detailed descriptions of these tests in previous progress reports and papers⁽¹⁻⁻⁶⁾, listed in the References, have been summarized in this final report. The reader is directed to these references for information not presented herein, such

as additional data, the method of testing, and the theoretical methods of analysis.

2. General background of the problem. The problem of openings in plates has been dealt with in a number of papers, and solutions⁽⁷⁻⁻¹²⁾ are available for the elastic stress distribution for cases exactly the same as or similar to those investigated here. These solutions assume plane stress conditions, which actually are not realized or even approached in the case of many types of reinforcement, especially those with an appreciable width in the direction of the body plate thickness. The assumptions made in these solutions concerning the interaction of the reinforcing ring and the body plate are also important. For example, when the reinforcing ring becomes sufficiently rigid, it begins to act in the manner of a rigid inclusion in the body plate⁽¹⁵⁾. In this experimental investigation no particular correlation was found between the parameters developed by the theory of elasticity and the ultimate strength and energy absorption to maximum load of the plates with openings.

Theoretical analyses⁽¹¹⁾ based on the theory of elasticity have shown that reinforcement of an opening in a plate cannot restore the strength to that of the prime plate. Recent analyses⁽¹³⁾ based on the theory of plasticity indicate that yield strength can be re-established with well designed reinforcement, provided that the material is ductile.

Only a small number of theoretical solutions are available for the problem of the reinforced opening, primarily because of its difficulty. The simplifying assumptions sometimes necessary to permit a solution for this case often impair the usefulness of the solution.

III. TESTS OF PLATES WITH OPENINGS

1. Specimen material and specimens. All specimens were fabricated from the same heat of plain carbon semikilled steel, a grade meeting ASTM Designation A 7-49T, in the as-rolled condition and called "Steel U as-Rolled" in this report. Plate thicknesses of 1/4, 1/2, 3/4, and 1 inch were used. Their mechanical properties are shown in Table I. The plates used in the fabrication of each specimen are listed in Table III. The transition temperature as determined from one plate of each thickness was as follows:

Plate Thickness Inches	Tear Test Transition Temperature °F	Temperature for 15 ft-lb Energy (Charpy Keyhole Test) °F	Average ASTM Grain Size
1/4	-40		8
1/2	40	-24	6
1	120		5

TABLE I

MECHANICAL PROPERTIES OF PLATES OF DIFFERENT THICKNESS, SEMIKILLED STEEL U AS ROLLED

Plate No.	Thick-ness In.	Temp. of Tensile Test °F	Used for Spec. No.	Direction of Test, Parallel or Transverse to Rolling	Tensile Properties					Tear Test Trans. Temp. °F
					Upper Yield Point psi	Ultimate Strength psi	Elong. in 8 in* per cent	Reduction of Area per cent	Poisson's Ratio in Plastic Range	
<u>Room Temp. Tests</u>										
1	1/2	Room Temp.	35	P	36,600	62,400	28	56	0.45	
3	1/2		69,70	P	34,900	61,200	33*	62	0.47	
4	1/2		38	P	34,900	60,200	32*	61	0.45	40
				T	35,200	61,500	†	52		
5	1/2		52,56	P	39,900	61,900	27	45		
6**	1/2		49,50,51	P	38,800	59,500	27	43		
10	1		55,56,70,71	P	32,800	61,100	33	56		120
15	1/4		32,34,99	P	44,200	63,400	28	45		
16	1/4		17,23,31	P	44,100	65,300	29	51		
17	1/4		8,19,21	P	44,300	65,200	29	52		
18	1/4	1,7,10	P	45,100	65,800	29	51		-40	

* Percentage elongation in 12 inches where noted.

** Tensile specimen made from normalized sample out of permanently strained specimen.

† Tensile specimen broke outside of gage length.

All tear test specimens of full-plate thickness.

Chemical Composition

C	Mn	P	S	Si	Ni	Cr	Cu	Mn/c
0.22	0.47	0.010	0.028	0.05	0.07	Tr.	0.066	2.14

TABLE I (Cont.)

MECHANICAL PROPERTIES OF PLATES OF DIFFERENT THICKNESS, SEMIKILLED STEEL U AS ROLLED

Plate No.	Thick-ness In.	Temp. of Tensile Test °F	Used for Spec. No.	Direction of Test, Parallel or Transverse to Rolling	Tensile Properties				Tear Test Trans. Temp. °F
					Upper Yield Point psi	Ultimate Strength psi	Elong. in 8 in* per cent	Reduction of Area per cent	
19	1/4	Room Temp. ↓	9,20,22	P	44,000	65,900	28	52	
20	1/4		5,14,16	P	44,700	66,000	28	50	
21	1/4		13,15	P	44,500	66,000	29	50	
22	1/4		11,12,18	P	43,800	65,600	29	50	
23	1/4		4,6	P	45,400	66,100	30	50	
24	1/4		2,3	P	44,800	65,800	29	50	
25	1/2		55A	P	36,500	61,800	31	54	
26	1/2		37,55	P	36,900	61,800	35*	62	0.48
				T	36,500	61,100	31*	54	0.46
<u>Low Temperature Tests</u>									
1	1/2	-46	96	P	42,300	69,900	23*	51	0.48
3	1/2	-46	71	P	44,300	70,900	29*	56	0.50
	1/2	-46		T	43,900	72,000	†	49	0.51
4	1/2	-20	38	P	42,500	68,500	32*	60	0.51
		-20		T	38,900	64,100	†	50	0.46
24	1/4	-46		P	55,100	73,600	28*	50	40

TABLE II
DESCRIPTION OF SPECIMENS

Spec. No.	Opening		Size of Reinforcement	Percentage of Reinf.	Cross-Section Area		Gage Length In	Test Temp. F
	Shape	Corner Radius In.			Gross In ²	Net In ²		
<u>36" x 1/4" Plain Plates (No Opening)</u>								
1				100	9.07	9.07	36	81
23				100	9.14	9.14	36	76
<u>36" x 1/4" Plates with Unreinforced Openings</u>								
2	Circular			0	9.21	6.92	36	76
3	Square	1/32		0	9.18	6.82	36	72
4	Square	1-1/8		0	9.15	6.87	36	78
<u>36" x 1/2" Plates with Unreinforced Openings</u>								
37	Square	1-1/8		0	18.00	13.50	36	76
38A	Square	1-1/8		0	18.00	13.50	36	0
38	Square	1-1/8		0	18.00	13.50	36	-20
69	Circular			0	18.00	13.50	36	76
95	Square	1/32		0	18.00	13.50	36	76
96	Square	1/32		0	18.00	13.50	36	-46
<u>36" x 1/4" Plates with Openings Reinforced by a Face Bar</u>								
5	Circular		2 x 1/4	40	9.11	7.76	36	74
6	Circular		1 x 1/4	17	9.15	7.25	36	73
7	Square	1/4	2 x 1/4	40	9.11	7.72	36	75
8	Square	3/16	1 x 1/4	16	9.02	7.13	36	74
9	Square	1-1/8	2 x 1/4	40	9.13	7.74	36	72
10	Square	1-1/8	1 x 1/4	16	9.15	7.22	36	85
99	Square	1-1/8	2 x 1/4	40	9.00	7.74	36	-46
31	Square	1-1/8	1 x 1/4	16	9.00	7.22	36	-46
<u>48" x 1/2" Plates with Openings Reinforced by a Face Bar</u>								
49	Square	1-1/8	2 x 1/2	33	24.32	21.24	48	70
50	Square	1-1/8	2 x 1/2	33	24.32	21.24	48	-20

See Fig. 1 for location of gage length and dimensions of body plate, and Fig. 2 for details of reinforcement.

TABLE II

DESCRIPTION OF SPECIMENS (Cont.)

Spec. No.	Opening		Size of Reinforcement	Percentage of Reinf.	Cross-Section Area		Gage Length	Test Temp.
	Shape	Corner Radius			Gross	Net		
		In			In ²	In ²	In	F
<u>36" x 1/4" Plates with Openings Reinforced by a Single Doubler Plate</u>								
11	Circular		18"D x 1/4	102	9.11	9.13	36	75
12	Circular		13 1/2"D x 1/4	50	9.14	7.99	36	73
13	Square	1/32	18 x 18 x 1/4	104	9.17	9.21	36	76
14	Square	1/32	13 1/2 x 13 1/2 x 1/4	51	9.14	8.02	36	71
15	Square	1-1/8	18 x 18 x 1/4	103	9.13	9.16	36	76
32	Square	1-1/8	18 x 18 x 1/4	103	9.00	9.16	36	-46
16	Square	1-1/8	13 1/2 x 13 1/2 x 1/4	52	9.13	8.01	36	73
<u>48" x 1/2" Plates with Openings Reinforced by a Single Doubler Plate</u>								
51	Square	1-1/8	18 x 18 x 1/2	96	24.17	24.01	48	74
52	Square	1-1/8	18 x 18 x 1/2	96	24.00	24.01	48	-46
<u>36" x 1/4" Plates with Openings Reinforced by an Insert Plate</u>								
17	Circular		12-3/4"D x 1/2	39	9.11	7.71	36	74
18	Circular		10-1/2"D x 1	50	9.13	8.08	36	75
19	Square	1/32	15"D x 1/2	33	9.04	7.55	36	76
20	Square	1/32	12-3/4 x 12-3/4 x 1/2	39	9.13	7.72	36	72
21	Square	1-1/8	15"D x 1/2	62	9.02	8.17	36	77
34	Square	1-1/8	15"D x 1/2	62	9.00	8.17	36	-46
22	Square	1-1/8	12-3/4 x 12-3/4 x 1/2	39	9.04	7.66	36	73
<u>48" x 1/2" Plates with Openings Reinforced by an Insert Plate</u>								
55	Square	1-1/8	15"D x 1/2	66	23.63	22.09	48	70
55A	Square	1-1/8	15"D x 1/2	67	23.58	22.10	48	69
56	Square	1-1/8	15"D x 1/2	66	24.00	22.09	48	-46
70	Square	1-1/8	12-3/4 x 12-3/4 x 1	39	24.00	21.38	48	76
71	Square	1-1/8	12-3/4 x 12-3/4 x 1	39	24.00	21.38	48	-46
<u>36" x 1/4" Plate with Opening Reinforced by a Combination of Face Bar and Insert Plate</u>								
85	Square	1-1/8		78	9.00	8.50	36	76

TABLE III

LIST OF PLATES USED FOR FABRICATION OF EACH SPECIMEN

Specimen Number	Plate Number Used for	
	Body Plate*	Reinforcement
1	18	--
2	24	--
3	24	--
4	23	--
5	20	20
6	23	23
7	18	18
8	17	17
9	19	19
10	18	18
11	22	21
12	22	21
13	21	21
14	20	21
15	21	21
16	20	21
17	16	25
18	22	10
19	17	25
20	19	25
21	17	25
22	19	25
23	16	--
31	16	21
32	15	21
34	15	26
37	26	--
38A	4	--
38	4	--
49	6	25
50	6	25
51	6	25
52	5	26
55	26	10
55A	25	10
56	5	10
69	3	--
70	3	10
71	3	10
85	11	11, 1
95	--	--
96	1	--
99	15	21

*Mechanical properties of plates are given in Table I. Sketches of specimens are in Figs. 1 and 2.

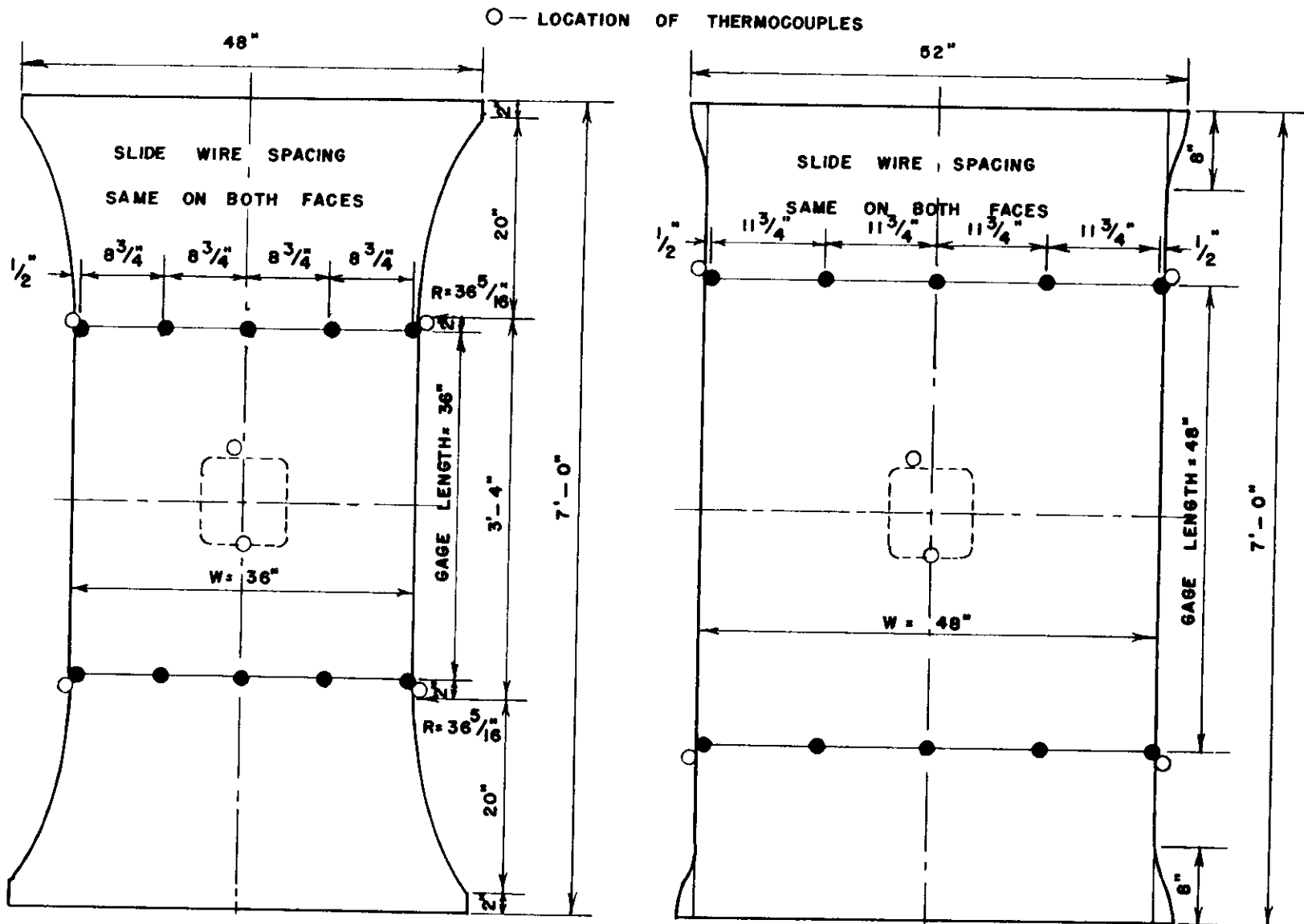


FIG. 1. DETAILS OF BODY PLATES OF 36" X 1/4", 36" X 1/2" AND 48" X 1/2" SPECIMENS.

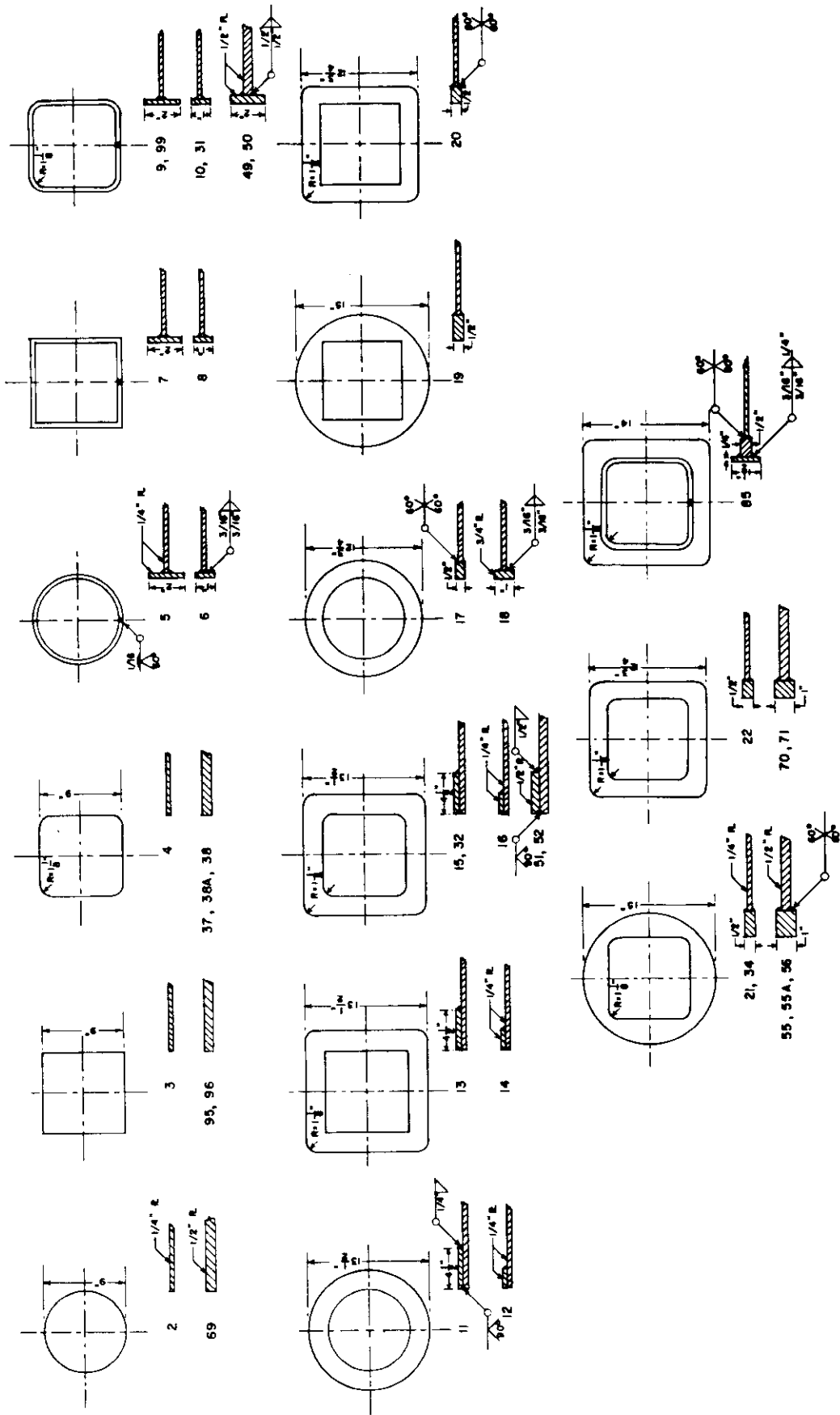


FIG. 2. DETAILS OF OPENING AND REINFORCEMENT

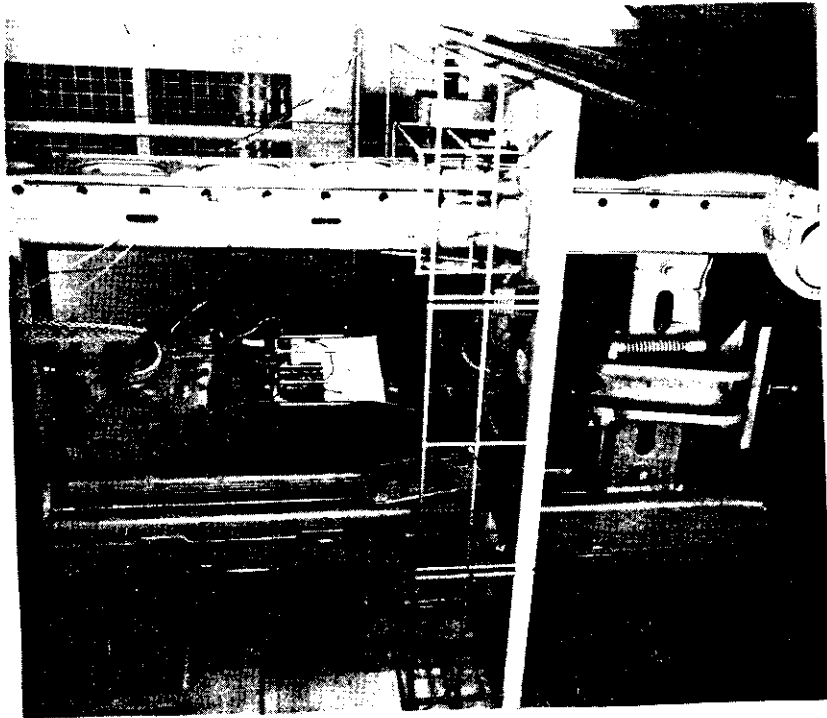
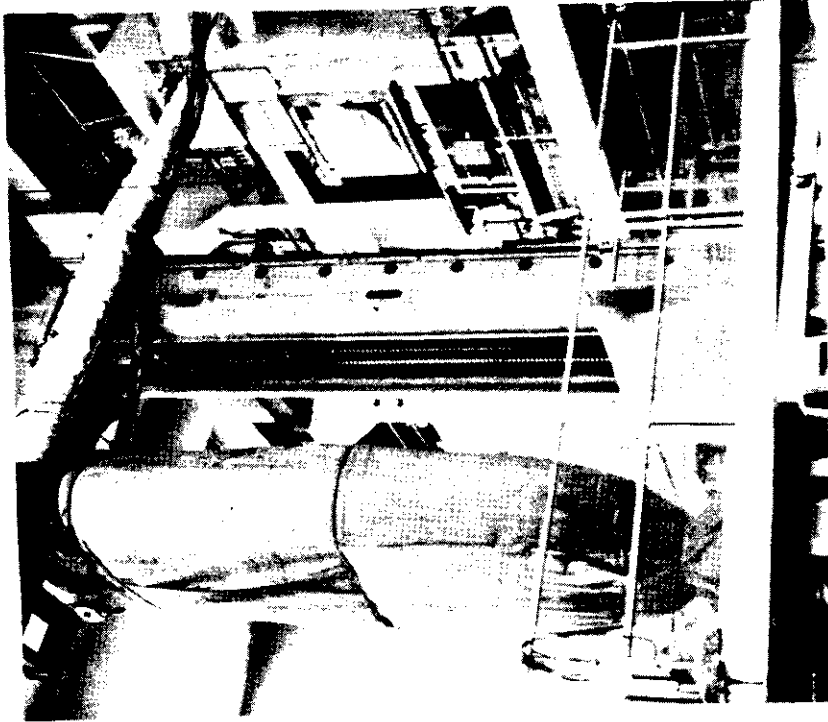


FIG. 3. Specimens for Room and Low Temperature Tests in 2,400,000-lb. Testing Machine

The details of the specimens, including the size of the body plate, the shape of opening, and the type of reinforcement are given in Figs. 1 and 2 and Table II. Three sizes of body plates were used: 36-in. by 1/4-in., 36-in. by 1/2-in., and 48-in. by 1/2-in. The edges of the specimens were flame cut and ground smooth. The reinforcement was welded in accordance with U. S. Naval General Specifications, Appendix 5 (Navships 451). The electrodes met AWS Specification E-6010. No specimen was tested until at least seven days after the welding was completed.

2. Method of testing. All specimens were loaded as shown in Fig. 3 in a 2,400,000-lb. universal hydraulic testing machine with their longitudinal centerline parallel to the rolling direction of the plate. Three types of gaging were used on all specimens to make the following measurements: the overall elongation by slide-wire resistance gages on a gage length equal to the width of the plate and straddling the area of the opening, the strains in the elastic range on one quadrant of the plate by SR-4 strain gages, and the temperature of the plates by thermocouples. The deformation in the plastic range of an area containing the opening was intensively studied in the case of seven plates^(2,4). The elongations were measured by a slide-wire gage grid system specially devised for those tests. The specimens for the low-temperature tests were

enclosed in an insulated bag through which chilled air was circulated to bring the temperature of the plate to as low as -46°F , as shown in Fig. 3.

3. Definition of terminology. Some terms used in this report are defined below. The elongations measured over a gage length equal to the plate width at five points across the width as shown in Fig. 1 were averaged to give the average elongation. The term "load at general yielding of the specimens" refers to the load at the point where a definite elbow appeared in the plot of the total load on the plate against the average elongation. The area under this curve, or any portion of it, represented the energy absorption of the specimen up to the point under consideration. Two values of the energy absorption have been reported, the energy to ultimate load and the energy to failure.

The ultimate load (the maximum load sustained by the specimen) was divided by the original net cross section area of the specimen to give the maximum average net stress or ultimate strength of the plates. The three shapes of opening are referred to as circular, square with rounded corners, and square; and the plates without openings as plain plates.

The unreinforced plates with openings were considered as having zero percentage of reinforcement. For reinforced plates the percentage of reinforcement was computed as the ratio in

STRENGTH AND ENERGY ABSORPTION OF SPECIMENS

Spec. No.	Opening		Percent Reinf. In	Test Temp. F	General Yielding			Ultimate Strength			Energy Absorption in 1000's in-lb to Ultimate Failure Load	
	Shape	Corner Radius			Load lbs	Average Stress		Load lbs	Average Stress		Ultimate Load	Failure
						Gross psi	Net psi		Gross psi	Net psi		
<u>Plain Plates (36" x 1/4")</u>												
1			100	81	380,000	42,000	42,000	585,500	65,390	65,390	4,018	6,276
23			100	76	390,000	43,300	43,300	583,000	64,780	64,780	4,062	6,779
<u>Plates with Unreinforced Openings (36" x 1/4")</u>												
2	Circular		0	76	291,500	32,400	43,200	440,000	48,900	65,150	1,136	1,164
3	Square	1/32	0	72	292,000	32,500	43,250	357,500	39,800	52,900	338	538
4	Square	1-1/8	0	78	292,000	32,500	43,250	421,000	46,700	62,350	717	899
<u>Plates with Unreinforced Openings (36" x 1/2")</u>												
37	Square	1-1/8	0	76	450,000	25,000	33,300	800,000	44,500	59,300	1,700	2,179
38A	Square	1-1/8	0	0	500,000	27,800	37,000	898,000	49,900	66,500	2,890	3,470
38	Square	1-1/8	0	-20	500,000	27,800	37,000	915,000	50,800	67,700	2,778	2,778
69	Circular		0	76	500,000	27,800	37,000	845,000	47,000	62,500	1,739	2,533
95	Square	1/32	0	76	477,500	26,500	35,400	710,000	39,400	52,600	1,100	1,597
96	Square	1/32	0	-46	550,000	30,600	40,700	648,000	36,000	48,000	486	486
<u>Plates with Openings Reinforced by a Face Bar (36" x 1/4")</u>												
5	Circular		40	74	324,000	36,000	42,500	517,000	57,400	67,800	1,277	1,420
6	Circular		17	73	324,000	36,000	45,500	457,000	50,800	64,200	725	910
7	Square	1/4	40	75	322,000	35,800	42,230	397,000	44,100	52,070	422	750
8	Square	3/16	16	74	288,000	32,000	40,420	391,500	43,500	54,950	447	780
9	Square	1-1/8	40	72	319,000	35,500	41,840	451,000	50,100	59,150	747	1,063

Where the energy to ultimate load is slightly larger than the energy to failure, the difference represents elastic recoil of the specimen during fracture.

TABLE IV (Cont.)

STRENGTH AND ENERGY ABSORPTION OF SPECIMENS

Spec. No.	Opening Shape	Opening Corner Radius	Percent Reinf.	Test Temp.	General Yielding			Ultimate Strength			Energy Absorption in 1000's in-lb to Ultimate Failure Load	
					Load	Average Stress Gross	Average Stress Net	Load	Average Stress Gross	Average Stress Net		
		in		F	lbs	psi	psi	lbs	psi	psi		
<u>Plates with Openings Reinforced by a Face Bar (36" x 1/4") (cont.)</u>												
10	Square	1-1/8	16	75	313,000	43,930	48,930	467,000	51,900	65,540	1,214	1,504
99	Square	1-1/8	40	-46	340,000	37,800	44,000	507,000	56,400	65,500	1,062	1,019
31	Square	1-1/8	16	-46	364,000	40,400	50,400	527,000	58,600	73,000	1,857	1,880
<u>Plates with Openings Reinforced by a Face Bar (48" x 1/2")</u>												
49	Square	1-1/8	33	70	740,000	30,400	31,800	1,255,000	51,600	59,000	3,510	4,710
50	Square	1-1/8	33	-20	880,000	36,600	41,500	1,410,000	58,800	66,800	5,892	5,610
<u>Plates with Openings Reinforced by a Single Doubler Plate (36" x 1/4")</u>												
11	Circular		102	75	360,000	40,050	40,050	555,000	61,670	61,670	1,358	1,569
12	Circular		50	73	331,500	36,900	42,100	488,000	54,200	62,000	771	983
13	Square	1/32	104	76	337,500	37,500	37,500	451,500	50,170	50,170	387	728
14	Square	1/32	51	71	300,000	33,300	38,100	406,000	45,100	51,600	328	621
15	Square	1-1/8	103	76	362,000	40,220	40,220	522,500	58,060	58,060	729	1,099
32	Square	1-1/8	103	-46	441,000	49,000	48,100	548,000	60,900	59,800	894	1,104
16	Square	1-1/8	52	73	300,000	33,300	38,100	487,000	54,100	61,900	779	1,154
<u>Plates with Openings Reinforced by a Single Doubler Plate (48" x 1/2")</u>												
51	Square	1-1/8	96	74	770,000	31,900	32,100	1,385,900	57,400	57,700	4,730	5,360
52	Square	1-1/8	96	-46	950,000	39,600	39,600	1,460,000	60,800	60,800	4,303	4,187

TABLE IV (Cont.)

STRENGTH AND ENERGY ABSORPTION OF SPECIMENS

Spec. No.	Opening		Percent Reinf.	Test Temp. F	General Yielding			Ultimate Strength			Energy Absorption in 1000's in-lb to Ultimate Failure Load	
	Shape	Corner Radius in.			Load lbs	Average Stress		Load lbs	Average Stress		Ultimate Load	Failure
						Gross psi	Net psi		Gross psi	Net psi		
<u>Plates with Openings Reinforced by an Insert Plate (36" x 1/4")</u>												
17	Circular		39	74	322,000	35,800	41,880	495,000	55,000	64,390	1,196	1,361
18	Circular		50	75	340,000	37,800	43,200	521,500	58,000	66,300	1,268	1,400
19	Square	1/32	33	76	301,000	33,400	39,660	362,000	40,200	47,690	229	548
20	Square	1/32	39	72	320,000	35,600	41,620	427,000	47,500	55,540	545	836
21	Square	1-1/8	62	77	300,000	33,300	36,360	478,000	53,100	57,940	1,155	1,484
34	Square	1-1/8	62	-46	376,000	41,800	46,000	551,500	61,300	67,500	1,652	1,542
22	Square	1-1/8	39	73	319,000	35,500	41,490	437,000	48,600	56,840	600	974
<u>Plates with Openings Reinforced by an Insert Plate (48" x 1/2")</u>												
55	Square	1-1/8	66	70	800,000	33,800	36,200	1,275,000	54,000	57,700	4,240	4,660
55A	Square	1-1/8	67	69	800,000	33,900	36,200	1,288,000	54,800	58,300	4,082	4,328
56	Square	1-1/8	66	-46	900,000	38,200	40,800	1,360,000	57,600	61,500	3,424	3,220
70	Square	1-1/8	39	76	800,000	33,300	37,600	1,276,000	53,100	59,700	3,362	3,699
71	Square	1-1/8	39	-46	800,000	33,300	37,600	1,176,000	48,800	55,000	2,084	2,084
<u>Plate with Opening Reinforced by a Combination of Face Bar and Insert Plate (36" x 1/4")</u>												
85	Square	1-1/8	78	76	295,000	32,780	34,710	493,000	54,780	58,000	1,442	1,747

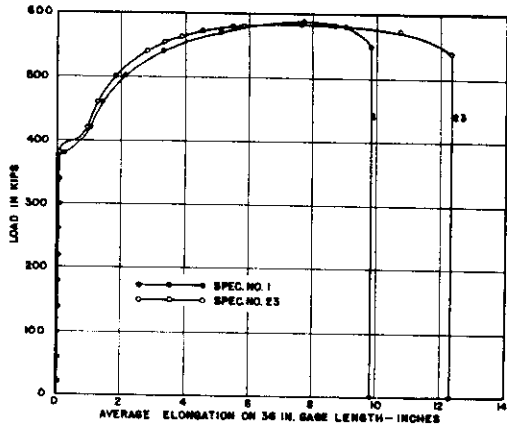
TABLE V
NATURE OF FAILURE IN SPECIMENS

Spec. No.	Shape of Opening	Per Cent Reinf.	Test Temp.	Fracture Per Cent			Location of Final Fracture
				Cleavage	Shear	Unbroken	
<u>Plain Plate (36" x 1/4")</u>							
1		100	81	0	76	24	
23		100	76	0	70	30	
<u>Plates with Unreinforced Opening (36" x 1/4")</u>							
2	Circular	0	76	0	80	20	Through Opening
3	Circular	0	72	0	60	40	Through Opening (Corner)
4	Square R.C.	0	78	0	59	41	Through Opening
<u>Plates with Unreinforced Opening (36" x 1/2")</u>							
37	Square R.C.	0	76	0	54	46	Through Opening
38A	Square R.C.	0	0	87	13	0	Through Opening
38	Square R.C.	0	-20	91	9	0	Through Opening
69	Circular	0	76	0	67	33	Through Opening
95	Square	0	76	0	89	11	Through Opening
96	Square	0	-46	100	0	0	Through Opening
<u>Plates with Openings Reinforced by a Face Bar (36" x 1/4")</u>							
5	Circular	40	74	0	58	42	Through Opening
6	Circular	17	73	0	63	37	Through Opening
7	Square	40	75	0	59	41	Weld to Rein.
8	Square	16	74	0	62	38	Through Opening
9	Square R.C.	40	72	0	44	56	Weld to Rein.
10	Square R.C.	16	75	0	69	31	Through Opening
99	Square R.C.	40	-46	97	3	0	Through Opening
31	Square R.C.	16	-46	75	25	0	Through Opening
<u>Plates with Openings Reinforced by a Face Bar (48" x 1/2")</u>							
49	Square R.C.	33	70	0	77	23	Weld to Rein.
50	Square R.C.	33	-20	99	1	0	Through Opening

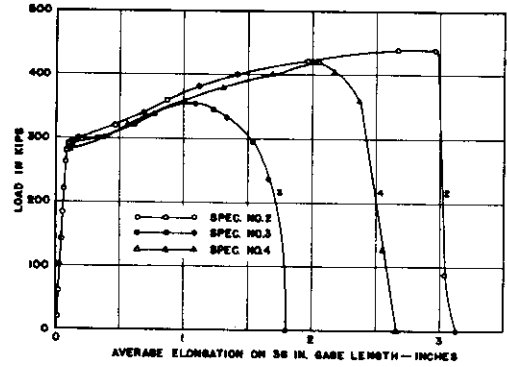
TABLE V (Cont.)
NATURE OF FAILURE IN SPECIMENS

Spec. No.	Shape of Opening	Per Cent Reinf.	Test Temp.	Fracture Per Cent			Location of Final Fracture
				Cleavage	Shear	Unbroken	
<u>Plates with Openings Reinforced by a Single Doubler Plate (36" x 1/4")</u>							
11	Circular	102	75	0	58	42	Through Opening
12	Circular	50	73	0	62	38	Through Opening
13	Square	104	76	0	58	42	Through Opening (Corner)
14	Square	51	71	0	50	50	Through Opening (Corner)
15	Square R.C.	103	76	0	65	35	Through Opening
32	Square R.C.	103	-46	63	22	15	Through Opening
16	Square R.C.	52	73	0	55	45	Through Opening
<u>Plates with Openings Reinforced by a Single Doubler Plate (48" x 1/2")</u>							
51	Square R.C.	96	74	0	81	19	Weld to Reinf.*
52	Square R.C.	96	-46	100	0	0	Through Body Plate*
<u>Plates with Openings Reinforced by an Insert Plate (36" x 1/4")</u>							
17	Circular	39	74	0	72	28	Through Opening
18	Circular	50	75	0	61	39	Through Opening
19	Square	33	76	0	54	46	Through Opening (Corner)
20	Square	39	72	0	62	38	Through Opening (Corner)
21	Square R.C.	62	77	0	66	34	Through Opening
22	Square R.C.	39	73	0	67	33	Weld to Reinf.
34	Square R.C.	62	-47	96	4	00	Through Opening
<u>Plates with Openings Reinforced by an Insert Plate (48" x 1/2")</u>							
55	Square R.C.	66	70	57	28	15	Through Opening*
55A	Square R.C.	67	69	0	79	21	Through Opening
56	Square R.C.	66	-46	100	0	0	Through Body Plate
70	Square R.C.	39	76	1	50	49	Through Opening
71	Square R.C.	39	-46	100	0	0	Through Opening
<u>Plate with Opening Reinforced by a Combination of Face Bar and Insert Plate (36" x 1/4")</u>							
85	Square R.C.	78	76	0	67	33	Weld to Body Plate

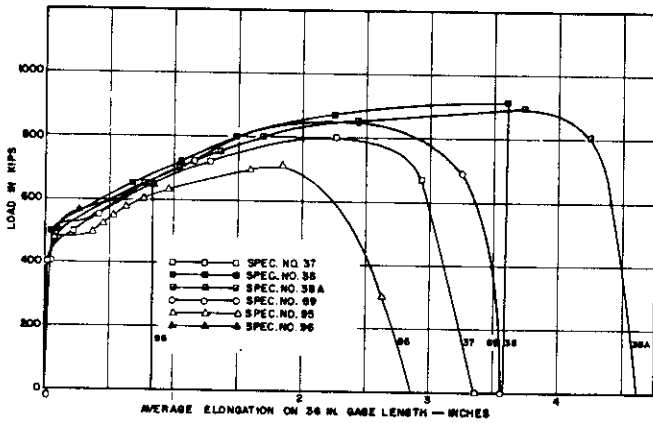
*Initial failure in pulling plates. Spec. No. 51 reloaded after 3 days, Spec. No. 52 after 9 days, and Spec. No. 55 after 10 days.



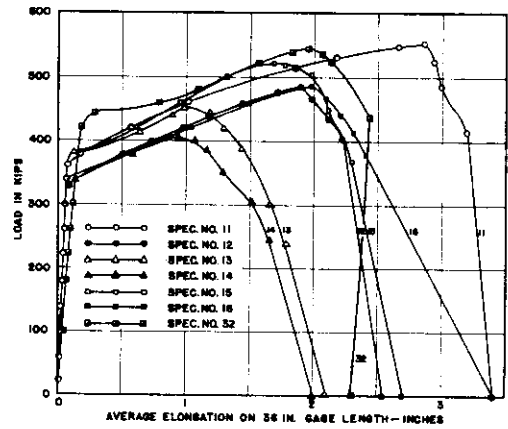
36" x 1/4" PLATE WITHOUT OPENINGS



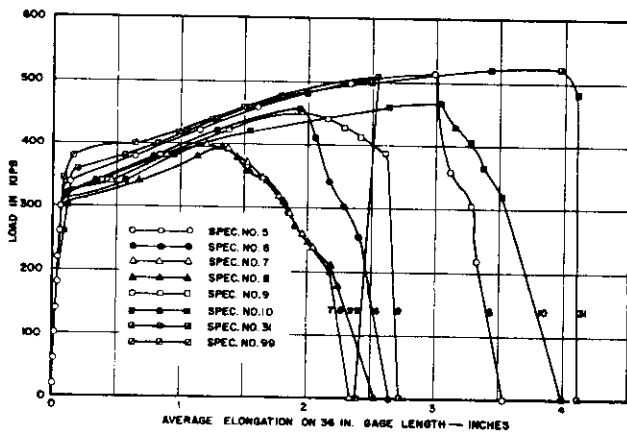
36" x 1/4" UNREINFORCED PLATE



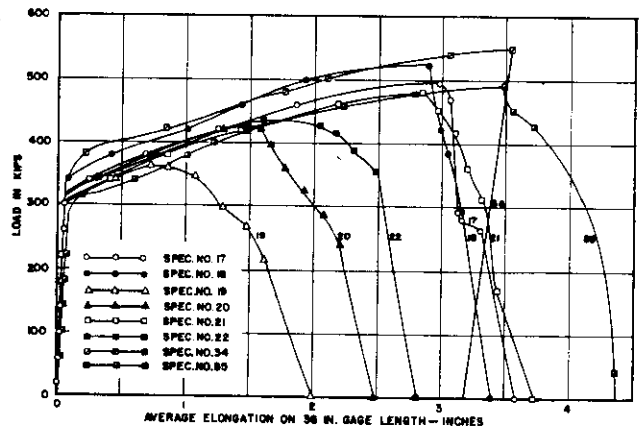
36" x 1/2" UNREINFORCED PLATE



36" x 1/4" R. REINFORCED BY SINGLE DOUBLER R.



36" x 1/4" R. REINFORCED BY FACE BAR



36" x 1/4" R. REINFORCED BY INSERT R.

FIG. 4. LOAD-AVERAGE ELONGATION CURVES FOR 36" x 1/4" AND 36" x 1/2" SPECIMENS.

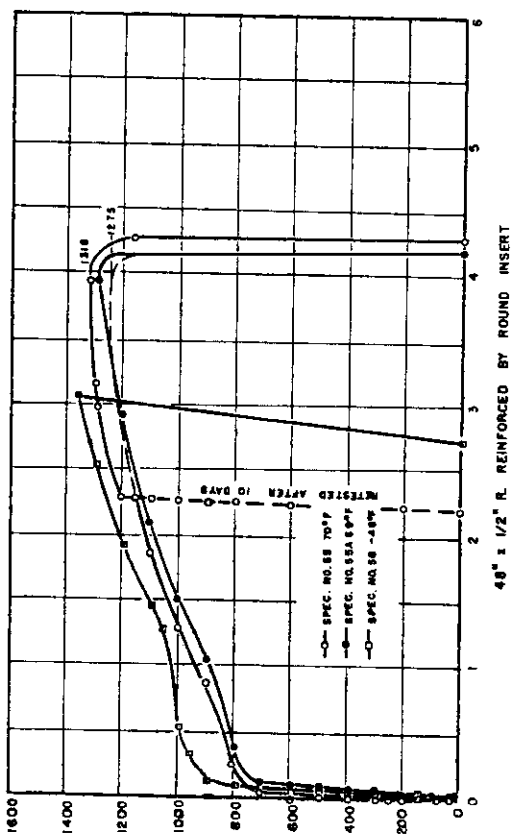
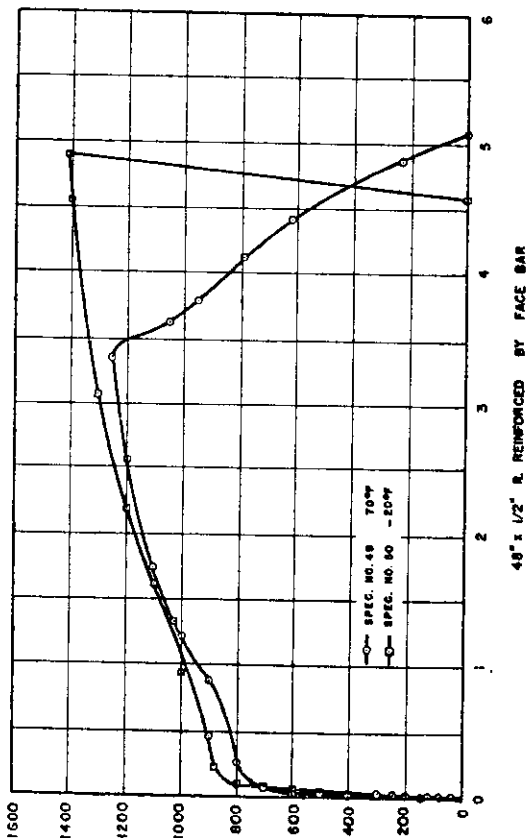
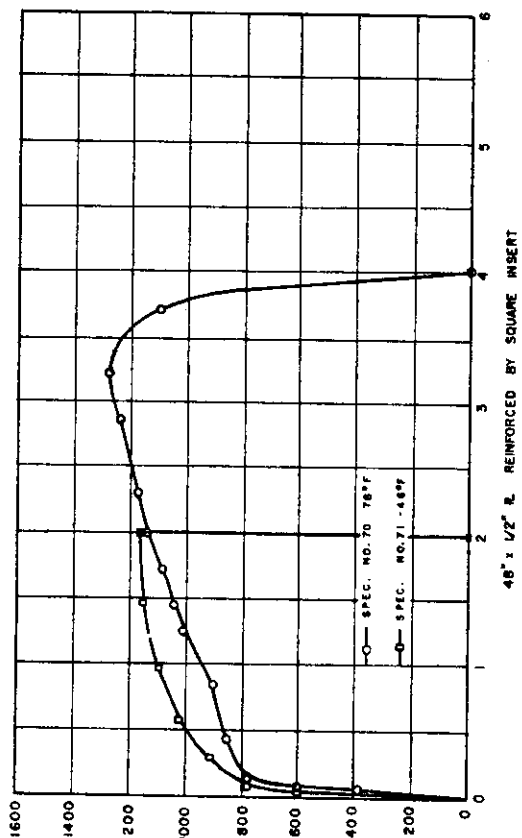
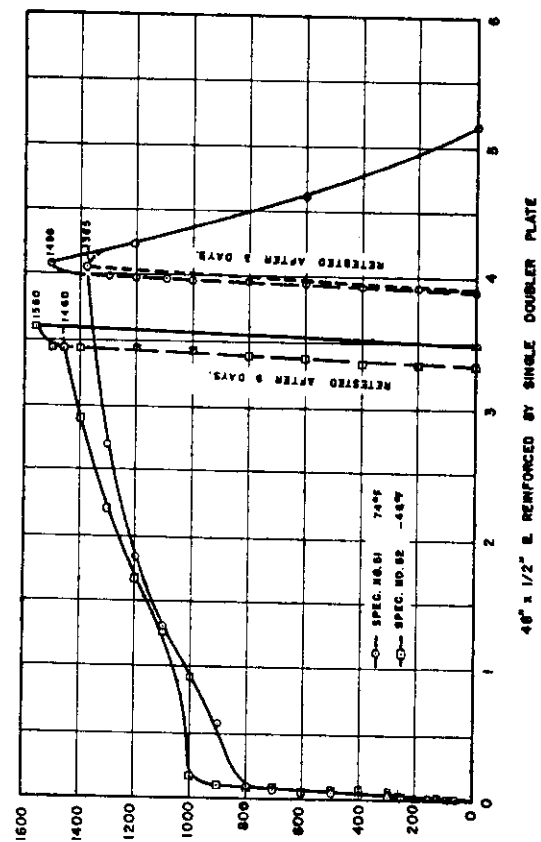


FIG. 5. LOAD-AVERAGE ELONGATION CURVES FOR 48" X 1/2" SPECIMENS.

per cent between the additional net cross section area added to the unreinforced specimen and the cross section area of the material removed from the body plate by the opening. Thus a reinforced plate with a net cross section area equal to the area of the plain plate would have a percentage of reinforcement of 100 per cent.

The percentage of cleavage or shear in the fracture was taken as the ratio in per cent of the cleavage or shear portion of the actual cross section, including any unbroken part, of the specimen along the fracture line.

4. General behavior during test and fracture of plates with openings. A detailed description of the results of these tests has already been presented in the previous progress reports⁽¹⁻⁻⁶⁾. Accordingly, only a summary of the data is included here.

A comparison of the applied load and the average elongation on a gage length equal to the width of the plate is shown for all tests in Figs. 4 and 5. A summary of the more important data and a description of the failure are given in Tables IV to VII, inclusive, and Figs. 7--18. The results of the tests and their significance will be discussed in the subsequent sections of this report.

IV. BEHAVIOR IN THE PLASTIC RANGE OF PLATES WITH OPENINGS

1. Theoretical elastic stress distribution. For purposes of comparison with the plastic stress distribution determined for certain specimens, the elastic stress distribution was computed by theory wherever a solution was available for a case similar to or the same as that of the specimens being tested. The results are presented in Fig. 6 in the form of elastic stress concentration contours. This figure indicates three important facts: first, that for those cases where the ratio of the width of the plate to the diameter or width of the opening is greater than about four, the solution for a plate of infinite width gives satisfactory results; second, for all practical purposes the shape of the opening affects the elastic stress pattern only in the vicinity of the opening; and third, the elastic stress concentration factor for a circular opening is 3.00 and for a square opening with a corner radius one-eighth the width of the opening 3.09. These facts are in accord with St. Venant's principle.

In the plates with a single doubler plate reinforcement, the SR-4 readings indicated a second peak of stress concentration in the body plate adjacent to the outer edge of the doubler. The theoretical stress distribution for an insert plate in Fig. 6 shows such a point.

2. Plastic stress distribution in plates with openings. The stresses in the plastic range of the steel were computed from the measured strains in the specimen by the tangent

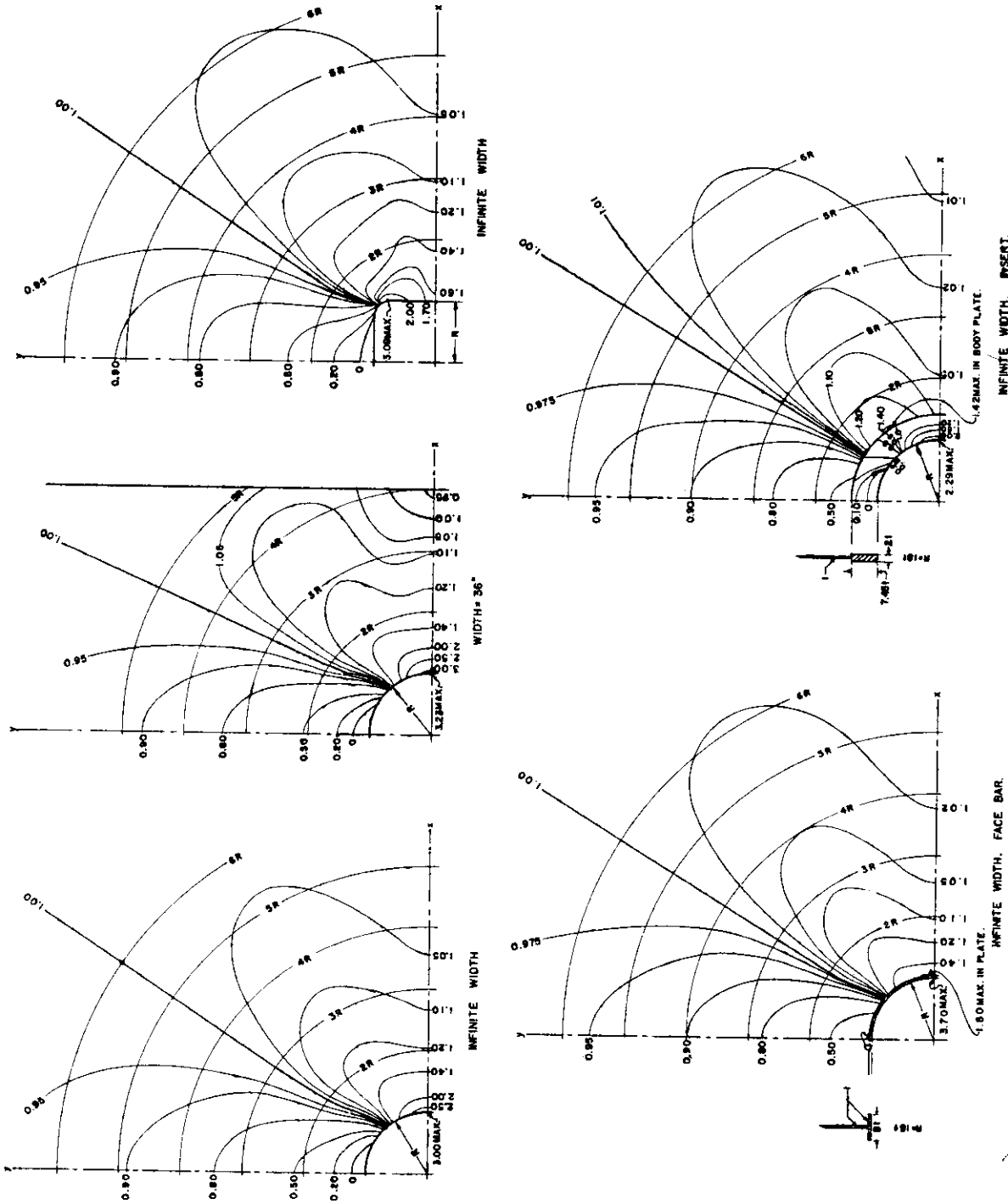
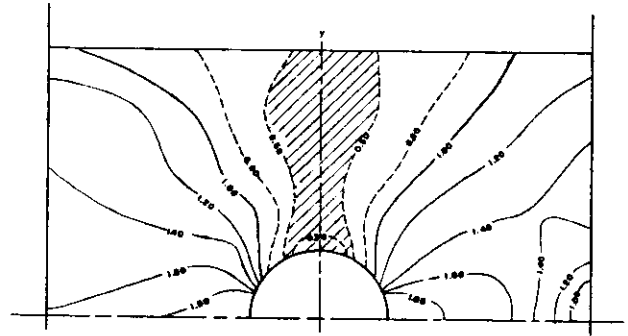


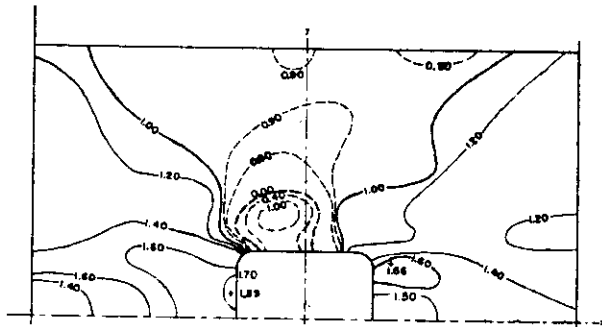
FIG. 6. STRESS-CONCENTRATION CONTOURS IN P-DIRECTION BY THEORY OF ELASTICITY FOR TYPICAL CASES.

modulus method of stress analysis^(2,6) developed by this investigation. The plastic stress concentration contours and distributions in Figs. 7 to 10, inclusive, give the ratio of the true stress at any point in the y-direction (the direction of the applied tension) to the uniform true stress on the gross area of the specimen in a region remote from the opening.

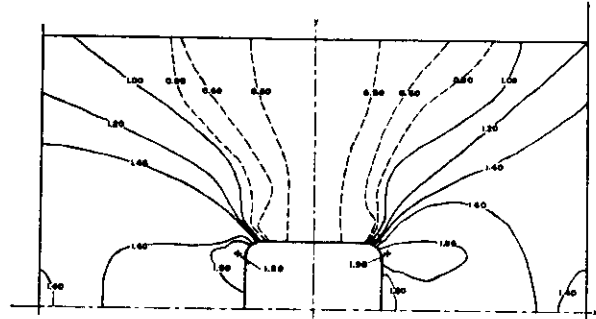
The transition from the elastic to the plastic stress state brought about no significant change in the general nature of the stress pattern but only in the relative values of the stresses themselves. As the load on the specimen was increased to the maximum, or ultimate load, there was a tendency for the plastic stresses across the section to approach uniformity, that is, for the specimen to develop a more efficient manner of carrying the stresses than existed in the elastic range. This trend towards a leveling out of high stress concentrations and consequently more nearly uniform stress distribution was most pronounced in the specimens with the lower elastic stress concentration factors. These tests showed why it is desirable in the design of openings and their reinforcement to remove causes of stress concentration to the greatest possible degree. When a severe stress raiser was present, the plastic stress gradients around the opening were steeper. Good design by reducing stress concentration results in a more



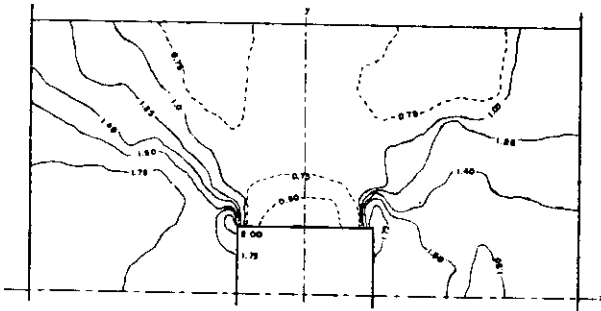
SPEC. NO. 69 76°F.



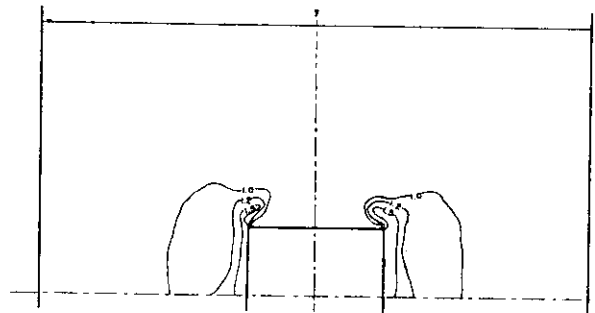
SPEC. NO. 37 76°F.



SPEC. NO. 38 -20°F.

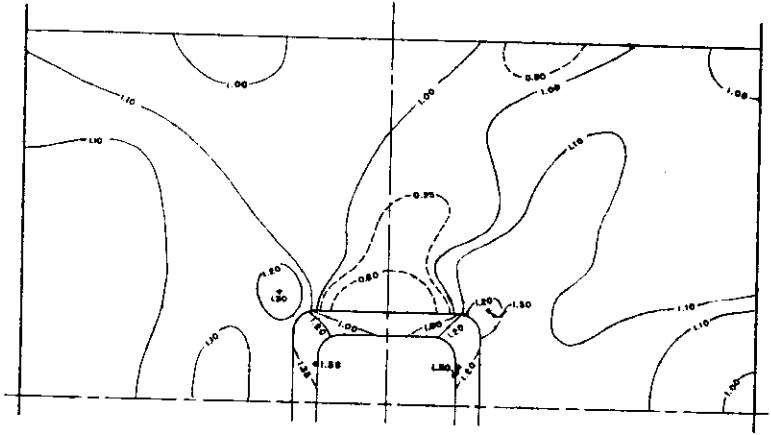


SPEC. NO. 95 76°F.

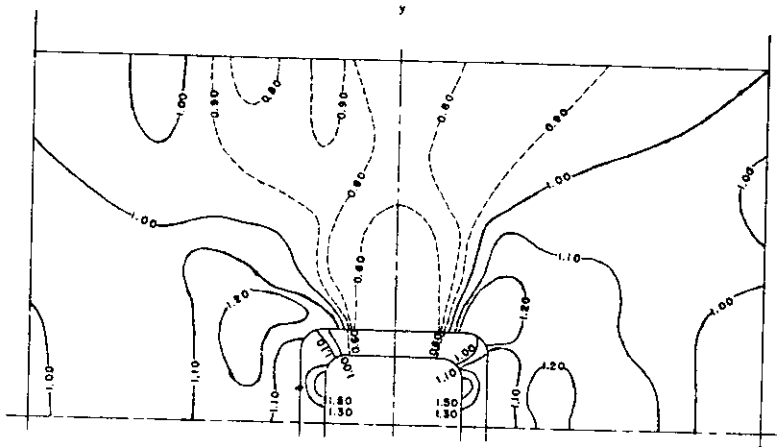


SPEC. NO. 96 -46°F.

FIG. 7. PLASTIC STRESS-CONCENTRATION CONTOURS IN y DIRECTION FOR UNREINFORCED PLATES AS DETERMINED FROM MEASURED STRAINS.

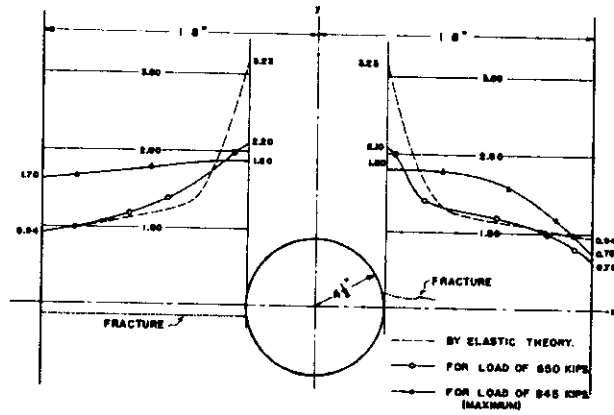


SPEC. NO. 70. 76° F.

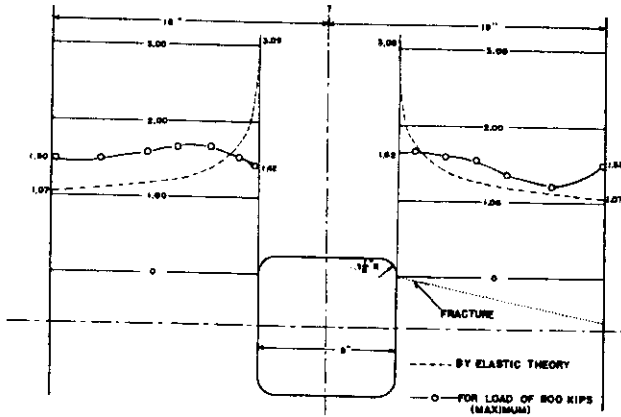


SPEC. NO. 71. -46° F.

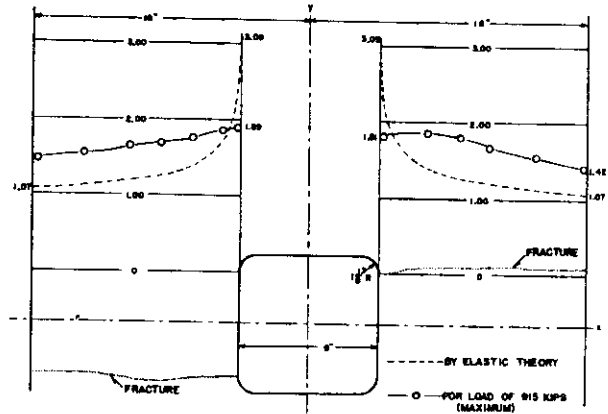
FIG. 8. PLASTIC STRESS-CONCENTRATION CONTOURS IN y DIRECTION FOR REINFORCED PLATES AS DETERMINED FROM MEASURED STRAINS.



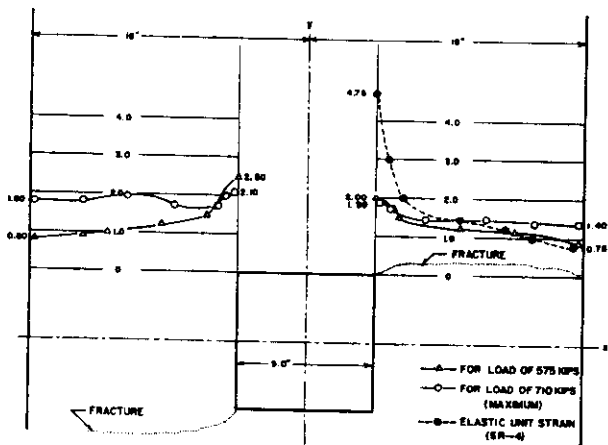
SPEC. NO. 69. 76° F.



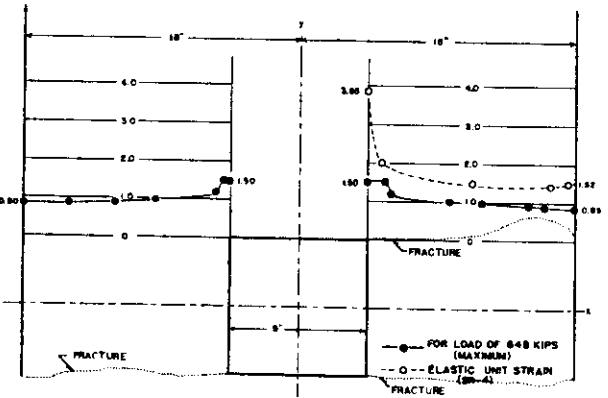
SPEC. NO. 37. 76° F.



SPEC. NO. 38. -20° F.



SPEC. NO. 95. 76° F.



SPEC. NO. 96. -46° F.

FIG. 9. COMPARISON OF ELASTIC AND PLASTIC STRESS-CONCENTRATION IN y DIRECTION ON NET CROSS SECTION OF UNREINFORCED PLATES

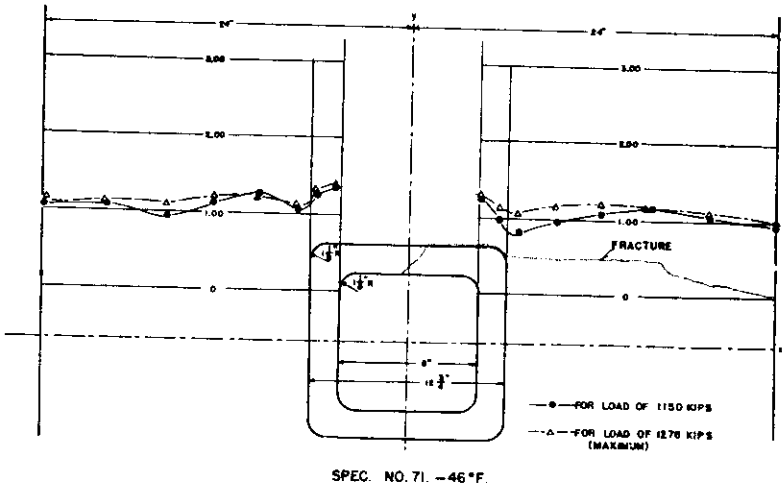
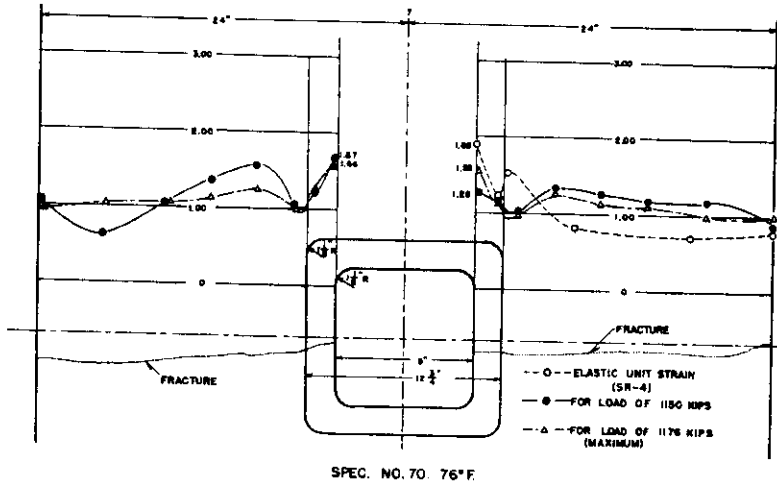


FIG. 10. COMPARISON OF ELASTIC AND PLASTIC STRESS-CONCENTRATION IN y DIRECTION ON NET CROSS-SECTION OF REINFORCED PLATES.

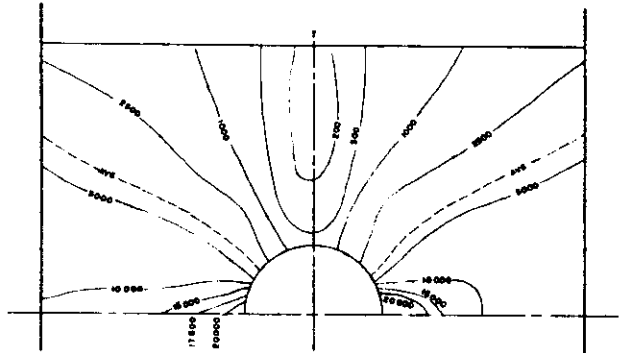
efficient plastic stress distribution and thereby a higher ultimate strength and energy absorption.

3. Plastic energy distribution in plates with openings.

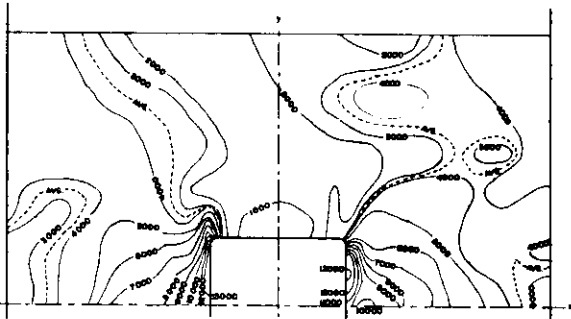
The unit strain energy distribution in the vicinity of the opening was computed from the measured strains in the specimens by the octahedral theory of A. Nadai⁽¹⁴⁾. Contour maps showing the unit energy distribution in the plastic range appear in Figs. 11 and 12.

It is interesting to point out that the contour line for the average unit energy absorption (the total energy absorption in the gaged area divided by the volume of the specimen within that area) fell in almost the same location in each plate as the contour line for unit stress concentration for both the elastic and the plastic stress states. Also, the higher values of the unit energy absorption appeared in the same area of the specimen where the higher values of the elastic and plastic stresses occurred.

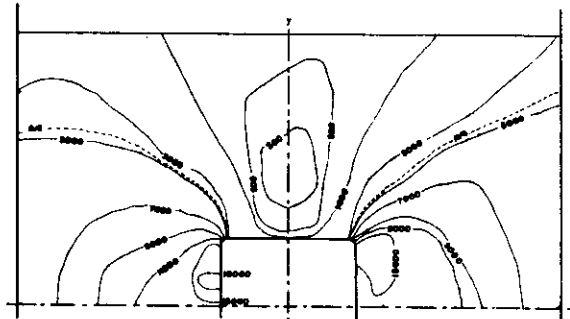
These few tests appear to indicate that one principal function of the reinforcement is that of reducing the spread between the maximum and the minimum values of the unit energy absorption. In respect to the unreinforced plates, Fig. 11 shows how decreasing the severity of the notch reduced the concentration of high values around the corner of the opening and caused a more nearly uniform distribution of the energy. Here again the importance of using generous notch



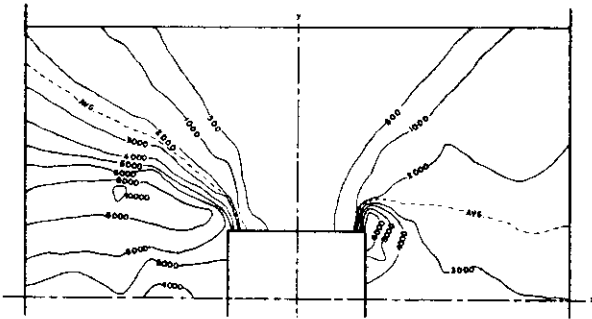
SPEC. NO. 69. 76° F.



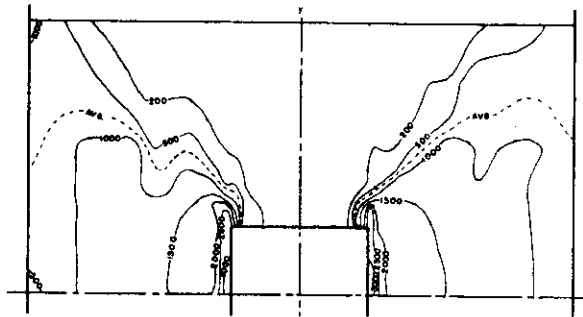
SPEC. NO. 37. 76° F.



SPEC. NO. 38. -20° F.

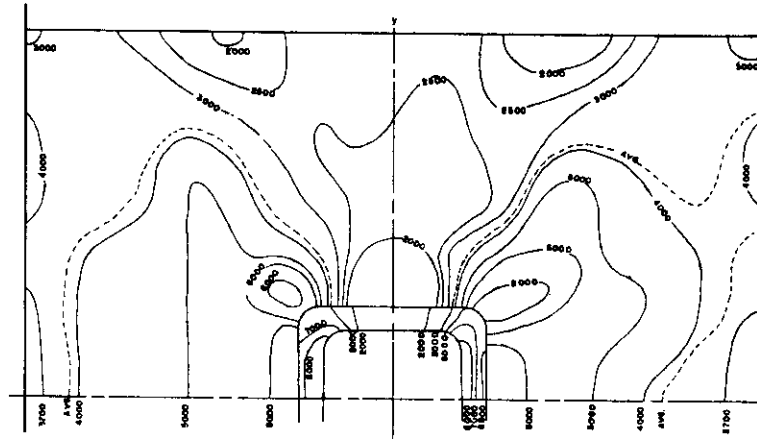


SPEC. NO. 95. 76° F.

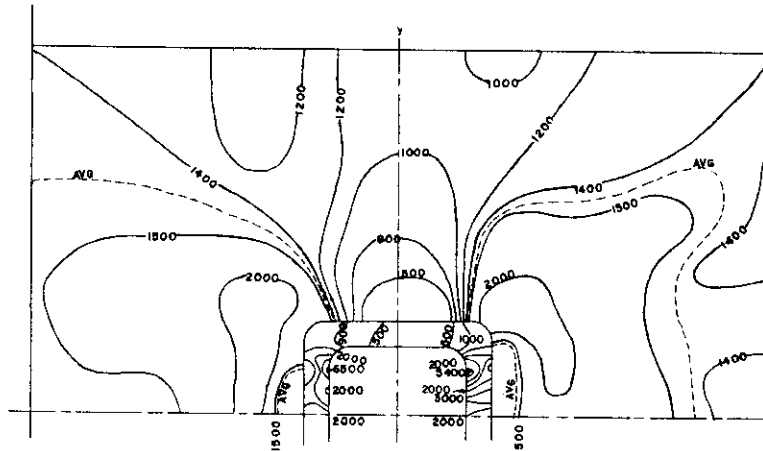


SPEC. NO. 96. -46° F.

FIG. 11. UNIT STRAIN ENERGY CONTOURS AT ULTIMATE LOAD FOR UNREINFORCED PLATES AS DETERMINED FROM MEASURED STRAINS.



SPEC. NO. 70. 76°F.



SPEC. NO. 71. -46°F.

FIG. 12. UNIT STRAIN ENERGY CONTOURS AT ULTIMATE LOAD FOR REINFORCED PLATES AS DETERMINED FROM MEASURED STRAINS.

radii in design was indicated. Similar statements could also be made concerning the plastic stress distributions shown in Figs. 7--10.

It was found^(2,6) that the unit plastic energy absorption at any given point in the specimen increased in accordance with the empirical equation,

$$u = e^{A+BP},$$

where e is the base of Napierian logarithms, A and B were numerical quantities, and P the applied load. The small quantity A was found to remain almost constant. The significant variable was B , the slope of the semi-logarithmic curve relating u and P . From semi-logarithmic plots of u against P for each of the many points of the grid system on the surface of the specimen, the values of B were obtained. A similar semi-logarithmic plot with respect to the average unit energy absorption u_{Av} for the entire gaged area gave the average value of B , or B_{Av} . The ratio $\frac{B}{B_{Av}}$ has been called the relative rate of increase of the unit energy absorption. Maps showing the contours of equal values of this ratio appear in Figs. 13 and 14. The fact that the experimental data were amenable to such a rationalization indicated that the energy absorption developed in a systematic and logical manner at all points of the specimen as the load increased.

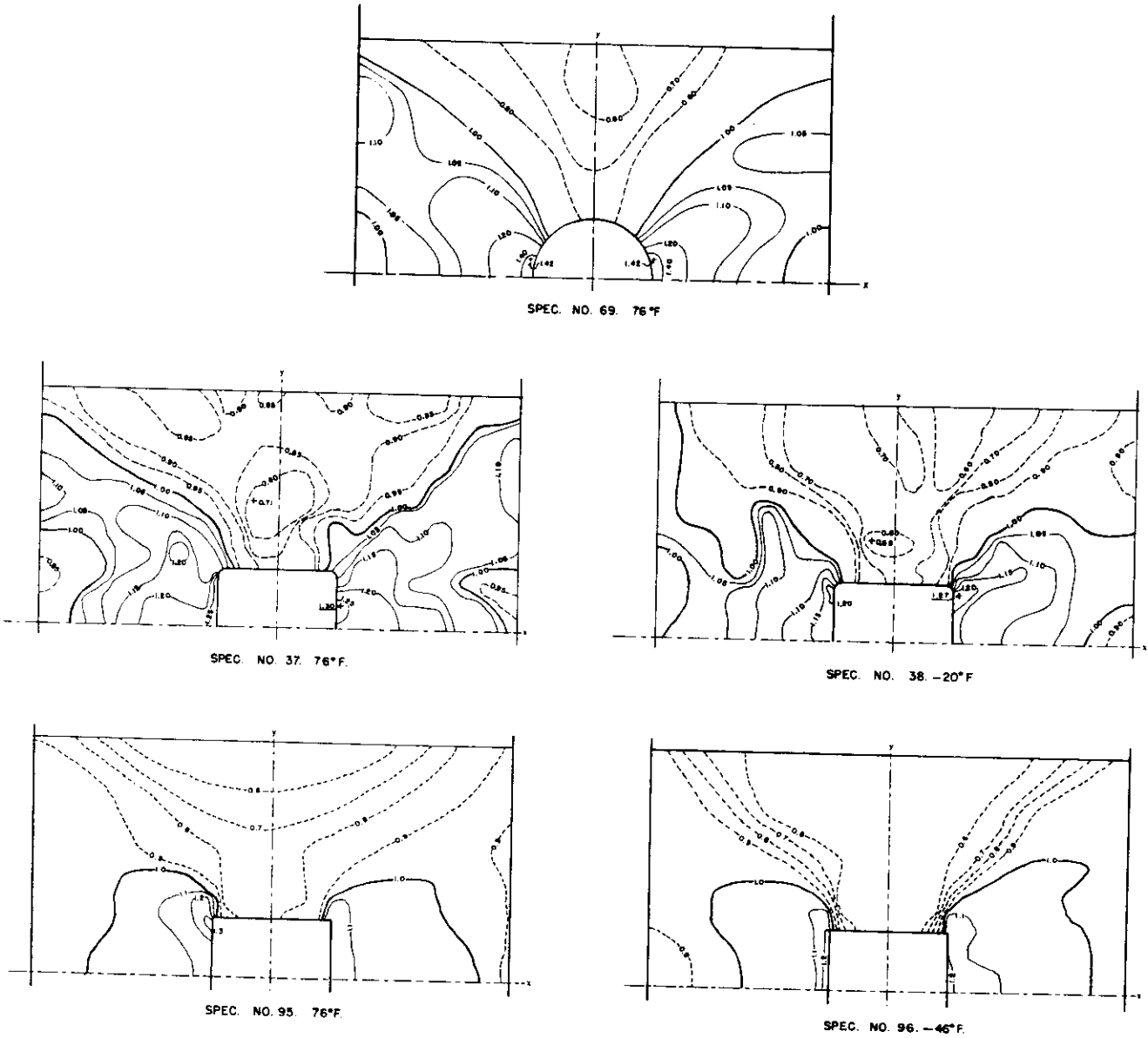
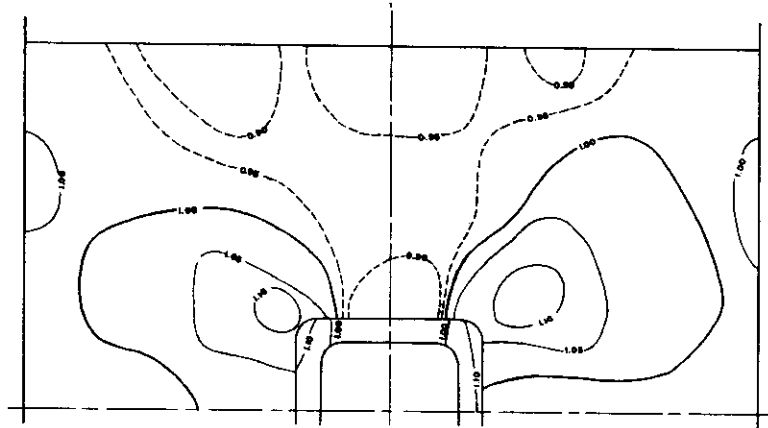
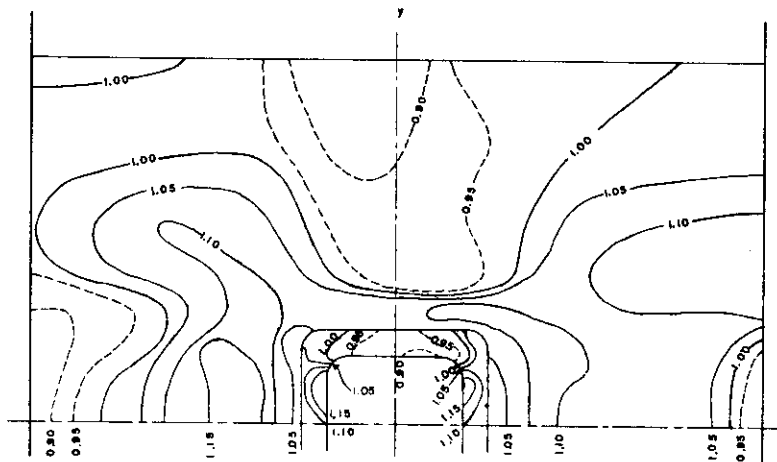


FIG. 13. Contours of Equal Relative Rate of Increase of Unit Strain Energy Absorption with Increase in Applied Load. Unreinforced Plates.



SPEC. NO. 70. 76°F.



SPEC. NO. 71. -46°F.

FIG. 14. Contours of Equal Relative Rate of Increase of Unit Strain Energy Absorption with Increase in Applied Load. Reinforced Plates.

4. Effect of testing temperature upon the plastic stress and energy distribution. The plastic stress distributions in Figs. 7 and 8 and the plastic unit energy distributions in Figs. 11 and 12 were examined by the application of statistical methods for the purpose of determining whether they could be correlated with the mode of fracture in any way. In each of these plots are shown the results for duplicate specimens tested at two different temperatures--one selected to produce shear fractures and the other predominately cleavage fracture, Specs. No. 37 and 38, and 95 and 96, and 70 and 71. It was found that in the plates with the latter mode of fracture the higher plastic stress and unit energy values were concentrated more closely around the opening than in the plates with the former mode of fracture; that is, the plastic stress and energy gradients were steeper. Cleavage fracture was accompanied by a less efficient stress and energy distribution than shear fracture. Moreover, this effect of testing temperature on the behavior of two identical specimens suggests that tests resulting in shear fractures cannot be used to give reliable predictions of the probable results of low-temperature tests which produce cleavage fracture.

5. Conditions for the initiation of fracture. In these tests it was observed that the fracture was initiated at the maximum, or ultimate load, whether it was of the shear or cleavage type, and in the region where the maximum values of

the true stress, unit energy, and unit strain were observed. The highest elastic stress and first Luders line were also found in this region.

The experimental data were examined for information which might describe the conditions under which fracture was initiated, such as the maximum true stress, the maximum unit energy absorption, and the maximum unit strain. It should be pointed out with respect to these maxima that the use of a grid system of 1-in. gage lengths may have resulted in small errors in determining the exact location or the true value of the absolute maximum, which always occurred near the boundary of the opening.

A considerable variation of the maximum true stress was observed in the seven specimens, the range being from 68.5 to 105.0 ksi. However, when the maximum plastic stress concentration factor was computed, the relations shown in the upper two diagrams of Fig. 15 were found. The stress concentration factor was always maximum in the elastic range, decreased as the plastic stress or load level increased, and approached a constant and also a minimum value as the ultimate strength of the plate was reached. This observation suggests that perhaps the low energy cleavage fracture of some welded members, which is often accompanied by low ultimate strength, may result in part because the amount of plastic flow which has occurred is not large enough to bring about a sufficient reduction in the plastic stress concentration factor.

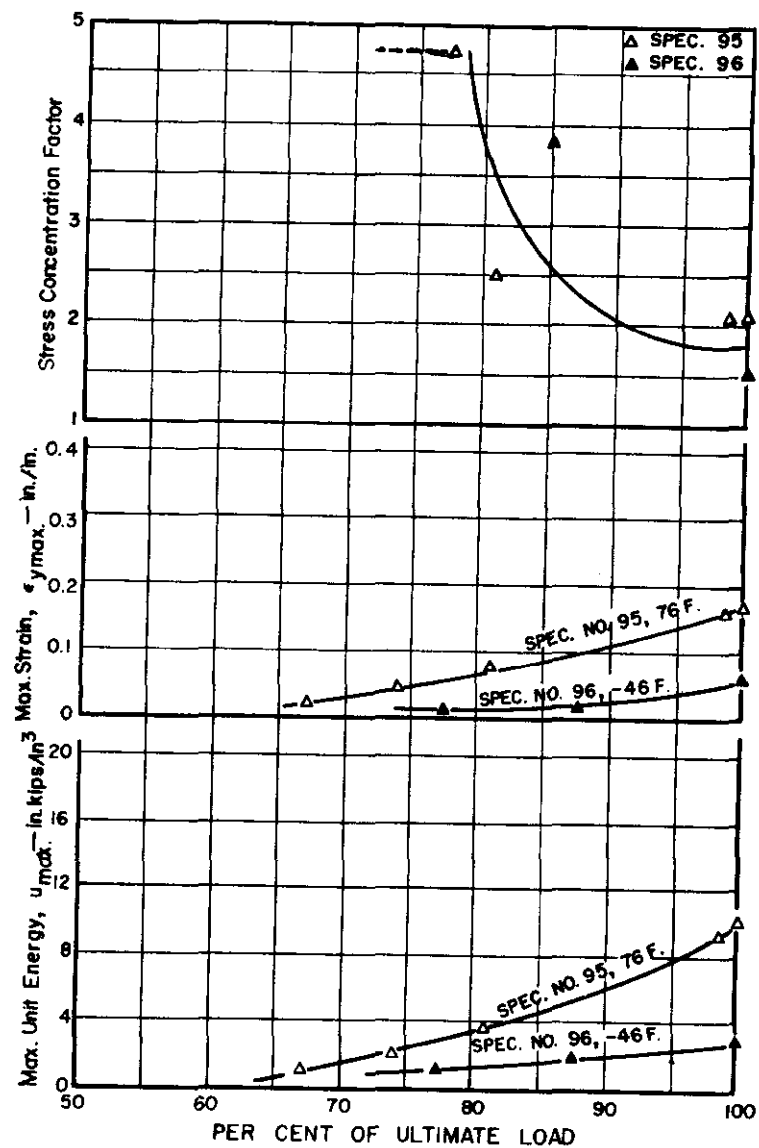
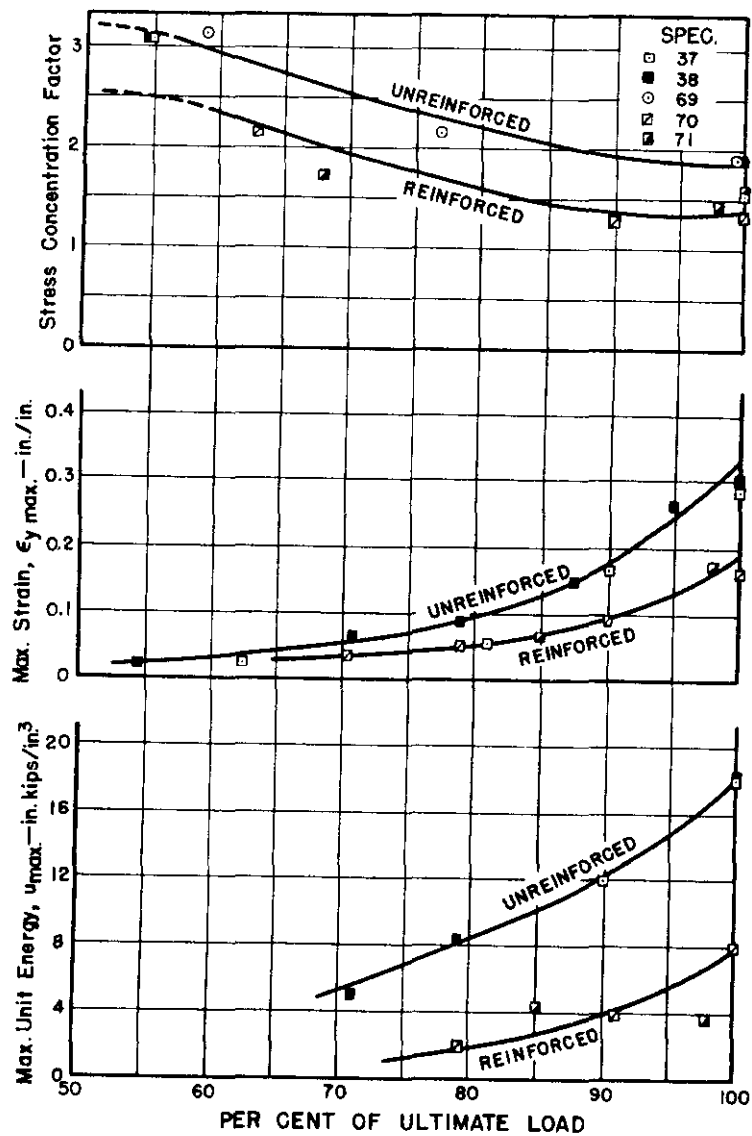


FIG. 15. PLASTIC STRESS CONCENTRATION, MAXIMUM UNIT STRAIN AND MAXIMUM UNIT STRAIN ENERGY AS ULTIMATE LOAD WAS APPROACHED.

The maximum unit nominal strains observed in the specimens are plotted in the middle diagrams of Fig. 15, and the maximum unit energy absorption in the lower diagrams.

While the plots in Fig. 15 show that certain of the maximum properties of the specimens followed a consistent relation, they also indicated that no single numerical value of any one of these properties could be used to predict the imminence of fracture. While the geometry of the specimen was an important factor in determining failure, other factors, such as the testing temperature, the mechanical properties of the steel before and after permanent deformation, and undoubtedly the many small stress raisers produced during the fabrication and welding of the specimens were also significant. The common theories of failure are related only to the geometry of the specimen.

6. Effect of the shape of the opening upon the properties of the plates with openings. In these tests it was found that the most important factor affecting the properties of the plates with openings was the notch severity of the opening, which depends primarily upon the notch radius. The notch acuity was expressed in terms of the ratio, $\frac{R_0}{R_N}$, where R_0 is the half-width of the opening and R_N the radius of the notch. The relations of various properties of these plates to this ratio are shown in Fig. 16. All the specimens in these plots sustained shear fractures.

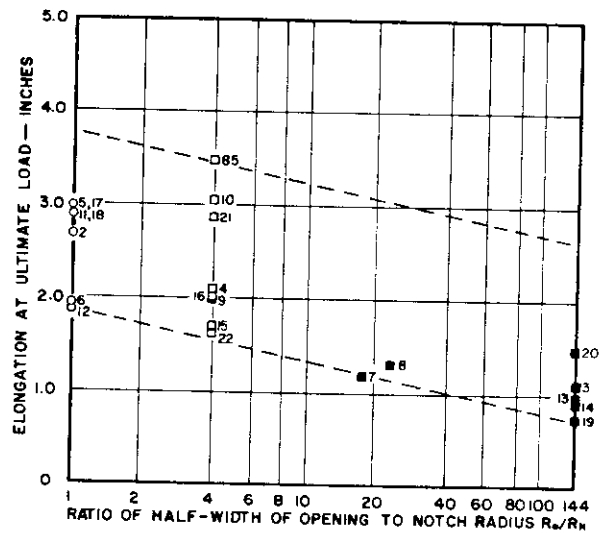
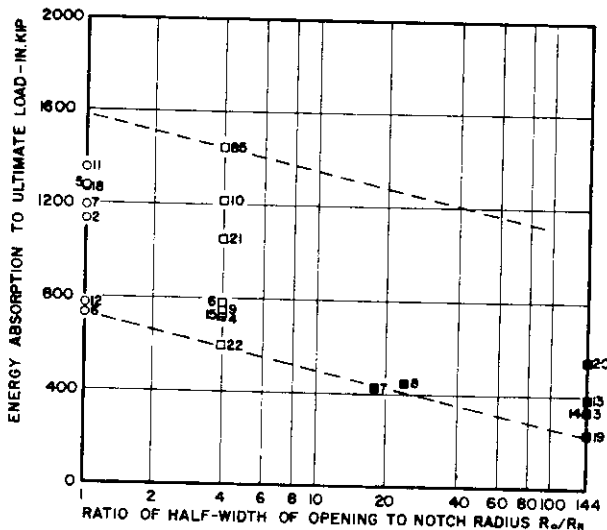
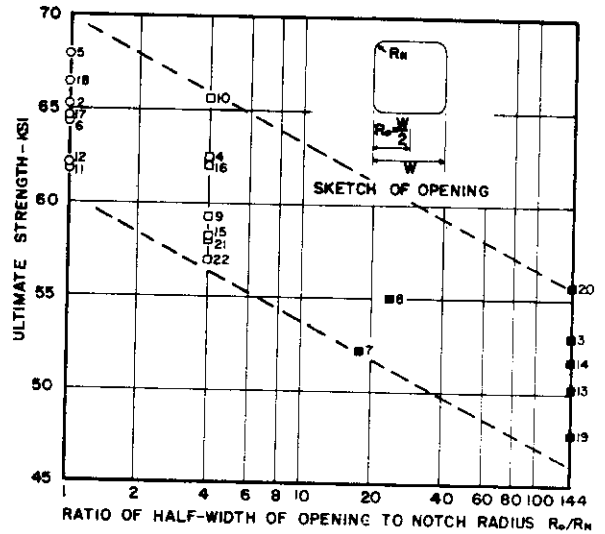
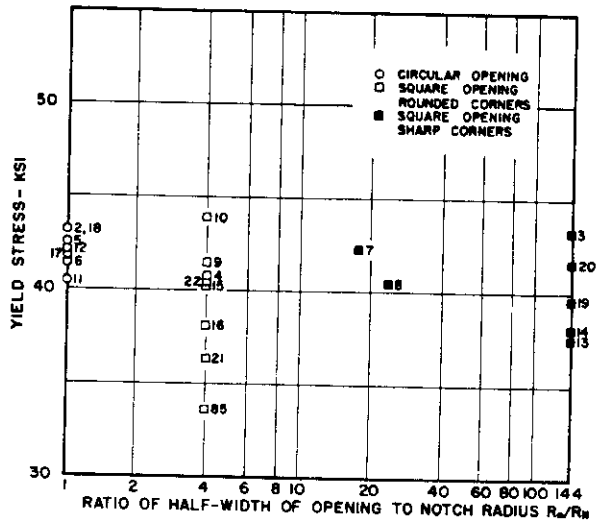


FIG 16. EFFECT OF NOTCH ACUITY UPON PROPERTIES OF PLATES WITH OPENINGS.

The average net stress at general yielding did not appear to be affected by the notch acuity to any appreciable extent. However, an increase in the ratio, $\frac{R_0}{R_N}$, which amounts to an increase in the notch severity, reduced the ultimate strength, the energy absorption to ultimate load, and the elongation to ultimate load in a manner which was linearly related to the logarithm of this ratio. The variation within the scatter bands in these plots represents the effect of the percentage of reinforcement and the geometric shape of the reinforcement.

In general, it was noted that the plates which developed the higher ultimate strengths absorbed the most energy.

7. Effect of the percentage of reinforcement upon the properties of the plates with openings. The effect of the percentage of reinforcement upon the properties of specimens sustaining shear fractures is shown in Fig. 17. A slight downward trend in the average net stress at general yielding and the ultimate strength and an increase in the ultimate load was found as the percentage of reinforcement increased. The load carrying capacity of the plates was increased by adding more reinforcement, but this improvement was accompanied by a small reduction in the ultimate stress carrying capacity of the plate. Thus the increase in load carrying capacity was not commensurate with the added amount of reinforcement. No significant change in the energy absorbing capacity of the plates was brought about by increasing the percentage of reinforcement.

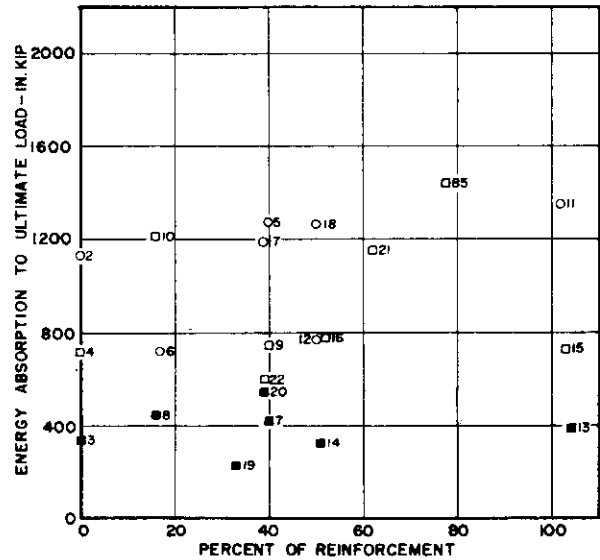
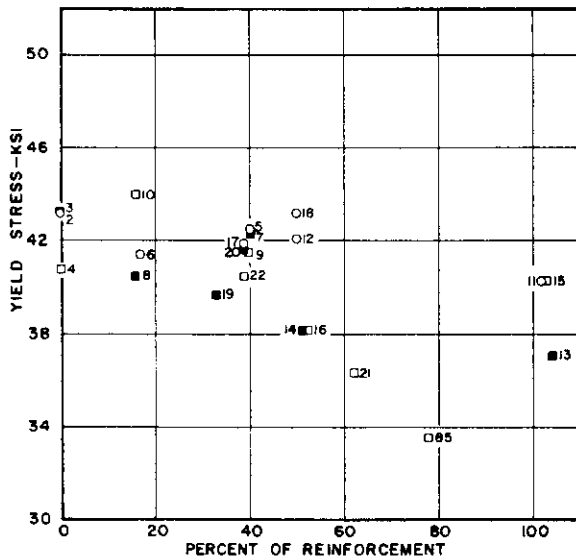
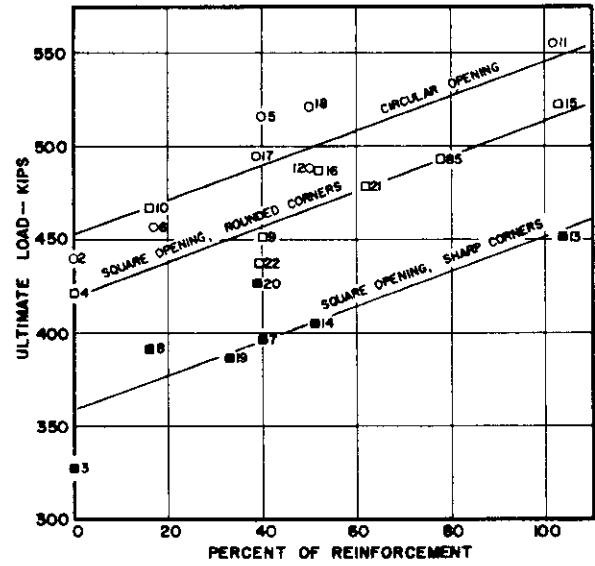
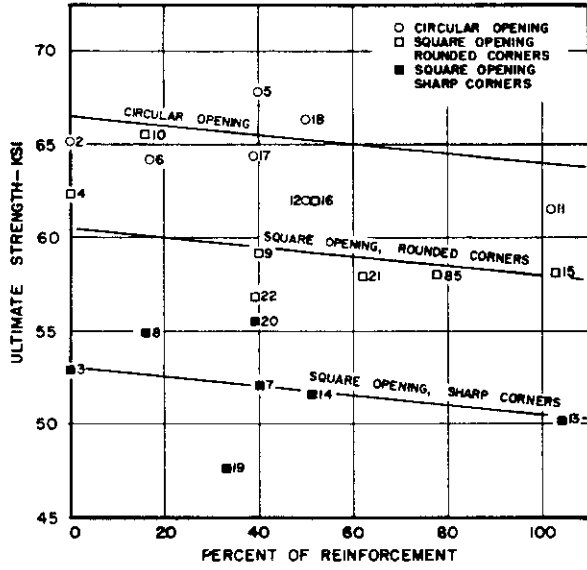


FIG. 17. EFFECT OF PERCENTAGE OF REINFORCEMENT UPON PROPERTIES OF PLATES WITH OPENINGS.

Fig. 17 shows the general trends for all the types of reinforcement. There existed for each type of reinforcement an optimum percentage below which the plates failed through the opening. Above this optimum percentage the reinforcement tended to act as a rigid inclusion in the body plate, and failure occurred by shear in the weld joining the outer edge of the reinforcement to the body plate. This latter mode of failure resulted in somewhat reduced strength and energy absorption.

This optimum percentage of reinforcement was different for each type of reinforcement. For example, it was around 35 to 40 per cent for a face bar, 95 to 100 per cent for a single doubler plate, and somewhere between 30 and 60 per cent for an insert plate. These values are tentative inasmuch as an insufficient number of tests were made to establish these values more definitely. However, they indicate that the doubler plate type of reinforcement would be most efficient for the higher percentages of reinforcement.

8. Effect of the geometric shape of the reinforcement upon the properties of plates with openings. The previous section showed that the optimum percentage of reinforcement varied for the different types of reinforcement. The reason for this variation was found to lie in the geometric shape of the cross section of the reinforcement, principally its width in the direction of the thickness of the body plate. Since

the solutions by theory of elasticity for the reinforced opening assume plane stress conditions and therefore do not fit the actual problem, it was necessary to develop an empirical parameter which would express the "shape factor" of the reinforcement. The square of the radius of gyration (the moment of inertia of the net section of the specimen about the transverse centerline of the plate divided by the area of the net section) was found to be a suitable parameter and will be referred to hereafter for brevity as k^2 . Various properties of the plates with openings are related to it in Fig. 18.

The average net stress at general yielding decreased as the value of k^2 increased to a value between 20 and 30, in which range the triaxiality of stress induced by the width of the reinforcement was maximum. For higher values the greater rigidity of the reinforcement, which tended to make it act as a rigid inclusion, increased the yield stress somewhat.

The relations of the ultimate load, the ultimate strength, and the energy absorption to ultimate load to the parameter k^2 were similar in nature. For the plates with the square opening and the square opening with rounded corners, there was an optimum value of k^2 , and the plotted points corresponding to higher values of this parameter represent those plates where fracture occurred in the weld at the outer edge of the reinforcement or in the body plate. However, no such failures took place in the plates with a circular opening, and for

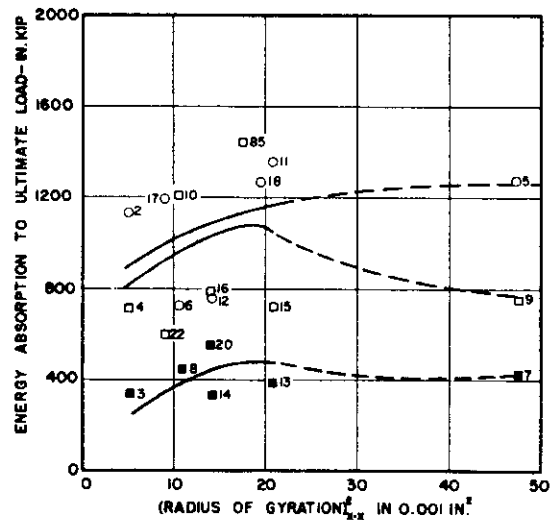
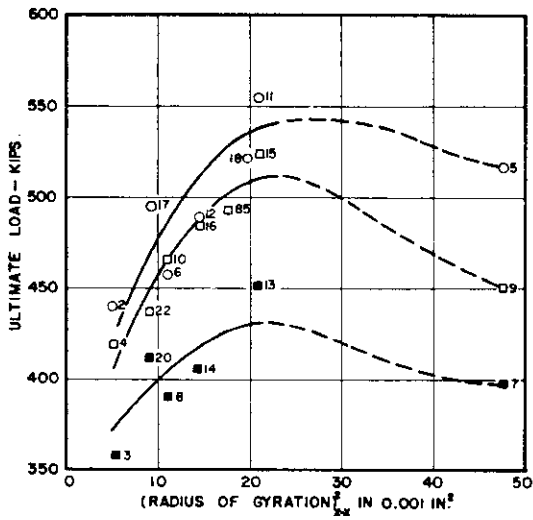
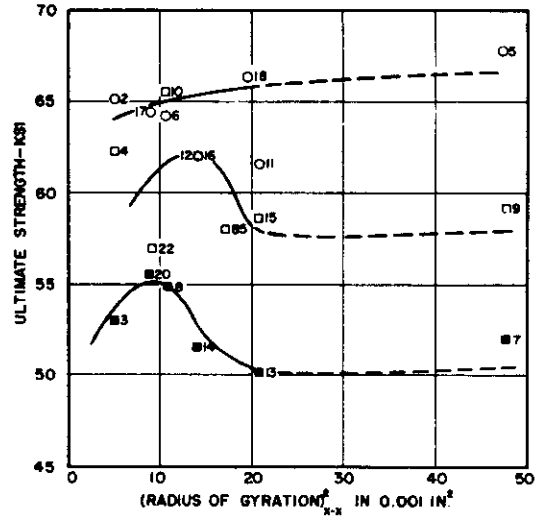
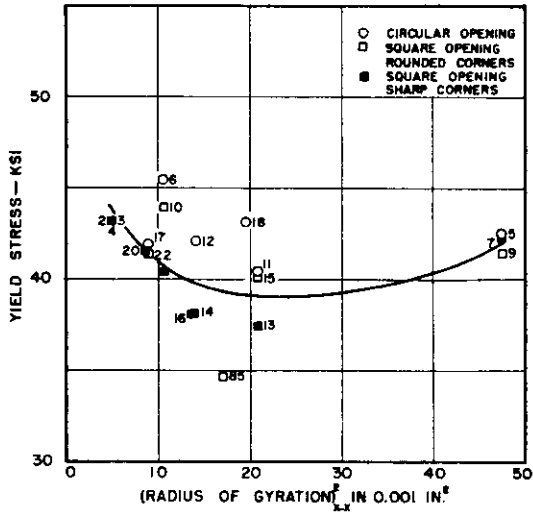


FIG. 18. EFFECT OF GEOMETRIC SHAPE OF REINFORCEMENT UPON PROPERTIES OF PLATES WITH OPENINGS.

this shape of opening no significant drop-off in strength or energy absorption was found for the higher values of k^2 . Thus this empirical parameter, the square of the radius of gyration of the net section of the plate, appeared to describe adequately and consistently the effect of the geometric shape of the reinforcement upon the ultimate properties of the plates.

There is good reason to believe that this parameter would be equally applicable to coamings, hatch corners, and other similar details.

The data of these tests were combined with the data of other tests of plates with reinforced openings⁽¹⁶⁾ in Fig. 19. Unfortunately, only the ultimate strength, and not the energy absorption of these latter tests, was recorded. A correlation similar to that in Fig. 18 was found here for the efficiency with respect to ultimate strength. In the Model Basin tests plates with square openings and values of k^2 larger than the optimum value almost failed in the weld at the outer edge of the reinforcement or in the body plate. Moreover, plates with a circular opening and a value of k^2 almost seven times the maximum value for any specimen in the present tests showed only a slight reduction in efficiency with respect to ultimate strength. This last observation suggests the possibility that it would be difficult to make a poor design of reinforcement for a circular opening. Again the extreme importance of the

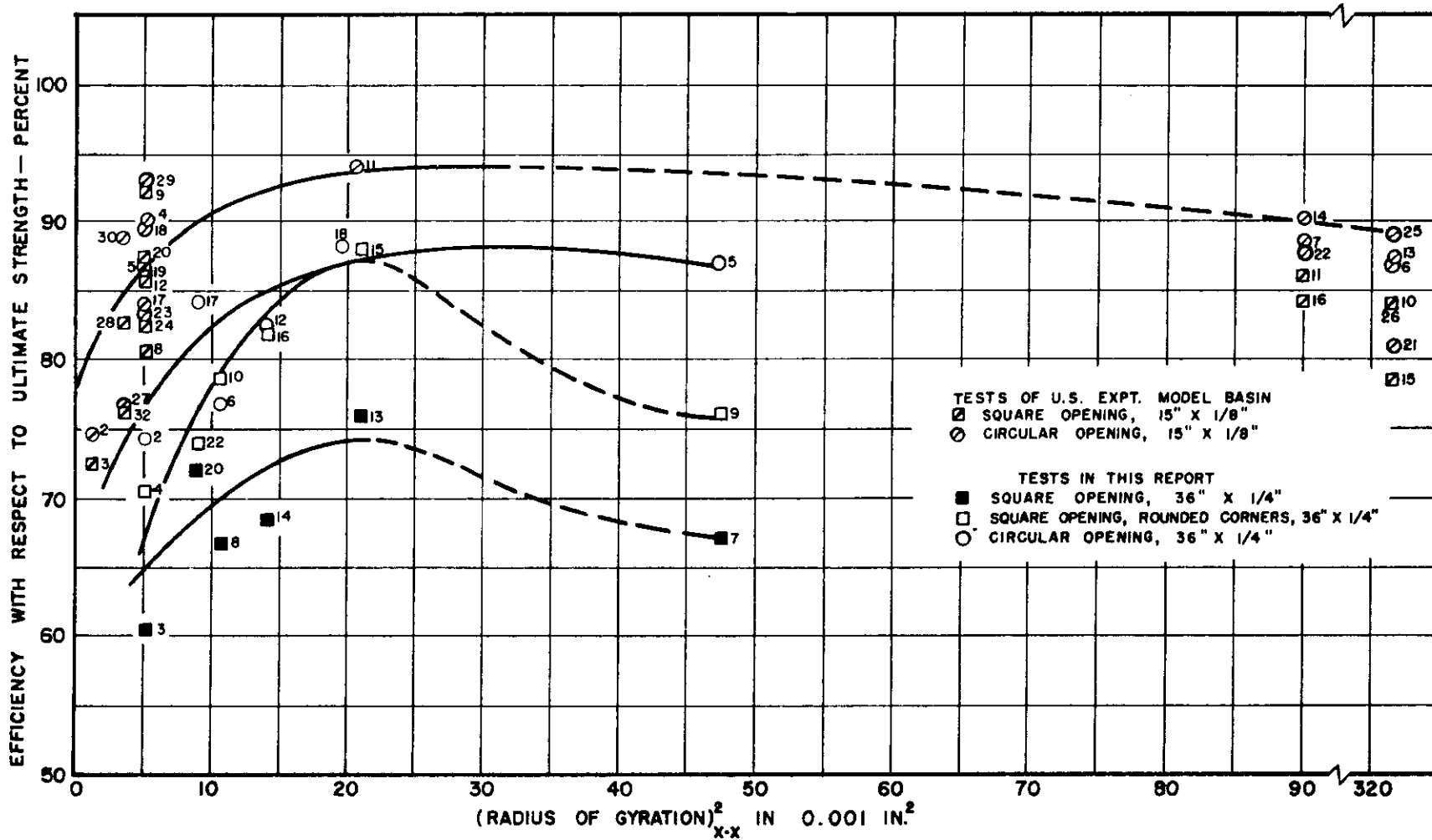


FIG. 19. EFFICIENCY WITH RESPECT TO ULTIMATE STRENGTH OF PLATES WITH OPENINGS SUSTAINING SHEAR FRACTURE.

shape of the opening is evidenced. Contrariwise, as the notch severity of the opening increases, the likelihood increases of losing some of the capacity to carry load or stress and absorb energy because of too much rigidity in the reinforcement in the direction of the thickness of the body plate.

9. Overall ductility of the plates with openings. The degree of ductility attained by the different specimens is summarized in Table VI. While the average unit strain to ultimate load in the plain plates exceeded 21 per cent, it ranged from approximately 2 to 11 per cent in the plates with openings. Most of the values fell between 2 and 6 per cent. The strain raising and ductility reducing effect of an opening in a structural member was made quite apparent by these tests.

10. Efficiency of the plates with openings. One purpose of the reinforcement is that of restoring to the greatest possible degree the properties of the plain plate. The ratio of the value of some particular property of a plate with an opening to the similar value for a plain plate may be called the efficiency with respect to the property under consideration. This ratio expresses the degree to which the reinforcement restores the qualities which would exist in the plain plate.

Table VII lists the values of the efficiency of the various 36-in. by 1/4-in. plates with openings. The average of the values for the two plain plates was used as the basis for each comparison. All the specimens in this table were tested

TABLE VI
 ELONGATION TO ULTIMATE LOAD AND FAILURE OF PLATES
 WITH AND WITHOUT OPENINGS

Spec. No.	Reinforcement Type	Per Cent	Test Temp. F.	Gage Length In.	Total Elongation in Gage Length to:		Av. Unit Strain in Gage Length to:	
					Ultimate Load In.	Failure In.	Ultimate Load In./In.	Failure In./In.
<u>Plates Without Opening (36" x 1/4")</u>								
1			81	36	7.75	8.83	21.5	21.5
23			76	36	7.70	12.35	21.4	34.3
<u>Plates With Square Opening with Sharp Corners (36" x 1/4")</u>								
3		0	72	36	1.08	1.80	3.0	5.0
7	Face Bar	40	75	36	1.19	2.35	3.3	6.5
8	Face Bar	16	74	36	1.31	2.54	3.6	5.1
13	Doubler	104	76	36	0.97	2.10	2.7	5.8
14	Doubler	51	71	36	0.92	2.01	2.6	5.6
19	Insert	33	76	36	0.73	2.00	2.0	5.6
20	Insert	39	72	36	1.47	2.00	4.1	5.6
<u>Plates With Square Opening with Sharp Corners (36" x 1/4")</u>								
4		0	78	36	2.07	2.67	5.7	7.4
9	Face Bar	40	72	36	1.97	2.73	5.5	7.6
99	Face Bar	40	46	36	2.55	2.38	7.1	6.6
10	Face Bar	16	75	36	3.04	4.00	8.4	11.1
31	Face Bar	16	46	36	3.93	4.16	10.9	11.6
15	Doubler	103	76	36	1.70	2.70	4.7	7.5
32	Doubler	103	46	36	1.93	2.30	5.4	6.4
16	Doubler	52	73	36	2.00	3.38	5.6	9.4
21	Insert	62	77	36	2.86	3.72	7.9	10.3
34	Insert	62	46	36	3.50	3.15	9.7	8.7
22	Insert	39	73	36	1.62	2.80	4.5	7.8
85	Insert & Face Bar	77.8	76	36	3.47	4.37	9.6	12.1
<u>Plates With Circular Opening (36" x 1/4")</u>								
2		0	76	36	2.67	3.12	7.4	8.7
5	Face Bar	40	74	36	2.98	3.53	8.3	9.8
6	Face Bar	17	73	36	1.94	2.64	5.4	7.3
11	Doubler	102	75	36	2.88	3.38	8.0	9.4
12	Doubler	50	73	36	1.88	2.55	5.2	7.1
17	Insert	39	74	36	2.98	3.59	8.3	10.0
18	Insert	50	75	36	2.88	3.40	8.0	9.4

TABLE VI (Cont.)
 ELONGATION TO ULTIMATE LOAD AND FAILURE OF PLATES
 WITH AND WITHOUT OPENINGS

Spec. No.	Reinforcement Type	Per Cent	Test Temp. F.	Gage Length In.	Total Elongation in Gage Length to:		Av. Unit Strain in Gage Length to:	
					Ultimate Load In.	Failure In.	Ultimate Load In./In.	Failure In./In.
<u>Plates With Square Opening With Sharp Corners (36" x 1/2")</u>								
95		0	76	36	1.81	2.81	5.0	7.8
96		0	-46	36	0.67	0.67	1.9	1.9
<u>Plates With Square Opening With Rounded Corners (36" x 1/2")</u>								
37		0	76	36	2.30	3.36	6.4	9.3
38A		0	-20	36	3.80	4.63	10.6	12.9
38		0	0	36	3.60	3.60	10.0	10.0
<u>Plates With Circular Opening (36" x 1/2")</u>								
69		0	76	36	2.45	3.57	6.8	9.9
<u>Plates With Square Opening With Rounded Corners (48" x 1/2")</u>								
49	Face Bar	33	70	48	3.36	5.08	7.0	10.6
50	Face Bar	33	-20	48	4.90	4.58	10.2	9.5
51	Doubler	96	74	48	4.04	5.15	8.4	10.7
52	Doubler	96	-46	48	3.58	3.58	7.5	7.5
55	Insert	66	70	48	3.97	4.28	8.3	8.9
55A	Insert	67	69	48	3.94	4.20	8.2	8.7
56	Insert	66	-46	48	3.05	2.75	6.3	5.7
70	Insert	39	76	48	3.25	4.0	6.8	8.3
71	Insert	39	-46	48	2.00	2.0	4.2	4.2

TABLE VII

EFFICIENCY OF 36" x 1/4" PLATES WITH OPENINGS AS COMPARED WITH PLAIN PLATES
TESTS AT ROOM TEMPERATURE

Spec. No.	Opening		Reinforcement	Efficiency Compared to Plain Plate - Per Cent					
	Shape	Corner Radius In.		Initial Yielding		Ultimate Strength		Energy Absorption	
				Load	Average Stress	Load	Average Stress	To Ult. Load	To Failure
<u>Plates with Unreinforced Openings</u>									
2	Circular	—	—	76	101	75	100	28	19
3	Square	1/32	—	76	101	61	82	8	9
4	Square	1-1/8	—	76	101	72	96	18	15
<u>Plates with Openings Reinforced by a Face Bar</u>									
5	Circular	—	2" x 1/4" Face Bar	84	100	88	104	32	24
6	Circular	—	1" x 1/4" Face Bar	84	106	78	99	18	15
7	Square	1/4	2" x 1/4" Face Bar	84	99	68	80	10	12
8	Square	3/16	1" x 1/4" Face Bar	77	95	67	84	11	13
9	Square	1-1/8	2" x 1/4" Face Bar	83	98	77	91	18	18
10	Square	1-1/8	1" x 1/4" Face Bar	81	103	80	101	30	25
<u>Plates with Openings Reinforced by a Single Doubler Plate</u>									
11	Circular	—	18" Dx 1/4" Doubler	94	94	95	95	34	26
12	Circular	—	13-1/2" Dx 1/4" Doubler	86	98	83	95	19	16
13	Square	1/32	18" Sq. x 1/4" Doubler	88	88	77	84	10	12
14	Square	1/32	13-1/2" Sq. x 1/4" Doubler	78	89	69	86	8	10
15	Square	1-1/8	18" Sq. x 1/4" Doubler	94	94	89	89	18	18
16	Square	1-1/8	13-1/2" Sq. x 1/4" Doubler	78	89	83	95	19	19

All specimens listed in this table sustained shear fracture.

TABLE VII (Cont.)

EFFICIENCY OF 36" x 1/4" PLATES WITH OPENINGS AS COMPARED WITH PLAIN PLATES

TESTS AT ROOM TEMPERATURE

Spec. No.	Opening		Reinforcement	Efficiency Compared to Plain Plate - Per Cent					
	Shape	Corner Radius In.		Initial Yielding Load Average Stress		Ultimate Strength Load Average Stress		Energy Absorption To Ult. Load To Failure	
<u>Plates with Openings Reinforced by an Insert Plate</u>									
17	Circular	—	12-3/4" Dx 1/2" Insert	84	98	84	99	30	22
	Circular	—	10-1/2" Dx 1" Insert	88	101	89	102	31	23
19	Square	1/32	15" Dx 1/2" Insert	78	93	62	73	6	9
20	Square	1/32	12-2/4" Sq. x 1/2" Insert	83	97	73	85	14	14
21	Square	1-1/8	15" Dx 1/2" Insert	78	85	82	89	29	25
22	Square	1-1/8	12-3/4" Sq. x 1/2" Insert	83	97	75	87	15	16
<u>Plate with Opening Reinforced by a Combination of Face Bar and Insert Plate</u>									
85	Square	1-1/8	1-1/2" x 1/4" Face Bar 1 1/4" x 1 1/4" x 1/2" Insert	76	82	85	85	36	29

-55-

at room temperature and sustained shear fractures. Since no 1/4-in. plain plates were tested at low temperatures or 1/2-in. plain plates at either room or low temperature, no direct comparisons could be made for the remainder of the specimens.

The efficiency with respect to the average net stress at general yielding did not vary through a very wide range. However, the ultimate strength and the energy absorption to ultimate load were greatly affected by the shape of the opening and to a lesser degree by the type and amount of reinforcement. The ten specimens which gave the best performance had either circular openings or square openings with rounded corners, and all had reinforcement. In this group were two with face bar reinforcement, three with insert plate reinforcement, and four with doubler plate reinforcement, and one with a combination of face bar and insert plate reinforcement.*

One interesting observation about the plates with square openings, either reinforced or unreinforced, is that their performance was worse in every case than that of the unreinforced plates with the square opening with rounded corners or the circular opening. Reinforcement could not improve a bad

*The results of these tests should not be applied to types of reinforcement and proportions of details of reinforcement which depart from those existing in these specimens. Quilted doubler plates, doubler plates on both faces of the strength member, wide coamings, and heavy insert plates are examples of types of reinforcement or proportions of details which differ considerably from those described in this report.

notch.

The inherent low capacity of a member containing an opening to absorb energy is indicated in Table VII. The efficiency with respect to energy absorption to ultimate load ranged from 6 to 36 per cent with only eight specimens having values in excess of 25 per cent. While it appears possible in the case of shear fractures to design welded reinforcement for openings that would develop the ultimate strength of a plain plate, the stress concentrations present in an opening and its reinforcement would prevent the member from absorbing more than a small fraction of the energy of a plain plate.

11. Modes of fracture in plates with openings. A summary of the modes of fracture of the different specimens as given in Table V is as follows:

No. of Specimens	Mode of Fracture	<u>1/4-in. Plates at</u>		<u>1/2-in Plates at</u>			
		Room Temp.	-46°F	Room Temp.	0°F	-20°F	-46°F
28	Shear fracture	22	0	6	0	0	0
1	One per cent cleavage	0	0	1	0	0	0
4	50--90 per cent cleavage	0	2	1*	1	0	0
8	90--100 per cent cleavage	0	2	0	0	2	4

*Initial failure in pulling plate. Specimen reloaded ten days later.

The transition temperature as determined by the Navy tear test for a representative sample of each plate thickness was -40°F

for the 1/4-in. plate and 40°F for the 1/2-in. plate.

It is interesting to relate the mode of fracture to the energy absorbed to failure as in the following tabulation:

Group of Specimens	Notch Radii in Group, in.	Percentages of Cleavage in Group	Energy to Ult. Load, in-kips	
			Minimum	Maximum
<u>36" by 1/4" Plates</u>				
Unreinforced	1/32, 1 1/8, 4 1/2	0	338	1136
Reinforced	1/32, 1 1/8, 4 1/2	0	328	1442
Reinforced	1 1/8	63--97	894	1857
<u>36" by 1/2" Plates</u>				
Unreinforced	1 1/8, 4 1/2	0	1100	1739
Unreinforced	1/32, 1 1/8	87--100	486	2890
<u>48" by 1/2" Plates</u>				
Reinforced	1 1/8	0--1	3510	4730
Reinforced	1 1/8	57--100	2084	4303

There are represented here four types of failure:

1. High-energy shear fracture.
2. Low-energy shear fracture.
3. High-energy cleavage fracture.
4. Low-energy cleavage fracture.

This tabulation shows that the minimum value of energy shown was always associated with the sharpest notch radius, regardless of the type of fracture surface.

Examples of the first type of failure can be found in Table V. The second type of failure appeared in at least the three of the 36-in. by 1/4-in. plates with a square opening which absorbed the least energy of all the plates, Specs. No. 3, 13, and 14. A number of examples of the third type of failure are those in which the energy absorption for predominately cleavage fractures was equal to or even greater than that developed for shear fractures. The fourth type, which was completely brittle in nature, was found in only two plates, Specs. No. 71 and 96. This unusual array of types of failure suggests to the designer that cleavage fracture of itself should not be regarded as undesirable, but rather low-energy fracture, whether cleavage or shear.

The third type of failure, high-energy cleavage fracture, occurred when the original notch in the specimen was not sufficiently sharp to initiate cleavage fracture at the testing temperature and if the testing temperature was below the transition temperature of the steel as determined by the Navy tear test. In this case the first crack to form was a shear fracture. This shear crack then became the predominate stress raiser and was immediately sufficiently severe to cause a cleavage fracture to pass completely through the plate. This cleavage fracture occurred in the same explosive manner as the low-energy cleavage fracture. The apparent explanation for this third type of failure is that the specimens displaying

this type fell in the fracture transition range and not in the ductility transition range for the particular combination of geometry and steel.

In Fig. 15, the plots on the left side of the figure include specimens with a circular opening or a square opening with a $1\ 1/8$ -in. corner radius. These plates sustained high-energy cleavage or shear fractures. These data fell in two groups--one for reinforced and one for unreinforced specimens--and certain favorable geometric characteristics of the plates, not testing temperature, were the decisive factor in determining the test results. However, in the diagrams on the right of Fig. 15 for the unreinforced plates with a sharp notch of $1/32$ -in. radius, shear fracture and brittle or low-energy cleavage were the modes of failure, and the plotted points were segregated according to the testing temperature. In the latter case with an unfavorable geometry, a sharp notch present, testing temperature became a decisive factor.

V. CONCLUSIONS

In view of the complexity of this problem, as well as its many interrelations with others not even touched upon by this investigation but no less important, some of the following conclusions may be modified by future research in this field. Such factors as the properties of different steels, states of stress other than pure tension, and dynamic loading, for example, deserve intensive study.

1. Conclusions with respect to plastic flow and fracture.

- a. Good design by reducing stress concentration results in a more efficient plastic stress and energy distribution and therefore a greater capacity for plastic deformation. This greater capacity is reflected in a higher ultimate strength and energy absorption.
- b. High energy absorption and high ultimate strength can be obtained in welded structures, together with either shear or cleavage fracture, if all stress raising effects are sufficiently reduced and if, in addition, the operating temperature of the structure is not far below the fracture transition temperature for the steel as determined by the Navy tear test.
- c. Cleavage fracture of itself should not be regarded as dangerous, but rather low-energy fracture, whether cleavage or shear.
- d. Cleavage fracture was accompanied by a greater concentration of the high values of plastic stress and energy around the opening than shear fracture. These differences in the plastic flow of two identical specimens give warning that tests resulting in shear fractures cannot be used to give entirely reliable predictions of the probable results of low-temperature tests which would produce cleavage fracture.

- e. Where cleavage fracture is anticipated rather than shear fracture, it is more necessary to reduce stress concentration because, as these tests indicated, the concentration of the high values of plastic stress and energy was greater in the case of the cleavage type of fracture.

2. Conclusions and recommendations with respect to the design of an opening and its reinforcement.

- a. It is reasonable to conjecture on the basis of these tests that the best possible practical design of an opening and its reinforcement would assure the development of the yield strength and the ultimate strength of the steel--but only about one-fourth to one-third of its potential energy absorbing capacity--and these properties only when the conditions of loading and temperature are favorable.
- b. The primary problem in the design of an opening and its reinforcement is the reduction of stress concentration. The shape of the opening was found to be more important in these tests than the amount or type of reinforcement. The present tests indicated that a corner radius equal to or greater than one-eighth of the width of the opening is desirable.
- c. While reinforcement can somewhat increase the load carrying capacity of a member with an opening, it does not appreciably improve its energy absorbing capacity.

- d. Increasing the cross section area of the reinforcement for an opening brought about increased ultimate strength and energy absorption only when the width of the reinforcement in the direction of the body plate thickness was kept small.
- e. The optimum percentage of reinforcement was around 35 to 40 per cent for a face bar, 95 to 100 per cent for a single doubler plate, and somewhere between 30 and 60 per cent for an insert plate.
- f. Above the optimum percentage of reinforcement, failure with reduced strength and energy absorption occurred in the weld at the outer edge of the reinforcement or in the body plate.
- g. That the optimum percentage of reinforcement increases as abrupt changes in the dimensions of the reinforcement in the direction of the body plate thickness are reduced was apparent in the present tests.
- h. The square of the radius of gyration of the net cross section of the plates with openings about the transverse centerline was a satisfactory empirical parameter for determining the best distribution of the reinforcement.

VI. ACKNOWLEDGMENTS

This research was under the guidance of Project SR-119 Advisory Committee of the Ship Structure Committee. The members of the Advisory Committee are:

Mr. John Vasta, Chairman	Bureau of Ships, Navy Department
Mr. W. G. Frederick	Maritime Administration
Mr. Hubert Kempel	Military Sea Transportation Service
Mr. M. J. Letich	American Bureau of Shipping
Mr. W. E. Magee	United States Coast Guard

The investigation was carried out at the University of Washington in the Structural Research Laboratory of the Department of Civil Engineering, of which Professor R. B. Van Horn is executive officer, as a project of the Engineering Experiment Station, of which Professor F. B. Farquharson is director. Their assistance in the administration of the project, as well as that of Dr. H. E. Wessman, Dean, College of Engineering, is appreciated.

The investigation was directed by Dr. R. A. Hechtman, Professor of Structural Research. Dr. D. Vasarhelyi, Assistant Professor of Civil Engineering, was project engineer and was assisted by Mr. Keith J. Kenworthy, Mr. Y. T. Yoshimi, and Mr. Robert McHugh, Research Assistants.

The authors express their appreciation to Mr. Vasta for his active interest in the research, to Dr. Finn Jonassen of the National Research Council for his many suggestions and encouragement, and to the many others who assisted on the project.

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