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Predicting Hull Bending Moments for Design

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ABSTRACT

This paper surveys current knowledge of the various hull longitudinal bending loads acting on seagoing ships, with particular attention to new developments in the last five years. These "demands" on the structure include still water bending, low and high-frequency wave bending and thermal effects. A probabilistic approach is followed wherever possible, and an attempt is made to generalize on the accepted basic approach to predicting a long-term cumulative distribution of wave-induced bending moment, showing that many apparently diverse methods are closely related. The problem of combining all loads to obtain a complete probabilistic picture of loads is discussed in relation to the probability of either ultimate failure or propagation of fatigue cracking to the point where repair is required. Consideration is also given to unexpectedly high loads that may be encountered under unusual circumstances at sea.

The paper concludes with a discussion of the application of the expanding understanding of demand to the problems of design. It is shown that the ultimate goal is to be able to match structural "capability" to demand by predicting failure probability, and finally to establish acceptable levels of such probability. The principal gaps in the ideal procedure that must be filled before this is possible are listed. Meanwhile, it is shown that the partial application of the probabilistic approach has already resulted in more rational longitudinal strength standards.

NOMENCLATURE

C	anticipated cost of damage
F	anticipated total cost of failure
$H_{1/3}$	significant wave height
I	initial cost of ship
m_n	nth moment of spectrum
L	total expected lifetime cost
p,q,r	probabilities
P,Q,R	cumulative probabilities
R	Rayleigh parameter
s	standard deviation
S	spectrum shape parameter
S_B	bending moment spectrum
S_ζ	wave spectrum
T	period
T_a	average period
T_e	period of encounter
x,y,z	variables
Y^2	response amplitude operator
ε	spectrum broadness factor
μ	angle, wave component to dominant wave
σ	standard deviation, or rms value
ψ	ship-to-wave heading angle
ω	circular frequency
ω_e	encounter frequency

INTRODUCTION

Background

For many years after the introduction of steel into ship construction, the design of the main hull girder was based on a nominal standard bending moment in association with an allowable stress that varied with ship length. As ships became larger and faster, and novel seagoing vehicles were developed, a more rational approach was needed. This led, over the past decade, to a great increase in knowledge of all types of loads acting on ships' hulls. At the same time, understanding of the nature of hull girder failure, as well as sophisticated new techniques of structural analysis, were developed.

It has become clear that the loads--particularly those caused by waves at sea--are highly variable in nature and can best be analyzed by statistics and described in probability terms. Furthermore, the strength of the hull is also variable, as a result of variations in scantlings, steel quality, workmanship, and other factors. Hence, the principles of reliability theory, developed in the field of civil engineering, are peculiarly adapted to the problem of ship longitudinal strength. The ideal goal has become to develop methods of expressing both the loads, or "demand," on the hull and its structural strength, or "capability," in terms of probabilities, so that the probability of failure can be calculated. A satisfactory and economical design will then be one in which failure probability has been reduced to an acceptably low value.

Meanwhile, the reliability approach has already clarified the longitudinal strength problem and has provided a rational basis for extrapolating the design loads for larger and larger ships. Until the ideal, fully probabilistic methods become available, partial probabilistic approaches are being used by all classification societies in evolving and updating their standards of strength for routine use in design--particularly for wave bending moments.

It is the stated purpose of this Symposium to consider the state-of-the-art in all aspects of structural response to extreme loadings, including attention to the reliability approach to design. Hence, it seems appropriate that this paper should review current knowledge of hull loads, particularly longitudinal bending moments. It may be considered a sequel to a survey paper on "Dynamic Loadings Due to Waves and Ship Motions," by the present authors, which was presented at the joint SSC-SNAME Ship Structure Symposium held five years ago [1].

Since the last Symposium, the basic principles discussed in the above paper have not changed and therefore need only brief summary at this time. Principal attention in the present paper will be given to outstanding new developments since 1975. For more detailed surveys reference is made to reports of Committees I.2 and I.3 to the ISSC of 1976 and 1979 [2][3][4][5]. But since the 1975 paper did not deal with specific procedures for calculating extreme wave-induced bending moments, an attempt will also be made here to generalize the basic principles underlying various available procedures that may appear more different than they really are because of differences in notation, emphasis and application. Attention will be directed to gaps in our understanding and uncertainties that at the present time still prevent full adoption of the reliability approach to the design of ships and other marine structures. The problem of combining loads in a form suitable for design and the possibility of unusually high bending moments under abnormal conditions, such as shoaling water, opposing currents, etc., will also be discussed.

Modes of Failure

Before attempting to consider methods of evaluating all the longitudinal bending loads and then combining them, it is well to discuss the different modes of hull failure that the designer must guard against. This should clarify the form of load information that is needed and which loads are critical. The following statement, largely from the 1975 paper, still stands.

Caldwell [6] considers ultimate failure as the complete collapse by buckling of the compression flange and simultaneous tensile failure of the tension flange. However, it is clear that a considerably less severe damage would be a serious matter, as indicated by such factors as necessity for major repairs, interference with normal ship operation and non-watertightness. Hence, for our purpose we may define damage as a structural occurrence that interferes with the operation of the ship to the extent that withdrawal from service for repair is required, such as:

- Local hull deflection: buckling and/or permanent set.
- Fatigue cracking.
- Brittle fracture, minor.

Failure is a severe damage that endangers the safety of the ship:

- Collapse of the hull girder, by buckling and/or permanent set.
- Extensive brittle fracture.

Considering the various types of damage (or failure) in more detail, excessive hull deflection is a rare occurrence, except locally, and complete failure or collapse is even rarer. This suggests that conventional standards of longitudinal strength are generally adequate--in fact, they may be excessive. Loads that can combine to threaten hull failure are still water bending moments, wave-induced bending moments (quasi-static), vibratory (high frequency) loads, and thermal effects [7].

Second is the possibility of fatigue cracking, which seldom constitutes failure but is important for two reasons: Fatigue cracks, which are fairly frequent, can grow to the point that they must be repaired, and fatigue cracks are notches that under certain circumstances can trigger rapid propagation as brittle fracture. Nibbering notes [8], "It is a favorable circumstance that fatigue cracks propagate very slowly in ship's structures." Cyclic loads to be considered include the same loads as mentioned above, with widely varying periodicities and mean values.

Brittle fracture, which was a serious problem with early welded ships during World War II, was long ago brought under control by insuring satisfactory "notch-toughness" of shipbuilding steel, as well as by eliminating severe design stress concentrations and by improving techniques for welding and inspection. However, brittle fracture can and does occur, and therefore the philosophy has been one of "fail-safe" design. Crack arresters, consisting of rivetted seams or strakes of steel having lower transition temperature are provided as standard practice. These have proven effective in limiting crack propagation and thereby restricting brittle fracture to a minor damage rather than a hull failure problem.

It can be argued then, that since fatigue cracking does not normally threaten the life of the ship and brittle fracture can be controlled, the primary criterion of rational ship structural design should be one of ultimate strength--reducing the probability of excessive deflection through buckling or plastic flow to an acceptable level, as defined subsequently. The secondary criterion of rational design is fatigue strength--reducing the number of cases of cracks that propagate to a size requiring repair to a cost effective level, also discussed subsequently. Accordingly, the extreme and cyclic loads that affect both ultimate and fatigue strength will be given particular attention here. As mentioned above, these consist of:

- Still water bending moments.
- Wave-induced bending moments (quasi-static).
- Vibratory bending moments.
- Thermal effects.

In the next section separate consideration will be given to each of these bending moment components, and how they can be predicted.

BENDING MOMENT COMPONENTS

Still Water Loads

The longitudinal bending moments experienced by a ship in port can vary widely, particularly with the amount and distribution of cargo. Hence, they represent an important load, both as an addition to wave bending moments that could cause ultimate failure and as a mean value about which the cyclic loading oscillates. Although still water bending moments are easily calculated when the distribution of cargo and other weights is known, it has been found that records are seldom kept and, therefore, it is difficult to assemble statistical data on actual values in service [7] [9].

However, it appears tentatively that still water bending moments can be described by two (or more) normal probability density functions, one representing outbound and the other inbound conditions [7]. See Fig. 1. The validity of this approach has been confirmed by recent work of Ivanov [10] and Soeding [11]. Typical conditions, such as full load, ballast, light load, etc. are first established. Calculations are then made--which can be verified by service records--of both the average value of bending moment and the standard deviation for each basic condition, assuming normal distribution functions. This approach is desirable because ships are never loaded exactly in accordance with the designer's loading manuals, and it is consistent with an overall probabilistic approach to design. However, Maniar points out in discussion of Committee I.3 report, ISSC 1979 [5] that operators will avoid extreme conditions, and therefore these distribution functions may be truncated, as shown in Fig. 1. See also [9].

The bending moment created by a ship's own wave when moving at high speed (above Froude No. of 0.20) can be determined by model test, or from published model test results [12].

Concern is often expressed regarding residual stresses in steel plates and shapes and stresses "locked-in" by the welding process. It is possible that, where such stresses exist in combination with other stresses at a weld defect or notch, they might under certain conditions contribute to the inception of a brittle fracture. However, as noted in [7], "For other types of failure it seems reasonable to consider them to be of minor significance to longitudinal strength, since they tend to be eliminated by 'shakedown' or adjustment in service. That is, an occasional high longitudinal wave bending load--in combination with other loads--may cause local yielding in the high residual stress region. Upon termination of this high wave load, the structure will tend to return to a condition of reduced residual stress."

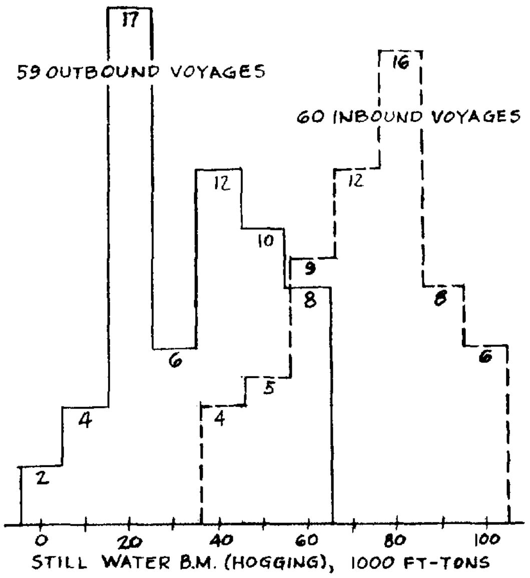


Fig. 1 Histogram of Still Water Bending Moments for Containership New Orleans [7]

Quasi Static Wave Bending

When a ship encounters irregular waves or swells with wave components in the range of 0.5 to 2.0 times the length of the ship (shorter at oblique headings), significant bending moments are developed, acting at the comparatively low frequency of wave encounter. These wave-induced bending moments were first determined by model tests in waves [12] [13]. Subsequently, the development of the strip theory approach to the calculation of ship motions by Korvin-Kroukovsky [14] led to methods for calculating stress and bending moment in regular waves [15] [16] [17].

The extension of regular wave results to predicting various ship responses to short-crested irregular seas was accomplished by St. Denis and Pierson [18], on the assumption that both the irregular waves and the ship short-term responses are stationary stochastic processes, where stochastic means random but following definite statistical laws. By short-term is meant periods of time of typically a few hours during which sea conditions remain essentially constant, statistically speaking, so that the character of both the sea and the ship's response can be assumed to be "stationary". Hence, under these assumptions, the bending moment response can be predicted for any ship for which response amplitude operators (RAOs) are available in any known or assumed sea spectrum. Fig. 2 shows how the directional wave spectra are multiplied by the appropriate RAOs to produce the directional response spectra,

$$S_B(\omega,\mu) = S_\zeta(\omega,\mu)\,|Y(\omega,\mu)|^2, \qquad (1)$$

where S_B is the bending moment spectrum, S_ζ is the wave spectrum, Y^2 is the RAO, ω is circular frequency and μ is the angular direction of a wave component relative to the dominant direction. When these components are integrated over wave direction a single response spectrum is obtained, whose area and shape define the bending moment response,

$$S_B(\omega) = \int S_B(\omega,\mu)\, d\mu. \qquad (2)$$

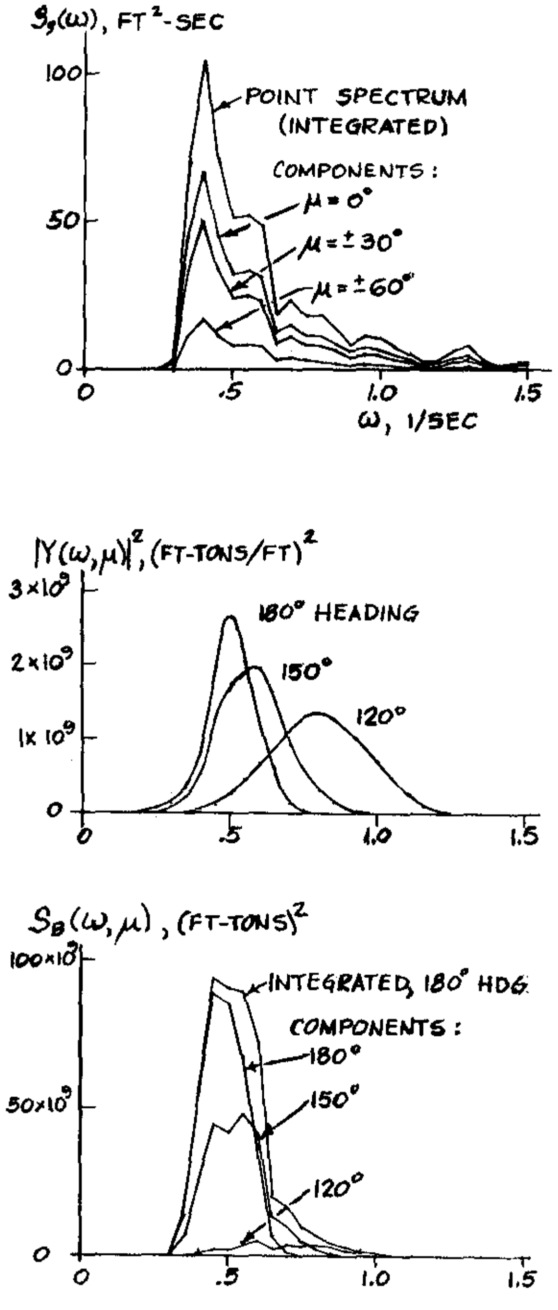


Fig. 2 Short-Crested Sea and Typical Bending Moment Response

Short-term statistics, applicable to periods of time from ½ hour to several hours (while sea conditions remain relatively unchanged), can be derived from the response spectrum by taking the various moments of $S_B(\omega)$,

$$m_n = \int \omega^n\, S_B(\omega)\, d\omega. \qquad (3)$$

First, the area under the spectrum, m_0, defines σ^2, the variance of the short-term process, i.e. the mean-square value of the deviations from the mean at equal (or random) intervals, and σ is the root-mean-square value. Thus,

$$\sigma^2 = m_0$$

$$\sigma = \sqrt{m_0}$$

Furthermore, it is well known that if the response spectrum is narrow-band, the short-term peak-to-mean amplitudes are Rayleigh distributed,

$$p(x) = \frac{2x}{R}\, e^{-x^2/R} \qquad (4)$$

In this case the parameter of the Rayleigh distribution, R, is given by,

$$R = 2m_0 = 2\sigma^2$$

Various statistical properties of the peak-to-mean amplitudes can then be determined. For example,

Significant amplitude
(average of highest 1/3) = $2.0\sqrt{m_0}$

Average "apparent" period, based
on zero crossings = $2\pi\sqrt{m_0/m_2}$

For a broad spectrum, of course, the Rayleigh distribution is not applicable, and a more generalized distribution is required, involving the spectrum broadness parameter,

$$\varepsilon = \left(1 - \frac{m_2^{\,2}}{m_0\, m_4}\right)^{\frac{1}{2}} \qquad (5)$$

This generalized distribution function is a complicated expression [9] which reduces to the Rayleigh form when $\varepsilon = 0$.

An important study of the applicability of the Rayleigh distribution was made by Dalzell, Maniar and Hsu [9] on midship stress (bending moment) records. They found that for an ocean-going dry bulk carrier (Fotini L, $C_B = 0.84$), "the assumption that the short-term wave-induced moment fits the Rayleigh distribution would be reasonable, if not always true. The magnitudes of compression and tension are symmetrical." For a large, high-speed container ship (SL-7, $C_B = 0.53$) the bending moment maxima "at slow speed fit the Rayleigh distribution reasonably well." But at normal operating speeds, "a relatively small portion of the data fits the Rayleigh distribution." A much better fit was obtained with the generalized Rayleigh distribution, with broadness parameter ε "approaching unity for over half the data" (i.e. approximating a normal distribution), particularly in quartering and following seas. "The ratio of compression to tension of wave-induced bending moment maxima is approximately 1:1.2 for extremes and 1:1.06 for rms." It is concluded that "if it is assumed for the simplification of analyses that the short-term wave-induced bending moment maxima are always Rayleigh distributed, the resulting long-term prediction is likely to be conservative to a degree which is not possible to judge based on the work performed." This means that we may accept the Rayleigh assumption at this time, considering the many uncertainties that still remain, but in the long-run further refinement will be needed, particularly for high-speed ships.

Combined Vertical and Lateral Wave Bending

A ship sailing obliquely into a train of waves will be subject to unsymmetrical bending about a neutral axis that continually shifts its angular position. This can be accounted for by considering that there is a lateral (horizontal) as well as a vertical component of longitudinal bending moment. (Note that the directions are not really horizontal and vertical when the ship rolls.) RAOs for lateral bending moment can be determined either by experiment [19] or by theoretical calculation [17]. Lateral bending moment is mainly a dynamic effect, since the only gravity forces are proportional to the sine of the roll angle. If one is concerned only with extreme stresses--as occur at the deck-sheer strake intersection and at the bilge--an effective vertical bending moment can be determined [7] [20]. This effective bending moment is that simple vertical moment that would produce the same stress all across the deck as the maximum deck edge stress resulting from combined vertical and horizontal bending. It depends on the ratio of the section moduli for vertical and lateral bending.

However, for ultimate strength we are not interested in the stress at the deck edge (or bilge), since this structure is not likely to fail under compressive loading. We are more concerned with the behavior of deck (or bottom) plating panels, and the above effective bending moment is unrealistic because it assumes that the deck edge stress continues uniformly across the deck. The panel loads will be greater or less than those calculated on the assumption of vertical bending alone, and, of course, the loads will have a non-uniform distribution across the section. It is difficult to deal with this problem without considering the local longitudinal stresses at critical points in the section, and this can be done for a specific midship section by calculating RAOs for stresses at the critical points, using the RAOs for vertical and lateral bending and their phase angles. Then short and long-term probabilities can be calculated in the same manner as for vertical bending moments, as discussed in subsequent sections. Another approach described by Dalzell [21] is to make use of so-called cross spectra. He gives an example of combining the effects of vertical bending moment, horizontal bending moment, torsional moment, shearing forces and axial force by this method. See also [20] and [22].

The above procedures, however, can only provide probability information on stresses at separate, specific points, not the simultaneous stresses at different points. It is clear, therefore, that this problem of providing detailed combined bending moment information in probability format for use of the structural designer requires further research.

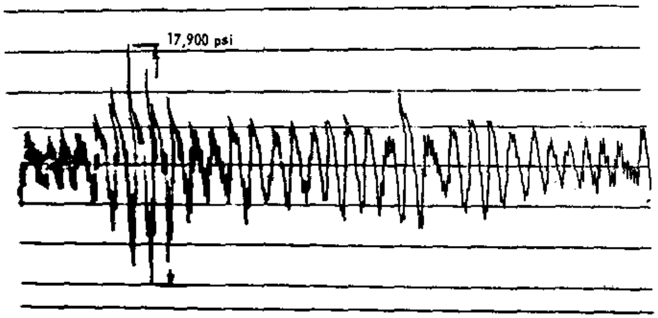
Meanwhile, for the present we have two practical alternatives: 1) to confine the major calculation efforts to vertical bending, with an empirical allowance for the effects of combined vertical and lateral bending; 2) to substitute a single extreme regular wave (at different headings) for the irregular seaway, which can provide an approximate deterministic solution.

Torsional moments are also of interest, mainly in respect to their effect on local stresses, such as at hatch corners. RAOs can be calculated or obtained experimentally and, hence, probabilities of stresses at specific points can be calculated. It is not expected that torsional loads will have a significant effect on ultimate strength of the hull girder, however.

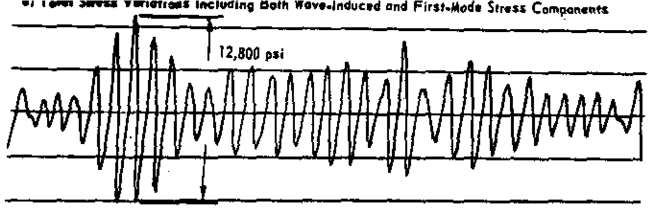
Dynamic Vibratory Bending

The vibratory loads excited by waves at sea are of relatively high frequency, corresponding to the natural frequencies of hull vibration. As indicated in our 1975 paper [1], these loads are either transient or cyclic in nature. The former category is generally described by the terms slamming and whipping, where slamming refers to the initial effect of a wave-ship impact and whipping to the consequent vertical hull vibration in one or more modes. Wave-excited cyclic responses are generally referred to as springing. Both the transient and cyclic hull responses can, in principle, be handled by the theory of vibration of a free-free beam. Since fairly complete discussions of the nature of these vibratory loads and problems of dealing with them in design are given in [1] and [7], only some highlights will be considered here. Examples of actual stress records showing both low and high-frequency effects (as shown in Fig. 3), with extensive analyses for a dry bulk carrier and a high-speed container ship, are given in [9]. Mention should also be made of the work of Professors Bishop and Price [23] [24] in developing an integrated theory of ship dynamic response, covering both low and high frequencies, that clarifies short-term behavior. Since the high-frequency effects are not always present, it is desirable here to consider them separately from low-frequency responses for long-term predictions. See Fig. 3.

Springing can be treated as a stochastic process over the short-term, and methods are available for calculating the RAOs, assuming that only the fundamental (2-noded) vertical mode need be considered. A method of calculation was first presented and applied by Goodman [25] and later extended--particularly for Great Lakes bulk carriers--by Hoffman and van Hooff [26] and Stiansen, et al [27].



a) Total Stress Variations Including Both Wave-Induced and First-Mode Stress Components



b) Wave-Induced Stress Variations (Frequency Approx. 0.1 Hz)

c) First-Mode (Springing) Stress Variations (Frequency Approx. 0.70 Hz)

Fig. 3 Typical Record of Midship Stress Variation, M.V. Fotini L, Showing Filtered Low and High-Frequency Stresses [7]

Having the RAOs, response spectra can be calculated in the normal manner, and hence a long-term cumulative distribution can be determined. One difficulty is that wave spectral data may not be sufficiently well defined in the range of encounter frequencies corresponding to the natural frequency of hull vibration. Another is the abundant evidence [28] [29] that springing is also excited by longer waves--at encounter frequencies that are whole fractions (sub-harmonics) of the natural frequency--as well as by short waves at resonant frequencies. Pending the development and confirmation of a suitable non-linear theory to explain these effects, it may be possible to make an approximate allowance for them.

Most difficult of all to deal with is the transient phenomenon of slamming--either associated with bottom impact or flare immersion. A great deal of excellent work has been done on bottom impact slamming by Ochi [30] [31] and others, which shows that it is possible in principle to predict the frequency of occurrence of slams under specified conditions of sea state, ship drafts, speed and heading. Similarly, Kaplan [32] and others have dealt with flare entry slamming. Following the slam, vibratory whipping usually occurs which decays in accordance with the damping characteristics of the ship. A study by van Hooff in [7] confirms the general opinion that slams do not occur at random but within a fairly narrow range of phase angles to the wave excitation. Typically, the whipping that adds to the first wave-induced hogging moment peak after the slam will produce a higher total bending moment than the slam itself. See Fig. 4.

Full-scale strain recording on board S.S. Wolverine State has captured hull girder slam and whipping stress variations experienced during rough sea operations of the vessel. Evaluation of data [33] obtained in various sea states has shown that the severity of slam response--when it occurs--tends to be somewhat insensitive to sea condition. Fig. 5 shows histograms of midship slam stress variation for the Wolverine State when exposed to different Beaufort Nos. The similarity of the high-stress portions of these histograms can be explained by noting that though slamming frequency and severity are increasing functions of sea state, and vary with ship speed and heading, the latter two are under the control of the ship master who will tend to alter course and speed so as to keep the frequency and severity of such slam loadings within reasonable bounds, according to his experience.

Presumably, continued research and study may eventually permit us to predict accurately the frequency of slamming and the resulting additional bending moments expected under different conditions. However, the factor that makes this goal not only elusive but also in a sense unnecessary of attainment is the above practice of the master to alter course or speed to prevent slamming impacts exceeding a certain level of subjective severity. These changes can be quantified only by obtaining statistical data on attainable speed on different types of ship at different headings to a wide range of sea conditions. However, what is really needed is simply full-scale data on the additional bending moment resulting from the most severe slamming and whipping that are allowed to happen. These can be obtained by taking midship stress records and translating them into bending moments, as was done for the Wolverine State in [7]. Of course, the severity of slamming that is permitted will vary among individual captains and with ship type, hull form, etc. Hence, data should be collected from several ships of various types, but not necessarily over long periods of time. Records that include several storms in which slamming occurs should suffice.

The problem may be different for naval ships that may be called upon to maintain high speed into head seas in spite of local or hull girder damage. Here the problem may be to predict the highest impact loading to be expected at a certain maximum speed in a particular limiting sea state, in order that the structure can be designed accordingly.

Some data on maximum whipping stress in relation to quasi-static bending stress were presented in the report of Committee 3 to the ISSC 1973 [34] and quoted in [1]. Full-scale data on four different ships, varying in length from 130 - 230 m. (426 - 754 ft.) are given by Aertssen [35].

Thermal Effects

Records of midship stress obtained on five bulk carriers [36] [9] indicated surprisingly high thermal effects. These showed a consistent diurnal variation, with magnitudes of 3-5 kpsi in some cases. The temperature gradients that

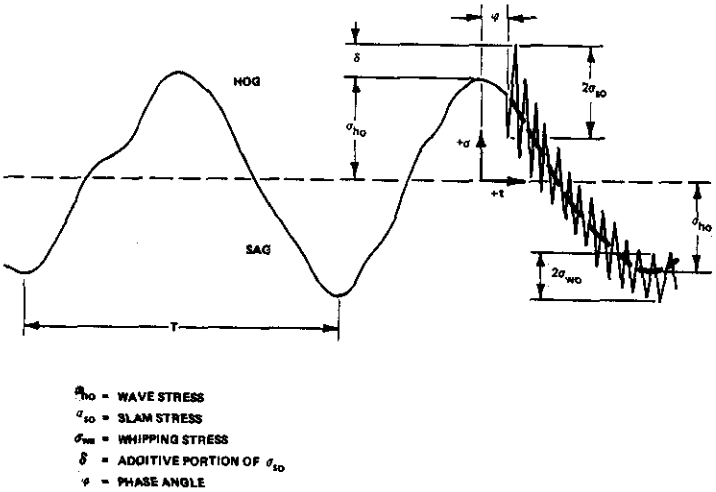


Fig. 4 Definition of Stresses and Phase Angles Involved in Slamming [7]

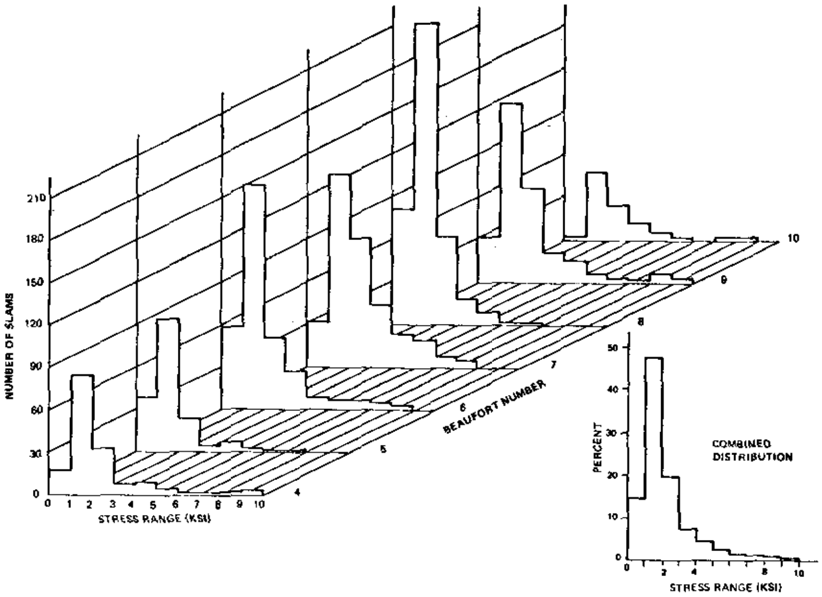


Fig. 5 Peak-to-peak Slam Stress Distributions in Different Weather Conditions, S.S. Wolverine State [33]

produce such thermal stresses may not be, strictly speaking, loads but they are considered as loads here because they have similar effects.

Although it often happened that high thermal stresses occurred at times of low wave bending stresses (sunny weather), and vice versa (stormy and cloudy weather), this was not always the case [36]. The exceptions are presumably times when a heavy swell was running while the weather was clear.

It should be noted that the thermal stress changes recorded here were overall averages, since they were based on combined port and starboard readings. Because of the effect of local shading, it can be expected that even larger thermal stresses would be experienced. However, it can be assumed that such local high thermal stresses can be ignored when considering ultimate strength.

In order to include overall thermal effects in design calculations, two distinct steps are required: estimating the magnitude of the effect under different conditions of sun exposure (Fig. 6) and estimating the frequency of occurrence of these different conditions in service. A procedure for dealing with this problem was presented by Lewis, et al [7] and summarized in the 1975 paper [1]. Little further work has been done on the subject.

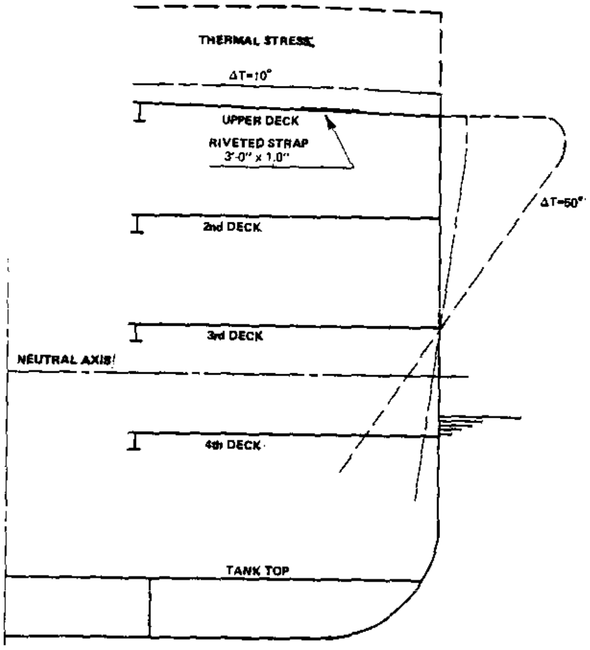


Fig. 6 Calculated Thermal Stresses, S.S. Wolverine State [7]

Local Loads

Finally, consideration must be given to local loads, caused by cargo and internal or external fluid pressures, that would be applied to main hull girder components such as the bottom shell. Insofar as the primary load criterion is concerned, the principal local load to be considered is that of hydrostatic pressure on the double bottom. As discussed by Evans [37], this secondary loading results in a bending moment at the middle of each hold and a larger one in way of each bulkhead. The former causes significant compression in the bottom plating and tensile stress in the inner bottom, both of which would be superimposed on the longitudinal bending stresses. These local stresses are higher in the vicinity of longitudinal girders in transversely framed bottoms, but would be more uniform across the ship in the case of the more common longitudinal double bottoms. Since the bottom pressure is higher when wave crest is amidships than when wave trough is amidships, this effect is greater in hogging than in sagging and would increase the compressive stresses. Pressures can be approximated on the basis of static head for the present purpose, although methods have been developed for taking into account the dynamic effects of ship motions. See Hoffman [38].

Although local stresses can be added to longitudinal bending stresses, the true effect of combined local and longitudinal bending on ultimate strength is a complex matter, to be discussed in a later section.

LONG-TERM PROBABILITIES

Evidence From Full-Scale Tests

A sound approach to the application of probability methods to predicting extreme hull loads--particularly those due to waves--requires the study of statistical data on actual loads measured in service. Fortunately, considerable full-scale stress data have been collected in recent years under projects sponsored by SSC, ABS and others [36] [39]. A summary of ships involved in such tests is given in Appendix 2.

Various analyses and studies of the above full-scale results [40] [41] have revealed a number of basic facts:

- During a short-term observation (i.e. periods of time in which statistical measures of response do not vary significantly) the responses can be considered to be stochastic processes, treated statistically in a similar manner as ocean waves.

- Consequently, short-term responses can be described approximately by Rayleigh probability density functions having the form shown in Fig. 7. (Such functions are handy because they are completely described by a single parameter, as previously explained.)

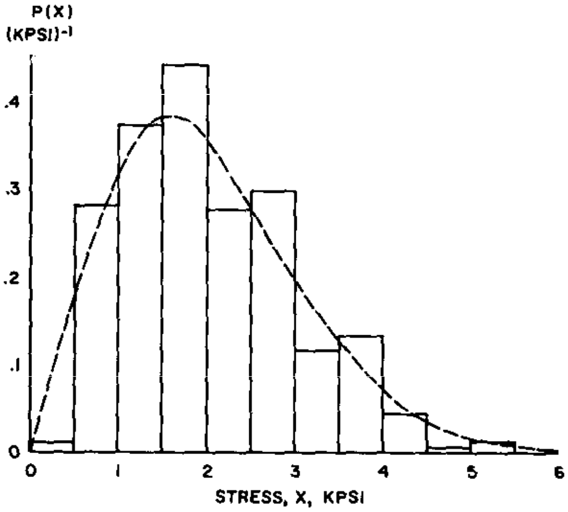


Fig. 7 Typical Fit of Rayleigh Density Function to Wave Stress Data [41]

- Bending moment response varies with the following factors, in order of decreasing importance: significant wave height, ship heading to waves, wave spectral shape or apparent period, ship loading, ship speed.

- Hence, statistical analysis is facilitated by separating data in accordance with these factors, particularly significant wave height (or other measure of sea severity, such as the indirect measure of Beaufort No. or wind speed).

- The statistical long-term distribution of bending moments (derived from stresses by means of static "calibrations" in port) can be

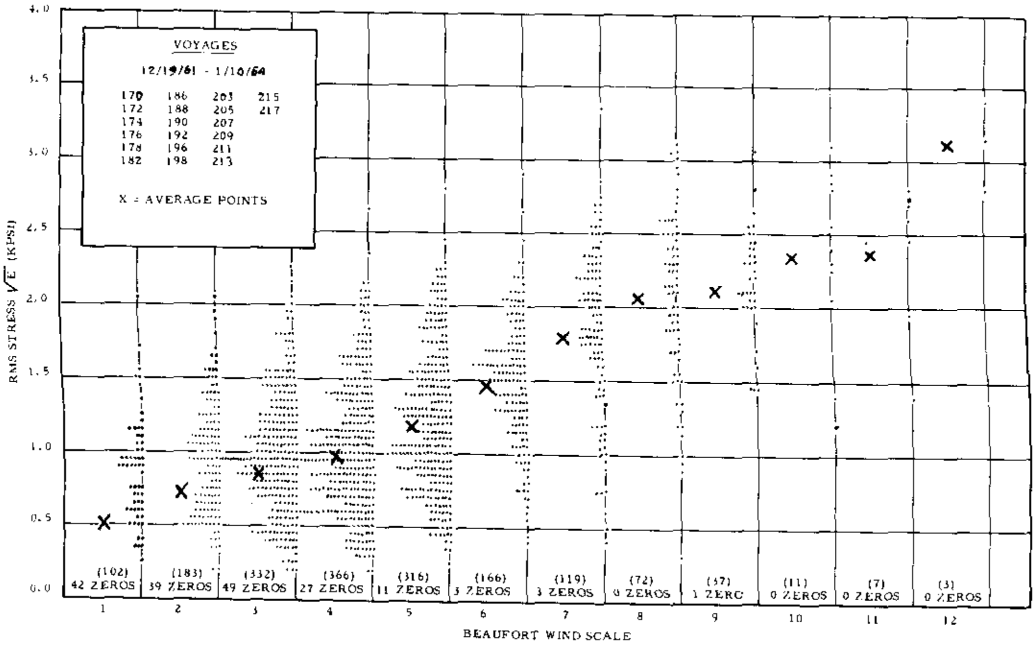


Fig. 8 RMS Stresses from Short-Term Records Plotted vs. Beaufort No., S.S. Wolverine State in North Atlantic Service [41].

interpreted as a summation of many short-term distributions.

The accompanying Fig. 8 presents samples of full-scale results obtained by Teledyne Materials Research for the S.S. Wolverine State [41]. The plot shows Rayleigh parameters (rms-values) divided into different Beaufort Nos. (wind speeds), each point representing one 20-minute stress record. The rms data show roughly normal distributions of the parameters within each Beaufort No. (except above Beaufort 8, where data are relatively scarce). The scatter is due mainly to the factors previously mentioned--variation in spectral shape, ship heading, ship loading, and ship speed. In addition, the variability in wave height vs. Beaufort No. is an important factor here (See Compton [42]).

Fig. 9 shows the result of a similar analysis of available records for the SL-7 containership, divided into Beaufort No. groups. Visually the fit of the Rayleigh parameters to a normal distribution seems reasonably good. (These results are from an unpublished Webb Institute report to ABS).

On the basis of the preceding discussion of observed loads, we can now proceed to consider suitable ways to describe the wave loads for design. It has been shown that damage can occur either on the basis of a single very large bending moment causing failure or as a result of cumulative damage resulting from many cycles of repeated loads. To predict the first type of ultimate failure, we can make use of either a cumulative distribution of all cycles of load, as discussed above, or we can focus attention on only the highest loads by means of extreme value theory or order statistics [Ochi, 43]. For the second type of damage, fatigue, only

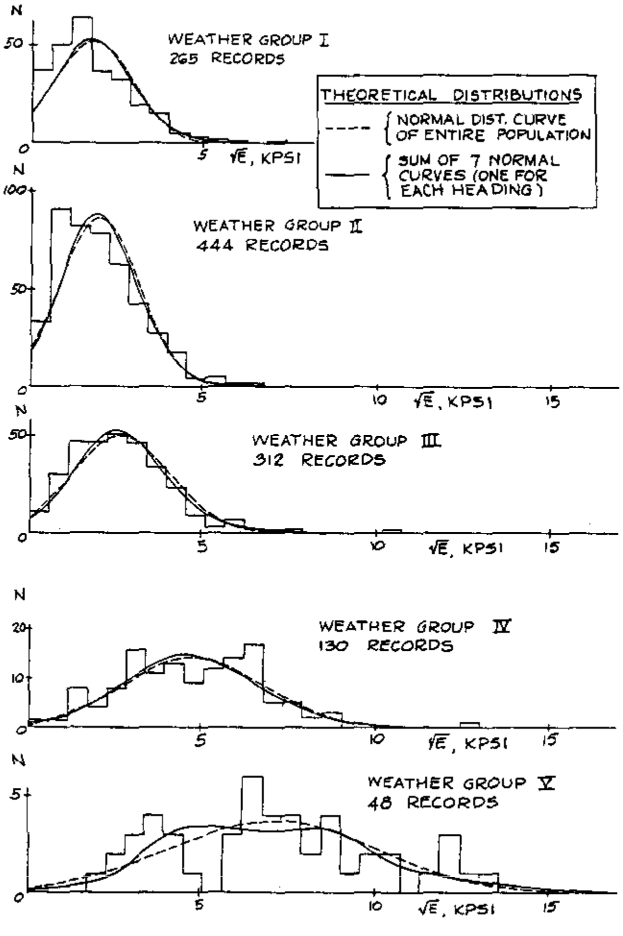


Fig. 9 Comparison of Stress Histograms With Normal Distributions, SL-7 Container Ship.

a complete picture of all load cycles will suffice, as provided by a long-term cumulative distribution.

Statements to the effect that we must use order statistics to find the highest expected load are not correct, since a cumulative distribution of loads will tell us the value that is expected to be exceeded once in a ship's lifetime--or the lifetimes of 100 ships. This is as good a measure of extreme load as the expected highest value. As a matter of fact, the two values should be approximately equal. The former method has been in use for comparative sutdies for many years by all classification societies and many researchers. It can, in principle, provide complete loading information for both ultimate strength and fatigue.

The extreme value approach has been applied, particularly in recent years, to offshore structures when fatigue is not under consideration. It has been developed into a rigorous design method by Ochi, who has found that it is a simple and useful method when only the ultimate or most severe lifetime load is required [43] [44]. He has shown that when properly carried out, the two approaches lead to comparable results [43].

This paper will deal mainly with the long-term cumulative distribution method, which involves first the problem of short-term probabilities previously discussed.

Cumulative Probabilities

The long-term cumulative approach was first developed by Bennet [45], Band [46], Lewis [40], and Nordenstrøm [47] as a means of analyzing full-scale stress data obtained over periods of one to three years and extrapolating to longer periods, corresponding to the lifetime of a ship--or of many ships. The approach was then applied to calculating predicted long-term probabilities of exceedance for bending moment (or stress), for design use.

Different writers have presented many variations of the basic long-term prediction procedure, including Compton [42], Lewis, et al [7], Nordenstrøm [47], Fukuda [48], Soeding [49], Goodman [50], Dalzell, et al [9] and Ochi [43]. They are all based on the idea of predicting short-term probabilities and then combining them on the basis of assumed lifetime service profiles to obtain long-term probabilities. Some variations in the various methods:

- Choice of wave spectra.
- Sequence of dealing with various factors.
- Whether or not component and final distributions are fitted to specific mathematical formulations.

The first step in all methods is the selection of suitable sea spectra covering a wide range of both severity (i.e. significant height or spectral area) and spectral shape. The simplest and most common method of doing this is to adopt a mathematical spectral formulation--such as the 2-parameter ISSC formula [51]--which for each significant height (spectral area) allows for a wide variation in location of the spectrum peak. See Fig. 10. This approach leaves something to be desired in the investigation of wave loads, where the extreme rather than average conditions are most important, because actual sea spectra show far more variety in shape than this idealized family. A method that allows for shape variation is to use a random sample of wave spectra, grouped by their areas, so that a variety of realistic shapes are utilized [40]. Fig. 11. A family of 8 to 10 spectra having the same significant height will provide a means of determining a mean and standard deviation of rms response (for fixed ship speed and heading). This approach to selecting spectra has been made more explicit by Ochi [52], who developed a formulation of double-peaked spectra, established descriptive parameters, and then used statistical wave data to arrive at suitable families or groups of spectra to use in the calculations. This method may be more complicated than results justify. In any case, five or six different significant heights are needed. Ochi [43] has also developed an interesting weighted sample technique for selecting spectra.

The next step is to obtain RAOs for bending moment by either model test or calculation. Model experiments in regular waves involve an extensive program to cover all wave lengths and headings, plus several speeds and at least two conditions of loading. Hence, theoretical computer calculations are preferable for conventional ship forms. However, the need for continuing verification of theory by model testing, especially for new or unconventional forms, must not be overlooked. A number of computer programs are available for calculating the RAOs at all headings, typically in increments of 30 or 45° [53] [17].

Having the RAOs, the bending moment response spectra can be calculated by superposition for all of the selected wave spectra. Sometimes the spectra are assumed to represent long-crested seas for simplicity. But this results in

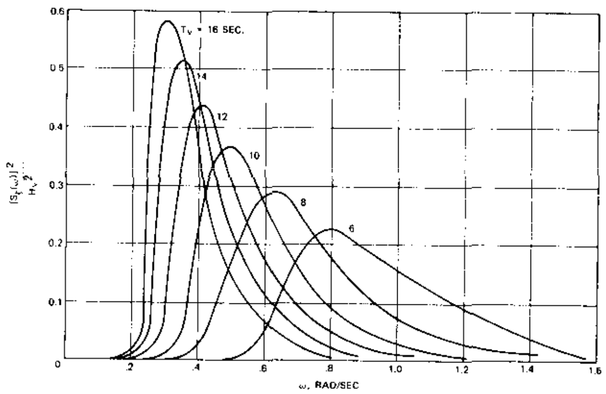


Fig. 10 ISSC Ideal Wave Spectrum Family

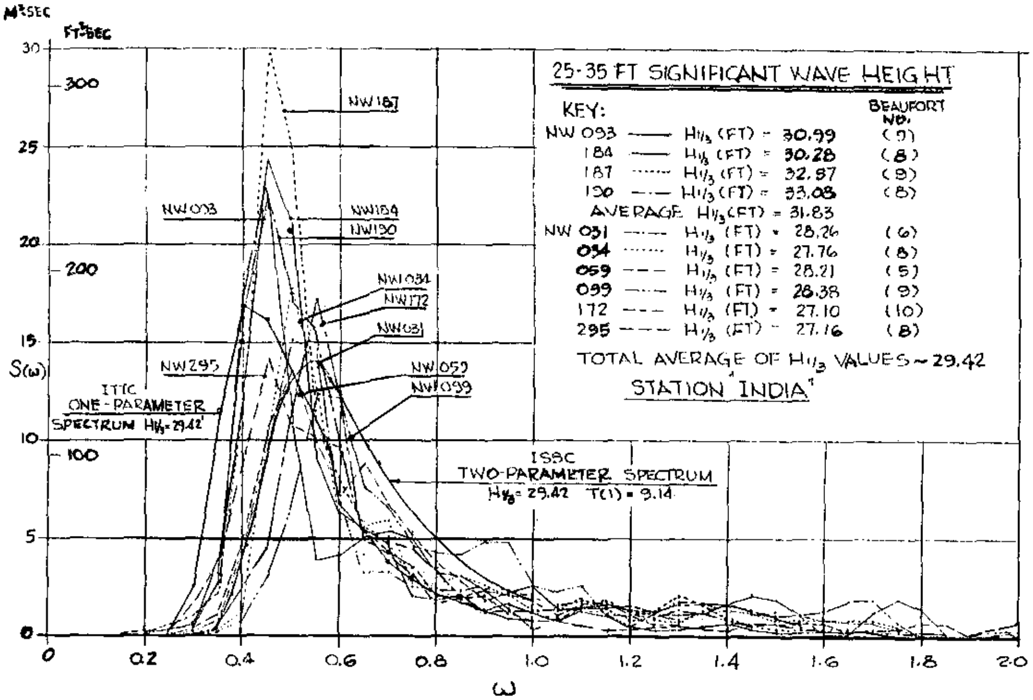


Fig. 11 Example of 10 Sample Wave Spectra Having a Significant Height of 30 ft. [95]

consistent over-estimates, and therefore it is preferable to introduce short-crestedness (the result of differences in direction of component waves in a storm sea) by means of a suitable "spreading function", as discussed subsequently.

Almost all methods in general use for predicting a long-term cumulative distribution of bending moment amplitudes begin with the assumption that a Rayleigh distribution is an adequate statistical description of the short-term response--either the single (peak-to-mean) or double (peak-to-trough) amplitudes of bending moment. For greater accuracy, the previously mentioned generalized distribution involving the band-width parameter, ε , could be used but would add greatly to the complexity of the calculations. Until all other aspects of design calculations are brought to a higher level of precision, this refinement is considered unnecessary.

The Rayleigh probability density function for single amplitudes, x, as derived from (4) is,

$$p(x) = x/m_0 \, e^{-x^2/2m_0} \qquad (6)$$

and integrating,

$$P(x_i) = \text{prob } (x \geqq x_i) = \int_{x_i}^{\infty} p(x)\, dx =$$

$$e^{-x_i^2/2m_0} = e^{-x_i^2/2\,\sigma^2} \qquad (7)$$

where σ^2 is the mean-square value (variance) of deviations from the mean and

$$\sigma^2 = m_0$$

where m_0 is the area under the response spectrum.

In order to predict the long-term distribution, we must combine many conditional distributions--assumed to be Rayleigh--representative of all the situations expected to be encountered in a ship's lifetime. These different situations can be described by a number of parameters, first those that refer to wave conditions:

Significant height.
Spectral shape.
Directional characteristics of
- one wave system, or
- combinations of two or more storm seas or a sea and swell.

In order to simplify the problem, it is customary to neglect wave systems coming from more than one direction and to assume one particular form of spreading function to describe the short-crestedness of a single storm sea,

$$f(\mu) = \frac{2}{\pi} \cos^2 \mu \qquad (8)$$

where μ is the angle between an angular wave component and the dominant wave direction. If the one-dimensional or point spectrum is $S_\zeta\ (\omega)$, the directional spectrum is assumed to be, $S_\zeta\ (\omega, \mu) = S_\zeta\ (\omega)\ f(\mu)$.

Variations in significant height are defined by variations in the area under the spectrum. Variations in spectrum shape can be defined either by use of families of randomly selected measured spectra or by using a formulation that specifies families on the basis of characteristic periods, as previously explained.

The following parameters refer to ship conditions:

Loading (drafts).
Speed.
Heading to the seaway.

The most important of these variables to consider is the ship heading relative to dominant wave direction. Ship speed, which has a relatively small effect on wave bending moment, can be eliminated as a variable by assuming either the design sea speed or the highest practicable speed for the particular sea condition and ship heading under consideration. The effect of amounts and distribution of cargo and other weights, which in turn affect draft and trim, transverse stability, longitudinal radius of gyration, etc., can be a complicated problem. Usually, however, it can be simplified by assuming two representative conditions of loading, such as normal full load and ballasted, or typical outbound and inbound. Then completely independent short and long-term calculations can be carried out for both, or attention can be concentrated on the condition giving the higher values of bending moment.

With the above simplifications, we are left with the following variables, or parameters, to be considered in any one probability calculation: significant wave height, $H_{1/3}$; spectrum shape, S; and ship heading to the seaway, ψ. S indicates a generalized quantity which can be defined in different ways.

Response spectra are calculated then for one or two conditions of loading, for all ship headings and for all sea spectra, using an appropriate ship speed for each. Each response spectrum is then integrated and the resulting rms values, σ, are the parameters that characterize all of the short-term responses.

The final step in the calculations combines all of the above Rayleigh distributions, weighted by the frequency of occurrence of the different spectrum shapes, ship headings, and significant wave heights. An attempt will be made to describe the basic principles in the most general terms possible, so that the various procedures now in use can be understood as variations of the general approach. First of all, it must be clearly stated that in what follows the variable under consideration, x, is the peak-to-mean amplitude of bending moment, i.e. the deviation of successive peaks (usually the highest maxima between zero crossings) from the mean value. (It is usually assumed that the response is symmetric about the mean and therefore that the statistics of sagging and hogging are the same). It should be noted that in some cases the double amplitude, peak-to-trough values have been considered the prime variable.

The long-term probability density function is obtained as a summation of many short-term density functions, weighted by the frequency of occurrence of each. Initially we assumed that each short-term sample has the same number of amplitudes or peaks. The short-term functions are conditional probabilities, since it is assumed that each applies while the ship is operating in a particular steady sea condition at constant heading to the sea (as well as at constant loading and speed). Each function is defined by its Rayleigh parameter, $\sigma^2 = m_0$, or $\sigma = \sqrt{m_0}$, determined by the variables $H_{1/3}$, S and ψ, which are assumed to be mutually independent. The long-term formulation may be expressed in many ways, but it is basically a joint probability of x and σ, expressed as,

$$q(x, \sigma) = p(x|\sigma)\ p(\sigma), \tag{9}$$

where $p(x|\sigma)$, the probability of x for a given σ, is the conditional density function of x with respect to σ, assumed to be Rayleigh. Thus

$$p(x|\sigma) = (x/\sigma^2)\ e^{-x^2/2\sigma^2} \tag{10}$$

and $p(\sigma)$ is a complicated function--or group of functions--that defines the probabilities of σ on the basis of different combinations of the variables, $H_{1/3}$, S, ψ; i.e. σ is a function of $H_{1/3}$, S, ψ.

The density function of x is $q(x) = \int q(x,\sigma)\ d\sigma$, and the cumulative long-term distribution, which is of particular interest to us, is obtained by a second integration, so that,

$$Q(x > x_i) = \int_{x_i}^{\infty} \int_0^{\infty} q(x, \sigma)\ d\sigma\ \ dx$$

$$= \int_{x_i}^{\infty} \int_0^{\infty} p(x|\sigma)\ p(\sigma)\ d\sigma\ dx. \tag{11}$$

Since one of the integrals, the cumulative Rayleigh distribution, is

$$\int_{x_i}^{\infty} p(x|\sigma)\ dx = e^{-x_i^2/2\sigma^2}$$

the above becomes,

$$Q(x > x_i) = \int_0^{\infty} e^{-x_i^2/2\sigma^2}\ p(\sigma)\ d\sigma \tag{12}$$

where $\sigma = \sigma(H_{1/3}, S, \psi)$.

The problem is to define the complicated manner in which σ varies with the parameters $H_{1/3}$, S, ψ, i.e. to determine $p(\sigma)$. One approach is to assume that $p(\sigma)$ can be replaced by a joint density function of $H_{1/3}$, S, ψ, all assumed to be independent,

$$p(H_{1/3}, S, \psi) = p(H_{1/3})\ p(S)\ p(\psi) \tag{13}$$

This is equivalent to the other expression, $p(\sigma)$, since for each combination of values of the parameters there will be a specific value of σ to be incorporated into the integration. In either case, three separate steps are required, and different researchers have made different assumptions and simplifications in carrying out these steps. (Appendix 1 summarizes the assumptions made by various classification societies in 1973). They all arrive at weighting functions or probabilities indicating the contribution of each combination of parameters to the total probability density function of σ, defining the Rayleigh distributions. (Initially each of these is assumed to cover the same number of peaks, or x-values.)

In practice, the above short and long-term probabilities are calculated by numerical summation instead of by integration. For example, one might make use of 5 significant wave height bands, 10 different spectral shapes or mean periods and 12 bands of ship headings.

The procedure in use at ABS and Webb Institute is based on the study of full-scale data which suggested that if data are stratified into different sea (or wind) groups, the Rayleigh parameters of the response within each group follow approximately a normal density function (truncated at 0), as shown in Figs. 8 and 9. Hence, Band [46] showed that the single function, p(σ), can be separated into the product of two functions, one being the probability density function of σ within individual wave height groups and the other the probability with which the ship encounters each wave group. Hence,

$$p(\sigma) = p(H_{1/3}) \cdot p^{*}(\sigma) \qquad (14)$$

where σ in $p^{*}(\sigma)$ depends on S and ψ, and $p^{*}(\sigma)$ is assumed to be a normal distribution on the basis of full-scale observations previously discussed. Spectrum shape is accounted for by using families of measured wave spectra having equal areas, and equal probability of all headings is assumed. See Lewis [40].

Then, for the assumed normal distribution of σ in each wave group and ship heading,

$$p^{*}(\sigma) = (1/\sqrt{2\pi s^2} \; e^{-(\sigma - m)^2/2s^2}, \qquad (15)$$

where m is the mean value and s is the standard deviation of σs for each combination of S and ψ. Hence, for all conditions combined,

$$Q(x > x_i) = \sum_{H_{1/3}} \sum_{\substack{S \\ \psi}} e^{-x_i^2/2\sigma^2} \; p(H_{1/3}) \cdot$$

$$(1/\sqrt{2\pi s^2}) \; e^{-(\sigma - m)^2/2s^2} \qquad (16)$$

The summation with respect to $H_{1/3}$ is done last, so that a series of long-term curves for different wave groups is an intermediate step (Fig. 12). The final long-term result, determined by numerical summation, can be presented graphically (Fig. 12), although in some procedures it fitted to some specific function, such as a Weibull distribution. (The 3-parameter Weibull density function, which includes the Rayleigh as a special case, has a wide range of applications, both short and long-term. Fitting to this function can be done by plotting on special Weibull probability paper.) However, it should be emphasized that it is unnecessary to fit the final calculated long-term probabilities to any explicit distribution function.

Another approach is that of Nordenstrøm [47], who left the summation over ship heading to the last step and expressed all other probabilities in the Weibull format.

Fukuda [48] also carried out the summation over ship heading as the last step, but he retained the Rayleigh distribution to describe the short-term response, with parameters $H_{1/3}$, S and ψ constant. He made use of an idealized wave spectrum formulation in which T_a, the average period, is the shape parameter, and a tabulation of observed wave heights and periods defined the probabilities, from which the joint probability distribution of the Rayleigh parameter, σ, as a function of $H_{1/3}$ and T_a, was derived numerically.

Another difference among procedures is that, whereas most methods assume that all short-term samples cover the same number of cycles of load, Ochi [43] points out that this assumption is not correct. Hence, he has shown that it is important to allow for the fact that different sea spectra, ship headings and ship speeds will lead to different numbers of cycles (or peaks) per unit time. This correction has also been introduced by Soeding [49].

A few simple calculations for a ship of 500-ft length in regular waves of its own length (which give near-maximum bending moment) will show the magnitude of speed and heading effects on number of cycles. See Table I.

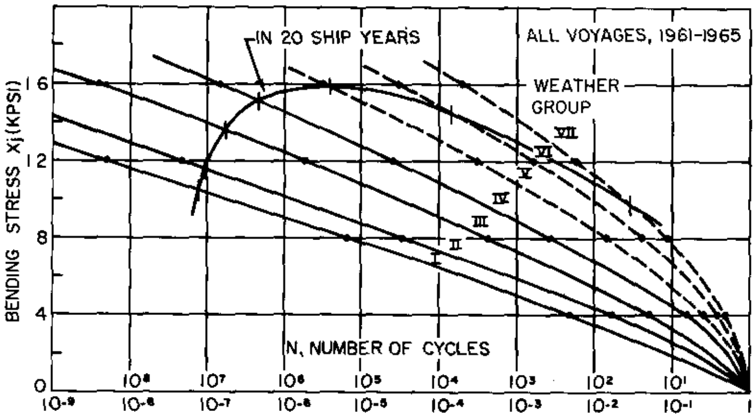


Fig. 12 Predicted Long-Term Probability of Exceeding Peak-to-Mean Stress Values in Different Weather Groups, S.S. Wolverine State and Hoosier State [41]

Table I Effects of Speed and Heading on Cycles of Bending Moment (Regular Waves)

16 Knots

	T_e sec.	n = cycles in 20 mins.
Head Seas	6.7	179
Beam Seas	10	120
Following Seas	20	60

20 Knots

	T_e sec.	n = cycles in 20 mins.
Head Seas	6.0	200
Beam Seas	10	120
Following Seas	28.5	42

Suppose account is taken of variations in the number of amplitudes in a given period of time, as proposed by Ochi. It is necessary then to compute, in addition to the areas of response spectra, m_0, the second moments, m_2, as well. The number of crests per unit time (i.e. maxima between 0-crossings) is equal to the number of zero up-crossings, or $\frac{1}{2\pi}\sqrt{m_2/m_0}$ (for each short-term case). A weighting function must then be applied whereby each σ is multiplied by the ratio,

$$\frac{\text{No. of crests per unit time, short-term}}{\text{Ave. no. crests/unit time, long-term, n}} = \frac{1}{2\pi n}\sqrt{\frac{m_2}{m_0}} .$$

The average number of crests per unit time for all σs,

$$n = \int_0^\infty \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}\; p(\sigma)\, d\sigma \tag{17}$$

and the corrected cumulative probability distribution becomes,

$$Q(x > x_i) = \frac{\frac{1}{2\pi}\int_0^\infty \sqrt{\frac{m_2}{m_0}}\; e^{-x_i^2/2\sigma^2}\; p(\sigma)\, d\sigma}{\frac{1}{2\pi}\int_0^\infty \sqrt{\frac{m_2}{m_0}}\; p(\sigma)\, d\sigma} \tag{18}$$

Expressed as a summation for calculation purposes,

$$Q(x > x_i) = \frac{\sum\limits_{H_{1/3}} \sum\limits_{\substack{S\\ \psi}} \sqrt{m_2/m_0}\; e^{-x_i^2/2\sigma^2}\; p(\sigma)}{\sum\limits_{H_{1/3}} \sum\limits_{\substack{S\\ \psi}} \sqrt{m_2/m_0}\; p(\sigma)} \tag{19}$$

Since this more accurate expression involves a great deal of calculation, it was of interest to determine whether or not it would have a significant effect on the long-term prediction. Accordingly, computer calculations were carried out of wave bending moment amidships for the SL-7 container ship at all headings to one family of short-crested seas. Assumed conditions:

Ship Speed	25 knots
Significant wave height	24.5 feet
Equal probability of all headings	
RAOs from Davidson Lab. model tests	

The results showed a wide range of n-values (from 210 cycles in 20 minutes in head seas to 28 cycles in following seas). However, the mean value of bending moment came out to be almost identical (0.4% difference). The reason for this small effect appeared to be that the bending moment is almost the same for head and following seas (although lower for other headings). Hence, the difference in weighting of head and bow seas had little effect on the average. The actual calculations are summarized in Appendix 3.

It is believed that this result is typical of what would be expected for any ship when bending moment only is considered. In the case of pitching motion or vertical acceleration at bow, both of which show much higher response in head than in following seas, it would be expected, however, that this correction would have a significant effect. It is concluded then that for calculating long-term bending moments, the introduction of number of cycles can be omitted.

Lifetime Probabilities

Up to this point, the variable x being considered is the amplitude of a peak or half-cycle of bending moment, hog or sag, between 0-crossings. Hence, the probabilities arrived at may be termed probabilities per cycle. However, ultimately we are concerned with probabilities to be related to the probabilities of strength (capability). In the latter, consideration is given to the effect of material quality, scantlings, workmanship, etc., on the variability of load-carrying ability of the structure, i.e. the variability in strength of many similar ships. Neglecting the effects of fatigue and corrosion for the present, the strength probability has no relation to the number of cycles of load, and in fact, represents a fixed characteristic of the structure that is independent of time. In other words, neglecting corrosion and fatigue, it represents the probable failure load at any time in the ship's lifetime. Hence, a compatible form for the probability of loads (demand) would be to consider many similar ships and to determine the density function for the bending moment expected to be exceeded once in the lifetime of any one ship.

Of course, on the average the bending moment value to be exceeded once in a ship's lifetime can be read from the cumulative distribution at the value of $N_L = 1/Q$, where Q is the probability per cycle, and N_L is the number of cycles in a lifetime. To obtain the number of cycles expected in a ship's lifetime, a rough estimate may suffice. But an accurate value may be determined from the previously given equation (17) for n and total time t (sec.), since $N_L = n\,t$.

But when many ships are considered, the lifetime exceedance value would be higher for some and less for others. In fact, there is theoretically a probability of 0.67 that this average value will be exceeded by any one ship. The meaning of this probability is that 67% of a large fleet of similar ships would be expected to experience a bending moment greater than the above average in their lifetimes. Hence, the bending moment corresponding to N_L is not a satisfactory design value, although it is useful for comparative purposes.

To obtain a meaningful design value for the extreme bending moment to be expected in service, we must consider what may be termed the "lifetime probability." This is the probability that a given level of bending moment will be exceeded by any one of a large number of ships (e.g., a probability of .01 means one ship in a hundred, .001 means 1 in a thousand, etc.). The lifetime probability can be determined in several ways, but the most convenient is simply to extend the long-term cumulative distribution (Fig. 16) to much lower probabilities (larger N) than would otherwise be required. Karst [54] has shown that for many load cycles,

Lifetime probability of exceedance =

$$\text{(cumulative prob./cycle) (number of cycles in lifetime)} = Q \cdot N_L \qquad (20)$$

For example, if there are 10^8 cycles in a ship's lifetime, and we read a certain bending moment value at $Q = 10^{-10}$, then the lifetime probability of exceeding that value is

$$10^{-10} \times 10^{8} = 10^{-2}$$

(equivalent to Ochi's risk parameter, $\alpha = 0.01$)

Similarly, lifetime probabilities can be determined for other values of bending moment, as shown by the upper scale of Fig. 24, which is discussed in a later section.

However, it should be noted that the above is an approximation which breaks down when cumulative probability per cycle equals the reciprocal of the number of cycles. At this point, the correct value for lifetime probability is 0.67 instead of 1.0. The value approaches 1.0 as load approaches zero, but this range is of no interest to us.

Lifetime Cyclic Loading (based on [7])

From the fatigue viewpoint the type of loading is one of irregular cyclic load reversal, usually with fluctuating mean load and possible occasional overload at points of stress concentration. It is further complicated by diurnal thermal stress variations. These loads are tabulated below along with the estimated cycles of load reversal for each in a typical ship's lifetime:

Still water	340
Wave bending	10^7 - 10^8
Dynamic	10^6
Thermal	7000

The fluctuating mean load is the so-called still water bending moment, discussed in a previous section. In general the specification of two probability curves, one for outbound (A) and the other for inbound (B) conditions, will provide the information needed for fatigue design. However, one additional item is needed: the time that the ship operates in condition A before changing to B, time operating in condition B, etc. In general both times will be equal simply to one-half the total round voyage time and will be measured in weeks. To be more accurate the effects of consumption of fuel and additions of salt water ballast should be included.

The cyclic loading consists of the low-frequency wave-induced bending moments and the high-frequency dynamic bending moments previously discussed. Their phase relationship is perhaps of less significance for fatigue than for brittle fracture. At any rate, long-term cumulative distributions of low-frequency loads should be available as part of the load determination for ultimate loading. However, the earlier discussion of dynamic loads was directed mainly toward the problem of their possible contribution to ultimate failure. To determine the cyclic load spectra of dynamic loads, additional information is needed. One must estimate or predict:

a) The frequency of occurrence during a ship's lifetime of episodes of slamming and springing.

b) The number of cycles of whipping and rate of decay following a slam, or

c) The average number of cycles of springing in one episode and the corresponding distribution function.

In principle, it should be possible some time in the future to make predictions of the above for a specific ship on a particular trade route. For the present, estimates on the basis of measured data on similar ships should suffice, since there are so many uncertainties in determining fatigue strength. Hence, a long-term distribution of each high-frequency load can be constructed.

From each of these distributions one can obtain a cyclic load spectrum in the following manner. The reciprocal of the probability is the number of cycles, N. For a ship's lifetime of N_L cycles, a scale of $N_F = N_L - N$ is then constructed on the distribution plot. Then N_F gives the number of cycles expected in the ship's lifetime of any desired level of bending moment. See Figure 13, which deals with wave bending effects only [55].

Report SSC-240 [7] gives sample cyclic loading "spectra" for the S.S. Wolverine State, as shown in Fig. 14. The curves for low-frequency wave bending were obtained from the long-term cumulative distribution of bending moment by converting to stress and reversing the scale, as previously explained. The curve for dynamic loading (slamming and whipping) was obtained by first estimating a lifetime histogram of stress cycles, using available voyage records as a guide, then integrating.

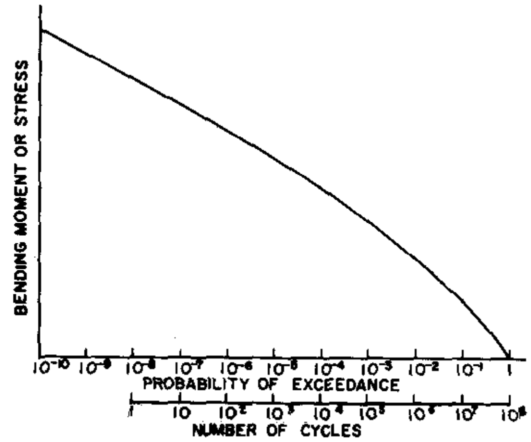


Fig. 13 Long-Term Distribution of Bending Moment or Stress, with Reversed Scale Showing Number of Cycles of Different Stress Levels in One Ship Lieftime (10^8 Cycles) [7, 55].

Finally, information should be provided on the expected diurnal variation of thermal effects, as previously noted.

The above information should provide the data needed by the stress analyst to evaluate the different types of cyclic loadings, as well as the variation in mean stress, as discussed subsequently.

COMBINING LOADS

One of the most difficult problems--or series of problems--to be solved in attaining a rational approach to hull structural design is that of combining the different loads previously discussed into one complete picture in probability format. As pointed out before, the various loads that are superimposed have widely varying frequencies. Furthermore, it is often difficult to separate loads from stresses. High-frequency loads usually evince themselves as stress (or deflection) waves throughout the length of the ship, originating in local hydrodynamic pressures. Local loads (forces and moments) on panels and structural components can best be determined from the stresses acting around the boundaries. Hence, many schemes for combining loads actually deal with combining stresses. This problem was discussed previously in connection with the consideration of combining lateral and vertical longitudinal bending components, where the two components are at the same frequency.

Unfortunately one simple, practical solution is not available. Methods of combining loads or stresses based on statistical methods of combining variances are inadequate unless the form of the corresponding probability density function is known. Hence, for practical purposes, it seems best to consider different combinations of loads separately, and a review of the situation will show where further research is needed.

The combination of low-frequency wave-induced bending moments with still water moments is discussed by Lewis, et al [7]. Here it is assumed that for most ships the still water bending moment can be described by two normal distributions, one corresponding to a full-load condition and the other a ballast or light-load condition, corresponding to (A) outbound and (B) inbound, or vice versa, voyages. Fig. 15. It is then recommended that complete short and long-term calculations be carried out for both conditions A and B. To obtain single long-term curves for hogging and sagging--including still water bending--requires that the wave bending moments be first expressed as a probability density function (instead of a cumulative distribution). The functions for still water and bending moments can then be combined on the basis of joint probability, since the two phenomena are essentially independent [7] [34].

Let x be a random variable describing the wave-induced bending moment (hog or sag). The density function of x will be called $p_W(x)$. Let y be a random variable describing the still water bending moment, with density function $p_S(y)$, which will be assumed to be normal. We are interested in the cumulative distribution function for the random variable $z = (x + y)$, which is given by the convolution integral [7],

$$P(z) = \int_{-\infty}^{\infty} p_W(x)\, p_S(z - x)\, dx \qquad (21)$$

Since $p_W(x)$ is not known in explicit form, the above integral cannot be evaluated analytically. However, it can be determined numerically for any specific case. Fig. 16.

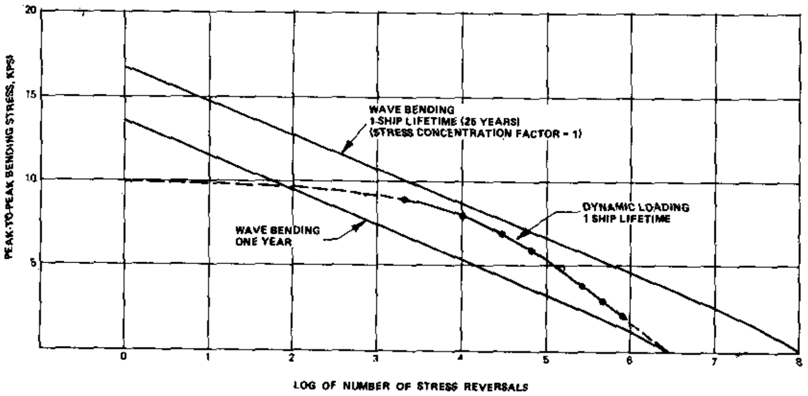


Fig. 14 Estimated Cyclic Loading Spectra for S.S. Wolverine State [7]

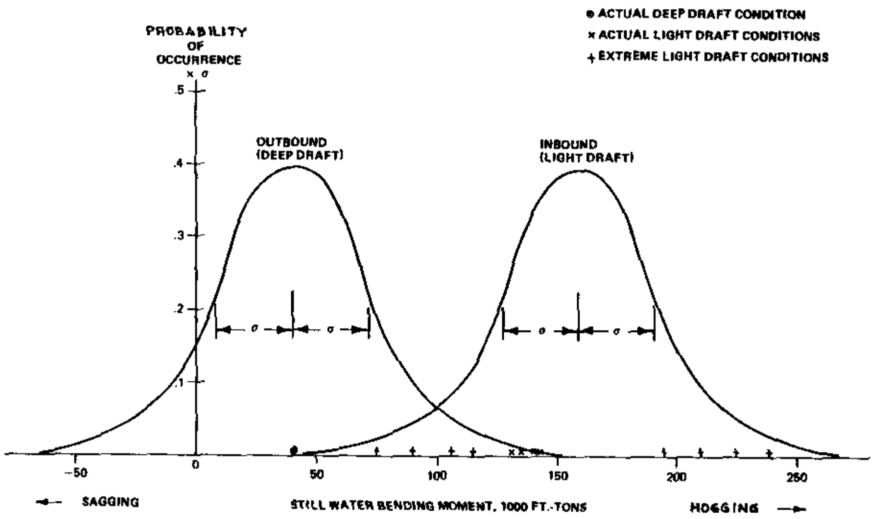


Fig. 15 Estimated Probability Density Functions of Still Water Bending Moment for S.S. Wolverine State [7]

We come now to combining the thermal effects, which can be interpreted in terms of a zero bending moment at night when thermal gradients are small and an effective sagging bending moment in daytime--especially if the sun is shining. To simplify this problem of combining loads, we can make the conservative assumption that the thermal effects are always present in day time and then have a constant value. All wave data can be roughly divided into two classifications--those that occur at night with no thermal effects and those that occur in daytime, with thermal effects superimposed. This would lead to two long-term curves, as shown in Fig. 17. We can conclude then that a safe, approximate treatment of thermal effects is to shift the base line by one-half the amount of the average total change in effective thermal "bending moment."

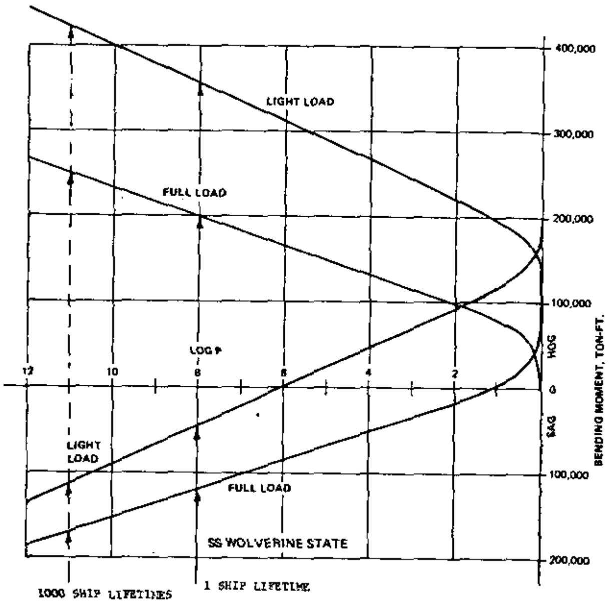


Fig. 16 Long-Term Distribution of Combined Bending Moments: Wave Bending (Vertical and Lateral) and Still Water Bending [7]

Finally, another low-frequency effect--that of local loads--is usually calculated in terms of stresses. From the viewpoint of structural design, however, more importance attaches to the effect of such loads on ultimate strength. For example, the hydrostatic pressure on the under side of the double bottom may affect the longitudinal buckling load of the double bottom structure. Hence, information on such secondary loadings should be supplied to the structural designer.

Thus, insofar as the possibility of damage or ultimate failure by buckling or permanent set is concerned, the complete low-frequency hull loading picture can often be presented in the form of two sets of long-term curves, one for outbound and the other for inbound--as in Fig. 16--adjusted for thermal effects.

The combining of dynamic or vibrational loads with those previously discussed poses a number of more difficult problems. Not only do these loads act at different frequencies, but some are transient (slamming and whipping) and others are steady state but intermittent (springing, which may occur in head and beam seas, for example, and not in following seas).

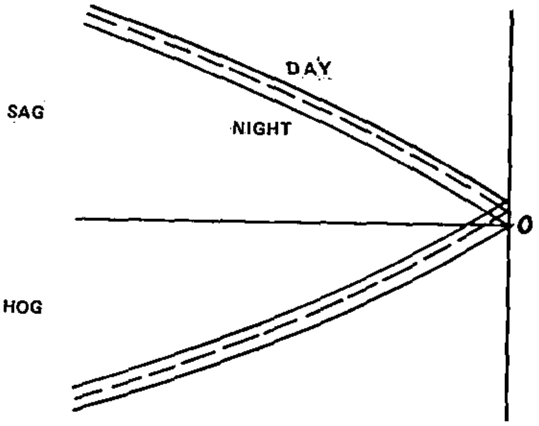


Fig. 17 Typical Long-Term Distribution of Wave Bending Moment, Sag and Hog, with Thermal Effect Superimposed [7]

Dalzell, et al [9] made an extensive analysis of stress records showing vibrational effects for a typical ocean-going bulk carrier and a high-speed container ship.

Considering springing first, it can be assumed, as previously discussed, to be a stationary stochastic process--when it occurs. Hence, a long-term cumulative distribution can be calculated in a manner similar to that used to determine such a distribution for low-frequency wave bending moment. But serious problems arise in attempting to combine these two distributions directly, since account must be taken of the manner in which the low and high-frequency bending moments combine. Furthermore, these distributions must be compared with great caution because the meaning of probability changes when mixed frequencies are involved. For example, when both low and high-frequency (springing) bending are present, the springing oscillations will be superimposed on the low-frequency bending (Fig. 3), and there will be many more bending moment maxima. The response spectrum for a combined record will have two distinct peaks, as shown in Fig. 18, and will therefore be a type of broad spectrum. Hence, a Rayleigh distribution will not apply to the peaks. The customary practical solution has been to assume that use can be made of a generalized Rayleigh distribution with broadness parameter calculated from spectral moments. Such a distribution defines all maxima (or minima), not just the principal peaks between zero crossings. To obtain the statistics of the principal peaks, a different distribution applies, as discussed by Battjes [56], who refers to the principal peaks as crests. See Fig. 19. So far as is known, this distribution has not been applied to the problem.

Another alternative, that also apparently has had only limited application, is to make use of the probabilities of the highest maximum (or minimum) in a period of say 20 - 30 minutes instead of the distribution of all maxima. The highest expected maximum in 20 minutes of combined low and high-frequency bending can be derived from the generalized Rayleigh distribution. Van Hooff [57] has shown how the resulting long-term distribution, based on the highest maximum in 20 minutes, can be interpreted in comparison with one based on rms, σ.

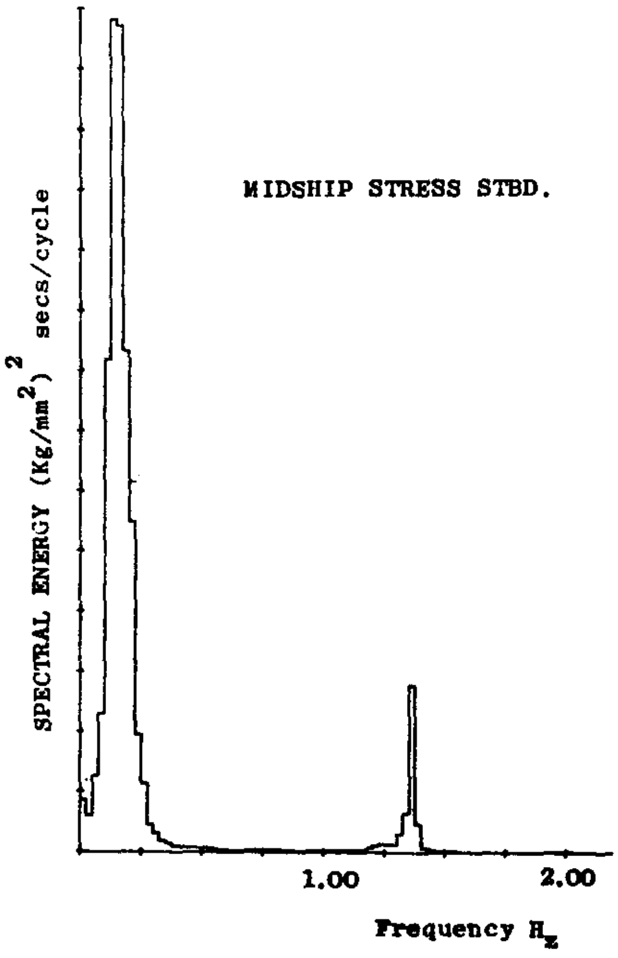


Fig. 18 Typical Stress Response Spectrum for Great Lakes Bulk Carrier, Showing Two Distinct Energy Peaks [4]

Probably the best available approach for combining low-frequency wave bending and springing, for ships having a significant amount of springing, is to calculate a long-term distribution of combined bending moment. To do this requires that the RAOs for the combined response be used, in conjunction with a generalized Rayleigh distribution that takes account of the spectrum broadness parameter for the double-peaked response spectra [58].

As previously noted, the number of springing maxima in a given period of time will be much greater than the number of low-frequency bending maxima. Hence, the resulting long-term distribution of combined bending will represent probabilities governed by the high-frequency, or springing, response. Fig. 20 [34] shows a comparison of long-term distributions for wave bending, springing, and combined loads, and indicates how the latter two must be shifted on the probability scale so that they will all be comparable in time. However, if the Battjes [56] formulation could be used, which counts only the highest maximum between zero crossings, no adjustment of the probability scale for the

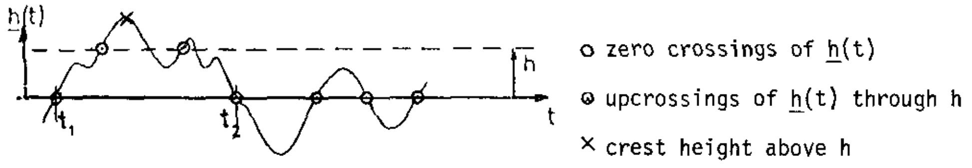


Fig. 19 Sketch Showing Relations Among Stress Maxima, "Crests," and Level Crossings [56]

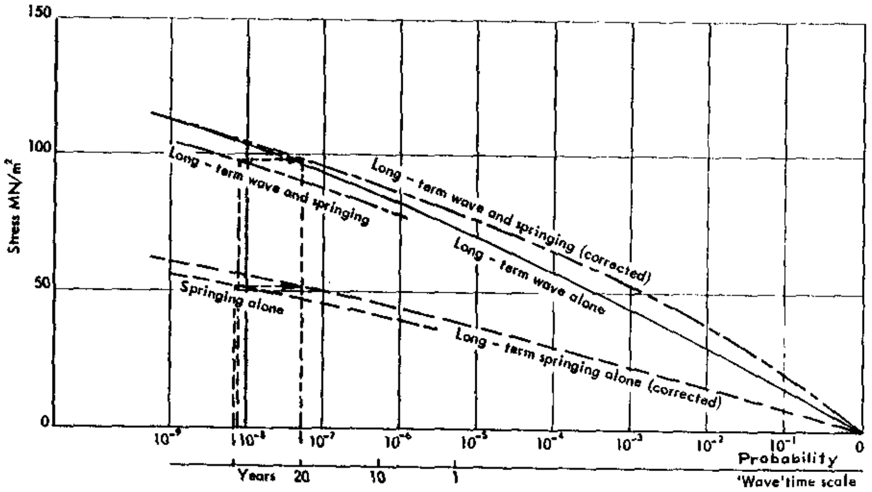


Fig. 20 Theoretical Long-Term Distributions of Stress for Wave (Low-Frequency), Springing (High-Frequency), and Wave and Springing Combined, Showing Shifts to Bring into a Common Time Scale [34]

combined curve will be required.

As a practical matter, springing loads are moderate in most ships and reach high levels only in very full, flexible ships such as Great Lakes bulk carriers.

Slamming also occurs under certain conditions only--depending on sea severity and on ship speed, heading and draft forward. But when it does occur it is transient and occurs at irregular intervals. Furthermore, it is followed by a number of cycles of whipping that die out as a result of damping. It does not combine with wave-induced bending moment in a random way, for there is a fairly limited range of phase angles between a slam and the preceding wave bending moment peak [7]. Hence, not only must the occurrence of slamming be predicted, but the phasing must be allowed for when it does occur. And account must be taken of the fact that speed and course will be adjusted by the master to prevent slams exceeding some subjective level of severity. For the present our only recourse seems to be, as previously noted, to make use of full-scale ship measurements to determine an allowance for the amount by which peak bending moments are permitted to increase because of slamming and whipping on different ship types. See Aertssen [59], Lewis et al [1] [7], and Dalzell, et al [9].

A further problem within the domain of the structural designer must be considered in relation to the effect of high-frequency loads on ultimate strength. Even if one knows the magnitude of a peak of combined low and high-frequency bending moment, is this a realistic load to be considered in relation to possible ultimate failure? Such failure involves absorption of a large quantity of energy by the structure, but the short duration of the high-frequency load may mean that its effect is significantly reduced. Hence, for the moment the problem is to furnish as complete information as possible on the low and high-frequency loads, leaving it to the structural researcher to determine their combined effect.

Finally there is the problem of combining cyclic load spectra in relation to fatigue. Since the still water bending moment stays realtively constant for long periods of time--between cargo changes or major ballast shifts (often an entire one-way voyage)--it has the effect of periodically changing the baseline about which the other bending moments vary. This constitutes, as previously noted, "irregular cyclic load reversal with fluctuating mean load." A crude overall picture of the total cyclic load spectrum can be obtained by direct superposition of low and high-frequency load spectra, provided they are all calculated for the same lifetime period. The problem of obtaining more realistic spectra for fatigue testing of critical structural details is given by Nibbering and Scholte [60], as discussed subsequently.

UNUSUAL BENDING MOMENTS

So far, consideration has been given only to ship behavior in the open sea assuming linearity of waves and response. Hence, predictions of long-term wave bending moments are valid only for such conditions. Therefore, in order to establish the most extreme loads for design, it is important to give attention to possible wave conditions that would cause even more severe bending moments. Some such conditions may be mentioned:

- Steep, breaking (non-linear) waves (multi-directional).
- Shoaling water (depths of the same order as ship-length waves).
- Opposing currents.
- Wave reflection from steep, rocky shores, or jetties.
- Effects of earthquakes.

The mathematical model of the ocean wave environment discussed earlier in this paper assumes simple storm-generated waves uncontaminated by seas or swells from other storms, and without influence of shallow water, currents or nearby shores. The combination of point wave spectra of different shapes and areas, with a spreading function to account for shortcrestedness, seems to provide a reasonably good representation of this simplified situation. Only when conditions become very severe does the assumed linearity come into question. St. Denis [61] has warned that "When the sea attains such a severity that its surface takes on the appearance of mountains in turmoil, no representation based on linear or quasi-linear theory can possibly suffice to describe it and one would need have recourse to a fundamentally non-linear theory....." However, it is fortunate that the non-linear trend as waves become higher and steeper is in the direction of sharper crests and flatter hollows. (Note the favorable "Smith effect" on calculated bending moment in going from a sine wave to a "trochoidal" or Stokes wave form). Work of Dalzell has shown in fact that when experimental bending moments obtained in regular waves in a model basin are plotted against wave height, the values usually tend to fall below the linear trend when wave heights are high [62]. Fig. 21. Hence, the assumption of linearity is on the safe side. Dalzell also found that much steeper waves--and therefore, higher bending moments--could be generated in the model tank than are ever measured on full-scale ships at sea. Although non-linear wave theories are available for regular waves, a non-linear model of irregular waves becomes highly complicated, as shown by St. Denis [63].

One problem with very steep waves is the breaking crests, which can be dangerous for small craft and can cause damage to deck structure and outfit, as noted by Buckley [64], even though longitudinal bending may not be as severe.

Assuming that a linear storm sea model is generally acceptable, consideration must be given next to the fact that simple storms seldom develop and decay in an otherwise undisturbed ocean. Usually the sea's surface is already disturbed by the waves remaining from a previous storm or by a swell from a distant storm or storms. Lewis [40] attempted to account for this effect by making use of samples of actually measured spectra which showed irregularities and double peaks that could be accounted for only by the superposition of different storms. Ochi [43] carried this further by developing a mathematical formulation of families of point spectra with double peaks. However, neither of these approaches allow for the fact that usually the superimposed storms have different dominant directions. Fig. 22. As noted in the report of Committee 1, Environmental Conditions, to the ISSC in 1973 [65] "Sea waves and swell both from different directions are the rule, rather than the exception, on the ocean surface... However, the climatology of such situations of sea and swell combinations is inadequate... The most desirable solution is the measurement of directional spectra. Experience has shown this to be very difficult and very expensive--justifiable only for a limited number of research situations... Wave prediction techniques offer the best potential for an adequate accounting of the mixture of sea and swell which normally exists on the ocean surface..."

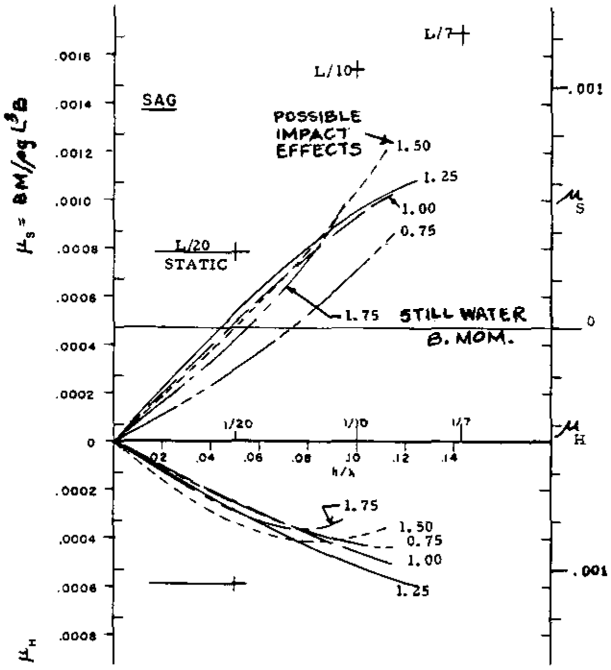


Fig. 21 Model Test Results for Heave, Pitch and Bending Moment Coefficient, Mariner Model in Regular Head Seas (Wave length, λ = 1.0L; Froude No. = 0.12 - 0.14) (Davidson Lab. Report R-926, Part II)

Since that time (1973), as a matter of fact, considerable progress has been made in the application of forecasting techniques to the ship designer's problem. In particular, the ocean wave spectrum model developed at the Fleet Numerical Oceanographic Center (FNOC), Monterey, Calif. (Lazanoff and Stevenson, 1975) [66] has been expanded to cover the North Atlantic and North Pacific Oceans, as well as the Mediterranean Sea, and will be extended to the southern hemisphere. Not only are the resulting routine operational forecasts of wave spectra of great value to ship operators, but the accumulation of data over many years is of potentially inestimable value to ship designers.

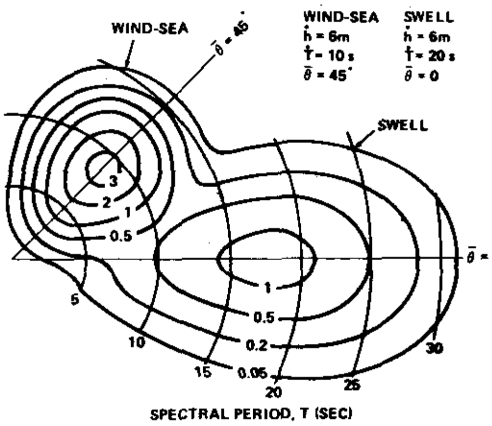


Fig. 22 Directional Sea Spectrum, Showing Superposition of Wind Sea and Swell [61]

Table II Typical Hindcast Directional Spectrum [68]

WAVE FREQUENCY

DATETIME
9Z 31 MAR 68

LOCATION
58.292N 12.297W

WIND DIRECTION AND SPEED, WHITE CAP PERCENTAGE, FRICTIONAL WIND VELOCITY
WIND DIR 267.6 WIND SPD 22.6 WHITE CPS 0 USTR .89

FREQ IN HZ	.308	.208	.158	.133	.117	.103	.092	.081	.072	.067	.061	.056	.050	.044	.039		DIR (FROM)
	.00	.00	.01	.00	.00	.01	.00	.02	.06	.00	.00	.00	.00	.00	.00	.1	6.58
VARIANCE ENERGY	.03	.08	.16	.15	.17	.07	.00	.03	.00	.00	.00	.00	.00	.00	.00	.7	336.58
	.05	.15	.27	.25	.30	.22	.01	.04	.00	.00	.00	.00	.00	.00	.00	1.3	306.58
	.06	.21	.38	.38	.56	.42	.31	.00	.00	.00	.00	.00	.00	.00	.00	2.3	276.58
	.06	.19	.34	.30	.40	.06	.18	.11	.00	.00	.00	.00	.00	.00	.00	1.6	246.58
	.04	.12	.19	.14	.14	.00	.01	.03	.00	.00	.00	.00	.00	.00	.00	.7	216.58
POINT SPECTRUM	.24	.75	1.34	1.22	1.57	.78	.51	.23	.06	.00	.00	.00	.00	.00	.00	6.7	WAVE DIRECTIONS

H1/3 10.35 FT
SIGNIFICANT WAVE HEIGHT

TOTAL ENERGY

To make the data available in useable form, a comprehensive project was undertaken some time ago to develop a worldwide climatology as a joint DTNSRDC/FNOC project (Bales and Cummins [67]), using the computer bank of meteorological data to "hindcast" the wave spectra over a period of many years. The principal advantages of such hindcast spectra (Table II) for design use are:

- Wide coverage of the world's oceans.
- Availability of input wind data for many years in the past.
- Inclusion of directional properties of seas and cross seas or swells.

There are certain limitations to be considered:

- Data are calculated rather than measured, a disadvantage that can be overcome by systematic and extensive comparison with actual measurements.
- Accuracy is of necessity reduced in areas where wind observations are scarce.
- Range of wave frequencies covered may not be adequate for all purposes.

Recent papers by Cummins and Bales [68] [69] give some preliminary results and suggest the potential value of this work. Of particular importance is the need to select suitable parameters to describe the directional spectra and then to give statistical data on these parameters, in order to reduce the vast number of hindcast wave spectra to manageable form.

Special consideration also needs to be given to the waves generated by hurricanes and typhoons. These severe storms that move at relatively high speeds along erratic paths cannot be accounted for in the normal collection of routine wave observations, forecasting and hindcasting. Not only are the winds unusually strong, but they change direction continually, so that the resulting high, confused seas do not conform to the ideal model. Some studies have been made of hurricane seas [70], but much more work needs to be done. Buckley has called attention to the strong correlation between ship damages and the occurrence of unusual or freak sea conditions [64].

Other conditions that may lead to more severe, i.e. steeper, waves are shallow water and opposing currents. The report of ISSC Committee I.1, Environmental Conditions, 1976 [71], discusses these effects in Chapter 9 on Abnormal Waves (based on a contribution by R. Wahab). It is stated that "much attention has been paid recently to abnormal waves occurring off the east coast of South Africa... Other areas where unusually high waves have been observed include the Sea of Casquets (N.W. France) and the vicinity of the Dogger Bank in the North Sea." The South African example is explained on the basis of storm seas or moderate swells opposing the Agulhas Current that runs along the coast. The North Sea example is explained as being caused by wave refraction by shelving and shoaling water (Pierson [72]). Other locations in which shallow water has apparently been a factor in ship losses or damages are the Continental Shelf approaching the English Channel, the Grand Banks, the Continental Shelf in the vicinity of Cape Cod, and the bar at the mouth of the Columbia River in Oregon.

The report of Committee I.1, Environmental Conditions, to the ISSC, 1979 [73], states, "Conditions which are somewhat similar to those off the African coast, as far as the possible occurrences of freak waves are concerned, occasionally prevail along the north wall of the Gulf Stream northeast from Cape Hatteras, when north-easterly winds generate a sea running counter to the Gulf Stream (James [74]). An additional disturbing factor is the instability of the initially cold polar air masses, heated by the relatively high and gusty winds near sea level, resulting in a rougher sea." The losses of the tanker Texaco Oklahoma and bulk carriers Anita and Norse Variant in 1971 and 1973 may have been related to such abnormal waves.

Another condition of unusually steep waves is the standing waves caused by the reflection of an oncoming sea by vertical cliffs and breakwaters. Such steep waves have been reported in the Bay of Biscay off the coast of Portugal.

Earthquakes are known to cause severe waves called tsunamis or "tidal waves." Little is known about the magnitudes and forms of such waves, but on the other had, it may not be necessary or desirable to design for such rare and unusual occurrences.

Although the incidence of abnormally high or steep waves in certain areas of the world's oceans has been known for a long time, comparatively little has been done by way of direct measurement of such waves that would be useful to the ship structural designer. In recent years, large moored buoys have collected meteorological and wave spectral data for NOAA, a few of them giving directional spectra. Unfortunately the buoys have been located in coastal waters and generally not on main shipping routes [75]. The potentiality exists for making more extensive use of such moored buoys to provide systematic wave data at critical locations. A data buoy was deployed for "at least a year" at the edge of the Continental Shelf at the entrance to the English Channel [76], but results of data collection are not yet known to be available. No matter how good our open sea wave data and our methods of predicting long-term trends of wave bending moments in the open sea may become, we can never be sure that the resulting design loads are acceptable until we know more about the possibly more severe loads that may be encountered under unusual circumstances.

APPLICATION TO DESIGN

Ultimate Strength

Having reviewed the loads affecting the ultimate strength of the hull girder and considered methods of predicting and combining them in probabilistic form, it will be well to consider how these loads can be applied in structural design. Of course, this involves consideration of strength or capability, which is the subject of other papers at the Symposium. But there is a need to try to link demand with capability at some time early in the meeting.

Considering classification society methods first, there has been a definite trend toward more rational approaches in recent years. The Rules of all Societies have adopted procedures that embody the following features:

- Separate determination and addition of still water and wave bending moments.
- Determination of wave bending moment in terms of a coefficient (or "effective wave height") arrived at by comparative calculations on the basis of wave spectra and probability theory.
- Establishment of allowable stress values that are essentially constant (a small variation with ship length in some Rules is explained by varying amounts of absolute values of corrosion allowance) and include an empirical factor of safety.

Required section modulus can then be determined directly from the Rules, taking into account the following factors: ship type, length and breadth, block coefficient and material of construction.

It is significant that all classification societies have also made provision for a more detailed approach to design, taking account of special features, expected sea conditions in the intended service, more advanced techniques, etc. Attention is focussed on a proabilistic approach to design bending moment, while retaining an empirical value of allowable stress. As explained in the ABS Rules (1981) [77] (Section 6), "Consideration will be given to the wave-induced bending moment calculated by means of a statistical [probabilistic] analysis based on the ship motion calculation in realistic sea states." Lloyds' Register Rules (1978) [78] (Chapter 4) amplify this: "In direct calculation procedures capable of deriving the wave induced loads on the ship, and hence the required modulus, account is to be taken of the ship's actual form and weight distribution.... The Society's direct calculation method involves derivation of response to regular waves by strip theory, short-term reponse to irregular waves using the sea spectrum concept, and long-term response predictions using statistical distributions of sea states." This approach has become quite general among classification societies (Appendix 1).

Thus classification societies have advanced to the point of accepting a probabilistic approach to demand (loads) while retaining a deterministic approach to capability (strength). Furthermore, strength is viewed basically in terms of allowable stress rather than ultimate strength (although separate consideration may be given to avoidance of local buckling). A number of writers have pointed out that the capability of the structure to carry loads, or what Vasta called "load-carrying ability", is a very different thing from the load corresponding to material yield or ultimate stress.

It is significant that the classification societies, as well as ISSC, are interested in developing even more advanced, rational, direct method. Quoting Aldwinckle [79], "The statistical [probabilistic] analysis of the demand/capability method continues to show great promise in moving towards a completely rational approach to structural safety... It is apparent that further research and development is needed to study the effects of variation in constructional standards ... With the improved shipbuilding technology in the world... it should be possible within the next decade to introduce the reliability approach... to benefit both the shipbuilder and the shipowner."

For present purposes the reliability approach refers to the concept of designing for an acceptable level of failure probability. It was developed in the field of Civil Engineering over many years, under the leadership of A. M. Freudenthal [80] and has since been applied in Aeronautical Engineering [81] and many other engineering fields.

When suitable probabilistic expressions for hull structural capability become available, the probability of failure can be explicitly derived. See Fig. 23. Abrahamsen et al. [82] have shown that a double integration of demand and capability is involved. Since our normal procedure is to integrate to obtain the lifetime probability of demand exceedance, only one more integration, involving an assumed normal density function of capability remains to obtain the probability of failure.

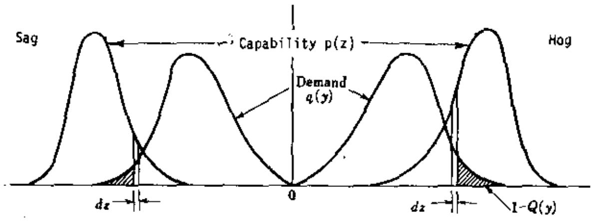


Fig. 23 Probability of Failure Determined from Density Functions of Demand and Capability

If y is demand and z is capability, the probability of failure may be defined as the probability that the ratio z/y is less than unity. Since z/y is a sort of ideal safety factor, we can also say that the failure probability is the probability that the safety factor is less than unity. It can be shown that the cumulative distribution of the safety factor is

$$P\left(\frac{z}{y} \geq \frac{z_0}{y_0}\right) = \int_0^\infty Q(y)\ p(z)\ dz \tag{22}$$

where, Q(y) is the lifetime probability of demand exceedance,
p(z) is the probability density function of capability.

The integral can be evaluated numerically and then its value at $z_0/y_0 = 1$ determined.

In a discussion by the present authors of Committee V.I. Report (ISSC, 1979) [83], an example was given of a tentative application of this approach to a typical tanker, assuming a constant (deterministic) value of still water bending moment and neglecting high-frequency response. Use was made of a density function of capability based on values of average strength and coefficients of variation (standard deviations as percent of mean) for critical panels in buckling, from Mansour and Faulkner [84]. The lifetime curve for demand was obtained from data in the same paper on stresses vs. wave height, on the basis of the binomial or Poisson model, as described by Karst [54], as given by Equation (20). See Fig. 24.

For the tanker under consideration the low-probability portion of the demand curve calculated on the above basis is shown in Fig. 25. Using the demand curve in conjunction with the capability curves shown leads to lifetime failure probabilities for grillage failure of 0.017 and strut-panel failure of 0.047. In a subsequent section on choice of probability levels, figures given will show that these are somewhat high to be acceptable (i.e., 2 to 5 ships in 100).

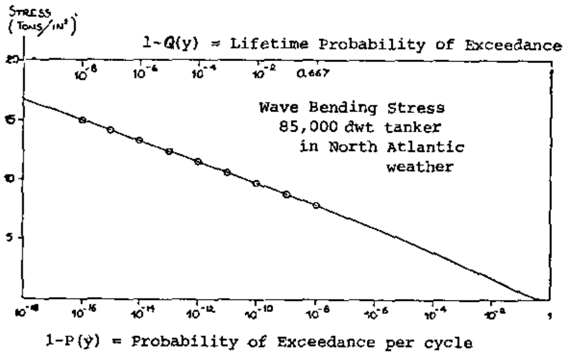


Fig. 24 Determination of Lifetime Probability from Probability Per Cycle [83]

However, the assumed standard deviations of structural capability appear to be too severe. If the coefficients of variation were reduced from the calculated value of 11.2%, the results for grillage failure would be:

Coeff. of var.	11.2%	8%	5%
Failure Prob.	0.017	0.0026	0.0002

Assuming that a failure probability of 0.0026 is acceptable (2.6 ships in 1000), a coefficient of variation of about 8% would make the design satisfactory. The figures show how sensitive the failure probability is to the coefficient of variation of capability, and therefore the importance of further work to define the latter--as well as the mean value--more accurately.

It was noted [83] that available data on probabilistic aspects of capability, in general, seem to apply to local panel failure rather than complete failure of the entire girder flange in compression or tension. For example, for a tanker such as that under consideration here, the local compression buckling of a deck panel in the center tank would shift the load to the top of the side tank structure--deck, side shell and longitudinal bulkheads. This structure would probably carry considerable higher load before there would be further buckling, such that ultimate failure or collapse could be said to have occurred. As stated by Faulkner in discussion of the ISSC Committee II.2 report (1979) [85], "A better understanding of the ultimate moment, M_C, which allows for the spread of elasto-plastic instability, etc., through the cross section is arguably the most important outstanding item to complete the capability modeling."

An outstanding contribution to the determination of a realistic value for average capability (ultimate strength) of the hull girder is that of Smith [86], whose procedure considers the simultaneous stress-strain behavior of the various structural elements, including "hard corners," as compressive load is incrementally increased. It takes account of the shift of load from buckled components to the more rigid members in a reasonable way, and allows for the shift of elastic neutral axis. Billingsley [87] has made a valuable contribution to this approach and focusses on how the buckling and post-buckling behavior of individual elements is related to "the response of the overall section." The distinction he draws between the aims and methods of the ship structural designer in contrast to the "marine safety structural analyst" should, we believe, be gradually eliminated as emphasis in design is shifted from stresses to load-carrying ability.

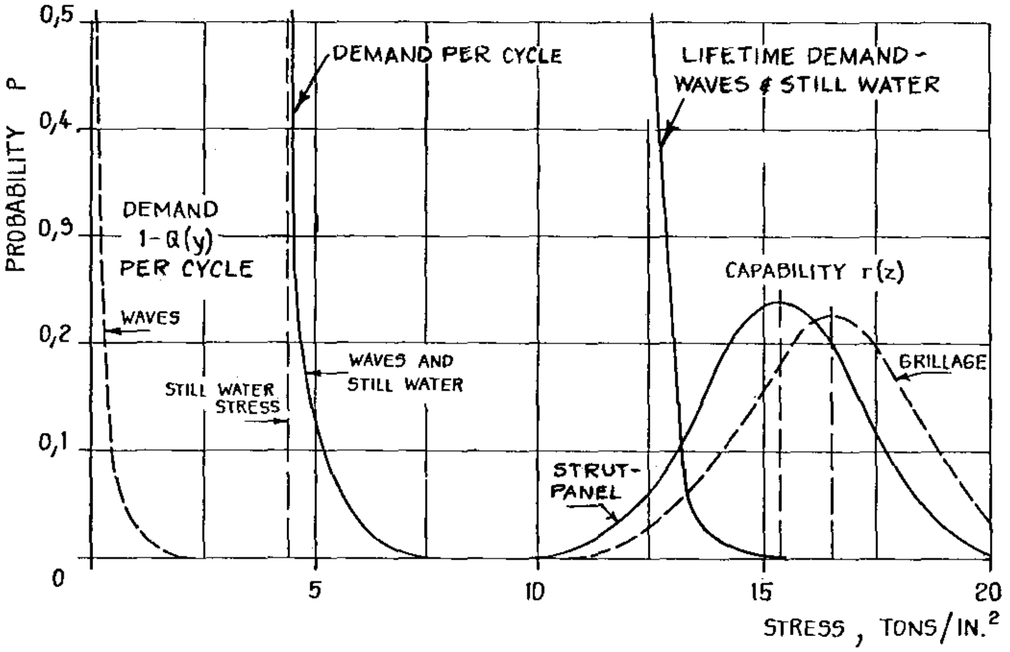


Fig. 25 Plot of Probability Data for Calculation of Probability of Failure for Typical Tanker [83]

The recent paper by Mansour and Thayambilli [88] gives a great deal of valuable information on ultimate strength. But the above writers do not deal with the question of variability, and this remains perhaps the area of greatest need for research. Billingsley does suggest certain variable factors indirectly when discussing his analytical methods:

- Conditions of edge fixity, and
- "Quality of construction in terms of plating fairness, residual stress, and alignment."

To these can be added other objective uncertainties such as dimensions, scantlings, material properties, plus subjective uncertainties associated with the reliability of theories and experimental data utilized. Some data are already available (Stiansen, et al) [89] and a new SSC project on Major Sources of Uncertainties should further clarify what is known and what remains to be investigated.

Attention must also be given to human error. Ractliffe mentions errors "at the level of design, or construction, or maintenance" as causes of most major structural failures, and Yamamoto mentions ship handling (discussion of Joint Session 2, ISSC 1979) [90]. Another consideration is the increasing use of shipboard load or stress monitoring equipment, as discussed in the report of Committee I.3, ISSC 1979 [91], which can potentially reduce the probability of structural failure if properly used.

Fatigue

In general, it appears that fatigue is not specifically considered in all classification society design procedures. But presumably fatigue is considered indirectly in establishing allowable stresses (as in the case of ABS Rules for aluminum ships, for example) and in evaluating specific structural details during plan approval.

One approach to structural design relative to the fatigue loading is simply to make sure that the probability of exceeding the yield point at critical areas of stress concentration is at an acceptable level, considering the cost of repairing nuisance cracks. Presumably, low-cycle fatigue would thereby be avoided. However, although this can be readily done in relation to major points of concentration, such as hatch corners, it is difficult to accomplish in respect to all of the many welded structural details in longitudinal structural members. In any case, this approach can lead to excessive scantlings, and therefore it appears that a complete picture of cyclic loadings should be obtained for the use of the structural analyst (and researcher).

For the fatigue viewpoint, the type of loading (discussed earlier) is one of cyclic load reversal, usually with fluctuating mean (still water) load. Lifetime spectra for cyclic loadings of different frequencies can be provided in useable form on the basis of available techniques, as explained in a previous section.

Again it appears that the more difficult problem today is the determination of the capability of the ship's structure. In general, the materials used in shipbuilding are not subject to fatigue failure when unflawed samples are subjected to cyclic loadings of the number corresponding to low and high frequency bending in a ship's lifetime. But the longitudinal structure of a ship contains many points of stress concentration, arising either from gross design discontinuities, such as hatch corners, or from local discontinuities occurring in structural details and at welding flaws. Of particular concern is a concentration factor of 2 or 3 at a location such as a hatch corner which results in mean stresses well below yield point being magnified beyond yield. A comparatively small number of stress reversals (several hundred) can then lead to so-called "low cycle" fatigue cracking. The effect may be exaggerated by effects of corrosion. See Fig. 26 from Nibbering [55].

Ideally design for fatigue needs full-scale fatigue tests of all of the critical structural details in the ship, using an irregular loading spectrum and varying mean value corresponding to the ship's expected lifetime experience [60]. This is obviously not feasible, but much can be learned from such tests of representative details corresponding to situations of local stress concentrations. Here the problem is primarily one of controlling crack propagation, since--as pointed out by Nibbering and Scholte [60]--tiny cracks are almost always present at critical points and can seldom be detected at an early stage. Fortunately, such cracks propagate slowly and can be detected and repaired before damage becomes serious. (Guidance to ship operators is needed on this point.)

A current SSC project by Professor W. H. Munse is assembling data from structural engineering sources on details applicable to ships. Results should go a long way toward defining the capability of local structure to withstand cylic loading. (See his paper at the present Symposium.)

The report of the ISSC Committee V.1, Design (1976) [92] develops a theoretical probabilistic approach that is intended to be a substitute for fatigue tests for checking fatigue strength of "primary structural members, such as deck, side shell plating, ... in terms of crack initiation."

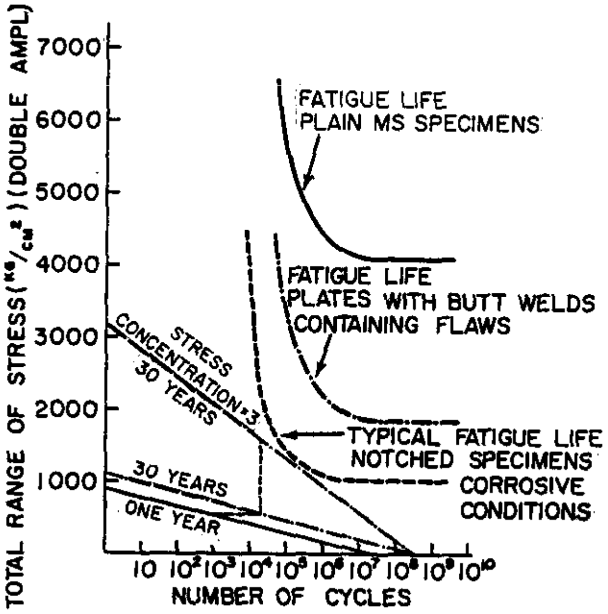


Fig. 26 Example of Application of Cyclic Loading Curves to Study of Fatigue [55]

Attention is focussed on welded butt joints, and use is made of experimental SN-curves (taking into account initial imperfections and corrosive atmosphere), Goodman's correction for mean stress, and Miner's law for assessing cumulative damage to arrive at a "capability function" (defining fatigue strength).

The analysis then proceeds to predict fatigue failure (crack initiation) probability on the basis of cyclic demand consisting of an assumed normal distribution of still water bending moment (stress) and a comparatively simple exponential distribution as an approximation of wave-induced moment (stress). Results for this ideal case are presented for a particular ship in the form of a graph of probability of fatigue crack initiation vs. characteristic extreme value of wave stress in 10^8 cycles. A table shows the effect of variations in certain random variables on the predicted probability.

The above procedure involves a number of uncertainties, such as doubt regarding the applicability of Miner's law, focussing on crack initiation rather than propagation, lack of consideration of high-frequency loads in the demand spectrum, etc. But it represents a promising approach for defining fatigue capability that presumably could be extended to the case of gross design discontinuities such as hatch corners. Soeding [49] has also dealt with fatigue on a probabilistic basis, considering cumulative damage with respect to both crack initiation and propagation, including hatch corner effects.

Note that if the general application of more rational design standards should in time result in reduced hull scantlings, then the incidence of fatigue cracking might increase to an unacceptable level. In this case some modification in strength standards by classification societies would be expected. The problem is to balance initial cost against costs of crack repair, as discussed in the next section.

Choice of Probability Levels

The final step in establishing a completely rational probability approach to design for both ultimate strength and fatigue will be the determination of an acceptable level of failure (or damage) probability that will give a satisfactory balance between demand and capability.

One approach is simply to ascertain what levels of failure have occurred in the past and then to select a suitable design value that is as good or better. This was discussed at length in report SSC-240 [7]. On the basis of an informal study by J. F. Dalzell of some Lloyds' statistics on merchant ship losses (presumed to be due to structural failure) it was concluded that a figure somewhere between 0.003 and 0.006 would be an estimate for the probability of failure that has been tacitly accepted over the past twenty years for large ocean going ships. In proposing a specific figure for a new design criterion, Dalzell suggests 0.001. This implies that merchant ships should be designed with a probability of ultimate--or catastrophic--failure of no greater than 0.001, i.e., that a new ship would have a chance of not over one in a thousand of failure during a normal life span.

A similar approach can be taken to damage to longitudinal structure requiring repair (from all causes, including fatigue). Hence, there is obviously a need for collecting more data on ship structural damages and failures in reliability format. This requires the compilation of statistics on ships suffering hull failures--or ships lost in which structural failure is suspected--and relating these to ship-years of service. Such damage data must be analyzed to:

a) Separate damages affecting longitudinal strength from all others.
b) Classify these as to type--fatigue, brittle fracture, or buckling.
c) Relate the data for a specific time period to number of ships in service during that time.
d) Collect data on estimated costs of repair of damages.

Another more basic approach discussed in [7] to determining the probability level to be used in a design criterion is that of "expected cost", which has been summarized in a convenient form by Freudenthal and Gaither for application to maritime structures in general [80]. It is based on the principle that the best design is one that minimizes the expected total cost, where the latter consists of the sum of initial cost, possible cost of failure, and cost of repairs of less serious damages--such as fatigue cracks. All damages not involving the main hull girder are excluded, since presumably they would not be affected by any change in the main hull structural design.

Expressed as an equation, the total expected cost to be minimized is,

$$L = I + p_1 F + (1 - p_1) \Sigma C p_2 \qquad (23)$$

where I = initial cost of the ship (or structure),
p_1 = probability of failure (in a lifetime),
F = anticipated total cost of failure (replacement cost + cargo loss + temporary charter of replacement ship + loss of business from customer reactions + cost of pollution or other environmental effects, etc.),
C = anticipated cost of damage or "failure of function of surviving structure" ("the success cost"), i.e., cost of repairs and other associated costs,
p_2 = the expectation or expected number of such damages.

The probability, p_1, of a failure that can lead to the complete loss of the ship is very low. The expectation of other damage, p_2, that would require more or less extensive time out of service for repair depends on any one of the modes of failure previously discussed--particularly, fatigue cracking and local buckling. In fact, a ship might experience a number of such damages in several modes during its lifetime.

After the probability of failure in a lifetime is estimated for each failure mode, and for each damage mode the expectation of damage, they should be multiplied by their respective costs in the above equation. In principle, the total expected cost, L, can be evaluated for several alternative hull designs and the optimum design determined graphically.

The above analysis assumes that safety of human life is not a problem. But if it cannot be assumed that lifesaving equipment will adequately protect human life, then statistics on loss of life in different industries and modes of transportation can be used to determine risk values to be used in design [93] [94].

CONCLUSIONS

It is concluded that great progress has been made in recent years toward the goal of rational design of the ship hull girder. Methods have been developed to predict the probability per cycle of extreme bending moments or stresses, and refinements of these methods continue to be made. One such refinement, adjusting short-term responses for variations in the number of cycles per unit time for different ship headings, has been shown here to be of negligible importance in the case of wave bending moment response. However, there are a number of serious gaps in the procedure that must be filled before such an approach can be used in any other than a partial or comparative fashion. One of the objectives of this paper has been to describe the ideal overall approach, and in so doing it has called attention to the principal gaps. These will be listed below as a guide to further research and study:

1. More complete ocean wave data on rough weather routes, particularly clarifying directional properties with two or more seas combined.
2. More data on sea spectra for abnormal conditions of shoaling water, opposing currents, etc.
3. More statistical data on still water and dynamic (vibrational) loads in service, both as to increases in extreme loads affecting ultimate failure and dynamic cyclic loads affecting fatigue.
4. Improved techniques for combining loads of differing frequencies that do not always occur simultaneously.
5. Further development of the probabilistic approach to structural capability--including, (a) prediction of ultimate strength and its variability; (b) prediction of fatigue strength and its variability in relation to critical details.
6. Collection by classification societies and/or regulatory bodies of ship damage and failure data in reliability format.

Successful completion of the above would permit the calculation with some confidence of probability of ultimate failure and of fatigue crack propagation (beyond critical length) plus acceptable levels of these probabilities for different ship types in the most severe services for which they are intended to be certified.

It should be emphasized that the complete ideal procedure for evaluating the longitudinal strength in reliability format envisioned here is not intended for routine application to individual ship designs--except in the case of unusual ships, or other marine structures, for which no empirical data are available. Its main value should be in continuing the development of more rational techniques for routine design--in particular, the refinement and increased rationalization of the rules of classification societies. This involves applying the reliability principles to many different ship types, over a wide range of sizes, and correlating predictions of failure probability against actual experience in terms of hull failure per ship-year "at risk." Similarly, the frequency of fatigue crack propagation to a size requiring repair should be correlated with predictions.

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APPENDIX 1

	ABS – Webb	R.I.Na.	Lloyds	Norske Veritas
Sources of Information on Procedure	Lewis-Montreal – 1967	R.I.Na. Report No. 128, by A. Gennaro, Dec. 1971	R. Goodman, Southhampton Symposium, 1970 Murray, Trans. N.E.C.I. 1965	Abrahamsen, 'Recent Developments in the Practical Philosophy of Ship Structural Design', SNAME, Spring, 1967; or N.V. Publ. No. 77, 1971
Wave Data				
Source:	Pierson-Moskowitz for North Atlantic	Walden data for North Atlantic (N.V.)	Hogben & Lumb; other data when available.	Roll's & Walden's data for North Atlantic
Format:	Observed spectra plotted in groups of constant sign. ht.	Tabular distribution of periods and heights.	Tabular distribution of periods and heights.	Data expressed in form of Weibull distributions.
Wave Spectrum Formulation	Use random sampling of actual spectra for North Atlantic	N.V.	N.V.	Modified Pierson-Moskowitz
Short-crestedness	Spreading function $\frac{2}{\pi}\cos^2\mu$	Spreading function $\frac{2}{\pi}\cos^2\mu$	Spreading function $\frac{2}{\pi}\cos^2\mu$ or modification	Spreading function $\frac{2}{\pi}\cos^2\mu$
RAO's				
From:	Model tests or calculations. Calculations preferred.	Strip theory calculations or model tests (generally calculations)	Prefer model tests (systematic or random with regression analysis) Strip theory calculations	Strip theory calculations or model tests.
All headings?	yes	yes (7+)	yes, up to 14	yes
Assumed distr. of headings	Equal probability	Equal probability	Variable probability	Normally equal probab.
No. of wave lengths		13+	up to 51	17 (normally)
Short-term Predict.				
Results in terms of rms Response for:		Standard (Subject of variation):		
Ship speeds	1	1+	4	2 (max. 3)
Ship headings	7	7	14	5 (max. 7)
No. of spectra per wave height	10	8		Any number. 20 values of $\bar{T}\sqrt{g/L}$
Sig. wave heights	5	16		
Ship lengths	4	5	1	
Long-term Predict.	Integrate probabilities over ship heading at constant wave height; integrate over vave ht.	Integrate probabilities over wave periods, wave hts., ship headings.	Integrate probabilities over wave periods, wave heights, & ship headings; and fair result graphically.	Integrate probabilities over wave periods, wave hts.; fit to Weibull distr. & integrate over ship headings.
Probability level (or Number of cycles) for Design.	$N = 10^8$ for comparative purpose only. Ultimate value depends on factor of safety, etc.	Generally $N = 10^2$ and 10^8	Depends upon response under study.	$N = 10^8$ for comparative purposes. Ult.value depends on factor of safety, etc.

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PROCEDURES FOR PREDICTING LONG-TERM WAVE LOADS [34]

G.L.	N.K.K.	Bureau Veritas	Register of Shipping of USSR
H. Söding, Schiff und Hafen, Oct. 1971, H.-G. Schultz, JSTG, 1969.	Shipbuilding Research Association of Japan, Report No. 69, Sept. 1970.	J. M. Planeix, M. Huther, R. Dubois, 'Solicitations externes et internes des navires a la mer,' ATMA, 1972.	A. J. Maximadji, Proceedings of the Research Institute of Sea fleet. 1971, N 134 and 1972, N 140 V.V. Kozljakow, Sudostroyeniye 1966, N 8
Roll & Walden for North Atlantic	Walden for North Atlantic	Roll; Hogben & Lumb - N.A. Hogben & Lumb; Yamanouchi & Ogawa - Pacific	Roll's data for North Atlantic
distrb. of sig. hts. & periods for N.A., presented graphically as cumulative distr.	Tabular distribution of periods and heights.	Fr. Meteor Office data Tables of periods & hghts. Tables of spectral ords.	Integral curve of the probability of exceeding of the wave states as a function of $H_{3\%}$.
I.S.S.C., with $H_V = H_{1/3}$	I.S.S.C.	$S(\lambda) = \frac{\pi}{g^2}\frac{H_{1/3}^2}{T^2}\lambda \exp\left(-\frac{4\pi}{g^2}\frac{\lambda^2}{T^2}\right)$ $H_{1/3}$ is sign. ht. T it means app. period λ is wave length	I.S.S.C., but based on observed $H_{3\%}$ ($H_{3\%} = 1.33\, H_{1/3}$)
Spreading function $\frac{2}{\pi}\cos^2\mu$	Spreading function	Varies with wave frequency	Spreading function $\frac{2}{\pi}\cos^2\mu$
Strip theory calculations	Strip theory calculations	Generally strip theory calculations	Strip theory calculations
yes	yes	yes (6)	yes
Equal probability	Equal probability	From wave data	Equal probability
22 (normally)	20	15 - 20	
5	7	1 - 6	
7	7	6	5
22	8	Usually 1	8
15	1	Usually 6	8
8	5		6
Integrate probabilities over wave periods, wave hts. and ship headings.	Integrate probabilities over wave periods, wave hts. and ship headings.	Determine short-term probability for each angular spectral comp., multiply by probs. of spectrum & heading. Integrate(36 terms)	Integrate probabilities over wave periods, wave heights, ship headings.
$N = 10^6$	$N = 10^6$ for comparative purposes only	About $N = 10^7$	$N = 10^8$

APPENDIX 2

Full-Scale Midship Stress Measurement Programs

This list includes ships on which the most comprehensive stress measurement and analysis programs have been done. It is not an exhaustive list of all such efforts. An unpublished Webb literature search in 1977 identified 124 papers describing measurements taken on about 150 ships, most of which contain more limited data. A summary of some of the other available data was given in [34].

	Ship	Type	Service	LBP	Beam	C_B	Instrumented Sea Time
USA-ABS	Universe Ireland	Tanker	PG/NE	1076.0	174.9	0.860	11 voy.
	Idemitsu Maru	Tanker	PG/Jap.	1069.3	163.3	0.830	14 voy.
	Esso Malaysia	Tanker	PG/NE	1000.0	154.8	0.830	13 voy.
	R. G. Follis	Tanker	PG/ECUS	754.6	104.5	0.820	18 voy.
	Fotini L	Bulk	Pacif.	800.0	106.0	0.840	18 voy.
USA-SSC	Wolverine State and	Cargo	N. Atl.	496.0	71.5	0.654	44 voy.
	Hoosier State	"	N. Pac.	"	"	"	
	California Bear	Cargo	N. Pac.	528.5	76.0	0.625	5 voy.
	Mormacscan	Cargo	N. Atl.	458.0	68.0	0.630	
	Sealand Mclean	Cont.	N. Atl.	880.5	105.5	0.590	3 seasons
	Boston	Cont.	N. Atl.	496.0	71.5		2 seasons
USA-SNAME	Edward L. Ryerson	Ore	G. Lakes	711.8	75.0		4 seasons
Belgium	Jordaens	Cargo	N. Atl.	479.5	65.9	0.692	
	Roi Baudoin	Car Ferry	N. Sea	362.9	49.9	0.542	
	Mineral Seraing	Bulk	NE/SA	715.3	105.0	0.800	10 months
	Dart Europe	Cont.	N. Atl.	715.3	100.0	0.600	14 voy
United Kingdom	Encounter Bay Class	Cont.	NE/Aus.	745.5	100.0		2 years
Japan	Japan Ace	Cont.	Pacif.	574.2	82.7	0.566	
	Wakahata Maru	Ore	Pacif.	787.4	120.8	0.82	13891 records
	Chidoruan Maru	Ore	Pacif.	912.0	146.0	0.82	7352 records
	Japan Adler	Bulk	Pacif.	689.0	104.9	0.80	10517 records

Abbreviations: ECUS-East Coast U.S.
SA- South America
NE- North Europe
PG-Persian Gulf
Aus.-Australia

APPENDIX 3

Short-term B.M. Response, SL-7 at 25 Knots, H 1/3=24.5 ft.

(1) Heading Angle		(2) Mean B.M. Response, FT-TONS	(3) For Equal Cycles Each Heading $p(\sigma)$	(4) $(2)\times(3)$	(5) For Equal Time Each Heading $p(\sigma)^*$	(6) $(2)\times(5)$
		10^5x		10^5x		10^5x
0	Following Seas	1.7664	.0833	.1472	.0216	.0382
30		1.6845	.1667	.2808	.0545	.0918
60		1.4827	.1667	.2471	.0968	.1435
90	Beam Seas	1.3616	.1667	.2269	.1770	.2410
120		1.4947	.1667	.2491	.2437	.3643
150		1.7412	.1667	.2902	.2690	.4684
180	Head Seas	1.8611	.0833	.1551	.1374	.2557
				1.5964		1.6029

Avg. Response for equal no. cycles each heading = 1.5964×10^5 FT-TONS
Avg. Response for equal time each heading = 1.6029×10^5 FT-TONS

Note: $p(\sigma)^*$, column (5), is $p(\sigma) \times \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}}$