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Considerations of Probability-Based Fatigue Design for Marine Structures

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ABSTRACT

Fatigue is a major failure mode in marine structures which respond dynamically to random wave and wind loading. Oscillatory stresses produce fatigue at points of stress concentration, typically the welded joints. Because all fatigue design factors are subject to significant uncertainty, a reliability approach to management of such uncertainty seems appropriate. This article summarizes some studies in fatigue reliability research and demonstrates how reliability methods can be effectively utilized by designers to avoid fatigue in marine structural components. Included are (1) a description of fatigue damage under variable amplitude stresses employing the characteristic S-N and fracture mechanics models, (2) models for reliability assessment relative to fatigue and the use of these models to derive design criteria, and (3) an elementary application of system fatigue reliability analysis to establish component design criteria.

INTRODUCTION

In general, reliability methods seem particularly appropriate for application to the marine structure design process because of uncertainties in the ocean environment and the historical use of statistical descriptions of that environment. But, structural component strength (particularly fatigue) data have considerable scatter. Thus, both structural loads and capacities are subjected to uncertainty.

Summarized in this paper are considerations of reliability analysis to fatigue in marine structures. Specific goals of such analyses are (1) reliability assessment of an existing or proposed designs and (2) development of fatigue-avoidance criteria appropriate for inclusion in design criteria documents or codes. Demonstrated are how reliability methods can be used as an effective design tool. It is implicitly assumed herein that fatigue will occur at welded joints, but the models are generic and could apply to other details as well.

Recommended as general references are the Fatigue Handbook [1] published as a Norwegian effort, an "encyclopedia" of tubular joint design by the British [2], SSC-318 by Munse et al. [3], SSC-326 by Burnside et al. [4], a paper by Marshall and Luyties [5], and a book by Gurney [6].

FATIGUE STRENGTH: ELEMENTARY ENGINEERING MODELS

The characteristic S-N curve

$$NS^m = A \qquad S > 0 \tag{1}$$

is commonly employed to describe fatigue strength of structural components, where m and A, determined empirically from constant amplitude S-N data, are the fatigue strength exponent and fatigue strength coefficient, respectively. As an extension of this basic model, a more general two-segment S-N curve as shown in Figure 1 reflects experimental data which tend to show an improvement in fatigue strength at lower stresses.

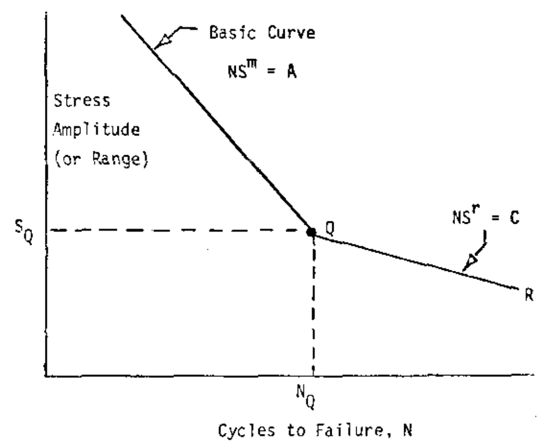

Fig. 1 Two-segment S-N curve

The fracture mechanics fatigue model is described in References [1], [2], and [7]. Assume that the crack propagation law is as defined in Figure 2. The Paris law is assumed to be valid for subcritical crack growth; K_c is fracture toughness and ΔK_{th} is the threshold stress intensity. Furthermore, we now assume that stress is a random process. Assuming an "equivalent stress" approach and after some analyses, it can be shown that [7]

$$N = \frac{1}{C S^m} \int_{a_0}^{a_f} \frac{da}{G(a) Y^m(a) (\pi a)^{1/2}} , \tag{2}$$

where a = crack depth; a_0 and a_f = initial crack

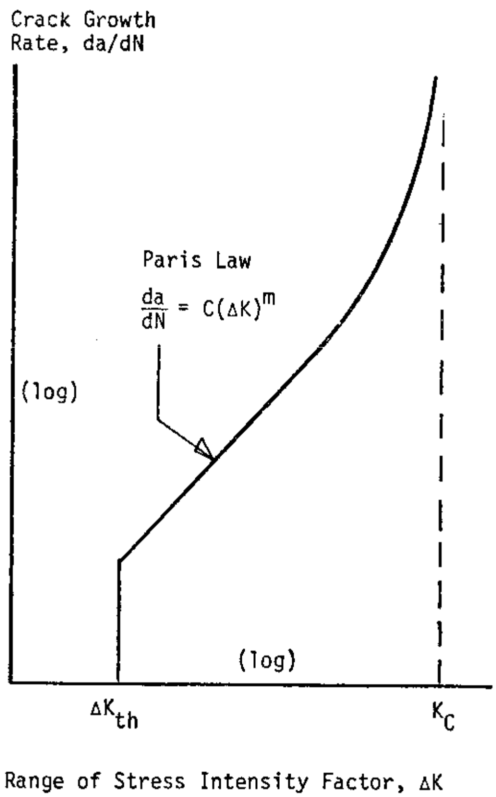

Fig. 2 A model for fatigue crack growth

depth and crack depth at failure, respectively; Y is the geometry factor; and

$$G(a) = \frac{\overline{S_0^m}(a)}{\overline{S^m}} \qquad \overline{S^m} = E(S^m) \tag{3}$$

$$\overline{S_0^m} = \int_{s_0(a)}^{\infty} s^m f_S(s)ds$$

$$s_0(a) = \frac{\Delta K_{th}}{Y(a)\sqrt{\pi a}} .$$

By multiplying both sides of Eq. 2 by $\overline{S^m}$, the characteristic S-N form of Eq. 1, $N\overline{S^m} = A$, results; $\overline{S^m} = (S_e)^m$, where S_e is equivalent stress and A is a function of all of the parameters in the crack growth law. Thus, the fracture mechanics-based fatigue crack growth law can be cast in a characteristic S-N format.

FATIGUE DAMAGE UNDER VARIABLE AMPLITUDE LOADING

Assuming that fatigue strength is defined by Eq. 1 and that Miner's rule works, fatigue damage D can be written as

$$D = \frac{n}{A} B^m E(S^m) , \tag{4}$$

where S is a random variable denoting estimated stress range for a single cycle, n is the total number of cycles applied, and E(·) is the expected value. B is introduced as a bias factor on stress range in recognition that the actual stress differs from the estimated stress because of modeling error resulting from assumptions made in the stress analysis. The event of failure is defined as $D > 1$.

It is convenient to write D as

$$D = \frac{TB^m\Omega}{A} , \tag{5}$$

where $T = n/f_0$ is time of exposure, f_0 is the lifetime average zero up-crossing frequency of the stress range, and $\Omega = f_0E(S^m)$ is the stress parameter. Models which are routinely employed by the marine industry are summarized in Table I [8, 9].

The Weibull model for S is commonly employed for a simplified fatigue assessment and for derivation of design criteria. The distribution function is

$$F_S(s) = 1 - \exp\left[-\left(\frac{s}{\delta}\right)^{\xi}\right] \qquad s > 0 . \tag{6}$$

where ξ and δ are the Weibull shape and scale parameters, respectively. This model idealizes the long-term distribution of stress range.

By defining a "design" stress S_0 as

$$P(S > S_0) = \frac{1}{N_T} , \tag{7}$$

where N_T is the total number of cycles in the service life, S_0 is then the value exceeded by S on an average of once every N_T times. The scale parameter δ can be written in terms of S_0, ξ, and N_T as

$$\delta = S_0[\ell n\ N_T]^{-1/\xi} . \tag{8}$$

The Weibull distribution function is plotted in Figure 3 in a form useful for designers. A key role is played by the shape parameter which describes implicitly both the environment and the structural system ξ. Some typical values are $\xi = 0.5$ for Gulf of Mexico platforms, $\xi = 0.5$ to 0.7 for template platforms outside the Gulf without significant dynamic amplification, and $\xi = 1.0$ for semi-submersibles and gravity platforms. Figure 4 (from Munse et al. [3]) shows $\xi = 0.7$ to 1.3 for hull girder stresses in ships (which, as a warning, may not be directly related to the most troublesome fatigue failures in ship structures).

FATIGUE DAMAGE: PIECEWISE LINEAR S-N CURVES

Damage expressions of Eq. 4 and 5 depend upon the assumption that fatigue strength is defined by Eq. 1. But the two-segment S-N curve of Figure 1, which provides an improvement in fatigue strength at lower stresses, is specified by API [10] and the UK DEn [11]. Extrapolation of $NS^m = A$ into the high cycle range produces conservative results. Differences in damage estimates between the two-segment and linear

Table I. A summary of the expressions for fatigue damage.

Fatigue Damage at Time T

$$D = TB^m\Omega/A$$

where

m, A = parameters from S-N curve (Eq. 1)

B = factor to account for uncertainties in estimating fatigue stresses from oceanographic data

Ω = $f_0 E(S^m)$, stress parameter

Stress Parameter Using Various Approaches to the Stress Distribution

- Wave Exceedance Diagram (Deterministic Method)

  Ω = $f_0 \sum_i \zeta_i S_i^m$

  f_0 = average frequency of stresses

  S_i = stress range

  ζ_i = fraction of total stress ranges of S_i

- Spectral Method (Probabilistic Method)

  Ω = $\lambda(m)(2\sqrt{2})^m \Gamma(m/2 + 1) \sum_i \gamma_i f_i \sigma_i^m$

  $\lambda(m)$ = rainflow correction [8, 9]

  $\Gamma(\cdot)$ = gamma function

  γ_i = fraction of time in i^{th} sea state

  f_i = frequency of wave loading in the i^{th} sea state

  σ_i = RMS of stress process in the i^{th} sea state

- Weibull Model for Stress Ranges

  Ω = $f_0 S_m^m [\ell n\ N_T]^{-m/\xi} \Gamma(m/\xi + 1)$

  S_m = largest "once in a lifetime" stress range

  ξ = stress range parameter

  N_T = total number of stress ranges in design life

- Nolte-Hansford Model [39] (Extension of the Weibull Model)

  Ω = $f_0 \delta^{m\phi} \psi^m \Gamma(m\phi/\xi + 1)$

  Terms same as Weibull except ϕ, ψ = parameters from empirical equation $S = \psi H^\phi$, where H is wave height

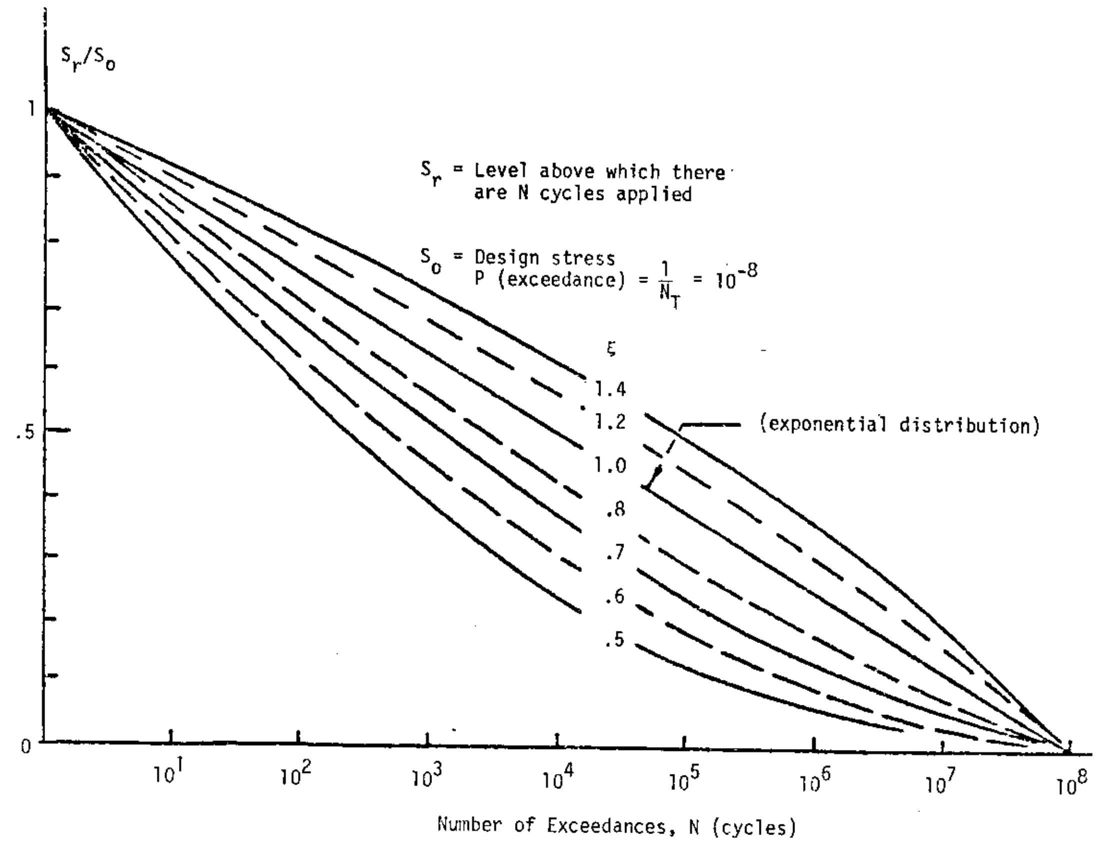

Fig. 3 Long-term distribution of stress ranges; Weibull distribution function

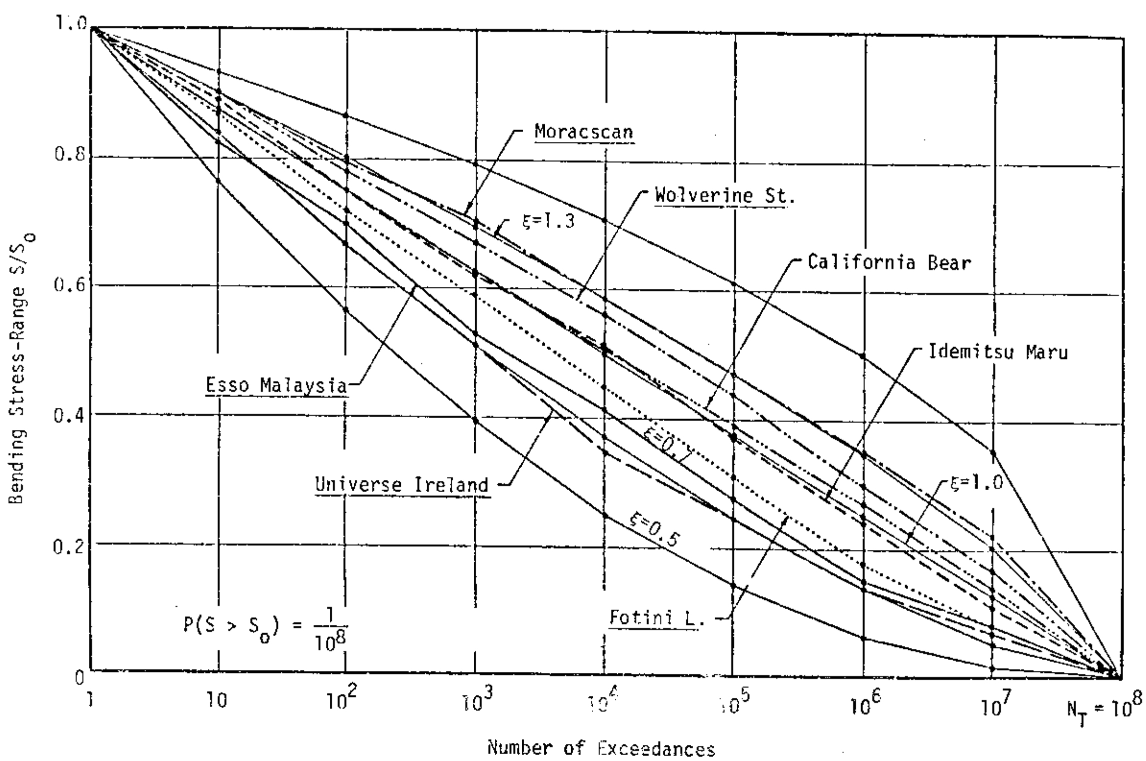

Fig. 4 Loading histories of large tankers, bulk carriers, and dry-cargo vessels compared with Weibull (after Munse [3])

cases were studied in an unpublished article [12] and are summarized as follows.

Assume that the long-term distribution of stress ranges, S, is Weibull (Eq. 6, 7, and 8). Given the form of Figure 1, it can be shown that fatigue damage for the two-segment case is

$$D_B = \Lambda D \ , \qquad (9)$$

where Λ is the bias factor given as

$$\Lambda = \frac{A\delta^{r-m}\Gamma_0(b,\ z)}{C\Gamma(a)} + \Psi(a,\ z) \qquad (10)$$

where

$$\Psi(a,\ z) = \frac{\Gamma(a,\ z)}{\Gamma(a)} \ , \qquad z = (S_Q/\delta)^{\xi} \ , \qquad (11)$$

$$a = \frac{m}{\xi} + 1 \ , \qquad b = \frac{r}{\xi} + 1$$

$\Gamma(x)$ is the gamma function, and $\Gamma(a,\ z)$ and $\Gamma_0(a,\ z)$ are incomplete gamma functions (integrals z to ∞ and 0 to z, respectively).

Two examples are presented in Figure 5 (bias factor for UK DEn-T curve) and Figure 6 (bias factor for API-X curve).

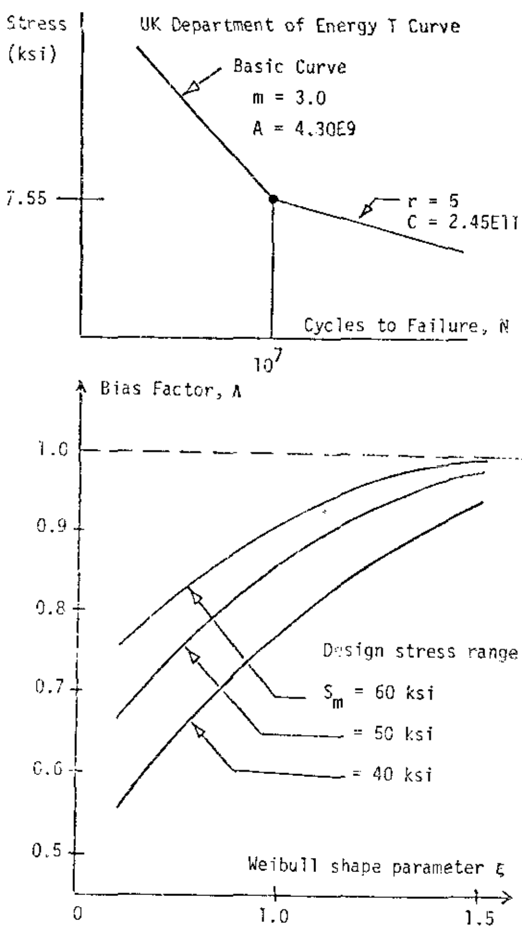

Fig. 5 Bias factor for DEn-T curve

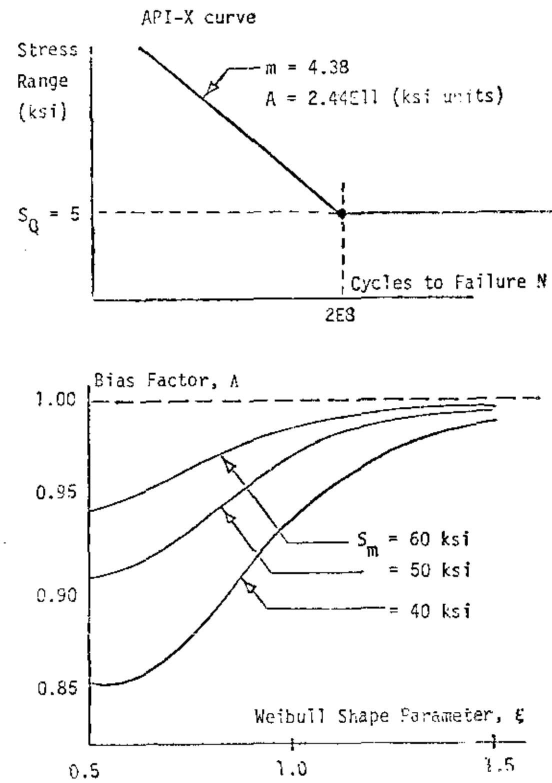

Fig. 6 Bias factor for API-X curve

In summary, it can be seen that for a well-designed joint (i.e., assumed to have S_Q = 60 ksi), the reduction in damage implied by the endurance limit of the API-X curve is modest. On the other hand, reduction in damage from the linear case implied by the high cycle segment of the UK DEn-T curve can be as high as 20% for the same joint.

Described above were engineering models of fatigue. Now, attention will focus on the probability problem in which uncertainties in the fatigue analysis processes will be translated into random variables of the design factors in such a way to make reliability assessment tractable.

FATIGUE RELIABILITY MODEL: SSC/MUNSE

The model used by Munse et al. in SSC-318 [3] for reliability assessment relative to fatigue in a component was originally derived by Ang [13]. The development of this model is well documented [3, 13] and only a summary is provided here.

Let N be a random variable denoting cycles to failure. Assume that N has a Weibull distribution. The shape and scale parameters are

$$\alpha = C_N^{-1.08} \qquad \gamma = \frac{\mu_N}{\Gamma\left(\frac{1}{\alpha} + 1\right)} \ , \qquad (12)$$

where μ_N is the mean life obtained from a least squares analysis of fatigue data and C_N is the coefficient of variation (COV) of cycle life. All uncertainty is included in C_N:

$$C_N = [C_f^2 + m^2 C_S^2 + C_C^2]^{1/2} \quad , \tag{13}$$

where all C's are COV's. C_S accounts for stress modeling error, C_C for workmanship uncertainty, and $C_f = \sqrt{\delta_f^2 + \Delta_f^2}$. δ_f and Δ_f are COV's representing scatter inherent in S-N data and uncertainties in Miner's rule, respectively.

Miner's rule is assumed, which implies that $E(S^m) = A/\mu_N$. Then it is easily shown that the probability of failure at the service life N_S is

$$p_f = \left[\frac{N_S E(S^m)\Gamma(1 + C_N^{1.08})}{\tilde{A}}\right]^{C_N^{-1.08}} \quad . \tag{14}$$

The use of this form to derive design criteria is demonstrated below.

FATIGUE RELIABILITY MODEL: API/WIRSCHING

To derive an expression for reliability relative to fatigue, Wirsching [8, 14] used the simple lognormal format for multiplicative limit state functions. A fundamental difference between the Munse and Wirsching approaches is the use of the Weibull and lognormal distributions, respectively, for N. The lognormal model is also well documented. A summary follows.

The fatigue strength coefficient A is defined as a random variable describing the inherent variability of the fatigue strength. The median $\tilde{A}$ defines the median S-N curve, and the COV, C_A, is the COV of N given S. Bias and uncertainty in Miner's rule are defined by Δ, the damage index at failure; the event of failure is $(D > \Delta)$. B was previously defined to describe stress modeling error. Δ, B, and A are assumed to have lognormal distributions with medians ($\tilde{\Delta}$, $\tilde{B}$, $\tilde{A}$) and COV's (C_Δ, C_B, C_A), respectively.

The probability of failure is

$$p_f = \Phi(-\beta) \quad , \tag{15}$$

where $\Phi(\cdot)$ is the standard normal distribution function and β is defined as the safety index,

$$\beta = \frac{\ell n(\tilde{T}/T_S)}{\sigma_{\ell n\ T}} \quad , \tag{16}$$

where T_S is the service life and $\tilde{T}$ is the median time to failure,

$$\tilde{T} = \frac{\tilde{\Delta}\tilde{K}}{\tilde{B}^m \Omega} \quad , \tag{17}$$

and

$$\sigma^2_{\ell n\ T} = \ell n\{(1 + C_\Delta^2)(1 + C_A^2)(1 + C_B^2)^{m^2}\} \quad . \tag{18}$$

The Munse model and the lognormal format are elementary reliability models, but more sophisticated approaches have been developed. For example, Madsen [29, 30] employs advanced reliability methods with a more general description of the fatigue limit state along with a more detailed and accurate description of the S-N statistics defining fatigue strength.

STATISTICAL DATA: EXAMPLES

Examples of supporting data for the reliability models are provided in Tables II-IV. The S-N fatigue data of Figure 4 are analyzed using a least squares model on a log-log basis. The relatively large COV's associated with cycle life demonstrate the high level of uncertainty in fatigue design factors, thereby supporting the claim that reliability methods are particularly relevant for fatigue.

Exercises which have attempted to quantify stress modeling errors (the random variable B) are summarized in Table III. The figures in each example, provided by expert testimony and some data, are highly dependent upon the nature of the system and how the analysis is performed. Therefore, the figures should not be used by themselves without knowledge of relevant details. It is interesting to note, however, that there is some coherence in these numbers. Designers seem to believe (a) that there is a slight conservative bias to stress analysis and (b) that uncertainties may range from about 20 to 50%, with lower figures typical of static designs.

Strength modeling error (the random variable Δ) is measured from random fatigue testing. Because fatigue behavior is influenced by so many factors, it is difficult to interpret the meaning of each result in the summary of Table IV. These figures also contain variability inherent in the material. But, again, there seems to be some coherence to the values. A slight non-conservative bias is suggested by recent tests on welded detail, and uncertainties of 30 to 60% seem to be typical.

DESIGN CONSIDERATIONS

Example 1

Reliability assessment of a component is possible using the Munse or lognormal form (Eq. 14 and 15). These equations can also be used to establish design criteria. As an example, assume that the stress distribution is Weibull (Eq. 6-8). Using the value of Ω from Table I, a maximum allowable design stress can be derived [3]

$$S_0 < S_N \cdot \psi \cdot R_F \quad , \tag{19}$$

where

$$S_N = (\tilde{A}/N_S)^{1/m}, \text{ mean fatigue strength at service life, } N_S \tag{20}$$

$$\psi = [\ell n\ N_S]^{1/\xi}\ \{\Gamma(1 + m/\xi)\}^{-1/m}, \text{ random load factor}$$

and

Table II. Examples of statistical data on S-N curves.

$NS^m = A$	m	Median of A $\tilde{A}$ (MPa/ksi)	COV of N* C_N (%)
WRC data from RP2A (1982) commentary [10]	4.38	2.16 E 4.60 E12	73
API-X [14]	4.42		136
Butt Welded Joints Munse et al. [3]	2.88	9.67 E11/3.72 E9	50
UK DEn S-N curves for welded joints [11]			
B	4.0	2.34 E15/1.04 E12	44
C	3.5	1.08 E14/1.25 E11	50
D	3.0	3.99 E12/1.21 E10	51
E	3.0	3.29 E12/1.00 E10	63
F	3.0	1.73 E12/5.28 E9	54
F2	3.0	1.23 E12/3.75 E9	56
G	3.0	5.66 E11/1.73 E9	43
W	3.0	3.68 E11/1.12 E9	44
T	3.0	4.79 E12/1.46 E10	67

*It is easily shown that $C_A = C_N$.

Table III. A summary of some efforts to quantify modeling error in stress analysis; the random variable B.

Study	Bias ($\bar{B}$)	COV (%)	Comments
Load and resistance factor design (LRFD) for onshore construction [15]	--	24	Live load effects for floor beams; a "small" part is objective uncertainty
National Bureau of Standards SP577 providing background for ANS A58, "Building Code Requirements . . ." [16]	--	20	Maximum live load for a 50-year reference period
LRFD proposed for offshore construction; Project API-PRAC-22 [17]	0.70	37	Maximum design load effect for fixed offshore structures
Modeling error for extreme wave loads on a pile; average conditions in the North Sea; NTH, Trondheim [18]	--	34-45	Using design wave approach, the uncertainty on the extreme life-time force on a pile
ABS survey to establish modeling error associated with design loads for cylinders and pontoons for TLP's [19]	0.90	25	Bias value selected by the committee is more conservative than the 0.70 actually found in the survey
Study of fatigue stress modeling error for offshore platforms by DnV [20]	0.84	14	COV seems low and out of line with other experiences
Study of fatigue stress modeling error for offshore platform; API fatigue reliability project API-PRAC-15 [8]	0.70	50	A major contributor to COV; uncertainty in description of sea state has strong influence on fatigue stresses
SSC-322 extreme wave and whipping loads of ship structures [21]	--	25-30	These figures do not include uncertainty in stress calculations

Table IV. Examples of statistical data on damage at failure, Δ.

	Median (Δ)	COV of Δ
Shin and Lukins [22]: Survey of variable amplitude fatigue data	0.90	0.67
Schütz [23]: Survey of random tests	1.00	0.60
Large quasi-static mean load changes	0.70	0.60
Schilling et al. [24]: Full-scale cover-plated steel beams	1.15	0.48
Gurney [25]: Longitudinal non-load carrying fillet welds	0.85	0.28
Eide and Berge [26]: Non-load carrying fillet welds	0.78	0.19
Berge and Eide [27]: Non-load carrying fillet welds	1.06	0.40
Holmes and Kerr [28]: Cruciform specimen	0.69	0.61

$$R_F = \begin{cases} \left[\dfrac{p_0 C_N^{-1.08}}{\Gamma(1 + C_N^{1.08})}\right]^{1/m} & \text{Munse reliability factor} \\[2ex] \dfrac{1}{B}\left[\dfrac{\tilde{\Delta}}{\exp(\beta_0 \sigma_{\ell n} T)}\right]^{1/m} & \text{lognormal reliability factor} \end{cases}$$

where p_0 and β_0 are the target p_f and safety index, respectively, which must be specified.

Examples of the use of the Munse and lognormal formats are given in References [3] and [8]. Because of the relatively "strong" left tail of the Weibull distribution for N, design stress ranges S_0 of the Munse criterion are smaller than those of the lognormal format and, sometimes, much more so. A study of the distribution of fatigue data in welded joints has indicated the lognormal to be generally a better model for N than the Weibull [31]. But it would be premature to suggest that the lognormal format provides improved design criteria.

Example 2

Frequently, design criteria documents specify a maximum allowable damage at failure defined as a target damage ratio, Δ_0. A restatement of the form of the lognormal approach can produce a Δ_0 having a probability basis [40].

Noting from Eq. 5 that median damage at service life T_S is

$$\tilde{D} = \frac{T_S \tilde{B}^m \Omega}{\tilde{A}} , \tag{21}$$

it follows from Eq. 16 and 17 that

$$\beta = \frac{\ell n(\tilde{\Delta}/\tilde{D})}{\sigma_{\ell n} T} . \tag{22}$$

Requiring that $\beta > \beta_0$, the target safety index, the requirement for a safe design can be written as

$$\tilde{D} < \frac{\tilde{\Delta}}{\exp(\beta_0 \sigma)} . \tag{23}$$

To develop a convenient design equation, consider a factored form of $\tilde{D}$. Let the design S-N curve be a lower bound specified by A_0, as shown in Figure 7. Define the scatter factor

$$\lambda = \tilde{A}/A_0 . \tag{24}$$

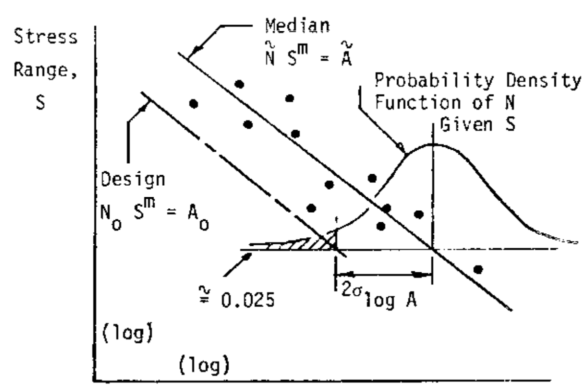

Fig. 7 Definition of the S-N design curve

The relationship between λ and the coefficient of variation of cycle life is given as

$$\lambda = \exp(2\sigma_{\ell n A}) , \tag{25}$$

where

$$\sigma_{\ell n\ A} = \sqrt{\ell n(1 + C_A^2)} \quad . \tag{26}$$

From Eq. 23 and 24,

$$\tilde{D} = (B^m/\lambda)D_0 \quad , \tag{27}$$

where

$$D_0 = T_S\Omega/A_0 \quad .$$

D_0 is nominal damage (damage as would be computed by conventional design procedures). Finally, the safety check expression can be derived from Eq. 23-28 as

$$D_0 < \Delta_0 \quad , \tag{29}$$

where the target damage ratio Δ_0 is

$$\Delta_0 = \frac{\lambda\tilde{\Delta}}{B^m \cdot \exp(\beta_0\sigma)} \quad . \tag{30}$$

As an example, consider the statistics in Table V. These values were assumed to be "reasonable" for development of TLP design criteria [32]. Upon substitution into Eq. 30, a plot of Δ_0 versus β_0 can be constructed as shown in Figure 8. Upon selection of an appropriate target safety index β_0, the value of Δ_0 can be established. An example of design criteria for deck and hull structural detail for TLP's is given in Table VI. Selection of β_0 is influenced by considerations of importance and inspectability.

ELEMENTARY CONSIDERATIONS OF SYSTEM RELIABILITY ANALYSIS: TLP TENDONS

Ultimately, it is hoped that technology will be available to perform (a) reliability assessments of a system of components or (b) given the target reliability of a system, derive component requirements. In general, the system reliability problem is extremely complex [33, 34, 35]. Stahl and Geyer [36, 37] have addressed the tendon system reliability problem considering both ultimate strength and fatigue.

Table V. Reference data for calculation of target safety index [40].

m	3.0
C_A*	0.50
$\tilde{\Delta}$	1.0
C_Δ	0.30
B	0.90
C_B	0.25

*Equal to C_N, the COV of cycles to failure.

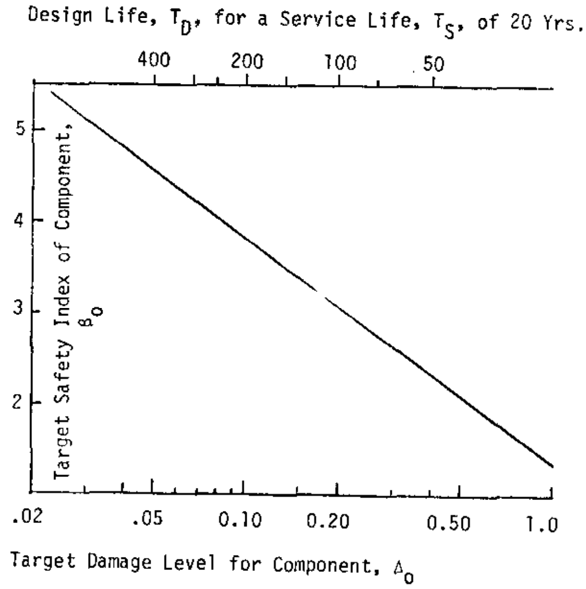

Fig. 8 Example: The target safety index as a function of the target damage level for a component

Table VI. Example: Fatigue design criteria for TLP deck and hull structure.

Target Safety Index, β_0	Application	Target Damage Level, Δ_0
2.0	The structure is redundant and cracks are easily inspected and repaired; used for deck structure, mating joints, main body of cylinders and pontoons, and production risers	0.55
2.5	For redundant and non-critical structure which is non-inspectable, i.e., non-inspectable deck structure	0.35
3.0	The structure is critical and, while inspection is possible, repairs are expensive; used for pontoon/cylinder interface, main braces, and for tension pile pullout	0.22

In the example which follows (a summary of [40]), TLP tendon fatigue design criteria are derived based on a host of simplifying assumptions.

A TLP tendon model is shown in Figure 9. It is assumed that (1) each tendon has n "components" or fatigue-sensitive points, (2) the axial force throughout the tendon is uniform, (3) fatigue will be the principal failure mode, (4) stress corrosion effects are ignored, and (5) there is no effective inspection program.

Fig. 9 Model of a TLP tendon

Let S denote the stress in a tendon and R_i denote the strength of the i^{th} element. The event of failure of the i^{th} component is

$$E_i = (R_i < S) . \quad (31)$$

The probability of failure of the i^{th} component is

$$p_i = P(R_i < S) . \quad (32)$$

The tendon then is a simple series system of m components. If E_i were assumed to be independent, an upper bound on the probability of tendon failure is [33]

$$p_T < np_i . \quad (33)$$

The exact probability of tendon failure

$$p_T = \int_0^\infty f_S(x)F_R(x)dx , \quad (34)$$

where

$$F_R(x) = 1 - [1 - F_{R_i}(x)]^n . \quad (35)$$

F_{R_i} and F_R are the distributions of component and tendon strength, respectively.

For fatigue, it is shown in Reference [38], using the lognormal format, that

$$S = B_S Se/\Delta^{1/m} \quad (36)$$

$$R_i = \frac{(A/N_S)^{1/m}}{B_{c,i}} , \quad (37)$$

where stress modeling error B is separated into components B_S for the structure as a whole and $B_{c,i}$ for error which varies from component to component. Characteristic statistics based on typical values are C_{R_i} = 0.20 and C_S = 0.23 [38].

Using these values, the relationship between component and system probability of failure is illustrated in Figure 10 for both the dependent failure mode case (Eq. 35) and the upper bound (Eq. 33). The safety index is related to p_f by Eq. 15.

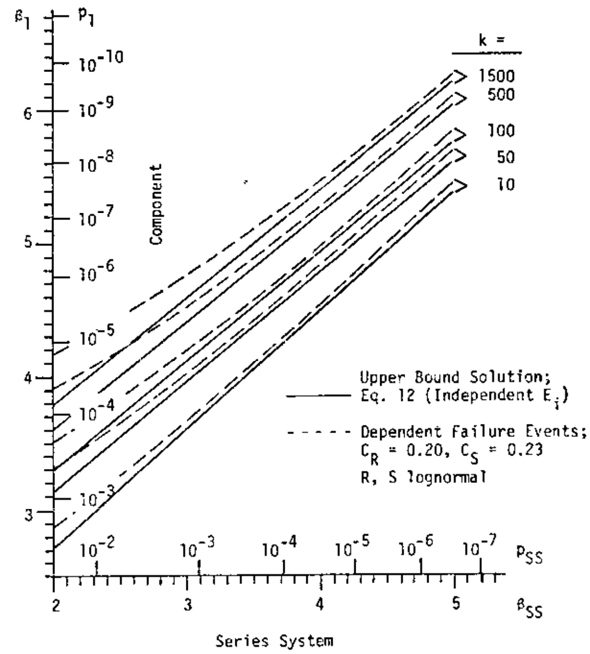

Fig. 10 Component reliability as a function of series system reliability; k elements

Using these results, Eq. 30, and the values in Table V, the target damage level Δ_0 can be derived as a function of the specified target safety index for a tendon β_T and the number of joints. Superimposed on the results (both the upper bound and dependent case) presented in Figure 11 is the recommendation of Δ_0 = 0.10 by the API Tendon Systems Design Task Group.

Conclusions from the exercise are that (1) Δ_0 is not a strong function of the number of components and (2) the upper bound solution produces only slightly conservative requirements.

Finally, it should be noted that specified target reliabilities relate to service lives. When, for example, the target tendon reliability is specified as β_T = 3.0, this value applies at time t = T_S. For all t < T_S during the service life, the actual reliability will exceed 3.0.

By direct application of the lognormal format described earlier, component reliability

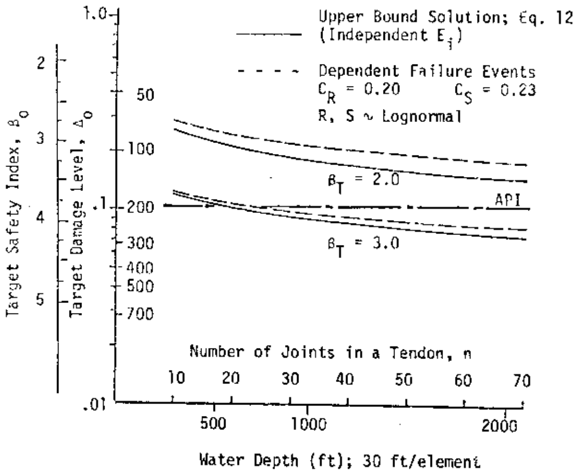

Fig. 11 Target fatigue damage level as a function of number of joints in tendon

β_1 for any time can be formulated as

$$\beta_1(t) = \beta_0 - \frac{\ln(t/T_S)}{\sigma_{\ln T}} , \tag{38}$$

where $\sigma_{\ln T}$ is given by Eq. 18. For the values in Table V, $\sigma_{\ln T}$ = 0.925. The relationship between risk and operating time of Eq. 38 is presented in Figure 12 for this value. This figure illustrates a rather dramatic degradation in structural integrity due to "aging." It also suggests conservatism in the way fatigue requirements are constructued; i.e., when reliability specifications are targeted to the service life, a higher reliability is realized during operation.

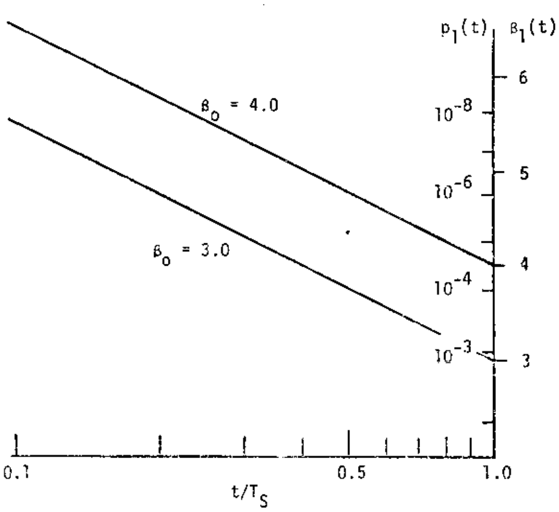

Fig. 12 Degradation in reliability as a function of time

CONCLUSIONS

Reliability mathematics can be useful as a tool for managing the large uncertainties associated with fatigue, thereby providing designers with a sound basis for decision making. Summarized in this paper are elementary methods of reliability assessment and design code development relative to fatigue. Research efforts continue worldwide on this important topic.

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