THE SOCIETY OF NAVAL ARCHITECTS AND MARINE ENGINEERS
601 Pavonia Avenue, Suite 400, Jersey City, New Jersey 07306 USA
Paper presented at the Marine Structural Inspection, Maintenance, and Monitoring Symposium
Sheraton National Hotel, Arlington, Virginia, March 18-19, 1991

# Uncertainties in Stress Analysis of Marine Structures

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## ABSTRACT

Uncertainties in stress analyses on both ships and offshore structures are studied. Emphasis is given to the effect of modeling uncertainties on extreme design loads. This study investigates uncertainties in calculating short and long term loads and load effects on ships, and offshore platforms, and also in fatigue analysis.

It is shown that some previous studies have seriously underestimated extreme design loads because they have not properly treated modeling uncertainties.

## 1.0 INTRODUCTION

Structural analysis of marine structures consists of the following steps:

a) description of the environment;

b) modeling of the applied loads;

c) load combination;

d) response analysis, where displacements, nominal forces applied to each structural member, and stresses are calculated;

e) fatigue analysis.

Uncertainties are always involved in all the steps of structural analysis. These uncertainties are due to the random character of the loading environment and the resulting loads, or due to inadequate knowledge of physical phenomena associated with loads.

Rational analysis and design of marine structures requires consideration of all the uncertainties involved in predicting load effects. In probabilistic methods, these uncertainties must be quantified in order to assess structural safety. Furthermore, the determination of the partial load and resistance factors, in the safety equation of a Load and Resistance Factor Design code, also requires quantification of all uncertainties [1,2].

The development of probabilistic analysis methods and design codes increased the importance of quantifying uncertainties. Recent studies on offshore [2-5], as well as ship structures [2, 6-8], investigated errors in evaluating loads and load effects. The results of these studies can be used to assess the relative importance of the various types of uncertainties.

Theory of reliability and structural analysis have reached a state of maturity but there are still gaps in the state of knowledge on quantifying loads and their effects. As part of the total effort associated with rational ship and offshore study design based on probabilistic methods of analysis, a project aimed at quantifying the uncertainties in determining loads and load effects in marine structures was established by the Ship Structure Committee. This paper describes some results of this project. The following issues are addressed in this paper:

a) what is the best way to model uncertainties?

b) what are the differences between random (natural) and modeling (subjective) uncertainties?

c) how do modeling uncertainties affect extreme loads?

d) how important are random uncertainties in fatigue analysis?

The information presented in this paper is organized as follows:

In section 2, we classify uncertainties into two categories, random (natural) and modeling, and study the basic differences between these two types. Emphasis is given to the effect of modeling uncertainties on extreme loads. Furthermore, we review various methods for modeling uncertainties.

Section 3 deals with uncertainties in loads and load effects. Most of the information is on uncertainties in short and long term stillwater and wave bending moments. Different ways for modeling uncertainties are compared. It is shown that we can dramatically reduce the variability, if we use the Guedes

Soares model for uncertainty and distinguish between different types of ships, and between hogging and sagging. However, although Guedes Soares' idea for reducing uncertainties is correct, we believe that it has not been properly implemented in [8] because modeling uncertainties have not been correctly treated in his study.

For offshore platforms, we study uncertainties in extreme global loads. Important factors, such as current velocity and marine fouling are also considered.

Section 4 focuses on fatigue analysis procedures. The study is confined to cumulative damage based approaches. We examine the contribution of the uncertainties, which are involved in all steps of fatigue analysis, to the overall uncertainty in fatigue damage. This allows to identify the most critical uncertainties. Finally, it is shown that the effect of random uncertainties on the cumulative damage is negligible for both ships and offshore structures.

## 2.0 TYPES OF UNCERTAINTIES

In this section we define two categories of uncertainties, random and modeling, and examine the differences between them. We also review various models for such uncertainties. Emphasis is given to the effect of modeling uncertainties on extreme design loads.

### 2.1 Classification

Uncertainties can be categorized into natural (random) and modeling ones. The former are due to the statistical nature of the loading environment and the resulting loads, and they induce scatter in predictions. The latter are due to the imperfect knowledge of various phenomena, and idealizations and simplifications in analysis procedures. These uncertainties introduce both bias and scatter.

An example of a natural uncertainty, is that associated with the wave elevation at a given position in the ocean. An example of a modeling uncertainty is the error in calculating the stresses in a structure, when the applied loads are known. For this case, the error is only due to the assumptions and simplifications in structural analysis.

Modeling uncertainties are information sensitive, in the sense that they can be reduced as the knowledge of the associated physical phenomena expands, and the mathematical models representing them become more accurate. This is not the case for random uncertainties which do not decrease as we gather more information on fundamental science, but only as we obtain more data.

Both random and modeling uncertainties must be quantified and accounted for in reliability analysis and development of probabilistic design codes.

### 2.2 Models for modeling uncertainty

Ang and Cornell [9] and Ditlevsen [10] proposed two different methods for treating modeling uncertainties. Ang's model is for both load and strength uncertainties. Ditlevsen's model was proposed for uncertainties associated with strength but it can also be applied to load variables.

In the following we present Ang's model.

Let $X$ be the actual value of some quantity of interest and $X_0$ be the corresponding value specified by a design code. Then,

$$X = B_I B_{II} X_0, \tag{2.1}$$

where $B_I$ is the ratio of the theoretically predicted value for this quantity, $X_p$, and $X_0$, and $B_{II}$ is the ratio of $X$ and $X_p$. $B_I$ is a measure of natural (random) variability, which is also called type $I$ uncertainty, and $B_{II}$ is a measure of modeling uncertainty. The mean values of random variables $B_I$ and $B_{II}$, $E(B_I)$ and $E(B_{II})$, are the biases corresponding to natural and modeling uncertainties, respectively. Assuming that the random and modeling uncertainties are statistically independent, and by using a first order second moment (F.O.S.M.) approximation, we can quantify the total uncertainty in $X$ as follows:

$$\begin{aligned} E(B) &= E(B_I)E(B_{II}), \text{ and} \\ COV_B &= (COV_{B_I}^2 + COV_{B_{II}}^2)^{\frac{1}{2}} \end{aligned} \tag{2.2}$$

where $B = B_I B_{II}$, and COV stands for the coefficient of variation of the quantity specified by the subscript.

Random variables $B_I$ and $B_{II}$ are also assumed to be independent of $X_0$.

An example of quantifying modeling uncertainties is illustrated in Fig. 1, which has been extracted from [11]. The quantity considered here is the maximum annual wave height in the northwest shelf of West Australia. The ratio of the measured over the predicted maximum wave height is shown in the horizontal axis. The maximum wave height is predicted using a hindcast method. The wind speed, which corresponds to the maximum wave height is assumed to be known. The vertical axis represents the probability that the value of the ratio is less than some given number. Based on the information provided in Figure 1, the mean of $B_{II}$, which represents modeling uncertainty, is 1.1 and its COV is 0.13. This means that, on the average, hindcast methods predict a value for the annual maximum wave height, which is 10% smaller than the actual value. Moreover, $B_{II}$ is lognormally distributed.

A random variable, such as the stress in a particular structural member, is a function of other random variables, such as the wave height and the average wave period. Besides the errors involved in calculating these variables, errors are also involved in calculating the stress given the values of the latter random variables. Ang and Cornell presented formulas for quantifying the uncertainty associated with the above errors [9].

The Ditlevsen's model is applicable to reduced random variables [1], which are independent gaussian. We can obtain these variables from the original ones by employing Rosenblatt transformation [12]. According to Ditlevsen [10], model uncertainty can be accounted for by the following equation,

$$X' = cX'_p + b \tag{2.3}$$

where $c$ is a constant, and $b$ is a gaussian random variable, which is statistically independent of $X'$. The prime indicates reduced random variables.

Ditlevsen, and Ang and Cornell models are compared in Table I.

Clearly, Ditlevsen model is more general. The main difference between these two models is that Ditlevsen model accounts for the statistical correlation between the error in predicting the value of a variable, $\epsilon = X' - X'_p$, and the value of the variable itself, while Ang's model assumes that these random variables are independent. This is demonstrated in Figure 2, which is for the special case that $X_p$ and $X$ are lognormal. The value of $X'$, which is equal to $lnX$, is plotted there as a function of $X_p$. The average of $X'$ or $lnX$, as well as regions corresponding to this average $\pm$ one standard deviation, are plotted in Figure 2. It is observed that the error between actual values and predictions for $lnX$, which is represented by the width of the shaded region, is independent of $lnX_p$ for the case of Ang's model.

Notes:

a) $\epsilon$ denotes the error between prediction and measurements, i.e. $\epsilon = X' - X'_p$ (reduced space), or $\epsilon = X - X_p$ (physical space).

b) $\rho_{\epsilon X_p}$ denotes the correlation between $\epsilon$ and $X_p$.

c) For the special case that $X_p$ and $X$ are lognormal, the Ditlevsen's model reduces to Ang's model, for $c = 1$.

Although Ang's model is not as general as the Ditlevsen model, it is preferable, because it is simpler. It requires less information in order to determine the statistics of its parameters, and it is very convenient to use for the case that the variables involved are lognormal. Moreover, it is expected that random variable $B$ is lognormally distributed, for most cases, because it is usually the product of several random variables. (Central Limit Theorem).

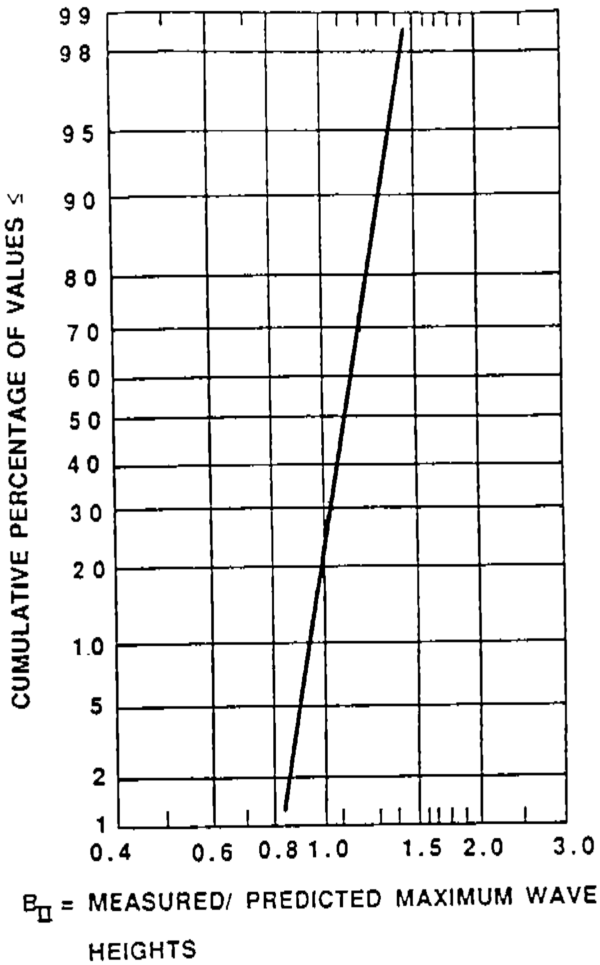
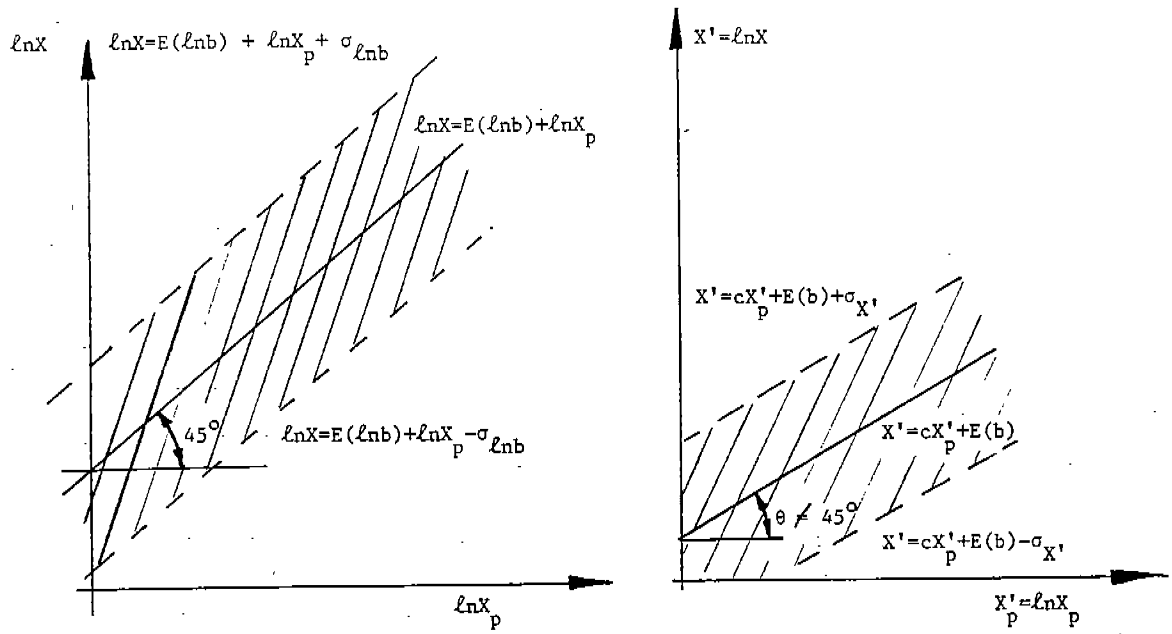



Fig. 1 Probability Distribution of Bias of Extreme Wave Height

### 2.3 Effect of modeling uncertainties on lifetime extreme loads

In both ships and offshore platforms, it is important to distinguish between natural and modeling uncertainties, and their effect on the maximum lifetime loads and load effects.

In contrast to random uncertainties, modeling uncertainties in extreme loads or load effects do not decrease with the length of the return period increasing. Indeed, these uncertainties are systematic. Consequently, the modeling errors corresponding to two or more load applications are perfectly correlated. Therefore, the modeling error corresponding to the maximum of these loads does not decrease with the number of load applications increasing, as it is the case for independent or weakly correlated errors. Therefore, uncertainties in lifetime loads may be grossly underestimated if we treat modeling uncertainties as random.




Ang Ditlevsen

Note: X is Lognormal

Fig. 2 Comparison Between Ang and Ditlevsen Models for Modeling Uncertainties

Table I. Comparison Between The Ang and Cornell and Ditlevsen Models for Modeling Uncertainty

| Characteristic | Ditlevsen | Ang |
|---|---|---|
| Equation: | $X' = cX'_p + b$<br>c: constant<br>b: random variable independent of $X'_p$ | $X = BX_p$<br>B: random variable independent of $X_p$ |
| Space in which model is applicable: | Reduced | Physical |
| Relation between statistics of actual and predicted values: | $E(X') = cE(X'_p) + E(b)$<br>$\sigma^2_{X'} = c^2\sigma^2_{X'_p} + \sigma^2_b$ | $E(X) = E(B)E(X_p)$<br>$COV_X = (COV^2_B + COV^2_{X_p})^{1/2}$ |
| Correlation between error and predicted value | $\rho_{\epsilon X'_p} = \frac{(c-1)\sigma^2_{X'_p}}{[(c-1)\sigma^2_{X'_p}+\sigma^2_b]^{\frac{1}{2}}}$ | $\rho_{\epsilon X_p} = 0$ |

We calculate uncertainties in the extreme value of some quantity $X$ according to the following rule: Let $X_1, ..., X_n$ be $n$ independent samples from a random variable and $X^{(n)}$ be their maximum value, i.e.

$$X^{(n)} = max\ (X_1, ..., X_n) \qquad (2.4)$$

Then, the COV of the maximum $X^{(n)}$ is:

$$COV_{X^{(n)}} = (COV_{II}^2 + COV_{I_{x(n)}}^2)^{\frac{1}{2}} \qquad (2.5)$$

where $COV_{I_{x(n)}}$ is the coefficient of variation of the maximum $X^{(n)}$ which corresponds to natural uncertainties. $COV_{II}$ is the coefficient of variation associated with modeling uncertainties.

Equation (2.5) implies that the two types of uncertainty, natural (random) and modeling, must be treated differently when studying the uncertainty in the extreme value of some load or load effect. Furthermore, the contribution of modeling uncertainties to the uncertainty in the maximum value, $X^{(n)}$, does not decrease as the number of samples, $n$, increases. In this paper, we have estimated uncertainties in extreme loads by employing eq. (2.5) for both ships and offshore structures. For most applications, this equation yields significantly larger uncertainties than those reported in the literature.

Olufsen and Bea [13], and Bea [11] have concluded in their work that uncertainties in maximum design loads and load effects have been seriously underestimated in the recently released API - PRAC 22 design code for offshore platforms. It is remarkable that the coefficients of variation of extreme global loads, which were derived from their studies, are almost 100% larger than those used by the developers of the API code. In our opinion, this should be attributed to the way in which uncertainties were treated in developing this code.

## 3.0 LOADS

In this section, we study uncertainties in loads and load effects. For ships, we examine loads applied to the main girder as well as hydrodynamic pressure. Uncertainties in both short and long term predictions are quantified. For offshore platforms, we quantify uncertainties in base shear and overturning moment.

### 3.1 Stillwater bending moments and shear forces on ships

Guedes Soares and Moan [14] analyzed stillwater bending moments and shear forces for various ship types. In this study, stillwater load effects were assumed to vary from voyage to voyage for a particular ship, from one ship to another in a particular class of ships, and also from one class of ships to another. The above sources of variability can be modeled as follows,

$$m_{ijk} = \quad m_o + m_k + m_j + \epsilon_i \qquad (3.1)$$

where,

$m_{ijk}$ is the bending moment or shear force, at the $i^{th}$ voyage, which is applied to the $j^{th}$ ship, which belongs to the $k^{th}$ class,

$m_o$ is the average load effect for all ships,

$m_o + m_k$ is the average load effect of all ships in the $k^{th}$ class,

$m_o + m_k + m_j$ is the average load effect for the $j^{th}$ ship of the $k^{th}$ class,

and $\epsilon_i$ represents the variation of the load effect from voyage to voyage. Accordingly, the following variances can be defined,

a) variance of the load effect for a particular ship: $\sigma_\epsilon^2$,

b) variance of the load effect for all ships in a particular class, which is specified by $k$: $(\sigma_\epsilon^2 + \sigma_j^2)^{1/2}$,

c) variance of the load effect for all ships: $(\sigma_\epsilon^2 + \sigma_j^2 + \sigma_k^2)^{1/2}$.

The generality of description increases from a) to c) by accounting for all ships in a class, or by accounting for all ships in all classes. Clearly, the variance increases with the generality of description increasing.

Tables II summarizes the results from statistical analysis of data on stillwater bending moments for seven types of ships. The values in this table have been normalized by the corresponding values which are prescribed by classification societies. The average stillwater load effect, and the variance of this load effect for one ship, and also for all ships in a given class, are presented in Table II. The results are based on the analysis performed by Guedes Soares and Moan [14], and all the numbers are normalized by the design values prescribed by classification societies. The data used in this analysis can be found in Guedes Soares and Moan [14] and in Guedes Soares [8].

Table II Variability in stillwater bending moments

| Type of ship | $(m_o + m_k)$ | $(\sigma_\epsilon)$ | $(\sigma_\epsilon^2 + \sigma_j^2)^{1/2}$ |
|---|---|---|---|
| Cargo | 0.50 | 0.28 | 0.30 |
| Containership | 0.72 | 0.16 | 0.20 |
| Bulk Carrier | -0.008 | 0.30 | 0.38 |
| OBO | 0.80 | 0.30 | 0.41 |
| Chemical Carrier | -0.005 | 0.22 | 0.36 |
| Ore/Oil Carrier | -0.44 | 0.22 | 0.37 |
| Tanker | -0.12 | 0.21 | 0.44 |

Notes:

a) Positive bending moments correspond to hogging, and negative ones to sagging.

b) The bending moments have been normalized by dividing by the values which are prescribed by classification societies.

It is observed that cargo and containerships experience large hogging moments. Tankers and Ore/Oil Carriers are subjected to sagging moments. Although the average stillwater bending moment is small for tankers, there is a large variability in this moment. This is attributed to the large variability of the stillwater bending moment from one tanker to another.

Kaplan [7] reported some results on stillwater bending moments obtained from Akita [15]. According to his study, the COV for containerships is 0.29, and for tankers it is 0.99 for ballast, and 0.52 for full load conditions. These values reflect variabilities from voyage to voyage and from one ship to another within a particular class. They indicate the same trend with Guedes Soares results. Indeed, the variability is considerably larger for tankers than for containerships. However, Guedes Soares reported a significantly larger variability for tankers (COV $\simeq$ 3.7) compared to that reported by Kaplan. This discrepancy might be due to the large spreading of sizes of the tankers which were considered by Guedes Soares. The COV reported for containerships are almost identical.

### 3.2 Uncertainties in short term vertical wave bending due to errors in response amplitude operators

Kaplan [7] compared model data against theoretical predictions of response amplitude operators for two Series 60 ships (0.70 and 0.80 block coefficients), and also for the WOLVERINE STATE. The data, which can be found in Kaplan and Raff [16], cover different speeds and headings in regular waves. Kaplan calculated the rms of the wave bending moment by using, a) theoretically calculated response amplitude operators obtained from the SCORES seakeeping computer code [17], and b) measured response amplitude operators. A reference wave spectrum was used, for which the value of the power spectral density function was constant with frequency. The bias due to errors in response amplitude operators was calculated by comparing the rms values of the wave bending moment, which were calculated by using experimental and theoretical response amplitude operators.

Based on the above approach, Kaplan found that the COV of the rms wave bending moment is 0.10. No information was provided on the probability distribution of the bias or its average value.

Guedes Soares [8], separated uncertainties in response amplitude operators into those due to nonlinearities and those due to all the other simplifications and idealizations. According to his approach the bias in the response amplitude operator is given by the following equation;

$$H(\omega) = B_L \; B_{S/H} \; H_P \; (\omega) \qquad \text{for any } \omega \qquad (3.2)$$

where, $B_L$ is the bias due to all uncertainties except nonlinearities, $B_S$ expresses the uncertainty in sagging, $B_H$ expresses the uncertainty in hogging, $H$ is the actual response amplitude operator, and $H_p$ is the value of the response amplitude operator as it is predicted by a linear strip theory based method. Errors due to the flexibility of the ship hull were found to be unimportant except for very long ($L \geq 350m$), fast ships. Therefore, this source of uncertainty was neglected. Guedes Soares examined the error associated with the Salvensen, Tuck and Faltinsen [18] (S.T.F.) method. Linear models were postulated for both $B_L$ and $B_{S/H}$, and the coefficients were found by regressing on data from model experiments. $B_L$ was assumed to be a function of the relative heading angle $\alpha$, the Froude number $V$, and the block coefficient $C_B$. The following relations were found for the bias, on the basis of regression fits.

$$\begin{aligned} &B_L = 0.00631\alpha + 1.22V + 0.657C_B + 0.064 \\ &\text{for } 0 \leq \alpha \leq 90°, \text{ and} \\ &B_L = -0.00495\alpha + 0.42V + 0.701C_B + 1.28 \\ &\text{for } 90° \leq \alpha \leq 180°. \end{aligned} \qquad (3.3)$$

The COV was found equal to 0.38 for both cases.

An alternative simplified approach was also followed, in which the linear bias, $B_L$, was assumed to be a function of the significant wave height, $H_S$, only. For this case, the bias was found to be,

$$B_L = 1.22 - 0.005H_S. \qquad (3.4)$$

The COV was found equal to 0.35. The bias in eq. (3.4) is defined as the ratio of the average values of the measured and predicted response amplitude operators over all heading angles and average wave periods.

The effect of nonlinearities was modeled by employing a linear model which involved the block coefficient $C_B$ as a parameter. The resulting equations, which were also derived by regression, are,

$$B_S = 1.74 - 0.93C_B \qquad \text{for sagging, and}$$

$$B_H = 0.26 + 0.93C_B \qquad \text{for hogging.} \qquad (3.5)$$

The COV was found equal to 0.12 for both equations.

The following conclusions can be extracted from [8] (eq. 3.3 - 3.5):

- S.T.F. method is unconservative, when it is used to predict sagging bending moments.
- The error of the S.T.F. method is larger for beam seas than it is for head and following seas. For example, the bias, $B_L$, for $\alpha = 90°$, $V = 0.2$, and $C_B = 0.8$, is 1.48, while it is only 1.03 for $\alpha = 180°$ and same $V$ and $C_B$.
- S.T.F. method underestimates sagging and overestimates hogging because the linear model does not distinguish between them.
- The error of the S.T.F. method due to nonlinearities is smaller for ships with large block coefficients. This is true because the assumption of vertical hull walls is realistic for ships with large $C_B$.
- The bias, $B_L$, decreases with $H_S$ increasing.

Although a large portion of the experimental data used by Kaplan and Guedes Soares are identical, a significant discrepancy is observed between their COV's. In our opinion, the above discrepancy should be attributed to the way by which uncertainties were quantified by Guedes Soares [8]. More specifically, Guedes Soares regressed on data for the ratio of measured and predicted response amplitude operators for various frequencies. This approach overestimates modeling error, because it uses data from test measurements which are contaiminated with experimental errors as well as concentrating on individual frequencies. In our opinion, a better way to proceed is the following,

a) postulate a linear model for the rms bending moment,

b) transform the data on transfer function into data on the rms bending moment by using some sea spectrum (for example, the ISSC spectrum) and by integrating over frequency,

c) regress on the data from b), or simply estimate the COV of the ratio of measured over predicted rms bending moments.

This procedure, which has been followed by Kaplan [7], allows to average out the experimental error as well as the individual frequency sensitivity by integrating over the frequency in step b). Therefore, the results obtained from this approach should be more realistic.

### 3.3 Long term induced bending moments

Kaplan [7] found that the COV of the extreme lifetime vertical bending moment is 0.19. The COV of random uncertainties was found 0.065. No information was provided on the probability distribution of the average value of the bias. The relative contribution of the uncertainties examined by Kaplan is presented in Table III.

Guedes Soares [8] estimated uncertainties in the most probable extreme long term vertical bending moment for different cases in which different amounts of information on the type of ship or bending moment is provided. The following cases were studied,

a) tankers ($C_B = 0.8$) in hogging,

b) tankers in sagging,

c) containerships ($C_B = 0.6$) in hogging,

d) containerships ($C_B = 0.6$) in sagging,

e) hogging in any type of ship,

f) sagging in any type os ship,

g) any type of ship and bending moment (hogging or sagging) is unknown.

The results from his study are shown in Table IV.

Clearly, the variability in load effects is smaller for cases that the type of ship and/or the type of moment are specified in the formulation. For example, the modeling bias for a ship with block coefficient of 0.8 is 1.13 and the COV is only 0.04. If the block coefficient is not specified, the bias is 1.10 and the COV is 0.15. This indicates that a design code, which distinguishes between various ship types and hull characteristics and specifies different load and strength factors for each case, allows to design more efficient ships.

Another conclusion from Guedes Soares study is that theoretical predictions are almost always unconservative. This is primarily due to the unconservative errors of linear strip theory in response amplitude operators. In particular, the error in the sagging bending moments is very large for ships with small block coefficients. For example the bias is 1.28 for containerships. The reason is that nonlinearity in response is significant for these ships, due to their nonvertical sides. This unconservative error must be accounted for in design because sagging can cause buckling of the deck plates, which is an important failure mode in ship hulls.

Table III Relative Contribution of Various Types of Uncertainties to Total Uncertainties in Extreme Bending Moment (source: Kaplan [7])

| Type of Uncertainty | Contribution (%) |
|---|---|
| Spectral shape variability | 61 |
| Uncertainty in transfer function | 27 |
| Random uncertainty | 12 |

Note: Contribution is defined as the square of the ratio of the particular uncertainty over the total uncertainty.

Table IV Uncertainties in Long Term Vertical Wave Bending Moment (source: Guedes Soars [8])

| Case | Modeling Uncertainty Exp. Bias/COV | Random Uncertainty COV | Total Uncertainty Exp. Bias/COV |
|---|---|---|---|
| Tankers in hogging | 1.13/0.04 | 0.07 | 1.13/0.08 |
| Tankers in sagging | 1.13/0.04 | 0.07 | 1.13/0.08 |
| Containerships in hogging | 0.88/0.05 | 0.07 | 0.88/0.09 |
| Containerships in sagging | 1.28/0.04 | 0.07 | 1.28/0.08 |
| Any ship in hogging | 1.0/0.15 | 0.07 | 1.0/0.17 |
| Any ship in sagging | 1.2/0.08 | 0.07 | 1.2/0.11 |
| Any ship/hogging or sagging | 1.1/0.15 | 0.07 | 1.1/0.17 |

Although Guedes Soares and Kaplan's results on long term bending moments are in good agreement, we believe that modeling uncertainties were not treated properly by the former. More specifically, Guedes Soares assumed that modeling errors in mean square bending moments are independent from one sea state to another or from one heading to another (eq. 4.57, p. 278 of [8]). This assumption is not realistic, because, as we mentioned in section 2, modeling uncertainties are systematic and as such, they are highly correlated from one sea state to another or from one heading angle to another. Therefore, the COV's reported in Guedes Soares might be lower than the actual values [8].

Faulkner [6], reported the following COV's for lifetime extreme vertical bending moments:

a) Modeling uncertainties:

0.15 for warships

0.10 for commercial ships

b) Random uncertainties:

0.12 for both warships and commercial ships.

Faulkner considered a SL-7 containership and a large tanker in his study. He found that the uncertainty is larger for the containership than for the tanker, which agrees with Guedes Soares' conclusions.

Finally, uncertainties in both vertical and horizontal bending moments were considered in [2], for a tanker with length equal to 160m. The bias of both bending moments was assumed to be normal with a mean of 0.95 and a COV of 0.1 for the vertical bending moment. The bias and COV for the horizontal bending moment are 0.85 and 0.15, respectively. The correlation coefficient between the two bending moments was assumed to be 0.70. Unfortunately, no information was provided in [2] on how these numbers were derived. Moreover, as it is mentioned in this report, these numbers are simply crude approximations.

The results from the studies considered in this section are summarized in Table V.

### 3.4 Uncertainties in hydrodynamic pressure

Chen et al [19], compared theoretically predicted hydrodynamic pressures on a ship hull against model tests results and full scale measurements. A linear strip theory based computer code (ABS/SHIPMOTION) was used to calculate pressures. Measurements were obtained for an SL-7 containership and a Great Lakes bulkcarrier. The total hydrodynamic pressure, the pressure component due to the incident and diffracted waves, and the pressure component arising from ship motions were considered in this study. Model tests were performed for head seas at Froude numbers 0.15, 0.23 and 0.32 over a range of ship length/wave length ratios from 0.65 to 1.65.

The following are the main conclusions from Chen's study.

- The calculated pressures due to ship motions correlated well with test measurements.
- Good agreement was also found between predictions and measurements for the pressure due to incident and diffracted waves.
- The agreement between predictions and measurements for the total hydrodynamic pressure was good except for the bow and stern regions. This should have been expected because three dimensional effects and nonlinearities are stronger in these regions.

Table V Summary of Results On Uncertainties in Long Term Extreme Bending Moments.

| Quantity | V. Bend. Mom. | H. Bend. Mom. |
|---|---|---|
| Source | $(X_1)$ | $(X_2)$ |
| Kaplan [7] | -/0.19 | - |
| Guedes Soares [8] | 0.88-1.28/ 0.08-0.17 | - |
| ISSC [2] | 0.95/0.1 | 0.85/0.15 |
| Faulkner [6] | -/0.19 warships -/0.16 container-ships | - |

Note: In ISSC study $X_1, X_2$ are normally distributed with correlation coefficient 0.7.

Table VI Bounds for Bias of Response Amplitude Operator for Hydrodynamic Pressure on SL-7 Containership (Source: Chen et al. [19])

| Froude number | Lower Bound | Upper Bound |
|---|---|---|
| 0.15 | 0.44 | 1.35 |
| 0.23 | 0.41 | 1.65 |
| 0.32 | 0.35 | 1.60 |

Using Chen's results, we found upper and lower bounds for the bias in the response amplitude operator for hydrodynamic pressure. (The response amplitude operator is the square root of the ratio of the spectral ordinates of hydrodynamic pressure and wave elevation at the same frequency.) The results are shown in Table VI and they are for the SL-7 containeship.

Clearly, the error in predicting hydrodynamic pressures is significantly larger than that in predicting global loads (bending moments and shear forces). This is true because global forces are obtained by integrating pressures over the hull. A large portion of the error is averaged out when integrating. Thus, the error in global forces is smaller than that in pressures.

### 3.5 Offshore platforms

In this section, we study uncertainties in loads on offshore platforms and their extreme values. The study focuses on global loads, i.e. base shear forces and overturning moments.

Uncertainties in analysis of fixed offshore platforms were studied in the context of the L.R.F.D. A.P.I. code (Moses, [20]). The maximum annual wave height was assumed lognormally distributed. Its COV ranges between 10 - 15% for the North Sea, 15% - 25% for the Gulf of Mexico and it is somewhat higher for offshore Alaska and California. This information was extracted from measurements reported by various authors and it is summarized on p. 2.23 of that report. It was shown that the effect of the length of the exposure time on the lifetime maximum load or load effect is to reduce its COV and to increase bias. However, no information was provided on natural and subjective uncertainties. Moreover, these two types of uncertainty were not distinguished when the lifetime distribution of the maximum wave height was derived from that of the annual one.

The lifetime maximum platform forces were assumed to be related to the maximum wave height according to the following relation,

$$F_N = AH_N^{\alpha} \tag{3.6}$$

where $A$ is called analysis coefficient, and the exponent $\alpha$ is 1 and 2 for inertia and drag dominated platforms respectively. For $N = 20$ years, the bias and the COV of the analysis coefficient were assumed to be 0.93 and 0.25 respectively. These results were based on measurements, which were obtained from the Ocean Test Structure (O.T.S.) (Anderson et al [21]). The latter is a drag dominated platform.

Olufsen and Bea [13], investigated the uncertainties in extreme shear force and overturning moment for two platforms located in the Gulf of Mexico and in the North Sea respectively. Uncertainties were categorized into random (type I), and modeling ones (type II).

An empirical model, which as obtained by regression, was used to derive global forces from the wave elevation. The following uncertainties were taken into account,

- errors in the procedure for deriving the force from the extreme wave height,
- error in predicting the extreme wave height, and
- uncertainties due to the effect of marine fouling.

It was stressed that the coefficient of variation in the extreme global forces is severely underestimated if modeling uncertainties are not treated properly. More specifically, modeling uncertainties, which are involved in the calculating of loads and their effects, are almost perfectly correlated from one load application to another. Therefore, in contrast to the random uncertainties, they do not decrease with the length of the return period increasing. Hence, if we do not recognize the difference between the ways that the above two types of uncertainties propagate, we will underestimate the coefficients of variation of the extreme lifetime loads. It is striking that the new L.R.F.D. A.P.I. design code is based on a value of 0.37 for the coefficient of variation of the 20 year extreme response of a platform, which is less than one half of the corresponding value which was reported by Olufsen and Bea [13] (0.73 - 0.98 for the Gulf of Mexico, and 0.65 for the North Sea).

Bea [11], also studied uncertainties for a platform located in the Northwest Shelf of Western Australia. The global force to the platform $F$, was calculated by the following formula,

$$F = k_d \, k_u \, H^{\alpha} \tag{3.7}$$

where $k_d$ is the coefficient in the relation between the kinematics of the water particles and $F$, and $k_u$ denotes the coefficient in the relation between the former and the wave height, $H$. Thus, the product $k_u k_d$ corresponds to the analysis coefficient A in eq. (3.6). Exponent $\alpha$ is 1 and 2 for inertia and drag dominated platforms respectively. Only uncertainties in annual maximum values were reported. These values are presented in Table VII. The bias and the coefficient of variation,which were obtained by combining the uncertainties in the quantities involved in calculating global forces (eq. (3.7)), were found to be in good agreement with the corresponding values estimated by comparing measurements against theoretical predictions.

The principal component of the uncertainty in $k_d$, as reported by Bea [11], is uncertainty in the drag coefficient $C_d$ in Morison's formula. Based on OTS data, Bea reported that the coefficients of variation of $k_d$ and $C_d$, which correspond to random uncertainties, are 0.10, and those due to modeling uncertainties are 0.23. These values incorporate the effect of marine fouling.

We derived the uncertainties in the lifetime maximum global forces from those of the annual maximum loads by using two approaches, in order to demonstrate how important it is to treat modeling uncertainties properly. In the first approach, we assumed that modeling uncertainties are perfectly correlated from one year to another and we used equations (2.2) to calculate the total uncertainty. In the second approach, we assumed that modeling uncertainties are independent from one year to another. The details of the calculation of the coefficient of variation are described in the Appendix. Return periods from 10 to 100 years were considered. We assumed that random variable $B_I$, which represents random uncertainties, follows the lognormal distribution. Thus, for long return periods, the maximum value of $B_I$ follows the asymptotic, type $I$, probability distribution.

The results are shown in Table VIII. It is observed that the second approach yields significantly lower estimates for the coefficient of variation than the first approach. This is because, in this approach, modeling uncertainties are assumed to be independent from one year to another. Therefore, the coefficient of variation of the latter decreases with $N$ increasing. On the other hand, the component of the total uncertainties due to modeling error does not change with $N$ in the first approach. It is observed that the coefficient of variation of the maximum force over a 20 year period is 0.66, which is significantly higher than the value which was used by the A.P.I. rules. According to the foregoing discussion, this discrepancy is due to the difference between the ways

Table VII Uncertainties in Annual Maximum Loads for Drag and Inertia Dominated Platforms

| Platform Type | Quantity | | Random (type I) | | Modeling (type II) | |
|---|---|---|---|---|---|---|
| | | | $EB_I$ | $COV_{B_I}$ | $EB_{II}$ | $COV_{B_{II}}$ |
| Drag Dominated | Wave Height | (H) | 1.0 | 0.30 | 1.1 | 0.13 |
| | Kinematics | ($k_u$) | 1.0 | 0.10 | 0.41 | 0.47 |
| | Force Coef. | ($k_d$) | 1.0 | 0.10 | 1.67 | 0.23 |
| | Global Force | | 1.0 | 0.62 | 0.83 | 0.58 |
| Inertia Dominated | Wave Height | (H) | 1.0 | 0.30 | 1.1 | 0.13 |
| | Kinematics | ($k_u$) | 1.0 | 0.10 | 0.93 | 0.20 |
| | Force Coef. | ($k_d$) | 1.0 | 0.10 | 0.65 | 0.3 |
| | Global Force | | 1.0 | 0.33 | 0.66 | 0.38 |

that modeling uncertainties are treated in [11] and in [20].

It should be noted that the actual numerical values for extreme load might be different than the values reported in Table VIII, because it is difficult to distinguish between random and modeling uncertainties, and to estimate the coefficients of variation for random variables $B_I$ and $B_{II}$. However, the trends observed in this table should be correct, and approach 1 is more appropriate than 2 for calculating uncertainties in extreme loads.

Table VIII Total Coefficient of Variation of Global Force as a Function of Return Period

| Years | Approach 1 | | Approach 2 |
|---|---|---|---|
| | $COV_{B_I}$ | $COV_B$ | $COV_B$ |
| 1 | 0.62 | 0.85 | 0.85 |
| 10 | 0.35 | 0.68 | 0.43 |
| 20 | 0.31 | 0.66 | 0.38 |
| 50 | 0.27 | 0.64 | 0.34 |
| 100 | 0.25 | 0.63 | 0.31 |

Wirsching [22], also studied uncertainties in loads applied to offshore platforms. He represented the uncertainties in loads by the product of two coefficients denoted by $B_S$ and $B_F$. $B_S$ corresponds to environmental uncertainties and $B_F$ accounts for the error in load calculation. The statistics of $B_S$ were presented in section 3. $B_F$ was assumed to be lognormally distributed with an average value ranging from 0.6 to 1.1 and a COV between 0.1 and 0.3.

Guedes Soares and Moan [5] considered the uncertainties in the extreme forces applied to a vertical pile in the North Sea. The extreme forces correspond to a return period of 100 years. Table IX presents the random variables which were considered in this study and their means and COV's.

The COV of the extreme load was found to be in the range between 0.34 and 0.45. The uncertainty in the extreme wave height was found to be the most important, because its effect on the global load was considerably larger than the effects of all the other uncertainties. This conclusion agrees with the conclusions from Wirsching and Bea. Therefore, the uncertainty in environmental description (the extreme wave height) is the most important for offshore platforms.

The results from the studies considered in this section are summarized in Table X.

Table IX Uncertainties Involved in Predicting Extreme Loads on a Vertical Pile in the North Sea (Source: Guedes Soares and Moan [5])

| Quantity | Mean value | COV |
|---|---|---|
| Extreme wave height (H) | 30m | 0.16 |
| Wave period ($T$) | 5.4 + 0.373 H | 0.14 |
| Water depth ($D$) | 80m | 2/D |
| Current velocity ($C$) | 1.25 m/sec | 0.35 |
| Pile diameter ($D'$) | 4.0m | 0.0 |
| Fouling thickness ($K$) | 0.175m | 0.45 |
| Surface roughness ($R$) | 0.02 | 0.4 |
| Drag coefficient ($C_d$) | Sarpkaya's data | 0.1 |
| Inertia coefficient ($C_M$) | | 0.1 |
| Wave Kinematics | Stokes theory | 0.25 |

Note: The following correlation coefficients were assumed for the above random variables:

$\rho\,(H,T) = 0.5$,
$\rho\,(H,C) = 0.4$,
$\rho\,(K,R) = 0.7$,
$\rho\,(R,C_D) = 0.5$,
$\rho\,(R,C_M) = -0.5$,
$\rho\,(C_D,C_M) = -0.9$.

## 4.0 FATIGUE

Fatigue is an important consideration in structural design. For many structural systems such as for offshore structures, fatigue is the most critical failure mode, and thus safety requirements associated with fatigue reliability dictate design decisions. Fatigue strength can be described by a characteristic S-N curve or by a fracture mechanics model.

A cumulative damage based approach for fatigue analysis consists of the following steps:

a) modeling the loading environment,

b) modeling loads,

Table X Uncertainties in Extreme Global Loads on Offshore Platforms

| Source | Bias/COV | Return Period (years) |
|---|---|---|
| Moses [20] | 0.7/0.37 | 20 |
| Table VIII | –/0.66[1] | 20 |
| | –/0.63[1] | 100 |
| Wirsching [22] | 0.4 – 1.3/ 0.41 – 0.67[1,4] | |
| Guedes Soares and Moan [5] | –/0.34 – 0.45 | 100 |
| Olufsen and Bea [13] | –/0.73 – 0.93[1,2] | 100 |
| | –/0.65[1,3] | 100 |

Notes: [1] Bias is lognormally distributed

[2] Gulf of Mexico

[3] North Sea

[4] Modeling uncertainties

c) evaluation of field stresses in the structure,

d) evaluation of stresses at all points of possible crack initiation (stress concentrations), and

e) evaluation of cumulative fatigue damage over the lifetime of the structure.

In this section, we quantify errors in calculating stress concentration factors. We also combine the errors involved in all the steps of fatigue analysis and quantify uncertainties in fatigue damage, for the case that a cumulative damage approach is used. Finally, we investigate the relative importance of the uncertainties in each of the steps a) to e), and also of the random and modeling uncertainties.

### 4.1 Uncertainties in stress concentration factor.

Wirsching [22] reported estimates of the uncertainties in stress concentration factors for tubular joints of offshore structures. These uncertainties are for stress concentration factors which are obtained from parametric equations, such as those by Kuang, Potvin and Leick [23]. According to Wirsching, the average bias in the stress concentration factor is in the range from 0.80 to 1.20, and the COV ranges between 0.1 and 0.50. The bounds for the bias and the COV in stress concentration factor are very wide, possibly because the parametric equations cover a large number of geometries and loading conditions.

Uncertainties in the stress concentration factor are large for other engineering structures. For example, the stress concentration factor for the fatigue analysis of a liquid propellant engine was assumed to follow the beta distribution. The stress concentration factor is in the range from 1.2 to 3.5, and that its COV is roughly 0.15.

### 4.2 Uncertainties in cumulative fatigue damage.

Studies on fatigue reliability of marine structures assume that the effect of random uncertainties is negligible. Thus, these studies account only for modeling uncertainties in stress evaluation procedures [26]. Although the effect of random uncertainties reduces with the number of load cycles increasing, to the best of our knowledge no study has proven that the effect of random uncertainties is negligible.

The objectives of the exercise presented in this section are to address the above issue, estimate uncertainties in the cumulative fatigue damage over the lifetime of platforms and ships, and study the relative importance of each uncertainty.

The following are the basic assumptions:

a) Fatigue life can be estimated by using the S-N curves. The slope of these curves is constant for any number of cycles, N.

b) Miner's rule can be used to estimate fatigue damage.

c) The stress amplitude distribution is known.

d) The mean and standard deviation of the cumulative damage, D, can be estimated by linearizing the expression relating D with all random variables around the mean values of these variables.

This is a crude approximation because the derivatives of the damage with respect to the values of the random variables are not constant. Advanced methods for fast probability integration are more accurate in this case [1]. However, the objective of this study is to identify the most important uncertainties and to obtain only rough estimates of the COV of D. Moreover, the estimates for the bias and the COV of the random variables involved in damage calculations are very crude. Thus, for this case, the benefits from using an advanced fast probability integration method are minimal. Due to the above reasons, we adopted assumption d.

Under the above assumptions, fatigue damage can be calculated by the following equation [26]:

$$D = \frac{B_{II}^{m} \Sigma S_i^m}{A} \tag{4.1}$$

where,

$B_{II}$ represents the modeling error in the stress at points of stress concentration,

$m$, is the exponent in the S-N curves,

$S_i$, is the predicted stress amplitude at the $ith$ load application, and

$A$, is the constant at the right hand side of the S-N equations.

The summation is for all load applications.

The modeling bias $B_{II}$ is given by the following equation:

$$B_{II} = B_M \cdot B_S \cdot B_F \cdot B_N \cdot B_H, \qquad (4.2)$$

where,

$B_M$ represents uncertainties in the geometry due to manufacturing imperfections,

$B_S$ represents uncertainties in seastate description,

$B_F$ represents uncertainties in wave load predictions,

$B_N$ is the bias for errors in structural analysis, and

$B_H$ is the bias for uncertainties in stress concentration factors.

By using a first order Taylor series expansion of the expression for $D$ about the mean values of all random variables, we obtain the mean value of $D$,

$$E(D) = \frac{E^m(B_{II})\, \Sigma\, E^m(S_i)}{E(A)} \qquad (4.3)$$

Assuming that the statistics of the predicted stress are the same for all load cycles, we have,

$$E(D) = \frac{E^m(B_{II})\, N \cdot E^m(S_i)}{E(A)} \qquad (4.4)$$

where $N$ is the number of cycles over the lifetime of the ship.

The coefficient of variation of fatigue damage $D$ in (4.1) is,

$$COV_D = (m^2 COV_{B_{II}}^2 + COV_A^2 + COV_{\Sigma S^m}^2)^{1/2}, \qquad (4.5)$$

where

$V_{B_{II}}$ is the COV of modeling bias,

$V_A$ is the COV of $A$, and

$V_{\Sigma S^m}$ is the COV of the sum $\Sigma S_i^m$.

Note that subscript $i$ has been dropped in equation (4.5).

The first term in the expression with the square root, on the right side of (4.5), represents the effect of modeling uncertainties. The second term is associated with uncertainties in S-N curves and the third represents the effect of random uncertainties.

As mentioned earlier, equations (4.4) and (4.5) are approximate. The reasons for using them have been mentioned earlier in this section.

### 4.3 Relative importance of random uncertainties.

Here, we compare the effect of random uncertainties on the fatigue damage against that of modeling uncertainties. We also investigate the effect of the correlation between the maxima of the stress process.

We considered two cases. In the first case, the maxima of the stress, $S_i$, follow the Rayleigh distribution, while in the second they follow the Weibull distribution. We assume that the correlation coefficient between the $ith$ and the $kth$ stress maxima, $S_i$ and $S_k$, is,

$\rho_{S_i S_k} = \rho_{S_i S_{i+1}}^{|i-k|}$, where $\rho_{S_i S_{i+1}}$ is the correlation coefficient between two subsequent peaks. In our study we considered different values for $\rho_{S_i S_{i+1}}$ in the range from 0. to 0.99. In the following discussion, subscripts will be dropped. After some algebra, the following equation was derived for the COV of random uncertainties:

$$COV_{\Sigma s^m} = \frac{COV_S m (\frac{1+\rho}{1-\rho} - \frac{2\rho}{N(1-\rho)^2})^{1/2}}{N^{1/2}} \qquad (4.6)$$

where $COV_S$ is the COV of a local maximum.

The COV increases with the correlation coefficient between subsequent maxima increasing. It is observed from (4.6), that the COV for random uncertainties decreases, with the number of load cycles, $N$, increasing. Moreover, it is almost zero for large values of $N$ (say $10^7$), for any value of $\rho$ less than one.

The COV for random uncertainties is presented in Table XI, for three cases. In the first case, the stress amplitude follows the Rayleigh distribution, while in the latter two cases, it follows the Weibull distribution [27],

$$F_X(x) = 1 - e^{-(\lambda x)^c} \qquad (4.7)$$

Coefficient c equals 0.7 and 1.0 for the last two cases.

It is observed that the effect of random uncertainties is small. Moreover, a similar calculation for $N = 10^8$, which is a typical number of load applications over the lifetime of a marine structure, showed that the COV due to random uncertainties is practically zero. Since the distributions considered for the stress peaks represent real life situations, we conclude that random uncertainties can be neglected in fatigue reliability analysis without losing any accuracy. This is true even for the case for which the adjacent stress maxima are strongly correlated. Hence, the value of $\rho$ is also unimportant provided that the number of load cycles is large (say $10^7$).

Table XI Coefficient of Variation of Cumulative Fatigue Damage Due to Random Uncertainties ($N = 10^6$)

| Distribution of Stress Amplitude | COV of Cumulative Damage | | | | |
|---|---|---|---|---|---|
| | Correlation Coefficient of Subsequent Peaks | | | | |
| | 0.0 | 0.5 | 0.8 | 0.9 | 0.99 |
| Rayleigh | 0.0 | 0.0 | 0.01 | 0.01 | 0.03 |
| Weibull (c = 1.0) | 0.01 | 0.01 | 0.01 | 0.02 | 0.06 |
| Weibull (c = 0.7) | 0.01 | 0.01 | 0.02 | 0.03 | 0.10 |

### 4.4 Uncertainties in cumulative fatigue damage.

The equations of the previous sections allow to quantify the uncertainties in the cumulative fatigue damage. Eq. (4.4) can be used to calculate the average bias while eq. (4.5) is for the COV We used the above equations to calculate the average bias and COV of fatigue damage for typical marine structures. We also studied the relative contribution of various uncertainties to the overall uncertainty in fatigue damage.

The data on various uncertainties, which are involved in fatigue analysis, are presented in Table XII. Exponent $m$ was taken to be 4.38.

Table XII Uncertainties Involved in All Steps of Fatigue Analysis

| Type of Uncertainty | COV | | |
|---|---|---|---|
| | [22,29] | [11] | [7] |
| $B_M$ | 0.2 | 0.2 | 0.2 |
| $B_S$ | 0.5 | 0.58[a] | 0.15 |
| $B_F$ | 0.2 | | 0.19[b] |
| $B_N$ | 0.3 | 0.3 | 0.12 |
| $B_H$ | 0.3 | 0.3 | 0.3 |
| $B_A$ | 1.0 | 1.0 | 1.0 |

Notes:

a) This COV refers to the cumulative effect of environmental and load evaluation uncertainties, ie. to the product of $B_S$ and $B_F$.

b) This COV represents modeling uncertainties in the combined wave and slamming bending moment. The estimate was based on COV's of 0.1 and 0.16 for the uncertainties in wave and slamming bending moments, respectively.

Wirsching's and Bea's data are for offshore platforms while Kaplan's data are for ships. Kaplan [7] and Bea [11] provided estimates for $B_S$ and $B_F$ only. In addition, the value of $COV_{B_N}$ for ships was separately calculated in [28]. The COV's for the other variables were assumed to be equal to the corresponding values provided by Wirsching [22,29].

There is uncertainty in stress calculations due to the fact that the peaks of the wave elevation do not follow the Rayleigh distribution. This uncertainty introduces conservative bias in the stress predictions.

Table XIII presents the overall uncertainty in cumulative damage, as well as, the relative contribution of all types of uncertainties to the cumulative damage. In this table $COV_M, COV_S, COV_F, COV_N$ and $COV_H$ represent the COV's of $B_M, B_S, B_F, B_N$ and $B_H$, respectively.

The following conclusions can be extracted from Table 13.

a) Uncertainty in cumulative damage is very large for both ships and offshore platforms. The reason is that fatigue damage is extremely sensitive to the amplitude of the applied stress. In other words, a small change in the amplitude results to a large change in the fatigue damage and the expected fatigue life.

b) As indicated by our example, uncertainty in fatigue damage is smaller for ships than for offshore structures.

c) Uncertainty in describing the loading environment is the most important for offshore platforms. This means that even a small reduction in this uncertainty will result to a large reduction in the overall uncertainty in fatigue damage.

d) For the case of ships, the uncertainty in the stress concentration factor is the most important. The next important uncertainty is that in A, which is the constant in the right hand side of the expression for the S-N curves.

e) The effect of random uncertainties is neglegible because these uncertainties are averaged out in the procedure for evaluating fatigue damage. Moreover, the statistical correlation between consecutive stress peaks is unimportant in fatigue.

Table XIII Uncertainties in Cumulative Fatigue Damage and Relative Contribution of Each Uncertainty in Table XII

| Source | $COV_D$ | $COV_M$ | $COV_S$ | $COV_F$ | $COV_N$ | $COV_H$ | $COV_{\Sigma S^m}$ | $COV_A$ |
|---|---|---|---|---|---|---|---|---|
| Wirsching [22,29] | 3.29 | 0.07 | 0.45 | 0.07 | 0.16 | 0.16 | 0.0 | 0.09 |
| Bea [11] | 3.42 | 0.07 | 0.55 | | 0.15 | 0.15 | 0.0 | 0.08 |
| Kaplan [7] | 2.21 | 0.16 | 0.09 | 0.14 | 0.06 | 0.35 | 0.0 | 0.20 |

## CONCLUSIONS

The following are the main conclusions:

1. In both ships and offshore platforms, it is important to distinguish between random and modeling uncertainties and between their effects on lifetime extreme loads and load effects. In contrast to the random uncertainties, modeling uncertainties do not decrease with the length of the exposure period increasing. Therefore, if we treat the latter uncertainties as if they were random, we may grossly underestimate uncertainties in extreme loads. In our opinion, approach 1 in Table VII is more appropriate than approach 2.

2. We believe that modeling uncertainties have not been treated correctly (i.e. according to the method described in Section 2) in the new Load and Resistance Factor Design code of the American Petroleum Institute. As a result, the coefficients of variation of load effects have been grossly underestimated.

3. An effective way to quantify uncertainties is to classify different types of ships with different characteristics and operational schedules, and determine uncertainties for each class, separately. The resulting uncertainties will be considerably lower than those determined by an approach which does not distinguish between different types of ships.

4. Moreover, we may consider the dependence of modeling errors on some parameters, such as the significant wave height, and the relative heading angle. Then, we can employ regression to determine relations between the error and the above parameters. From work dealing with this approach, it was demonstrated that such relations can be used to improve theoretical predictions of loads and load effects and reduce the associated uncertainties by a significant amount. However, this approach requires a sufficiently large database which contains results from analytical procedures and measurements on loads and load effects.

5. There is significant variability in still water load effects in different voyages of a ship. There is also significant variation between similar ships and between different ship types.

6. The coefficient of variation for extreme wave moments in ships is roughly 0.20. The bias (ratio of actual over predicted value) is greater than one, which means that theoretical estimates of wave loads are lower than the actual values. However, the magnitude of this exceedence above one depends upon the type of ship as well as consideration of the particular type of bending (ie. sagging or hogging).

7. It has been reported in the literature that the coefficient of variation in extreme global loads (overturning moments and base shear forces) on offshore platforms ranges between 0.60 to 0.90. While these numbers are large, and their precise magnitude may be questioned, the actual values are expected to exceed those in the new A.P.I. code. The trend indicated by these numbers appears to be proper.

8. Linear seakeeping methods cannot estimate hydrodynamic pressures on the ship hull with acceptable accuracy. This is particularly true in the vicinity of the bow and the stern of the ship hull. These methods are more effective in calculating global loads and wave bending moments.

9. Nonlinear effects are more important for ships with small block coefficients, such as containerships, than for ships with large block coefficients such as tankers and bulk carriers.

10. Random uncertainties are unimportant in fatigue reliability analysis of both ships and offshore platforms. Moreover, the statistical correlation between subsequent wave peaks is also unimportant.

The following conclusions refer to the relative importance of the uncertainties involved in stress analysis. Some of these conclusions are based on work, which was performed in the context of the SSC project but which has not been presented in this paper.

11. It is general consensus that, in offshore structures, the uncertainty in describing the loading environment is the most important. For those fixed offshore platforms, for which dynamic effects are insignificant, the largest part of this uncertainty is due to errors in estimating the long term maximum wave height.

12. Uncertainties in the value of the drag coefficient in Morison equation are also important.

13. Uncertainties in describing the loading environment (wave height and period) are the most important in fatigue analysis of offshore platforms. Errors in stress concentration factor and in structural analysis follow in terms of relative importance. It should be mentioned that errors in structural analysis are primarily due to errors in estimating the natural period of a platform.

14. The examples, which were studied in this paper, indicate that the uncertainty in stress concentration factor is the most important in fatigue analysis of ships.

**Acknowledgements**

The work described in this paper has been supported by Ship Structure Committee (Project No. SR-1326). The authors would like to express their appreciation to Mr. A. Engle, Dr. M. K. Ochi, Cdr. M.K. Parmelee, Mr. W. Siekierka, Mr. R. Sielski, and Dr. P. Wirsching for their comments and suggestions.

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## Appendix : Calculation of Uncertainties in Lifetime Maximum Loads or Load Effects

The bias $B$, of the annual maximum load has a lognormal probability distribution. Therefore,

$$f_B(b) = \frac{1}{\sigma\sqrt{2\pi}b} e^{-\frac{(\ell nb - \lambda)^2}{2\sigma^2}}$$

where,

$$\sigma = (\ell n(1 + COV_B^2))^{1/2},$$

$$\lambda = \ell n(EB) - \frac{\sigma^2}{2},$$

and $EB$ and $COV_B$ are the mean and the coefficient of variation of $B$, respectively. The lognormal distribution belongs to the exponential class of probability distributions because it satisfies von Mises' condition [27]. Therefore, the maximum value of $B$, over an $N$ year period $B^{(N)}$, follows the Type $I$ asymptotic extreme value probability distribution,

$$F_{B^{(N)}}(b) = exp\ (-e^{-a_N(b - \bar{B}_N)})$$

where, $\bar{B}_N$ is the most probable maximum over the $N$ year period, and

$$a_N = N \cdot f_B(\bar{B}_N).$$

The most probable maximum, $\bar{B}_N$, satisfies the following equation,

$$P(B \geq \bar{B}_N) = \frac{1}{N},$$

which is equivalent to,

$$P(\ell nB \geq \ell n\bar{B}_N) = \frac{1}{N}.$$

Therefore, since $\ell nB$ is normally distributed with mean $\lambda$ and standard deviation $\sigma$,

$$\bar{B}_N = exp\ (\Phi^{-1}(\frac{N-1}{N})\sigma + \lambda)$$

where $\Phi(\cdot)$ denotes the probability distribution function of a standard Gaussian random variable. Finally, the coefficient of variation of $B^{(N)}$ is

$$COV_{B^{(N)}} = \frac{\pi/\sqrt{6}}{a_N\bar{B}_N + \gamma}$$

where $\alpha$ is the Euler's constant (0.577).

## Stig Berge

Some questions related to the fatigue modeling:

You conclude that your example calculation indicates that the uncertainty in fatigue damage is smaller for ships than for offshore structures. This is surprising, not least because explicit fatigue design procedures are well established for offshore structures, whereas for ships there is no validated procedure available. How general do you feel your conclusion is?

In the fatigue analysis you appear to take into account environmental loading only. For ships it is known that other sources of loading (induced from machinery, changes in ballast condition) significantly affects fatigue life. Are you able to assess the effect of these loadings on your overall analysis?

Your analysis is based on SN curves essentially derived from small scale tests performed in air. In offshore structures design these curves have to be modified in order to take into account the detrimental effects of sea water, cf. ongoing revisions of the UK DEn guidance notes. For ship details, which often see an even more aggressive environment (intermittent water and air, high corrosion rates, these SN curves may be totally inappropriate as indicated by fatigue tests performed in sea water drip [1]. If an explicit fatigue design procedure for ships were to be formulated, what are the authors' comments on the choice of design SN curves?

Reference:

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## E. Nikolaidis

The first question regards the contention that modeling uncertainty in fatigue analysis is larger for offshore platforms than for ships. This is true because the error involved in calculating loads is significantly larger for offshore structures than for ships. Indeed, the coefficient of variation of global loads applied to offshore structures (base shear force, and overturning bending moment) ranges between 50% to 70% [5, 11, 13, 20, 26, 29]. The corresponding coefficient of variation of longitudinal wave bending moments applied to ship hulls is roughly 20% [7, 30], which is significantly lower than that for the case of offshore platforms.

The following is our response to the question on the effect of other than environmental loads (induced from machinery and from changes in ballast condition). We believe that the loads due engine vibration only affect the fatigue life of components that are located near the engine room, the propeller or the shafting system. Therefore, these loads are of limited interest in this study. Changes in ballast conditions should have a significant effect on the fatigue damage because:

- Static stresses in the ship hull depend on both ballast and loading conditions.
- Static stresses significantly affect the fatigue life of the hull.
- The coefficient of variation of still water bending moment and the resulting static stresses is large [8].

However, we believe that the effect of uncertainties in loading and ballast conditions on the cumulative fatigue damage over the lifetime of the ship is significantly smaller than that of the short term still water bending moment. Indeed, as it is explained in the paper (section 4.3), the effect of random uncertainties on the cumulative damage decreases with the length of the exposure period increasing.

We agree with the comment on the choice of S-N curves; the effect of corrosion is important and it should be taken into account in fatigue analysis. However, we did not take into account the effects mentioned in the previous two paragraphs because our objective was only to compare the significance of random and modeling uncertainties and to identify the most critical components in fatigue analysis rather than to derive exact estimates of the fatigue damage.

Reference:

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