SSC-96

NOTES ON THE INFLUENCE OF UNFAIR PLATING ON SHIP FAILURES BY BRITTLE FRACTURE

by

H. H. Bleich

SHIP STRUCTURE COMMITTEE

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ADDRESS CORRESPONDENCE TO:

SECRETARY
SHIP STRUCTURE COMMITTEE
U. S. COAST GUARD HEADQUARTERS
WASHINGTON 25, D. C.

March 15, 1956

Dear Sir:

In order to determine the influence of unfair plating on brittle fracture in ships, the Committee on Ship Structural Design of the National Academy of Sciences-National Research Council recommended in 1953 the initiation of analytical studies of the problem to be supported by Ship Structure Committee funds. This recommendation was concurred in by the Ship Structure Committee.

Herewith is a copy of the Final Report, SSC-96, of the investigation, entitled "Notes on the Influence of Unfair Plating on Ship Failures by Brittle Fracture" by H. H. Bleich.

Any questions, comments, criticism, or other matters pertaining to the report should be addressed to the Secretary, Ship Structure Committee.

This report is being distributed to those individuals and agencies associated with and interested in the work of the Ship Structure Committee.

Yours sincerely,

K. K. Cowart

K. K. COWART
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure Committee

Serial No. SSC-96

Final Report
of
Project SR-132

to the

SHIP STRUCTURE COMMITTEE

on

NOTES ON THE INFLUENCE OF UNFAIR PLATING ON SHIP FAILURES BY BRITTLE FRACTURE

by

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New York, New York

transmitted through

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Division of Engineering and Industrial Research
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March 15, 1956

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NOTES ON THE INFLUENCE OF UNFAIR PLATING ON SHIP FAILURES BY BRITTLE FRACTURE

I. INTRODUCTION

It was the primary purpose of this study to determine whether or not unfairness in deck or hull plates in transversely framed dry cargo ships may be a substantial contributory cause to the loss of such vessels due to brittle fracture. Two essentially separate questions were considered. The first one concerns the effect of unfair bottom plating in compression, shirking its load and increasing the tensile stresses in the deck. The second one concerns the increase of tensile stresses in the deck due to unfair deck plating. An appreciable increase of tensile stresses due to either cause would, of course, contribute to the danger of brittle fracture.

The claim has been made[4] that the tension failure of the bottom plating of certain transversely framed Norwegian tankers was due to prior buckling of the deck plating. It was claimed that the reduced effectiveness of the buckled deck plating reduced the overall section properties of the hull girder, increasing the tensile stresses in the bottom to the point of brittle fracture. The equivalent possibility of causing brittle fracture in the deck of a transversely framed dry cargo ship (where hogging stresses prevail) caused the present study. If the reduction of the effectiveness of the bottom (compression) plating by unfairness or buckling results in substantially larger tensile stresses in the deck plating, such unfairness or buckling might be the cause of some ship failures by brittle fracture. This question is studied in Section IIa for the typical case of a Liberty ship. As the discussion of the entire matter was started by the cases of the Norwegian tankers, these are shortly discussed in Section IIb. A study was also made to determine whether dynamic effects due to large deformations of the vessel caused by buckling of the bottom plating may aggravate the danger of brittle fracture of the deck plating. This study is presented as Appendix C and summarized in Section IIc.

Quite recently[6] attention has been drawn to the fact that the increase in tensile stresses in the deck due to unfair deck plating may contribute substantially to the danger of brittle fracture. The study of this question is begun in Section III. In order to determine the importance of this effect, it was required to delve at length into the subject of the increase of unfairness during operation of a vessel. This question is considered in Section IV on the basis of a new approach to the problem of unfair plating developed in Ref. 3; it was found necessary to extend this approach, particularly with respect to the effects of residual stresses. As a by-product of this study, Appendix B contains a number of comments on Ref. 3 concerning the explanation of large unfairness observed in the bottom of some transversely framed vessels.

At this point it can already be stated that none of the studies indicate any substantial effect of unfair plating on the brittle fracture problem. These notes should therefore be considered just as a record of the various investigations.

II. EFFECT OF UNFAIRNESS IN COMPRESSION PLATING ON THE DANGER OF BRITTLE FRACTURE

A. Transversely framed dry cargo ships. The effect of any unfairness in compression plating on the danger of brittle fracture of the hull girder depends solely on the increase of the stresses on the tension side due to the unfairness. To decide on the importance of such an effect for the brittle fracture situation, it is only required to find the increase in tensile stresses caused by the reduced effectiveness of unfair plating on the compression side.

The amount by which the reduced effectiveness of compression plating increases the stresses on the tension side of the hull girder can easily be examined for a typical Liberty ship. The following data for the S. S. "Philip Schuyler" (Ref. 1, Fig. 22):

moment of inertia:	$I = 424{,}170 \text{ in.}^2\text{ft.}^2$
neutral axis (from bottom):	$e = 17.16$ ft.
total depth:	$d = 37.33$ ft.

were used to compute the section moduli of the hull girder in the second column of Table I.

If, due to unfairness of the bottom plating, its compression effectiveness were reduced by 50% (amounting to a reduction of plate area by 169 $in.^2$), the section properties would be:

$$I = 371{,}200\ in.^2 ft.^2, \qquad e = 19.0\ ft.,$$

and the corresponding section moduli are listed in the third column of Table I.

TABLE I

Section Moduli $in.^2 ft.$ of Hull Girder

	Original Cross Section	50% Bottom Buckled	Difference %
Top Deck	21,050	20,250	3.7
Bottom	24,800	19,550	21.2

It is seen that the section modulus for the tension side, i.e., for the deck, is reduced by only 3.7%, a very small amount. There is, therefore, no foundation for the belief that buckling of the bottom (compression) plating, or its reduced effectiveness due to unfairness can increase the tensile stresses in the deck substantially, such an increase being the prime cause for brittle fracture. It is not even reasonable to consider such buckling as a major contributory cause; if the tensile stresses in the hull with fully effective bottom plating are already so high that an increase of the order of 5% produces brittle fracture, then the other circumstances which raised the stresses to this level and/or the low stress level at which brittle fracture occurs are the major culprits.

It might be noted that the conclusion that the tensile stresses in the deck will be only slightly affected by the reduced effectiveness of unfair bottom plating is in agreement with the finding by Murray[2] (discussion of Fig. 14).

One may ask if remedial measures to prevent the reduction in effectiveness of the bottom plating are warranted, disregarding all other reasons just to improve the brittle fracture situation to a slight extent. The full effectiveness of the plating could be maintained, e.g., by additional longitudinal stiffeners as suggested by Murray[2]. While such remedial measures would decrease the tensile stresses by 5% or less, it would seem that the same amount of material added to the top deck would reduce the tensile stresses more. From the viewpoint of preventing brittle fracture, it appears therefore not appropriate to recommend such additional stiffeners.

The above conclusions would not be affected by presence of residual stresses or if Horne's explanation of the origin of large unfairness of plating[3] is considered.

b. <u>Transversely framed tankers</u>. It may be of interest to discuss the case of several Norwegian transversely framed tankers whose loss has been stated, (Vedeler[4,5]), to have been caused by buckling of the deck plating although their tension bottom plating fractured. Information concerning a sister ship, available to the writer, shows 20 mm. deck plating, 17.5 mm. bottom plating, and 800 mm. frame spacing at the midship section. The conclusion drawn above for a Liberty ship, that reduced effectiveness of the compression plating increases the stresses on the tension side only slightly, applies also to the hull cross section of the tankers; and the reasoning ascribing the loss of these ships to buckling appears to the writer not conclusive.

The basis of the reasoning[4,5] is the fact that the compression plating of the hull will buckle at a load which produces tensile stresses of only about 60% of yield. Disregarding brittle fracture, the deck will therefore buckle prior to the bottom yielding in tension; the buckling, it is contended, will shift the neutral axis, increasing the tensile stresses and tearing the bottom plating. However, the present study shows that the assumed increase of tensile stresses does not occur and the sudden tension failure of the bottom* remains

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*It seems to the writer that failure of the deck plating in compression would have led first to jack-knifing of the hull without separation, followed by a gradual breaking up.

unexplained unless one assumes brittle fracture at a stress of about 60% of yield. Once one is forced to assume brittle fracture at a stress much less than yield, it follows that collapse would have occurred even without buckling. The sudden failure reported seems therefore to point to brittle fracture* as the cause, although the deck plating may have buckled at the same time.

c. Dynamic effects. It has occurred to the writer that the stress distribution in the hull might be affected appreciably by the dynamic effects if the bottom of a vessel fails in compression such that considerable permanent deformations of the hull occur. If these effects were important, the conclusions of the preceding paragraphs might not be correct. This question is therefore studied in Appendix C where the interesting quantity, the tension in the deck, is determined for the idealized case of a hull of uniform cross section and mass distribution.

Assume that a vessel, Fig. 11, is loaded in such a manner that the maximum static bending moment M produces compressive stresses in the bottom exceeding the capacity of the plating panel at point B. This panel will then buckle, and the ship will begin to jack-knife. The panel at B will not have lost all resistance but will still carry some load P_o. The tensile

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*It is not known to the writer if any parts of these ships were salvaged and examined.

side of the hull remains intact, and the permanent deformation of the vessel consists essentially of a relative rotation of the forward and aft portions with respect to a point marked A in Fig. 11. For analysis purposes it is assumed in the Appendix that the forward and aft parts of the vessel may be treated as rigid bodies. The interesting quantity to be obtained is the horizontal component P of the force exerted by the two parts of the hull on each other at point A; this force P is the total tensile force in the deck of this idealized hull and can be compared with the total tensile force in the hull due to the same moment M if the bottom had not failed by buckling.

It might be noted that, in the static case where the inertia forces vanish, the horizontal components of the resultants of the tensile and compressive forces in the section AB must be alike because the resultant of all loads acting to either side of AB is a pure couple, M. If this moment M exceeds the capacity of the section such that permanent deformations occur, the horizontal components of the tensile and compressive stresses, P and P_o, respectively, are in general not equal because the relative rotation of the portions of the hull with respect to point A results in inertia forces, the resultants of which do have horizontal components.

The analysis shows that the total tensile force P in the deck for the same applied moment M is necessarily smaller when the compression plating buckles and the hull deforms than when the hull remains intact. If the deck plating could carry the tensile stresses due to a moment M without danger of brittle fracture on the assumption that the compression plating will not buckle, then the deck is not in danger of brittle fracture if the compression plating actually does buckle.

It is therefore concluded that dynamic effects induced by failure of compression plating do not increase the danger of brittle fracture in the deck.

III. EFFECT OF PLATING UNFAIRNESS ON THE TENSION SIDE ON THE DANGER OF BRITTLE FRACTURE

Quite recently attention has been drawn by Evans[6] to the fact that the unfairness of plates in tension will result in such plates shirking part of their load which must then be carried by other longitudinal members, thus increasing the tensile stresses in such members. There can be no doubt that such an increase of stresses does exist, but it is not immediately apparent how large the increase is and if the contribution to the danger of brittle fracture is substantial.

Large amounts of unfairness have been reported in the bottom shell of dry cargo ships, not in the deck. However, even moderate unfairness, if present, reduces the plate effectiveness noticeably; and the question is worth investigating.

In order to evaluate the effect of an initial unfairness, s, Fig. 1, in a simply supported panel of thickness t, consider

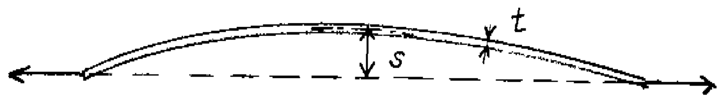

Fig. 1

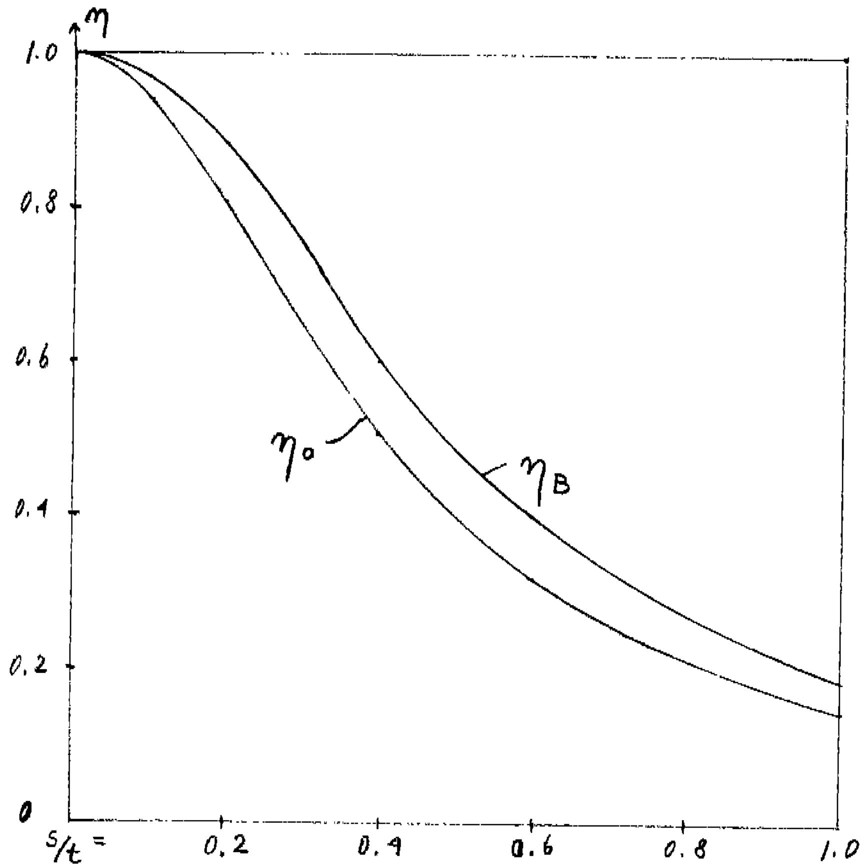

Fig. 2

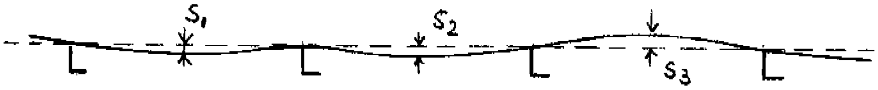

Fig. 3

the plate efficiency η, defined as the ratio of the stress in the plate required to stretch the panel by a certain amount, to the stress required to stretch a plane plate by the same amount. The efficiency η is a function of the ratio $\frac{s}{t}$ and of the average strain ε by which the panel has been stretched. Fig. 2 shows two non-dimensional curves for η as a function of $\frac{s}{t}$. The curve labeled η_o applies for small* strains and stresses, while the curve marked η_B applies when the average tensile strain in the panel is equal to the (compressive) strain at which the flat panel buckles according to the Euler theory. The curve for small strains is obtained from the equation,

$$\eta_o = \frac{1}{1 + 6\frac{s^2}{t^2}} \qquad [III.1]$$

which is easily verified for a sinusoidal initial buckle. The η_B-curve is taken from Ref. 3 (Fig. 5 for u = 1). For other values of the tensile strain, the efficiency η will be larger than the value η_o and will increase as the strain increases. For cases of interest the average tensile strain will rarely exceed the buckling strain, for which case the η_B-curve applies. As both curves do not differ vastly, Fig. 2 gives a good picture of the reduced effectiveness due to initial camber. It is seen that an initial unfairness of $s = \frac{t}{2}$

*Small compared to the average strain at which an identical flat plate buckles.

would reduce the efficiency of the panel to about 0.5, a very large reduction, indeed.

So far the efficiency of a single plate panel has been considered. If one considers more realistically a group* of panels 1 to n having differing unfairness s_n, Fig. 3, one can find their total efficiency η from the equation,

$$\frac{1}{\eta} = \frac{1}{n} \sum_{1}^{n} \frac{1}{\eta_n} \qquad [III.2]$$

where η_n is the efficiency of an isolated panel of unfairness s_n. This relation is only approximate because it applies strictly only to individually hinged panels. The crucial matter for the present problem is the overall efficiency, Equation [2], which does not depend for large n on the low efficiency of one badly distorted panel, but rather on the average value of the unfairness to be expected.

If the unfairness is only due to the unavoidable deviations from a true plane inherent in the limitations of the fabricating process, the average value of $\frac{s}{t}$ in deck plating may not exceed** 0.1, and the corresponding <u>efficiency</u> of 0.95 <u>is sufficiently high to ignore the whole matter</u>. On the other

*The necessity of considering the behavior of series of panels was first realized by Horne(3).

**Muckle(7) observed unfairness only up to 1/16-in. if the plate thickness was 1/2-in. or over, even in welded construction.

hand, the possibility of systematically caused larger unfairness must be considered.

One such possibility is that the initial unfairness produced by welding or other fabricating processes is further aggravated by residual stresses. Another possibility is that the relatively small unfairness increases during the operation of the ship. This may occur if compressive stresses, possible in combination with residual stresses, are large enough to buckle the unfair plate. To evaluate these possibilities, certain theoretical questions will be studied in the following section.

IV. THE EFFECT OF OPERATING STRESSES ON THE INITIAL UNFAIRNESS OF DECK PLATING OF DRY CARGO SHIPS--CONCLUSION IN CONNECTION WITH THE BRITTLE FRACTURE PROBLEM

In a recent paper[3] Horne made an attempt to explain the large unfairness observed in the bottom plating of transversely framed ships on a new basis. The paper deserves careful attention because it includes the effect of two physical factors whose bearing on the problem was not recognized previously. The first of these is the fact that successive alternate compressive and tensile loading of a plate panel may increase the deflections of the panel progressively. The second, no less important one, is the fact that if a unit of several panels is loaded, not only is the resulting unfairness in one panel much larger than in the case of a single panel, but the progressive increase occurs at a lower stress range.

The results of Horne's paper are not quite conclusive because he finds that the contemplated mechanism explains large unfairness only if the panels are subjected to stress cycles in which the sum of compressive and tensile stresses is of the order of 40,000 lb. per sq. in. As such high stresses are not believed to occur in normal operations, the theory so far does not seem to explain the large unfairness observed. However, there are certain points in which Horne's theory can be refined*, and it seems not at all unlikely that the theory will ultimately explain the facts fully. However, regardless of whether or not the large unfairness of panels observed in certain ships can be predicted by Horne's analysis, there can be no doubt that the <u>new effects</u> considered therein <u>exist</u> and ought to be included in any study of problems of unfair plating.

In the following consideration of the unfairness of deck plating, the stress range is assumed to be small enough to exclude the possibility of progressive buckling; but the effect of the action of a number of panels in series is included and will be seen to be essential. The approach follows Horne's paper except for one refinement, and his equations and graphs apply, a fact which is very convenient.

Consider first a group of n panels of span l as studied by Horne, Fig. 4. The transverse members AB and A'B' are

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*See the writer's comments in Appendix B.

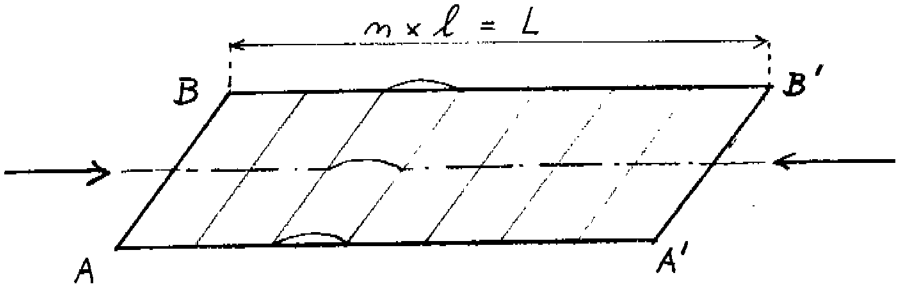


Fig. 4

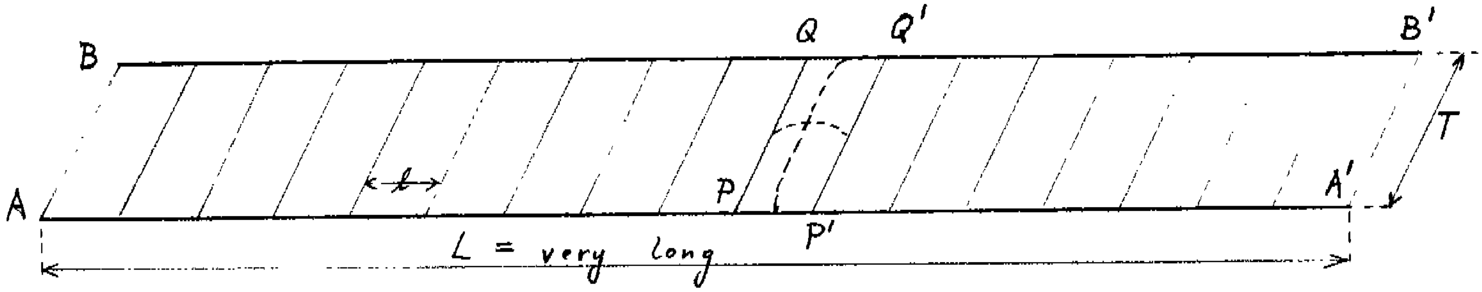


Fig. 5

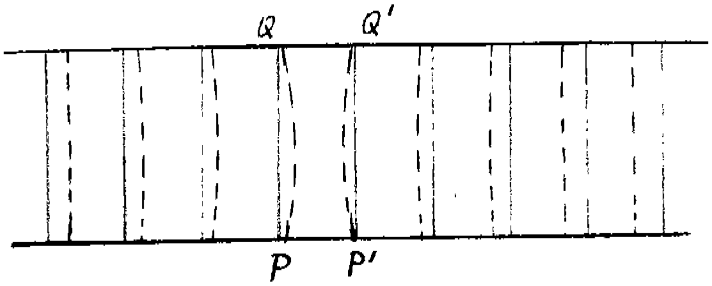


Fig. 6

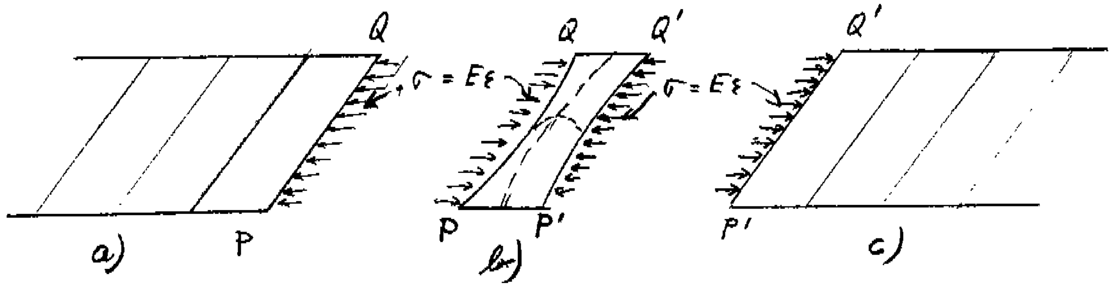


Fig. 7

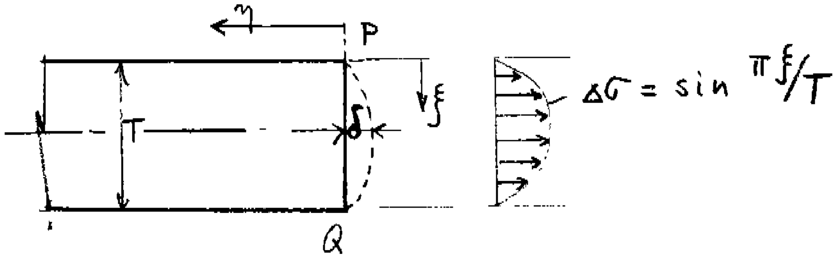


Fig. 8

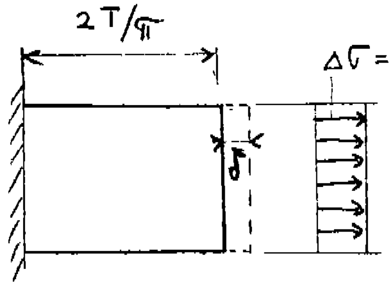


Fig 8a

assumed to be rigid in the plane of the plate, while the intermediate transverse members will resist forces at right angles to the plane of the plate only. Along the edges AA' and BB', the plates may slide in the longitudinal direction with respect to the longitudinal members AA' and BB'. If this unit consisting of plates and longitudinal members is loaded by longitudinal forces, the entire unit will shorten, the average strain being ε. If in the unstressed state one of the panels has a set s_r, the graphs in Horne's paper predict what will happen if a certain average compressive strain ε is imposed on the unit. There is a minor difficulty in applying this to the deck of a ship. One does not know the value of the strain ε corresponding to a contemplated loading, but rather the total longitudinal force S. To obtain the strain from the force S, the efficiency η of the plating must be known, and the relation between ε and S is therefore not only cumbersome; but to make matters worse, the relation is non-linear. Fortunately, for the present purpose it will be sufficient to know the order of the strain ε. We will need the effects of forces which produce nominal stresses σ of the order of 10,000 lb. per sq. in. if the plate were fully effective, corresponding to strains $\varepsilon = \frac{\sigma}{E}$. If the initial unfairness of the panel is not very large, as is assumed, the efficiency of a unit of many panels will be reasonably close to unity, and the actual strain will exceed $\frac{\sigma}{E}$ only by little, such that one may use $\varepsilon = \frac{\sigma}{E}$ as first approximation.

A major difference between the model, Fig. 4, and an actual deck of a ship becomes apparent if one tries to decide the number of panels n to be used in the analysis. In a transversely framed ship no members like AB which are rigid in the plane of the plate exist. The only element resisting motions in the plane of the plate <u>is the plate itself</u>. It provides, in a way, at each frame an elastic support, the rigidity of which depends essentially on the transverse distance between the longitudinal members AA' and BB'. The model, Fig. 4, is also not realistic in permitting sliding between the plate and the longitudinals along AA' and BB'. It appears that both unrealistic assumptions can be eliminated by a slightly different model.

Consider a model consisting of two longitudinal members AA' and BB' whose length L is very much longer than the transverse distance T between longitudinals, Fig. 5. The plate is subdivided in many panels of span ℓ, the transverse supports exerting no resistance against motions in the plane of the plate. One panel far from the ends A, B is initially bulged sinusoidally in the transverse <u>and</u> longitudinal directions. If a compressive strain ε is now imposed on the longitudinals AA' and BB', the plate will participate; and as L is much larger than T, the transverse panel supports which originally were straight lines will in general remain practically straight, except those near the ends AB, A'B' and in the vicinity of the bulged panel.

The behavior in the region of the bulged panel is shown in Fig. 6. The transverse members before loading are shown as solid straight lines, the dash lines indicating their position after loading. It is clear that at some distance from the panel PQ the transverse members will remain straight, but those in the vicinity must curve towards the bulged panel because this panel shirks the load. To obtain the shape of the edges PQ, P'Q', consider the free body diagrams of the semi-infinite pieces of plates to the left and right of the bulged panel and of the panel PQ itself, Fig. 7. If the longitudinal stress in the plate were uniform across the width and equal to the stress $\sigma = \varepsilon E$ in the longitudinals, the edges PQ and P'Q' of the semi-infinite flat plates, Fig. 7(a) and 7(c), would remain straight; the edges of the bulged panel, however, would be concave, Fig. 7(b). In the manner conventionally used in the theory of statically indeterminate structures, one can now apply unknown tensile stresses $\Delta\sigma$ to close the gap; as a first approximation the transverse distribution of these stresses is in the following assumed to be sinusoidal.

Fig. 8 shows the semi-infinite plate to the left of PQ under the action of such a sinusoidal corrective stress $\Delta\sigma = \sin\frac{\pi\xi}{T}$. The originally straight edge PQ will deflect by an amount $\delta \sin\frac{\pi\xi}{T}$ which depends on the boundary conditions at the longitudinal edges, i.e., on the cross sectional area of the longitudinal members. However, the dependency of the deflection on these boundary conditions is not very pronounced, and the value obtained for the simple case treated in Appendix A is sufficient for the present, essentially qualitative consideration. It is shown in Appendix A that the amplitude δ of the deflection of the semi-infinite plate, Fig. 8, is equal to the elongation of a strip of plate of length

$$L' = 2\frac{T}{\pi} \qquad [IV.1]$$

due to a stress $\Delta\sigma = 1$, Fig. 8(a).

The elastic behavior of the semi-infinite plate being equal to the behavior of a plate strip of equivalent length L', we conclude that the behavior of the bulged panel of width T, Fig. 5, will equal the behavior of the bulged panel in the case considered by Horne, Fig. 4, provided the number n of panels is such that

$$n\ell = 2L' + \ell \qquad [IV.2]$$

The problem of the panel of finite width is thus reduced to the case treated by Horne--at the price of an approximation--the assumption of sinusoidal shape of the bulge and of the corrective stress distribution. These approximations are really crude if $\frac{T}{\ell}$ is very large and the results are only qualitatively correct. In view of this it does not seem necessary to obtain a better value for L' than Equation [IV.1]. Substituting in Equation [IV.2] one obtains finally

$$n = \frac{4T}{\pi \ell} + 1 \qquad [IV.3]$$

In defense of the above approximations, it is important to realize that the results of Horne's analysis are <u>not</u> sensitive to changes in the number of panels. Even an error of as much as 50% in the number n would not affect the result substantially.

<u>Application to the upper deck plating of a Liberty ship.</u> The upper deck plating in a Liberty ship is supported longitudinally by the sides of the ship and by the hatch framing, and between hatches by longitudinal straps 16 in. by 3/4 in., making the dimension $T \simeq 15$ ft. Therefore with $\ell = 2.5$ ft.

$$n = 8.6 \simeq 8$$

At midship the plate thickness is $t = 3/4$ in., span $\ell = 30$ in., Young's modulus $E = 30{,}000$ kips per sq. in., and the yield stress $f_y = 33$ kips per sq. in.; the parameter μ (used in Ref. 3) for this case is

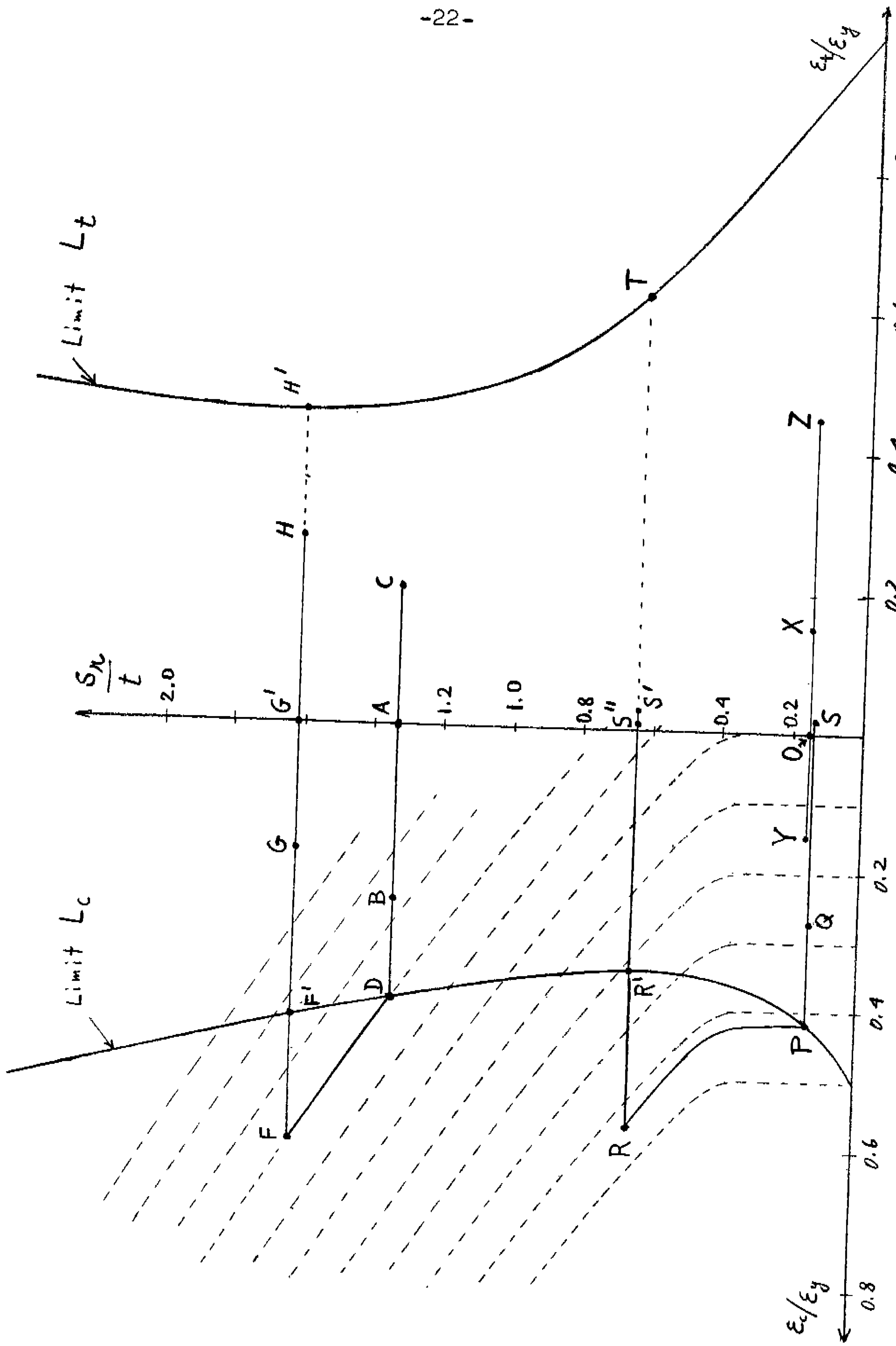
$$\mu = \frac{\pi^2}{12(1-\nu^2)}\left(\frac{t}{\ell}\right)^2 \frac{E}{f_y} = 0.52 \simeq 0.5$$

Ref. 3 contains certain graphs already for $\mu = 0.5$ which can be used. However the final graph required for the present purpose, equivalent to Figs. 23--25 of Ref. 3 is available* only for $n = 8$, $\mu = 0.2$ and 1.0; interpolation being not

*Note that part of the graph for $\mu = 0.2$, Fig. 23, is in error. See Appendix B, Item 5.

practicable, a new graph $n = 8$, $\mu = 0.5$ was constructed, Fig. 9. This graph is intended to find the changes of the permanent set at the center of the plate in the following manner:

The abscissas of Fig. 9 are the ratios of the average strains applied to the panel, to the yield strain ε_y. Compressive strains, $\frac{\varepsilon_c}{\varepsilon_y}$, are plotted to the left; tensile ones, $\frac{\varepsilon_t}{\varepsilon_y}$, to the right. The ordinates are the residual deflections s_r at the center of the plate divided by the plate thickness t. The "residual deflection" s_r is defined as that part of the actual deflection at a given instant which is permanent; if the plate panel were cut away from the rest of the structure and all external forces were removed, the panel would retain the deflection s_r. The diagram permits the prediction of the residual deflection s_r (not of the actual total deflection which is equal to s_r plus a further elastic component). Let the panel have an actual initial deflection s in the unloaded state and assume there are no residual stresses such that the panel is in an unstressed state. By definition the residual deflection equals the total deflection at this instant, $s_r = s$. The state of the system is then represented by a point A on the vertical axis having the ordinate $\frac{s}{t}$, see Fig. 9. If the panel is now compressed, it will behave elastically at first, such that s_r does not change; its state in the diagram will therefore be on a horizontal line through point A, the distance


Limit L_t
Limit L_c
$\frac{S_x}{t}$
2.0
1.2
1.0
0.8
0.4
0.2
$\varepsilon_t/\varepsilon_y$
$\varepsilon_c/\varepsilon_y$
0.2
0.4
0.6
0.8
H'
H
T
Z
C
X
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F'
R'
P
F
R

Fig. 9

$\overline{AB}$ indicating the strain. The panel will, however, become plastic if the strains exceed certain limits; these limits are shown by the heavy lines marked L_c and L_t in the diagram. If the panel is compressed only to a state as represented by point B in Fig. 9, it will not deform permanently further, and the residual deflection s_r will not change. Similarly, the panel may be subjected to tensile strains; if point C representing the state lies to the left of the heavy line L_t, no change of s_r will occur. In such a case even alternate cycles of compressive and tensile strain will not affect the residual deflection s_r.

If the originally unstressed panel represented by point A is subjected to a compressive strain exceeding the limit L_c, its state will follow the horizontal line only until point D in Fig. 9 is reached; beyond this point permanent deformation will occur, and the residual deflection s_r will increase. According to Ref. 3 the state of the system will follow a path which is parallel to the family of dashed curves shown in Fig. 9. If the panel is compressed until some point F is reached and then unloaded, it will act elastically again such that the value s_r reached at point F does not change. The path will therefore be the horizontal line FG. If the strain is decreased until the panel reaches zero stress, the compressive strain will <u>not</u> yet be zero. The respective state is represented by point G in Fig. 9, where $\overline{FG} = \overline{F'G'}$. If further tensile strain is applied, the panel will operate elastically only until a point H is reached, defined by $\overline{FH} = \overline{F'H'}$.

The fact that the panel on unloading reaches a state of no stress at point G, where the strain is still compressive, shows that when the strain finally becomes zero at point G', the panel will be in tension. The entire structure, consisting of plates and longitudinal members, Fig. 5, is therefore in a state of residual stress produced by the overloading. The residual tensile stress in the plate can be computed; it is equal to the strain represented by the distance FF' multiplied by $E\eta$ where the efficiency for small stress $\eta = \eta_o$, Fig. 2, may be used approximately.

Fig. 9 can now be applied to predict the behavior of the deck plating of a Liberty ship for a typical case.

Case A. Let the tensile stress in the deck in still water be 5 kips per sq. in. and find the effect of stress variations during operation of ±10 kips per sq. in. for an initial unfairness of $s_r = 0.15t$.

The initial state of the system on leaving the ways is given by $\frac{s_r}{t} = 0.15$, $\varepsilon = 0$, and is marked O in Fig. 9. The distribution of weights in still water produces a stress of 5 kips per sq. in. corresponding to a strain ratio of approximately $\frac{\varepsilon_t}{\varepsilon_y} = \frac{5}{33} = 0.15$; this is only approximately true because the plating efficiency is less than 100% such that the strain will be somewhat larger. It is, however, unnecessary to determine the strain more accurately because the result of the following consideration would not be affected. The state of the system in still water lies at point X on a horizontal line through point O because the strain $\frac{\varepsilon_t}{\varepsilon_y} = 0.15$ is much smaller than the yield limit given by the heavy line L_t.

The operational stresses of ±10 kips per sq. in. induce changes of the strain with respect to point X approximately of $\frac{\varepsilon}{\varepsilon_y} = \pm \frac{10}{33} = \pm 0.30$. The extreme states of the system in Fig. 9 are shown as points Y and Z, all lying on a horizontal line through point O. These points remain far away from the yield limits, and the initial permanent deflection s_r will therefore not change during operations.

This conclusion will remain correct for any similar stress level, even if the initial unfairness should be larger, because the yield limits L_t and L_c never do come close to the axis of the ordinates in Fig. 9.

Before drawing any practical conclusions from the example, it is necessary to consider also the effects of residual stresses which are likely to be present in a welded ship. While Ref. 3 does not specifically mention the fact, its procedure can be extended to the case of residual stresses in the manner shown hereafter.

Measurements of residual stresses on various ships are reported in Ref. 8. Compressive stresses up to 10 kips per sq. in. were found due to plate welding operations (Ref. 8, Figs. 5, 6); further stresses of the same order and somewhat larger ones were observed due to assembly procedures. It would, however, be erroneous to add such stresses without further thought; just as tensile stresses in a structure cannot exceed the yield point, compressive stresses in perfectly flat plate panels cannot exceed the buckling stress. If the panels are unfair, as we assume here, the compressive residual stresses are limited further, the maximum value being the one at which the critical section of the plate will yield; this maximum direct stress is necessarily smaller than the buckling stress. For the case just considered, $\mathcal{M} = 0.5$, $s_r = 0.15t$, $\sigma_y = 33$ kips per sq. in., the maximum possible residual stress would be* $\sigma_c \approx 13$ kips per sq. in. The observed values are not much smaller than this upper limit, and the following example will consider the extreme case that the residual stress equals the maximum which the panel can carry for the given unfairness.

<u>Case B</u>. Loading and initial unfairness are the same as in Case A, but the panel is assumed to have a residual compressive stress $\sigma_c^p \cong 13$ kips per sq. in. equal to its capacity for the given initial value of s_r. This initial residual stress occurs while the ship is still on its ways, at a time when the hull carries no external load. The resultant of all

*This value was found by appropriate use of Figs. 9 and 10 of Ref. 3.

the compressive residual stresses in the plate panel in this state must therefore be equal to the resultant of the residual tensile stresses in the longitudinal members.

To use a diagram, Fig. 9, of the type devised by Horne for the case of residual stresses, it is necessary to locate the point representing the initial state of the system. The strains ε_c and ε_t in Fig. 9 represent by definition the total strains applied to the unloaded panel. The panel under consideration, however, is already compressed carrying a stress σ_c^P. One can now visualize that a fictional tensile strain ε_1 can be applied to the system such that the stress vanishes; the residual deflection would still be $s_r = 0.15t$ and the state of the panel would again be point O in Fig. 9. Application of a compressive strain equal to the fictional strain ε_1 will then produce the actual initial state which will lie on a horizontal line to the left of point O. Because the compressive residual stress in this example is equal to the maximum the panel can carry, i.e., the one for which the critical cross section of the plate is fully plastic, the initial state must be represented by point P lying on line L_c indicating the yield limit.*

Having located the initial point P, the previously outlined procedure can be used again. To obtain the point Q

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*For any other smaller value of σ_c, the state would be represented by a point between O and P; the exact location would require some additional computation.

representing the still water state, the strain ratio $\frac{\varepsilon}{\varepsilon_y}$ is reduced by approximately 0.15. The operating stresses are equivalent to changing the strain ratio by ±0.30. Nothing happens if tensile strain is applied, the state being represented by point S; but if compressive strains are applied, point P which lies on the yield limit L_c is reached, and the yield limit is exceeded. Due to the excess strain the point representing the state of the panel travels to point R corresponding to a permanent set $s_r \approx 0.65t$. Subsequent tensile loading brings the system to point S'. Further changes of strain, however, produce no further yielding because the representative points of the system remain between points R and S', and the distance RS' is less than the distance between the yield lines R'T for $s_r = 0.65t$.

The overloading of the panel due to the combination of residual and operating stresses results in a substantial increase of the permanent set. This increase is accompanied by a decrease in the residual stress. At point R the plate is just yielding in its critical section, and the compressive stress in the panel can be easily computed from Ref. 3; it is only $\bar{\sigma}_c^P \approx 8$ kips per sq. in.

The point of interest in this study is the effect of all this on the participation of the plate panels in case of tensile strains, the states being represented by point S for the initial value of $s_r = 0.15t$, and by point S' for the increased value $s_r = 0.65t$. The direct stress carried by the panel at point S can be computed as follows: the stress at point P is $\sigma_e^P = 13$ kips per sq. in., and the stress at point S will be smaller by an amount $\sigma_y \eta \Delta$ where σ_y is the yield stress, η the efficiency, and Δ the difference of the strain ratios $\frac{\epsilon}{\epsilon_y}$ at points P and S. Using $\Delta = 0.45$ and $\eta = 0.48$,

$\sigma = + 13 - 33 \times 0.48 \times 0.45 = 5.8$ kips per sq. in. (compression). The direct stress corresponding to point S' is similarly found using $\sigma_c^P = 8$ kips per sq. in., $\eta = 0.18$, and $\Delta = 0.6$

$\bar{\sigma} = 8 - 33 \times 0.18 \times 0.6 = 4.4$ kips per sq. in. (compression). In spite of the tensile strain applied to the panel, direct stresses in the plate are in both cases still compressive, a fact which is due to the large initial residual compression. The tensile stress in the longitudinal members to which the panel is attached is the same at points S and S' because the total strains, which were prescribed in the above procedure, are identical. Compare now the total externally applied forces necessary to produce the two states of the system, i.e., panels and longitudinals, at points S and S'. The total force is in either case the sum of the forces in the longitudinals and the panels. The portion of the force due to the longitudinals is identical in both cases, say L, because their tensile stresses are equal. As the panels are still in compression, the total external force will be smaller than the tensile force L by an amount equal to the stress in the panel multiplied by its area. The compressive stress in the plate at point S being larger than at point S', the total tensile force carried will be larger at point S' than at S.

So far points S and S' having equal total strains were compared; however, it is really desired to compare cases of equal total external force. Retaining the state represented by point S, we ask which point will represent the system after the increase of the permanent set to $s_r = 0.65t$ if the total force equals the one at point S. Because the total tensile force at point S' is somewhat larger than at point S, a force equal to the one carried at point S will produce a state represented by point S" to the left of S' having a slightly smaller total strain. This in turn means that the tensile stress in the longitudinals for equal external loads is actually smaller at S" when the permanent set of the panel is large--$s_r = 0.65t$--than for the original small value of $s_r = 0.15t$. This surprising result is due to the fact that the decreased efficiency of the plate for the larger permanent set has been more than compensated by the decrease in the residual stress.

The overloading in compression of the panels of deck plating if residual stresses are present has, accordingly, the effect of reducing the worst tensile stresses in the longitudinals and <u>does not therefore aggravate the danger of brittle fracture</u>

The two examples, Cases A and B, permit the following conclusions:

1. The unfairness of the deck panels of a Liberty ship and other similar dry cargo ships will not increase in operation unless substantial residual stresses are present.

2. If residual stresses equal to the capacity of the panel with the largest unfairness are present, the unfairness will increase during operation. (It must be stressed that only the panel with the largest unfairness out of groups of about 10 will be affected). Even if this happens, the maximum tensile stress in the longitudinals will not be increased above its value before the unfairness was increased; the loss of efficiency (in tension) of the plating is overcompensated by a decrease in the residual stresses.

3. Observed residual stresses are smaller than the stresses visualized in the previous paragraph and considered in Case B. The effect of actual residual stresses will be intermediate between the results described in the two previous paragraphs. The important result remains that an increase in unfairness, if it should occur at all, will <u>not</u> increase the maximum tensile stresses in the longitudinals beyond the value corresponding to the initial unfairness.

4. In Section III it was found that the reduction of the tensile efficiency of deck plating due to the unfairness at the time of construction was small enough to be of no consequence. In the preceding paragraphs 1 to 3, it was concluded that this unfairness either will not increase during operation, or if it does, will not increase the maximum tensile stresses above the level associated with the initial unfairness. <u>It appears, therefore that the effect of initial unfairness of deck plating in dry cargo ships on the maximum tensile stresses and therefore on the brittle fracture problem is negligible</u>.

V. CONCLUSIONS FOR TRANSVERSELY FRAMED DRY CARGO SHIPS

1. It has been shown in Section II that the reduced effectiveness of warped or unfair bottom plating increases the tensile stresses in the deck only to an insignificant extent, less than 5%. Unfair plating can therefore not be considered a substantial contributory cause for brittle fracture.

2. Using only the viewpoint of reducing the danger of brittle fracture, it does not seem appropriate to provide additional longitudinal stiffeners to decrease the unfairness of bottom plating. While such stiffeners might reduce the tensile stresses by an amount possibly of up to 5%, it would seem that the same amount of material added to the deck would be more effective in reducing the tensile stresses.

3. The question has also been considered whether a vessel may fail by brittle fracture of the deck due to unexpectedly large tensile stresses caused by dynamic forces during the jack-knifing of a vessel whose bottom plates have buckled. Such a failure, while due to weakness of the bottom plates, might not be recognized as such and ascribed to brittle fracture. Section II and Appendix C show clearly that no such increase of tensile stresses occurs, and this possibility can therefore be dismissed.

4. The increase of tensile stresses due to unfair deck plating was also found to be insignificant for the small unfairness to be expected at the time of construction. In studying the effect of unfairness on the efficiency of plating, it is essential that the action of a number of panels in series be considered as indicated in Ref. 3. The above conclusion was obtained on this basis.

5. The effect of larger unfairness of some plate panels which might develop during the life of welded ships due to the combined action of operating stresses and residual stresses has also been studied. It was found that, in spite of gradually developing unfairness of deck plates, the maximum tensile stresses in the crucial longitudinal members of the deck do not increase above the value which the same loading combined with the initial residual stresses would have produced in the

hull at the time of construction (when the unfairness was small). This result is due to the fact that the reduced effectiveness of the more unfair plating is overcompensated by a reduction of the residual stresses as unfairness develops.

6. Summarizing, it was found that the unfairness of bottom or deck plating does not raise the tensile stresses to any significant degree. As any influence of unfairness on the danger of brittle fracture could be only through an increase of the tensile stresses, it is finally concluded that plating unfairness per se has no significant bearing on the problem of brittle fracture.

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APPENDIX A

Consider a semi-infinite plate of thickness t and width T, Fig. 8, loaded by longitudinal stresses

$$\Delta\sigma = \sin \frac{\pi\xi}{T}$$

on the finite edge PQ. The system of coordinates ξ and η is indicated in Fig. 8. As boundary conditions on the infinitely long edges, it is prescribed that the direct stresses vanish and further that the components of the displacements in the ξ-direction also vanish.

The stresses in the plate can be derived from a stress function

$$\sigma_\xi = \frac{\partial^2 F}{\partial \eta^2}, \qquad \sigma_\eta = \frac{\partial^2 F}{\partial \xi^2}, \qquad \tau = \frac{\partial^2 F}{\partial\xi\partial\eta} \qquad [A.1]$$

The expression for this stress function is

$$F = -\frac{T^2}{\pi^2} \sin \frac{\pi\xi}{T}\left(1 + \frac{\pi\eta}{T}\right)e^{-\pi\eta/T} \qquad [A.2]$$

The displacement of the edge PQ is sinusoidal, and its amplitude δ can be found in the following manner: The η-components of the displacement for large values $\eta \rightarrow \infty$ must obviously vanish. The amplitude δ must therefore be equal to the total elongation of a fiber on the centerline, $\xi = \frac{T}{2}$, of the plate. The stresses σ_η on the centerline are

$$\sigma_\eta \Big|_{\xi = \frac{T}{2}} = (1 + \frac{\pi\eta}{T})e^{-\pi\eta/T} \qquad [A.3]$$

and the elongation becomes

$$\delta = \frac{1}{E} \int_0^{\infty} (1 + \frac{\pi\eta}{T})e^{-\pi\eta/T} d\eta = \frac{2T}{E\pi} \qquad [A.4]$$

The amplitude δ of the displacement is therefore identical with the elongation of a plate strip of finite length $\frac{2T}{\pi}$ under a uniform stress $\Delta\sigma = 1$, Fig. 8(a). If one is principally interested in the displacement at the centerline, the semi-infinite plate under sinusoidal loading may be replaced by the finite plate strip under uniform stress.

APPENDIX B

Comments on Ref. 3

Ref. 3 contains two separate important contributions to the problem of unfair plating: 1) It demonstrates that cycles of compressive and tensile loading may produce progressively increasing unfairness. 2) It demonstrates that the mechanism of increasing the unfairness by overloading is changed if several panels in series are considered. These two new ideas explain the observed large unfairness in vessels qualitatively; yet the theory does not seem to give the final quantitative explanation because it requires the assumption of larger operating stresses than are believed to occur. It seems to the writer, however, that some relatively minor refinements of Ref. 3 are indicated and that an analysis including these refinements and, in addition, allowing for the effect of residual stresses may fit the observations.

1. One difficulty which arises when applying the method of Ref. 3 to a ship's bottom lies in the selection of the number n of plate panels in series. The analysis assumes that there are transverse members, AB and A'B', Fig. 4, which are rigid in the plane of the plate, while the other transverse members are flexible in this plane. This is not the situation in a ship. None of the transverse members or bulkheads by themselves have appreciable resistance in the plane of the

plate, any such resistance is supplied by a part of the plate itself which might be counted as part of an effective section. This difficulty can be overcome by analyzing instead a very long strip of plate containing the unfair panel, Fig. 5.

As shown in detail in Section IV above, this is with some approximations equivalent to using a number $n = \frac{2T}{l\pi}$, where T is the width of the panels. This approach leads to panel numbers of the same magnitude, 8 to 20, as used in Ref. 3 and does not change its conclusions at all. The proposed approach for selecting n seems rational and avoids an arbitrary selection of the number n.

2. In determining the limits of tensile and compressive strains at which yield occurs, Ref. 3 uses the value of the yield stress σ_y from standard tests. It appears likely that after the material has yielded for the first time the yield stress at reversal of strain will be smaller than the original value. This may also hold for all further cycles, although the writer knows of no factual evidence concerning this point. The adjustment to appropriately reduced values of σ_y in the later cycles does not require any changes in the basic theory; this modification will obviously lead to the conclusion that progressive increase of the unfairness occurs at smaller total stress variations than stated in Ref. 3.

3. In the early part of the analysis, it is assumed as an approximation that the residual deflection s_r does not change between the two states represented by point B in Fig. 13 of Ref. 3, where the yield stress is reached in an outer fiber for the first time, and point C, where the section is fully plastic. This approximation is perfectly justified for the purpose of determining the gradual increase of s_r if the strain is large; it is not justified if the applied strain lies between the values corresponding to points B and C because the assumption would then deny the fact that such a strain does increase the residual deflection, even if only slightly. It seems therefore that in the final conclusion of Ref. 3 the permissible strain range avoiding increase of unfairness should not be the sum of the compressive and tensile strains at which the plate gets fully plastic, but the corresponding sum of the strains at which <u>first yielding</u> of the plate occurs. It is quite true that the increase of the residual deflection per cycle can be expected to be quite small if the total applied strain lies between the limit proposed here and the limit stated in Ref. 3, but an increase must be expected because the non-linear mechanism which causes the progressive increase operates in a similar manner as in the case of larger strains. (This statement should, of course, be verified by a quantitative analysis.) Fig. 10 compares the

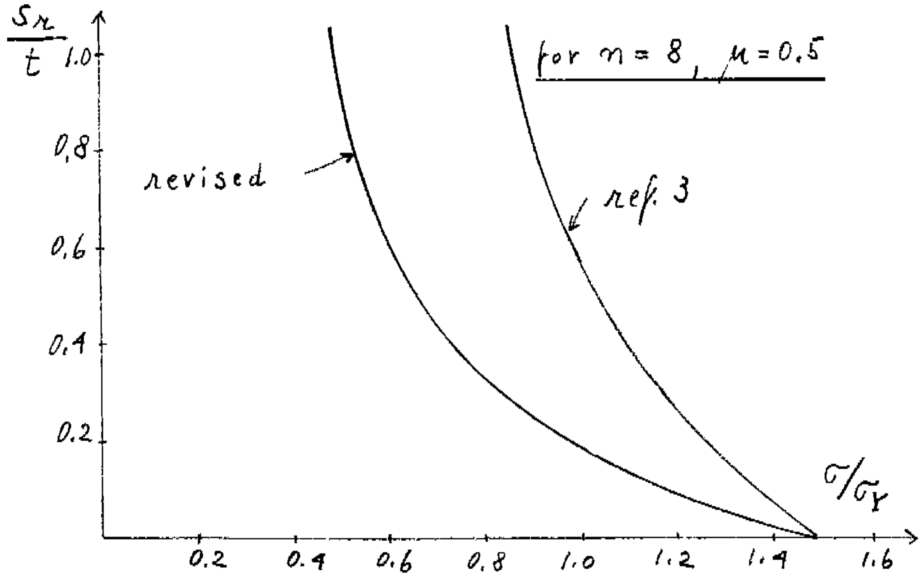

Fig. 10. Permissible stress range.

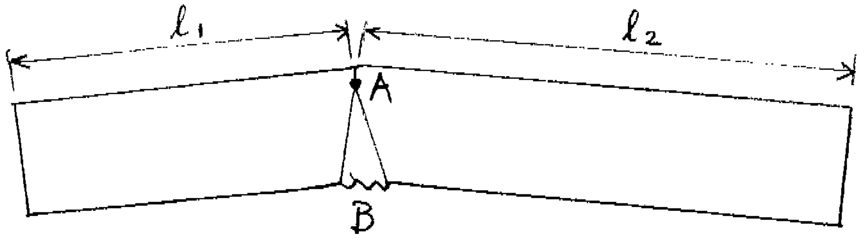

Fig. 11

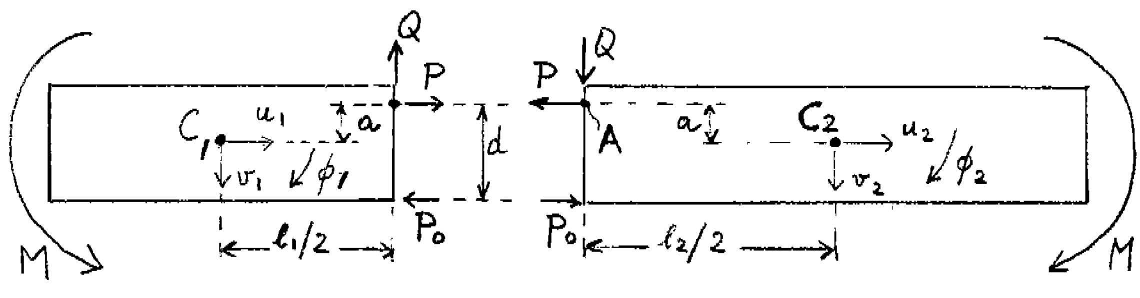

Fig. 12

total permissible strains for a typical case, $\mu = 0.5$, n = 8, found in the manner suggested here and according to Ref. 3. In the important range $0.2 < \frac{s_r}{t} < 1$ the revised values of the permissible strains are only about two-thirds of those found from Ref. 3, indicating that a stress variation of only one-half the yield stress, i.e., 17,000 lb. per sq. in., may already cause progressive development of buckles, presumably at a very slow rate.

4. There is also the possibility for the development of large permanent buckles, even if the stress range is quite small, if residual stresses are present. This explanation does not succeed if one considers one panel only but requires the assumption of a number of panels acting in series and is justified by the reasons set out* in 1. If residual compressive stresses are present, then a relatively small compressive operating stress combined with the residuals will make the plate yield and increase the permanent deflection. Such an increase will happen every time the previously largest stress is exceeded, and it may therefore take years for these buckles to develop fully. The details of the application of the analysis of Ref. 3 to such a case are demonstrated in the second portion of Section IV above for the case of upper deck plating. It is important that the proposed explanation does not depend on the magnitude of the stress variation but solely on the fact

*The points discussed in 2 and 3 become immaterial for the following.

that the residuals and the compressive operating stress may exceed the capacity of the originally slightly unfair plate. The vital element in this explanation is the mechanism involving plates in series first demonstrated by Horne in Ref. 3.

5. As a very minor point the writer believes that the dotted lines in the left half of Fig. 23 are incorrectly plotted; their values for s_r should be 10 times larger.

In conclusion, it seems likely that the occurrence of large buckles is due to a combination of the residual stress effect discussed in paragraph 4 and of cyclic compressive and tensile loading in the manner discussed in paragraphs 2 and 3. It seems now very obvious that welded ships which have large residuals should be more susceptible to unfairness than riveted ones.

It appears further that rules for permissible stress ranges as given in Ref. 3 can only be formulated concerning the unfairness due to cycles of opposite stress. Such rules will therefore protect only riveted ships which have small residual stresses. The part of the unfairness due to the combination of compressive stress and substantial residual stresses in welded ships cannot be covered by such rules unless a way is found to keep these residual stresses below definitely known and small values. At present this is hardly possible, and the writer concludes that the occurrence of large unfairness in transversely framed welded ships is not avoidable.

APPENDIX C

Dynamic Effects due to Compressive Failure of Bottom Plating

Fig. 11 shows a hull of length L of constant cross section and having uniformly distributed mass. Let the loads, including buoyancy, produce bending moments such that the maximum moment M at the section AB exceeds the capacity of the section because of plate buckling at point B. The hull will then jack-knife. To analyze this problem in the simplest possible manner, it is assumed that the forward and aft portions may be treated as rigid bodies connected at point A by a hinge and at point B by the buckling plate panel. To remain realistic, it is assumed that this panel is still able to carry some compressive load, P_o; the hinge at A is assumed to be able to transfer a direct horizontal tension P and shear Q. Fig. 12 shows free body diagrams of the two portions of the hull of length ℓ_1 and ℓ_2, respectively.

The equations of motion for this mechanism can be easily obtained. Fig. 12 indicates (with respective subscripts) the location of the centroids C and the coordinates u and v of their motion, the angles of rotation ϕ, and the acting forces P, Q, P_o as well as the moment M due to the loads and buoyancy. There are six relations between the horizontal vertical and angular accelerations and the forces,

$$m\ell_1\ddot{u}_1 = P - P_o \qquad m\ell_2\ddot{u}_2 = -P + P_o$$

$$m\ell_1\ddot{v}_1 = -Q \qquad m\ell_2\ddot{v}_2 = Q$$

$$m\ell_1 r_1^2\ddot{\phi}_1 = -M + aP - \tfrac{1}{2}\ell_1 Q + (d - a)P_o \tag{C.1}$$

$$m\ell_2 r_2^2\ddot{\phi}_2 = M - aP - \tfrac{1}{2}\ell_2 Q + (d - a)P_o$$

where m is the mass of the hull per unit length, r_1 and r_2 are the radii of gyration of the hull portions, and a and d dimensions defined in Fig. 12.

Two further equations result from the fact that point A is common to both portions of the hull and that its acceleration must be the same whether computed from the fore or aft portion. For small motions this gives,

$$\ddot{v}_1 + \tfrac{1}{2}\ell_1\ddot{\phi}_1 = \ddot{v}_2 - \tfrac{1}{2}\ell_2\ddot{\phi}_2 \tag{C.2}$$

$$\ddot{u}_1 + a\ddot{\phi}_1 = \ddot{u}_2 + a\ddot{\phi}_2$$

These eight equations may be solved for the eight unknowns, the forces P and Q, and the six components of acceleration. The only result of interest here is the force P

$$P = P_o + \frac{M - P_o d}{a + \frac{r^2}{a}} \tag{C.3}$$

where the quantity r^2 is defined by

$$r^2 = \frac{(4Lr_1^2 + \ell_1^2\ell_2)(4Lr_2^2 + \ell_1\ell_2^2) - \ell_1^3\ell_2^3}{4L(4\ell_1 r_1^2 + 4\ell_2 r_2^2 + L\ell_1\ell_2)} \qquad [C.4]$$

and $L = \ell_1 + \ell_2$.

Applying the result to the case of a ship's hull, the lengths ℓ_1 and ℓ_2 will be several times the depth of the hull, such that the approximations

$$r_1^2 = \frac{\ell_1^2}{12} \qquad r_2^2 = \frac{\ell_2^2}{12} \qquad [C.5]$$

can be used. Substitution in Equation [C.4] gives

$$r^2 = \frac{1}{3}\,\frac{\ell_1^2\ell_2^2}{(\ell_1 + \ell_2)^2} \qquad [C.6]$$

The result can be evaluated by consideration of several inequalities. If we assume that the lengths ℓ_1, ℓ_2 of the two broken portions of the vessel exceed twice the depth d, then the value r^2 according to Equation [C.6] is

$$r^2 > d^2 \qquad [C.7]$$

It is next necessary to select a value for the distance a, defining the location of the mass center. For two otherwise identical hulls Equation [C.3] will give a larger value for P if the denominator $a + \frac{r^2}{a}$ is as small as possible. The denominator would be an <u>absolute</u> minimum if

$$\frac{d}{da}\left(a + \frac{r^2}{a}\right) = 0 \qquad \text{or} \quad a = r \tag{C.8}$$

However, the centroid cannot lie outside the hull limiting the values of a to $0 \leqslant a \leqslant d$. As Equation [C.7] indicates that $r > d$, the absolute maximum at $a = r$ does not occur; within the permissible limits the denominator will be smallest for $a = d$. Using this value, Equation [C.3] yields the successive inequalities,

$$P < P_o + \frac{M - P_o d}{d + \frac{r^2}{d}} < P_o + \frac{M - P_o d}{d} = \frac{M}{d} \tag{C.9}$$

As $M - P_o d$ is necessarily positive, Equation [C.3] indicates $P_o < P$, and we have the ultimate result

$$P_o < P < \frac{M}{d} \tag{C.10}$$

The upper limit $\frac{M}{d}$ is the force which would occur in the static case when the plate panel at B does not buckle. Equation [C.10] indicates therefore that the total tensile force in the beam is reduced by buckling of the compression side.

It might be added that the force P is substantially smaller than the upper limit such that the approximations used are not likely to affect the result.